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47C (MMEA)

[17–HDGKACRM–M1]

MA 1st Semester Exam., 2017

ECONOMICS

[MEC 03]

(Mathematical Methods for Economic Analysis)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer any *five* from the following questions : 2×5=10

(a) Define a triangular matrix.

(b) Derive the marginal utility function from a total utility function.

(c) Find $\frac{\partial y}{\partial x_1}$ of the function $y = 4x_1x_2 + x_1^3 + 2x_2^2$.

(d) Given the total utility function $U = 2xy^2 + 3x^3y - 2x + 7y$. Compute the marginal utilities of x and y at $x = 1$ and $y = 3$.

(e) Evaluate $\int x^5(8x^6 + 7)^{15}\,dx$.

(f) Find out $\frac{\partial Q}{\partial L}$ and $\frac{\partial Q}{\partial K}$ for the production function $Q = 24KL - 10L^2 - 8K^2$.

2. Answer any *three* from the following questions : 4×3=12

(a) The average revenue function of a firm is given by $AR = 100 - 3q$. Find out the elasticity of demand when $q = 5$.

(b) Find the maxima and minima for the function $Z = 10x + 20y - x^2 - y^2$.

(c) Establish the relationship between average cost (AC) and marginal cost (MC) by using the product or quotient rule of differentiation.

(d) The total revenue (R) and total cost (C) functions of a firm are given as follows :

$$R = 10Q_1 + 4Q_2$$

$$C = 2Q_1^2 + \frac{1}{4}Q_2^2 + Q_1Q_2 + 5$$

Find profit maximizing output and maximum profit.

3. Answer any *three* from the following questions : 6×3=18

(a) Find the inverse of

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 1 & 4 \\ 1 & 3 & 5 \end{bmatrix}$$

(b) Discuss the concept of Euler's theorem. Why has it been criticized? 4+2=6

(c) Find the extreme values of the function

$$y = 5x_1^2 + 4x_2^2 - 50x_2 - 60x_1 + 10x_1x_2 + 20$$

(d) Solve :

$$\frac{dy}{dx} + 4x^3y = 0$$

4. Answer any *four* from the following questions : 10×4=40

(a) Discuss the major interpretations of derivative in economics. The cost function of a firm is given by $C = 10000 + 100q - 10q^2 + \frac{1}{3}q^3$. Find—

(i) the marginal cost function;

(ii) the slope of the marginal cost function;

(iii) the output at which marginal cost is equal to average variable cost. 5+5=10

(b) (i) A demand function of Engel's curve type is given as $D = AP^{\alpha} N^{\beta}$, where D is demand, P is price, N is income and α and β are parameters. Find the partial derivatives $\frac{\partial D}{\partial P}$ and $\frac{\partial D}{\partial N}$, and also interpret the values of α and β.

(ii) Compute the marginal utilities of x and y for the utility function $U = \frac{x^3 + y^3}{x^2 - y^2}$. 5+5=10

(c) Find extreme values of the following functions : 5×2=10

(i) $y = -3x^2 + 18x + 12$

(ii) $y = x^3 - 24x^2 + 20$

(d) A monopolist produces his product in two different plants and his total cost functions are $TC_1 = 10 - 2Q_1 + Q_1^2$ and $TC_2 = 15 - 6Q_2 + 2Q_2^2$. Average function is given by $AR = 50 - 2Q$, where $Q = Q_1 + Q_2$. Find *(a)* profit maximization output to be produced in plants 1 and 2 and *(b)* his maximum profit. 5+5=10

(e) Use Cramer's rule to solve the following equations :

$$2x_1 + 3x_2 - x_3 = 15$$
$$4x_2 + 2x_3 = 16$$
$$2x_1 + 2x_2 = 18$$

(f) Use Lagrange multiplication method and find extreme values of the function

$$y = 3x_1^2 - 4x_1x_2 + 2x_2$$

subject to $x_1 + x_2 = 22$

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