816 (MME)

[19-HRPGKOW-M1S(N) 20M]

MA 1st Semester (New) Exam., 2019 (W)

ECONOMICS

[PGEC S1–03]

(Mathematical Methods for Economic Analysis)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Answer the following questions in *1* or *2* sentence(s) each : 1×8=8

(i) Define square matrix.

(ii) Find the slope of the function $Y = 3 - 2x$.

(iii) Define cross-price elasticity.

(iv) Where does absolute extremum of a function occur?

(v) State the slope of the budget line.

(vi) What is meant by capital formation?

(vii) Find the degree of the following differential equation :

$$\frac{dy}{dt} = 2t + 3$$

(viii) What is meant by order of difference equation?

(b) Choose the correct option : 1×4=4

(i) MRS_{xy} implies

(1) $\frac{MU_x}{MU_y}$

(2) $\frac{MU_y}{MU_x}$

(3) $MU_x \times MU_y$

(4) $\frac{1}{MU_x}$

(ii) Optimization with equality constraint is known as

(1) matrix method

(2) Lagrange multiplier method

(3) linear programming

(4) game theory

(iii) If a differential equation is of the form

$$\frac{dy}{dt} + ay = 0$$

then it is said to be

(1) non-homogeneous

(2) homogeneous

(3) partial

(4) undefined

(iv) The difference between the amount of money a consumer is willing to pay and the amount that he actually pays for purchasing a given quantity of the commodity is known as

(1) producer's surplus

(2) elasticity of demand

(3) consumer's income

(4) consumer's surplus

(c) Fill in the blanks : 1×4=4

(i) $A^{-1} = \frac{Adj(A)}{____}$

(ii) $____ = \frac{d}{dL}(Q)$

(iii) TC – TFC = ____

(iv) The integral is obtained by ____ the process of differentiation.

2. Answer any *five* questions : 2×5=10

(a) Solve $y_{t+2} - y_{t+1} = 10$ with $y_0 = 10$.

(b) What is meant by Lagrange multiplier?

(c) Distinguish between equality of matrices and equivalence of matrices.

(d) Convert the following function into an implicit function :

$$y = f(x) = x^2 + 10$$

(e) A monopolist has the following average revenue function in two markets

$$AR_1 = 60 - 2Q_1$$
$$AR_2 = 45 - 3Q_2$$

and the total cost function of the monopolist is given by

$$TC = 3Q^2 - 6Q + 40$$

where $Q = Q_1 + Q_2$, then find the profit maximizing function.

(f) Find the norms of the following matrix :

$$A = \begin{bmatrix} 2 & 0 & 2 & 5 \\ 3 & 1 & 6 & 2 \\ 1 & 4 & 4 & 1 \\ 2 & 2 & 0 & 3 \end{bmatrix}$$

3. Answer any *four* questions : 3×4=12

(a) State the product rule of differentiation and find $\frac{dy}{dx}$ of the following function : 1+2=3

$$y = \frac{x+2}{2x+3}$$

(b) Find the total differential of the following function :

$$y = 2\log x_1^2 + 10 - x_1 x_2^2$$

(c) Show that $y = x^3 - 6x^2 + 12x + 10$ function has neither a maximum nor a minimum value at $x = 2$.

(d) Show the conditions for optimization with m explanatory variables for a function.

(e) State the structure of optimization with equality constraint.

4. Answer any *three* questions : 6×3=18

(a) Find the extreme values of the function

$$y = 4x_1^3 + 8x_1 x_2 - 4x_1^2 - x_2^2 + 10$$

(b) A producer desires to minimize his cost of production $C = 5L + 2K$, where L and K are the inputs, subject to satisfaction of the production $Q = LK$. Find the optimum combination of L and K in order to minimize the cost of production when output is 90.

(c) Evaluate the following integrals : 3+3=6

(i) $\int_1^e \log x\,dx$

(ii) $\int_2^3 x^2 \log x\,dx$

(d) Find the output level at which the average variable cost is minimum from the following total cost function :

$$TC = 2Q^3 - 20Q^2 + 18$$

(Continued)

5. Answer any *three* questions : 8×3=24

(a) Solve by Cramer's rule

$$x+6y-z=10$$
$$2x+3y+3z=17$$
$$3x-3y-2z=-9$$

(b) Derive and explain the relationship between AR, MR and elasticity of demand.

(c) State the Euler's theorem and prove that Cobb-Douglas (CD) production function satisfies the Euler's theorem. 2+6=8

(d) Find consumer's surplus from the following demand function :

$$Q=\sqrt{42-\frac{3}{4}P}$$

when price = 8.

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