47C (MMEA)

[18–HRGPKAC–M1S]

MA 1st Semester Exam., 2018

ECONOMICS

(MEC/MAEC 03)

(Mathematical Methods for Economic Analysis)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer any *five* from the following questions : 2×5=10

(a) Define a symmetric matrix.

(b) If the total cost of production is given by $C = 2000 + 4x + 0 \cdot 5x^2$, where x is the output, find the marginal cost.

(c) Find the differential dy for the following function :

$$y = \frac{x^2 + 10}{(3x - 2)^2}$$

(d) Write the necessary and sufficient condition of maximum value of the function $y = f(x)$.

(e) Find the value of

$$\int \left(2x^3 + x^2 + \frac{2}{x}\right) dx$$

(f) Find $\frac{\partial y}{\partial x_2}$ of the function

$$y = f(x_1, x_2) = (x_1^2 + 5x_1x_2^2 + x_2^3)^5$$

2. Answer any *three* from the following questions : 4×3=12

(a) Given the function

$$y = x_1^3 x_2 - 3x_2 x_3^2 + x_1 x_3$$

Find all nine-second order partial derivatives.

(b) Find the extreme values of the function

$$y = \frac{1}{3}x^3 - 3x^2 + 5x + 3$$

(c) Derive total cost function given the marginal cost function and total fixed costs

$$C'(Q) = 3x^2 - 4x + 3$$

when total fixed cost is 500.

(d) Solve the following differential equation :

$$x(2+4y)dy + 2y(1+y)dx = 0$$

3. Answer any *three* from the following questions : 6×3=18

(a) Find the extreme values of the following function :

$$y = x_1^3 - 8x_1 x_2 - x_1^2 + 2x_2^2 + 15x_1 + 50$$

(b) Maximize the function

$$y = 2x_1 + 2x_1 x_2 + x_2$$

subject to

$$3x_1 + 2x_2 = 12$$

(c) Given the marginal cost function

$$MC = C^1(Q) = 6Q^2 - 24Q + 20$$

Find out the output at which average variable cost (AVC) is minimum.

(d) Given the market-model

$$D = 100 - 5P$$
$$S = -10 + 2(P - t)$$
$$D = S$$

where D, S, P, t are demand, supply, price and tax rate. Evaluate whether an increase in the rate of tax (t) will increase tax revenue (T).

4. Answer any *four* from the following questions : 10×4=40

(a) Solve the following system of simultaneous equations by matrix inversion :

$$2x_1 - x_2 + 3x_3 = 9$$
$$x_1 + 3x_2 + 2x_3 = 13$$
$$2x_1 + 2x_2 + x_3 = 9$$

(b) Given the short-run total cost function

$$C = 2Q^3 - 15Q^2 + 30Q + 16$$

(i) Find out the level of output at which average variable cost (AVC) is minimum and also show that MC = AVC at the level of output.

(ii) Show that when output $Q = 4$, the average cost is minimum and MC = AVC. (4+2)+4=10

(c) The demand function of a monopoly in two different markets are given by

$$P_1 = 53 - 4Q_1$$
$$P_2 = 29 - 3Q_2$$

and the total cost function is $C = 20 + 5Q$, where P_1 and P_2 are the prices and Q_1 and Q_2 are the outputs in market–1 and market–2 respectively such that

$$Q = Q_1 + Q_2$$

Find—

(i) the profit maximizing output to be sold in first and second markets;

(ii) the equilibrium prices of first and second markets. 6+4=10

(d) A producer desires to minimize the cost of production $C = 16K + 4L$, where K and L are capital and labour respectively subject to the given production function $Q = 5K^{1/2}L^{1/2}$. Find out the equilibrium combination of inputs (K and L) in order to minimize the cost of production when output $Q = 40$.

(e) Discuss the major interpretations of derivative in economics. The total cost of a firm is given by

$$C = 50000 - 15q + 5q^2$$

(i) Find the level of output at which the slope of the average cost curve is zero.

(ii) Find the output at which the marginal cost curve cuts average cost curve. 5+3+2=10

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