[22 HKPD3SN-23F]

M.Sc IT 3rd Semester (New) Exam. 2022 (W)

MSCIT

[PGScIT S3-01]

(*Introduction to Discrete Mathematics and Formal Language and Automata*)

Full Marks : 70

Time : 3 hours

The figures in the right-hand margin indicate marks.

PART—A

1. Answer any *three* from the following questions : $2 \times 3 = 6$

(*a*) Let $A = \{a, b, c\}$
$R = \{(a, a), (b, b), (c, c), (a, b), (b, c)\}$
Examine if R is (*i*) reflexive, (*ii*) symmetric.

(*b*) If P is true and Q is false, then find truth values of (*i*) $\sim P \rightarrow Q$, (*ii*) $\sim (P \rightarrow Q)$

(*c*) How many diagonals are there in a polygon of 10 sides ?

(*d*) Draw a simple graph with 5 vertices each of degree 2.

2. Answer any *three* from the following questions : $3 \times 3 = 9$

(*a*) Let $X = \{a, b, c\}$. Let $P(X)$ be the power set of X. Show that $(P(X), \leq)$ is a poset where '$\leq$' means is a subset of i.e., '$\subseteq$'.

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(*b*) Let B be a Boolean algebra. Prove that

(*i*) the zero element 0 is unique

(*ii*) the unit element 1 is unique

(*c*) Let Q be the set of rational numbers. Show that '*' defined by $a * b = at\ ab$ is a binary operation on Q. Examine if * is (*i*) commutative, (*ii*) associative.

(*d*) Find the graph that have the following adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

3. Answer any *two* from the following questions : $6 \times 2 = 12$

(*a*) Define an equivalence relation. Let Z be the set of integers. Let $R = \{(a,b) : a,b \in z,\ a-b$ is divisible by $5\}$. Examine if R is an equivalence relation on Z. 2+4 = 6

(*b*) Define tautology and contradiction. With the help of truth table, prove the following : $(P \leftrightarrow Q) \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$ 2+4 = 6

(*c*) How many integers between 1 to 100 that not divisible by 7, 11, or 13 ?

4. Answer any *one* of the following questions : $8 \times 1 = 8$

(*a*) Define (*i*) simple graph, (*ii*) multigraph, (*iii*) Pseudograph, (*iv*) complete graph and given an example of each. 2+2+2+2 = 8

(*b*) Define a group. Show that the set G consisting of all 2×2 nonsingular metrices over reals is a group under matrix multiplication. 2+6 = 8

(*Continued*)

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PART—B

5. Answer any *three* from the following questions : $2 \times 3 = 6$

(*a*) Define domain and co-domain of function with suitable examples.

(*b*) What is Kleene closure and Kleene plus ? Define alphabet, string and Null string.

(*c*) Given a grammar G defined by the production rules

$$S \rightarrow AB$$
$$A \rightarrow Aa$$
$$B \rightarrow Bb$$
$$A \rightarrow a$$
$$B \rightarrow b$$

show that the word $w = a^2b^4 \in L(G)$, where L is a language determined by G.

(*d*) Define the regular expressions of the following languages :

(*i*) All strings of 0's and 1's ending with 100.

(*ii*) All strings of 0's and 1's.

6. Answer any *three* from the following questions : $3 \times 3 = 9$

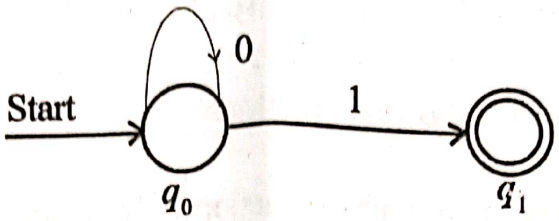
(*a*) Define NFA with ϵ-transition.

(*b*) Draw a DFA which recognizes the set of all strings on $\Sigma = \{a, b\}$ starting with the prefix *ab*. (*eq*, *ab*, *abb*, *abaa*, *ababb*, *abaaa* etc)

(*c*) Write the conditions for regular grammar. Give one example of regular grammar.

(*d*) What is the use of transition diagram in finite automata ? Write the regular

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expression for the automaton.

7. Answer any *two* from the following questions : $6 \times 2 = 12$

(*a*) Describe leftmost and rightmost derivation. For a given grammar

$$S \rightarrow A \mid AB$$
$$A \rightarrow a \mid Ab \mid AA \mid \in$$
$$B \rightarrow b \mid bc \mid Bc \mid bB$$

obtain the string *aabb* by leftmost derivation.

(*b*) Describe briefly the closure properties of context-free languages (CFL).

(*c*) Describe the types of relation with appropriate example.

8. Answer any *one* of the following questions : $8 \times 1 = 8$

(*a*) Define ambiguous grammar. Show that the grammar G with production is ambiguous $S \rightarrow a \mid aAb \mid abSb$
$A \rightarrow aAAb \mid bS$

(*b*) Write short notes on (any *two*) : 4+4 = 8

(*i*) Deterministic Finite Automata (DFA)

(*ii*) Chomsky Classification of Grammars

(*iii*) Pigeon Hole Principle.

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