48 (DS)

[19-HBPGKOW-M3S(O) 20M]

MCA/MScIT 3rd Semester (Old) Exam., 2019 (W)

DISCRETE MATHEMATICS

[MCA/MScIT (S3) 09]

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer any *five* from the following questions : 2×5=10

(a) Define bijective function with an example.

(b) Write truth table for 'OR' with two inputs.

(c) What is premise? Give one example.

(d) What is subgroup?

(e) What is a graph? Give one example.

(f) What is factor group?

2. Answer any *three* of the following questions : 4×3=12

(a) Let

$$A = \{1, 2, \cdots, 10\}$$
$$B = \{2, 4, 6, \cdots, 20\}$$
$$R = \{(2, 2), (4, 4), (6, 6)\}$$

Find the relation matrix and graph.

(b) Write the algorithm for finding complete sum of product forms. Express $x(y'z)'$ in full disjunctive normal form.

(c) Prove that a full k-ary tree with i internal vertices contains k_{i+1} vertices.

(d) Define permutation. Find the number of permutations if 4 objects are taken 2 at a time.

3. Answer any *three* from the following questions : 6×3=18

(a) Write the negative proposition for the following :

(i) Tomorrow is Monday

(ii) There is no pollution in New Delhi

(iii) 3 + 2 = 5

(b) What is Ackerman's function? Use it to find A (3, 3).

(c) Let A, B, C be three sets. Prove that

$$A \cap (B - C) = (A \cap B) - (A \cap C)$$

(d) Show that

$$p \to q,\ q \to r,\ \sim(p \wedge r),\ p \vee r \Rightarrow r$$

(Continued)

4. Answer any *four* of the following questions : 10×4=40

(a) If $(G, *)$ be a group, then prove that—

(i) the identity element is unique;

(ii) for any $a \in G$, a^{-1} is unique;

(iii) $a*b = a*c \Rightarrow b = c$;

(iv) $a*x = b$ and $y*a = b$ have unique solution.

(b) Define distributive lattice, modular lattice, bounded lattice. If $a, b, c \in L$, where $(L, \leq)$ is a distributive lattice, then $a \vee b = a \vee c$ and $a \wedge b = a \wedge c$ implies $b = c$.

(c) Using 1, 2, 3, 7, 8 and 9 numbers, each digit occurs only once, how many of them will be—

(i) divisible by 2;

(ii) all 4-digit numbers divisible by 2;

(iii) all 3-digit odd numbers;

(iv) all 4-digit even numbers?

(d) If H and K are subgroups of a group G, then prove that—

(i) $H \cap K$ is also a subgroup of G;

(ii) $H \cup K$ is not necessarily a subgroup of G;

(iii) $H \cup K$ is a subgroup of G iff $H \subseteq K$ or $K \subseteq H$.

(e) What is tree? Define a spanning tree. Name three algorithms to find the optimal spanning tree. Explain any one algorithm with a suitable example.

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