UG PH S1-01
(DSC/DSE)

FOUR YEAR BACHELOR'S DEGREE COURSE

(CBCS in Accordance with NEP 2020)

SEMESTER - I

LOGIC I

PHILOSOPHY

Surya Kumar Bhuyan School of Social Sciences

KRISHNA KANTA HANDIQUI STATE OPEN UNIVERSITY

Subject Experts

1. Prof. Sauravpran Goswami, Dept. of Philosophy, G.U.
2. Dr. Sabitri Devi, Associate Professor, Dept. of Philosophy, Cotton University
3. Dr. Naba Prasad Nath, Associate Professor Dept. of Philosophy, Nowgaong College

Course Coordinator : Dr. Bhaskar Bhattacharyya, Assistant Professor, Dept. of Philosophy, K.K.H.S.O.U

SLM Preparation Team

Units	Contributors
1	Prof. Bilkis Banu, Dept. of Philosophy, G.U.
2	Dr. Bhaskar Bhattacharyya
3, 4 & 5	Dr. Maina Sarma, B. Borooah College
6 & 8	Dr. Pranati Devi, B. Borooah College
7	Dr. Bhaskar Bhattacharyya
9 &10	Dr. Maina Sarma, B. Borooah College
11 & 13	Dr. Pranati Devi, B. Borooah College
12	Dr. Bhaskar Bhattacharyya
14 & 15	Dr. Bikash Bhattacharyya

Editorial Team

Content : Prof. Begum Bilkis Banu, Dept. of Philosophy, G.U.
Dr. Maina Sharma, Dept of Philosophy, B. Borooah College
Dr. Pranati Devi, Dept of Philosophy, B. Borooah College

Language Editor: Dr. Arpana Nath, Assistant professor, Dept of English, Cotton College, Guwahati

Format Editor: Dr. Bhaskar Bhattacharyya, Dr. Tejasha Kalita

July, 2023

This Self Learning Material (SLM) of the Krishna Kanta Handiqui State Open University is made available under a Creative Commons Attribution-Non Commercial-Share Alike 4.0 License (international) : http://creativecommons.org/licenses/by-nc-sa/4.0/

Printed and published by Registrar on behalf of the Krishna Kanta Handiqui State Open University.

The University acknowledges with thanks the financial support provided by the **Distance Education Bureau**, **UGC** for the preparation of this study material.

Headquarters: Patgaon, Rani Gate, Guwahati-781017
City Office: Resham Nagar, Khanapara, Guwahati-781022 Web: www.kkhsou.in

BACHELOR OF ARTS

LOGIC: I

CONTENTS

		Page
UNIT 1:	**Nature of Formal Logic** Formal logic: What it is, Logic and deductive argument, Symbolic logic.	**9 - 17**
UNIT 2:	**Argument, Argument forms, Truth and Validity** Logic: What it is, Nature of Argument, Kinds of Argument: Inductive and Deductive, Logic and Deductive argument, Argument and Argument forms, Truth and Validity.	**18 - 31**
UNIT 3:	**Logical form** Concept of Logical Form, Distinction between Form and Matter, Recognizing the form of an argument, Logical form and the use of symbols, Importance of logical form.	**32 - 50**
UNIT 4:	**Concept of Symbol and Use of Symbol** Symbolic logic and symbol, What is symbol, the difference between symbol and sign, Different types of symbols, symbols used in propositional logic, symbols used in predicate logic, different uses of symbols.	**51 - 63**
UNIT 5:	**Inference and Implication** What is implication, Different meanings of material implication, Features of implication, Material implication.	**64 - 77**
UNIT 6:	**Modern Analysis of Proposition** Modern Analysis of Proposition, Proposition and Sentence, Modern classification of Proposition, Singular and General Proposition, Comparison between modern and Traditional classification of proposition.	**78 - 96**
UNIT 7:	**Square of Opposition of Propositions** Existential import of A, E, I and O proposition, Traditional Analysis of Square of Opposition, Modern Analysis of Square of opposition.	**97 - 114**
UNIT 8 :	**Proposition and its Analysis: Traditional Analysis of proposition and its Drawbacks** Proposition, Structure of proposition, Nature of Copula, Proposition and judgment, Proposition and Sentence, Classification of Proposition, Categorical propositions and classes, Doctrine of Distribution of terms, Reducing sentences to logical	**115-137**

forms, Opposition of propositions, Drawbacks of traditional analysis and classification of proposition.

UNIT 9: Truth function **138 - 152**

Proposition: Simple and Compound, Variables, Logical constants, the scope of logical constants and the use of brackets, truth function, basic truth functions and their tabular representations, truth tables for basic truth-functions.

UNIT 10: Truth-table method **153 - 165**

Truth table method, determining the truth value of compound expressions, basic rules for determining the validity of arguments by truth table method, indirect truth table method or the method of reduction-ad- absurdum.

UNIT 11: Modern Analysis of Proposition **166 - 193**

Branches of symbolic logic, Development of propositional logic and predicate logic, Propositional logic, Forms of propositional logic, Sentence, proposition and statement, Simple and compound proposition, Logical constant, propositional variable and propositional constant, Truth function, Decision procedure, Formal proof of validity, ,Predicate logic, Need of predicate logic, Singular and general proposition, Propositional function, Quantifiers, Rules of inference, Difference between propositional logic and predicate logic.

UNIT 12: Quantification theory **194 - 209**

Propositional Logic and Predicate Logic, Singular and General Propositions, Propositional Function, Quantification: What it is, Types of Quantifiers, Quantification of Four-Fold Classification of Propositions, Square of Opposition Proving Validity and Invalidity: Rules of Quantification.

UNIT 13: Existential and Universal quantifier **210 - 224**

Predicate logic, Symbols used in predicate logic, Quantifiers and types of Quantifiers, Difference between universal quantifier and existential quantifier, Symbolisation of categorical statements with quantifiers, and Scope of quantifier.

UNIT 14: Concept of set **225 - 246**

Set Membership, Null Set, Finite and Infinite Set, Subset and Superset, Universal Set Power Set.

UNIT 15: Set operation **247 - 280**

Union of Sets, Intersection of Sets, Difference of Sets, Complement of a Set.

Reference

COURSE INTRODUCTION

This course deals with Logic: I. Logic is an important branch of philosophy. The word 'logic' is derived from the Greek word 'logos'. It means thought or reason or law. Logic is defined as the science of reasoning. This definition is not accepted as the real definition of logic because reasoning implies a special kind of inferring or inference. Again, we can define logic as the study of the methods and principles used in distinguishing correct from incorrect arguments. This helps us to make a distinction between correct and incorrect argument. That is why; it is clear that logic is a tool or strategy by which we can find out the distinction between good reasoning and bad reasoning. The study of logic, therefore, enhances our reasoning power to testify the arguments, whether they are valid or invalid. It also helps us in our domestic discourse to argue systematically with others.

This course consists of one block.

The first block consists of fifteen units. The first unit starts with 'Nature of formal logic'. After going through this unit, you will be able to know what is formal logic and also the relation of logic with deductive argument. It also gives us information of kinds of argument (inductive and deductive). Besides, this unit also discusses the concepts of logic and deductive argument, argument and argument form, and truth and validity.

The second unit is "Argument and Argument form, Truth and Validity. This unit basically includes the concepts like what is logic, nature of argument, kinds of argument: deductive and inductive, logic and deductive argument, argument and argument form, and truth and validity.

The third unit is 'logical form.' It discusses the concept of logical form, the distinction between form and matter, and how to recognize form of an argument. Besides, this unit also discusses the use of symbols and importance of logical form in logic.

The third unit, 'Concept of symbol and use of symbol' basically deals with symbolic logic and symbol. It also gives us knowledge of symbol, the difference between symbol and sign, and different types of symbols. Over and above, it discusses the subject of using symbols in predicate logic and also the different uses of symbols.

The fifth unit, "Inference and Implication" covers the concepts like what is implication, different meanings of implication, and features of implication.

The sixth unit is **"**Modern Classification of Proposition: Simple and Compound, Singular and General." It includes the concepts like proposition and sentence, modern classification of proposition, singular and general proposition, and comparison between modern and traditional classification of proposition.

The seventh unit is “Square of opposition of propositions”. This unit basically gives us knowledge of existential import of A,E, I, and O proposition, traditional analysis of square of opposition, and modern analysis of square of opposition.

The eight unit ‘Proposition and its analysis: Traditional analysis of proposition and its drawbacks” deals with the concepts like proposition, structure of proposition, nature of copula, proposition and judgment, and proposition and sentence. Moreover, it discusses the issues of classification of proposition, categorical propositions and classes, doctrine of distribution of terms, reducing sentences to logical forms, opposition of propositions, and drawbacks of traditional analysis and classification of proposition.

The ninth unit, ‘Truth functions’ discusses the concepts which are as followed: proposition: Simple and Compound, Variables, logical constants, the scope of logical constants and the use of brackets, truth function, basic truth functions and their tabular representations, truth tables for basic truth-functions.

The tenth unit, “truth table method” covers the concepts like what implication is, different meanings of implication, and features of implication.

The eleventh (11th) unit is **“**Propositional logic and Predicate logic.” The concepts included in this unit are: branches of symbolic logic, development of propositional logic and predicate logic, propositional logic, forms of propositional logic, sentence, proposition and statement, simple and compound proposition, logical constant, propositional variable and propositional constant, proposition and sentence, modern classification of proposition, singular and general proposition, and comparison between modern and traditional classification of proposition, branches of symbolic logic, truth function, decision procedure, formal proof of validity, predicate logic, need of predicate logic, singular and general proposition, propositional function, quantifiers, rules of inference, difference between propositional logic and predicate logic.

The twelfth (12th) unit is “Existential and Universal Quantifiers”. This unit basically gives us knowledge of propositional logic and predicate logic, singular and general propositions, and propositional function. It also deals with what is quantification, types of quantifiers, quantification of four-fold classification of propositions, square of opposition, and proving validity and invalidity through rules of quantification.

The thirteenth (13th) unit is Existential and Universal quantifier. It provides us the knowledge of predicate logic, symbols of predicate logic, the concept of quantifiers, and types of quantifiers. It also deals with the difference between universal quantifier and existential quantifier, symbolization of categorical statements with quantifiers, and scope of quantifier.

The fourteenth (14th) unit is ‘Concept of set’. It discusses the concepts which are: set membership, null set, finite and infinite set, subset and superset, universal set, and power set.

The fifteenth (15^{th}) unit is 'Operation of set'. The concepts included in this unit are: union of sets, intersection of Sets, difference of sets, complement of a Set**.**

While going through this course you will come across some boxes which are put on the left side or right side of the text. These boxes will give us the meanings of some words and concepts within the text. Apart from this, there will be some broad and short questions included under Activity and Check Your Progress in every unit. Activities will increase our thinking capacity because questions put in Activity are not directly derived from the text. But answers to the short questions are put in the section Answers to Check Your Progress. Besides, there are some text-related questions which are put in Model Questions. These questions will help you in selecting and mastering probable topics for the examination so that you can prepare for the examination with confidence.

UNIT 1 : NATURE OF FORMAL LOGIC

UNIT STRUCTURE

1.1 Learning Objectives
1.2 Introduction
1.3 Formal Logic: What it is
1.4 Logic and Deductive Argument
1.5 Symbolic Logic: A kind of Formal Logic
1.6 Logical form and Matter
1.7 Let Us Sum Up
1.8 Further Reading
1.9 Answers to Check Your Progress
1.10 Model questions

1.1 LEARNING OBJECTIVES

After going through this unit you will be able to:

- define, what is formal logic
- analyse the relation between logic and deductive argument
- explain the concept of symbolic logic
- describe the relationship of form and matter in logic.

1.2 INTRODUCTION

Logic is regarded as a branch of Philosophy. The term 'Logic' is derived from the Greek word 'Logos' which means thought or reasoning. Thus logic deals with reasoning. It is a study of reasoning. Reasoning is a special kind of thinking called inferring in which we arrive at a certain position on the basis of certain others propositions. The proposition which we arrive at is called the conclusion and the proposition or propositions from which we arrive at the conclusion are called the premise or premises. Thus reasoning is a kind of thinking in which conclusions are drawn from premises. Corresponding to every possible inference there is an argument. Every argument has a structure. It consists of 'premises' and 'Conclusion'. Premises provide support for the truth of the conclusion.

1.3 FORMAL LOGIC: WHAT IT IS

Aristotelian logic is formal logic. Formal logic is concerned exclusively with the principles of deductive reasoning and it is concerned with the forms rather than with the content of propositions. Deductive logic is formal logic as opposed to Inductive logic which is a non formal logic or material logic. Deductive logic is formal logic as the term validity which is applicable to any kind of deductive argument implies only formal validity. The concept of logical form is central to logic as the validity of an argument is determined by its logical form and not by its content on subject matter.

1.4 LOGIC AND DEDUCTIVE ARGUMENT

Deductive argument is characterized as one in which the conclusion follows necessarily from the premises. An argument is valid if the conclusion follows with strict necessity from the propositions known as premises. In a valid argument it is inconsistent to assert the premises as true and to assert the conclusion as false, i.e. to deny the conclusion. Logic is the science of deduction. Its aim is to find out whether inferences or arguments are valid or invalid. It aims to provide systematic means for telling whether given conclusion do or do not follow from given premises.

Logicians are primarily concerned with the correctness or incorrectness of arguments. The distinction between correct and incorrect argument is the central problem with which logic deals. They have developed certain methods and principles of logic primarily to make this distinction clear. The correctness or validity of arguments depends on the very form or structure of the arguments.

Logic was established as a formal discipline by Aristotle. Aristotle's system of logic is a formal logic. Aristotelian study of syllogism focused on arguments containing propositions of a special kind called categorical propositions. Categorical syllogisms consist of categorical propositions. Categorical propositions consist of subject, predicate and a copula (verb). In Aristotelian logic the letters S and P represent the subject and the predicate terms respectively. Thus letters 'S', 'P', 'M' etc are used as variables.

Universal affirmative proposition is of the form 'All S are P', Universal negative proposition is of the form 'No S are P'. Likewise, particular affirmative and particular negative propositions are of the form 'Some S are P' and 'Some S are not P' respectively. Different valid moods of different figures like Barbara, Celarent etc of Aristotelian logic are examples of formal validity.



CHECK YOUR PROGRESSS

Q 1: Fill in the blanks:

a) The term logic is derived from the Greek word……..

Q 2: What is reasoning?

.....

.....

Q 3: What is formal logic?

.....

.....

Q 4: What is deductive argument?

.....

.....

1.5 SYMBOLIC LOGIC: A KIND OF FORMAL LOGIC

Modern logic or symbolic logic is a formal logic. Symbolic logic is a developed form of Aristotelian logic. The Difference between symbolic logic and Aristotelian logic is not a difference of kind but a difference of degree but the difference in degree is huge. Symbolic logic makes an extensive use of symbols. Apart from variables like p, q, r, s, t etc. symbolic logic makes use of certain signs or symbols such as ~, ., v, ⊃, ≡ known as logical constants. Thus symbolic logic through the development of its symbolic language has become a powerful tool for analysis and deduction.

Propositional calculus is a basic part of logic and is usually taken as starting point for the study of symbolic logic. Propositions are taken as basic units and in propositional logic propositions are not divided into component parts.

In propositional calculus the letters of the alphabet like p, q, r, s, t and so on stand for propositions and these letters as used in propositional calculus are known as propositional variables because they can stand indifferently for any proposition. For example, the sentence, “It is raining now” is a proposition because this sentence can be either true or false. A proposition is defined as any sentence which is either true or false but cannot be both true and false at the same time. The sentences like (1) “Today is a holiday.”, (2) “Sky is blue”, (3) Arsenic is poisonous’ are all propositions. These three propositions can be symbolized as (1) p (2) q (3) r etc which are known as propositional variables. Words like ‘not’, ‘and’, ‘or’, ‘if— then’ etc are known as logical constants and these logical constants are represented by artificial symbols in the following way-

For “not P”, we write ‘~P ’

For ‘p and q’, we write ‘p.q’

For ‘p or q’, we write ‘p $\vee$ q’

For ‘if p then q’, we write, ‘p ⊃q’

Thus in propositional logic we symbolize the compound propositions by means of variables and constants. Thus in symbolic logic compound propositions are symbolized by means of variables and constants. The p.q, p $\vee$ q, p ⊃q, ~p are called truth functions because the truth or falsely of these functions depend on the truth and falsely of the constituent variable like ‘p’ and ‘q’. These ‘p.q’, ‘p $\vee$ q’, ‘p ⊃q’, p $\equiv$ q, ‘~p’ are logical forms of different compound propositions. Thus we can speak of the logical form of a statement or set of statements, constituting an argument and in doing so we distinguish the form or structure of statements or argument from its subject matter.

1.6 LOGICAL FORM AND MATTER

Two or more arguments may have different subject matter but all of them may share the same form. For example, the following two arguments have different subject matter but have the same form.

1) If it rains then the ground is wet.

It rains,

Therefore, the ground is wet.

2) If Ajay wins the game then his mother is happy.

Ajay wins the game

Therefore, his mother is happy.

These two arguments have different subject matter but they share the same form i.e.

p ⊃ q

p

∴ q

This argument form is valid and therefore all arguments that share this form will be valid no matter what their subject matters are. Thus the special symbols of modern logic help us to exhibit with greater clarity the logical structures or forms of propositions and arguments. Thus symbolic logic as formal logic is concerned with validity or invalidity of different argument forms rather than with the validity or invalidity of particular individual arguments. Symbolic logic offers methods of deciding the validity of types of argument. Symbolic logic or formal logic is the study of symbolic abstractions of the formal features of logical inferences. The different systems of formal logic e.g. Truth Functional System, Axiom System, Quantification System, Model Theory, Set Theory or Boolean algebra of classes are abstract study of propositions and of deductive arguments. Set theory or Boolean algebra of classes is also known as mathematical logic and as such it is purely a formal study. Thus Formal logic is the abstract study of proposition and of deductive arguments. It is not an empirical study and as such it is not concerned with empirical truth or falsity of propositions. It is concerned with formal truth and formal validity.



CHECK YOUR PROGRESS

Q 5: Why is Aristotle's system of logic formal?

.....

.....

Q 6: Do you find any difference between symbolic logic and Aristotelian logic?

.....

.....



1.7 LET US SUM UP

- The term 'Logic' is derived from the Greek word 'Logos' which means thought or reasoning. Thus logic deals with reasoning. It is a study of reasoning. Reasoning is a special kind of thinking called inferring in which we arrive at a certain position on the basis of certain other propositions. Reasoning is a special kind of thinking called inferring in which we arrive at a certain position on the basis of certain other propositions.
- Aristotelian logic is a formal logic. Formal logic is concerned exclusively with the principles of deductive reasoning and it is concerned with the forms rather than with the content of propositions.
- Deductive logic is formal logic as opposed to Inductive logic which is a non formal logic or material logic. Deductive logic is formal logic as the term validity which is applicable to any kind of deductive argument implies only formal validity.
- Logicians are primarily concerned with the correctness or incorrectness of arguments. The distinction between correct and incorrect argument is the central problem with which logic deals. Aristotle's system of logic is a formal logic.
- Aristotelian study of syllogism focused on arguments containing propositions of a special kind called categorical propositions. Categorical syllogisms consist of categorical propositions. Categorical syllogisms consist of categorical propositions. Categorical propositions consist of subject, predicate and a copula (verb).
- Modern logic or symbolic logic is a formal logic. Symbolic logic is a developed form of Aristotelian logic. The Difference between symbolic

logic and Aristotelian logic is not a difference of kind but a difference of degree but the difference in degree is huge.

- In propositional calculus the letters of the alphabet like p, q, r, s, t and so on stand for propositions and these letters as used in propositional calculus are known propositional variables because they can stand indifferently for any proposition.
- Thus in symbolic logic compound propositions are symbolized by means of variables and constants. The p.q, p ∨ q, p ⊃ q, ~p etc. are called truth functions because the truth or falsity of these functions depend on the truth and falsely of the constituent variables like 'p' and 'q'.
- Symbolic logic as formal logic is concerned with validity or invalidity of different argument forms rather than with the validity or invalidity of particular individual arguments.
- Formal logic is the abstract study of proposition and of deductive arguments. It is not an empirical study and as such it is not concerned with empirical truth or falsity of propositions. It is concerned with formal truth and formal validity.



1.8 FURTHER READING

1) Copi, M, Irving. *Symbolic Logic* (fifth edition) Prentice Hall of India (Private Ltd.) New Delhi
2) Jain, Krishna. 1998. *A textbook of logic* (fourth Revised Edition)
3) Singh, S. Shyam Kishore. *Modern Logic* (Vol.1) Lamyana Press
4) Copi, M, Irving & Cohen, Carl, & Jetli, Priyedarshi, & Prabhakar, Monica. *Introduction to Logic* (Twelfth edition) Dorling Kindersley (India) Pvt. Ltd.



1.9 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: Logos.

Ans to Q No 2: Reasoning is a kind of thinking which is also known as inferring in which we arrive at a conclusion on the basis of other propositions.

Ans to Q No 3: Formal logic is concerned with the forms rather than with the content of propositions.

Ans to Q No 4: Deductive argument is a kind of argument in which the conclusion follows necessarily from the premises.

Ans to Q No 5: Aristotelian syllogism is concerned with arguments which contain propositions of a special kind called categorical propositions. Categorical arguments consist of categorical propositions. It also consists of subject, predicate, and copula.

Ans to Q No 6: Symbolic logic is a developed form of Aristotelian logic. The difference between symbolic logic and Aristotelian logic is not a difference of kind but a difference of degree. Symbolic logic uses extensive use of symbols.



1.10 MODEL QUESTIONS

A. Very Short question

Q 1: What is argument?

Q 2: What is reasoning?

Q 3: What is deductive argument?

Q 4: Write a note on reasoning.

Q 5: What do you mean by premises?

Q 6: What is conclusion?

Q 7: Is inductive logic formal?

Q 8: What is formal logic?

Q 9: What is symbolic logic?

Q 10: What is propositional variable?

Q 11: Give an example of propositional variable.

Q 12: Give an example of logical form.

B. Short question (Answer in about 150 words)

Q 1: Write a note on formal logic.

Q 2: What is symbolic logic? Briefly explain.

Q 3: What is propositional variable? Briefly explain

Q 4: Do you find any relation between form and matter in logic? Briefly explain with example.

C. Long Questions (Answer in about 300- 500 words)

Q 1: How logic deals with deductive reasoning? Explain with examples.

Q 2: "The difference between symbolic logic and Aristotelian logic is not a difference of kind but a difference of degree, but the difference in degree is huge." Explain this statement

Q 3: Do you find any relation between symbolic logic and formal logic? Explain.

Q 4: Do you find any relation between logical form and subject matter in argument? Explain.

*** ***** ***

UNIT 2 : ARGUMENT AND ARGUMENT FORMS, TRUTH AND VALIDITY

UNIT STRUCTURE

2.1 Learning Objectives

2.2 Introduction

2.3 Logic: what it is

2.4 Nature of Argument

- 2.4.1 Kinds of Argument: Inductive and Deductive
- 2.4.2 Logic and Deductive Argument
- 2.4.3 Argument and Argument Form

2.5 Truth and Validity

2.6 Let Us Sum Up

2.7 Further Reading

2.8 Answers to Check Your Progress

2.9 Model Questions

2.1 LEARNING OBJECTIVES

After going through this unit, you will be able to—

- define logic
- define argument
- discuss the nature of argument
- explain inductive and deductive reasoning
- describe the relation of logic and deductive reasoning
- explain the distinction between truth and validity in logic.

2.2 INTRODUCTION

Argument and argument forms, truth and validity are the fundamental notions of logic. A proposition is the statement which is either true or false. As for instance, “Manmohan Sing is the present Prime Minister of India”, “Sachin Tendulkar is one of the best cricketers of the world” and “Men are four-footed animals” etc. are the examples of propositions. The first two

propositions are true, while the third one is false, because we know that men are not four-footed animals.

On the other hand an argument is either valid or invalid. An argument implies a logical relation of premise and conclusion. When a conclusion is necessarily derived from the premises, the argument is valid. On the other hand when a conclusion is not necessarily derived from the premises, the argument will be invalid. On the other hand an argument form is a symbolic representation of propositions. As for instance:

All historians are rationalists

Bipan Chandra is a historian

∴ Bipan Chandra is a rationalist

This is valid argument, because the conclusion is necessarily derived from the premises.

If he comes, I will go

He does not come

∴ I shall not go

This is invalid argument, because the conclusion is not necessarily derived from the premises. From the above examples it is clear that validity and invalidity are concerned with argument.

To be brief, truth and falsity are dealt with propositions and validity and invalidity are concerned with arguments.

2.3 LOGIC: WHAT IT IS

The word 'logic' is derived from the Greek word 'logos'. It means thought or reason or law. Logic is defined as the science of reasoning. This definition is not accepted as the real definition of logic because reasoning implies a special kind of inferring or inference. In inference we proceed from known to unknown. We can draw the conclusion that 'there is fire in the hill', which is derived from the premise that 'there is smoke in the hill'. But the important issue is that logic does not include the act of inferring or inference within the subject matter of logic.

Again we can define logic as the study of the methods and principles used in distinguishing correct from incorrect arguments. This helps us to

make a distinction between correct and incorrect argument. That is why, it is clear that logic is a tool or strategy by which we can find out the distinction between good reasoning and bad reasoning. The study of logic, therefore, enhances our reasoning power to test the arguments, whether they are valid or invalid. It also helps us in our domestic discourse to argue systematically with others.

Traditional logic is defined as the study of laws of thought. There are three laws of thought which are indispensable for correct reasoning. These laws are: i) the law of identity, ii) the law of excluded middle iii) law of non-contradiction. Apart from these laws, there are other laws of inference and deduction, which are also equally indispensable for correct reasoning. These three laws of thought refer to mental process, but every mental process is not considered reasoning. As for instance, remembering, imagining, memory, day dreaming etc. are not considered as reasoning. They are only considered as mental states. As logic deals with correctness or incorrectness, so, logic is defined as the science of valid reasoning. Logicians deal with validity or correctness and invalidity or incorrectness of arguments, while psychologists concern with mental process. That is why the traditional definition of logic as the study of laws of thought is not satisfactory from the logicians' points of view.



CHECK YOUR PROGRESS

Q 1: State whether the following statements are true or false:

a) Logic is derived from the Greek word 'logos'. (True/ False)

b) Validity and invalidity are concerned with propositions. (True/False)

c) Truth and falsity concern with arguments. (True/False)

d) A proposition is either true or false. (True/False)

e) There are five laws of thought. (True/False)

f) Logic is the study of the methods and principles used in distinguishing correct from incorrect arguments.

g) Logic deals with the product of reasoning that is argument

Q 2: What is argument?

Q 3: Give an example of argument.

Q 4: What is logic?

Q 5: What are the three laws of thought in traditional logic?

Q 6: What is conclusion?

Q 7: What is premise?

Q 8: Is the definition of traditional logic satisfactory? (Within 60 words)

2.4 NATURE OF ARGUMENT

As it is said that logic deals with the correctness or incorrectness of argument, so the question naturally comes to enquire, what is argument? And what are its kinds?

Arguments consist of a group of proposition, of which one is the conclusion and others are premises which are regarded as providing support or grounds for the truth of the conclusion. Premises are those propositions from which conclusions are derived. On the other hand conclusions are those propositions which are drawn from the premises. There is a necessary relationship between premise and conclusion. It will be clear when we go through examples.

All men are mortal
Socrates is a man
∴ Socrates is mortal

The first two propositions are premises because the last one is drawn from the first two propositions. The last proposition is known as conclusion because it is drawn from the first two propositions. So, in an argument there is a necessary relationship between premise and conclusion. If we do not notice a necessary relationship between premise and conclusion, the argument will be invalid. As for instance-

A. All men are rational
Shibaji is a man
∴ Shibaji is rational

B. If rain comes, we will not go out
Rain comes
∴ We will not go out

C. Most actors are celebrities
Amir Khan is an actor
∴ Amir Khan is a celebrity

The first two examples show that premises imply the conclusion and which is necessarily derived from the premises. And in the third one premises support the conclusion.

Logic is concerned with the product of reasoning that is argument. It is deals with the enquiry of whether an argument is correct (sound) or incorrect (unsound). The task of the logician is to find out methods and techniques which are essential for making this distinction.

2.4.1 Kinds of Argument: Inductive and Deductive

According to traditional logicians, there are two kinds of argument-deductive argument and inductive argument. And accordingly logicians have classified logic into inductive logic and deductive logic.

In deductive argument the conclusion is necessarily derived from the premises. Generally, we proceed from general premise to particular premise in case of deductive argument. So, the conclusion is less general than the premises.

As for instance:

All babies cry
Munmun is a baby
∴ Munmun cries

Here, the first proposition 'All babies cry' is a major premise. And the second proposition 'Munmun is a baby' is a minor premise. The last proposition 'Munmun cries' is the conclusion which is derived from the above two premises. So, this is called deductive reasoning. According to this reasoning, if the premises are true the conclusion must be true. It claims that premises provide absolutely conclusive

grounds for the truth of the conclusion.The technical terms 'valid' and 'invalid' are used in place of correct and incorrect in characterising deductive argument. A deductive argument is valid when its premises and the conclusion are true and it is absolutely impossible for the premises to be true and the conclusion to be false.

On the other hand, In case of inductive argument the conclusion is derived only with probability from its particular premises. The premises serve as evidence for the conclusion. Inductive arguments involve the claim that the premises provide some grounds for the truth of the conclusion. Here, we proceed from particular premise to general premise. So, in inductive argument the conclusion is more general than the premises. Inductive arguments deal with inductive logic.

As for instance:

Pinky is a baby and it cries
John is a baby and it cries
Paul is a baby and it cries
∴ All babies cry

In this inductive argument the conclusion is derived from particular premises. The premises are collected from experience. Therefore, they serve as evidence or ground for the truth of the conclusion. The conclusion- 'All babies cry' is a general premise which is derived from the particular premises like 'Pinky is a baby and it cries', 'John is a baby and it cries', 'Paul is a baby and it cries' etc. So, in inductive argument we derive a conclusion from particular premises. The argument is true because the premises provide some grounds or evidence for the truth of the conclusion. On the other hand, inductive argument will be false when its premises do not provide some support or evidence for the truth of the conclusion.

2.4.2 Logic and Deductive Argument

Logic specifically deals with the enquiry into the deductive reasoning in which the conclusion is necessarily derived from the premises. We often use the terms reasoning or argument for logic. Deductive logic is also known as *exact logic* or *formal logic*.

Logic deals with the enquiry whether an argument is correct (sound) or incorrect (unsound). The task of the logician is to find out methods and techniques to distinguish correct reasoning from incorrect reasoning. The correctness and incorrectness of argument is not dealt with the subject matter of reasoning.

For example,

(1) All cats are dogs
Raja is a cat
∴ Raja is a dog

(2) All Abbas are Babbas
Raja is an Abba
∴ Raja is Babba

The above argument is valid or correct. The correctness or validity of the above arguments is not concerned with the subject matter. They are valid because of their form and not because of their subject matter. The form of an argument implies the very structure of the argument. When we replace all the terms of an argument by letters of English Alphabet we get the form of the argument. For example, if we use the letters 'S' and 'P' in place of the terms of the above argument and the letter 'a' for the proper name then we get the argument form-

All S are P
a is S
∴ a is P

Here, this argument form is valid. Therefore, any argument with any subject matter will be valid only it conforms with the form of the argument as mentioned above. So, logicians deal with the argument form and not with the subject matter of argument.

2.4.3 Argument and Argument Form

In case of the discussion of the nature of argument we find a distinction between argument and argument form. We already know that argument contains propositions from which conclusion is necessarily derived from the premises. On the other hand, argument form is a symbolic formulation of propositions which contain variables. We can go to the following examples:

A
All men are rational
Ram is a man
∴ Ram is rational

B
If P then Q
P
∴ Q

The first example is an example of argument, while the other one is the example of argument form. It can be mentioned that logic is primarily concerned with the argument form. Argument form is nothing but the logical or symbolic structure of proposition.

Logic is concerned with the product of reasoning that is argument. It deals with the enquiry whether an argument is correct (sound) or incorrect (unsound). The task of the logician is to find out methods and techniques to distinguish correct reasoning from incorrect reasoning. The correctness and incorrectness of argument is not concerned with the subject matter of reasoning.

2.5 TRUTH AND VALIDITY

Truth and falsity, validity and invalidity are the fundamental notions of logic. A proposition is true when it corresponds to reality. As for instance, 'Grass is green' is a true proposition because we see that grass is green in reality. We nowhere see that grass is white. On the other hand the proposition 'All animals are immortal' is a false proposition because we never perceive immortal animals.

On the other hand in case of argument we use the terms 'valid' or 'invalid'. A deductive argument is valid when the conclusion is necessarily derived from the premises. So, the term validity is not applied in case of propositions. In a valid argument, the propositions which are considered as premises may be true or false. Therefore, a valid argument can have true premises or false premises. If the conclusion in an argument is false, still the argument can be valid, because the premises may imply the conclusion. Again, in a valid argument there can be false premises, yet the argument can be valid provided the conclusion follows from the premises. Truth and falsity are the attributes of propositions and validity and invalidity are the attributes of arguments.

The relations between true or false propositions and validity or invalidity of arguments are complex. We generally notice this relationship in deductive logic. An argument may be valid in spite of the fact that premises are false. Again, an argument may be valid even when its conclusion and one or its premises are false. The validity or invalidity of argument depends only upon the relationship of the premises and the conclusion.

We can take some examples in order to show that a valid argument can have false premises and an invalid argument can have true premises.

A. All dogs are cats
All cats are horses
∴ All dogs are horses

This argument shows that the premises are false but the argument is valid.

B. All dogs are animals
All cats are animals
∴ All cats are dogs

This argument is invalid, although premises are true.

C. All human beings are rich
All businessmen are rich
All businessmen are human beings.

This is an invalid argument because the premises in the argument are false.

From the combination of false premises and true conclusion we can decide whether the argument is valid or invalid, depending on the arrangement of premises and conclusion. For example:

A. All men are rational

No cats are rational

∴ No cats are men

This is a valid argument. This argument shows that the premises and the conclusion are true.

The above discussion holds that truth and validity are the basic concepts of logic. Validity and invalidity of argument become independent of the truth and falsity of the premises. From the above mentioned examples it is clear that a valid argument can have false premises; similarly an invalid argument can have true premises. Again, validity and invalidity of arguments are not concerned with whether the premises are empirically true or not. Irving M. Copy in this context states that "To determine the truth or falsehood of premises is the task of scientific enquiry in general, since premises may deal with any subject matter at all. But determining the validity or invalidity of arguments is the subject of deductive logic." (*Symbolic Logic* p-5). Logicians are concerned with the question of validity and invalidity of arguments depending on the logical relation of the premises and conclusion.



CHECK YOUR PROGRESS

Q 9: What are the two kinds of argument?

Q 10: What is deductive argument?

Q 11: What is inductive argument?

Q 12: Give an example of deductive argument?

Q 13: What do you mean by argument form?

Q 14: Give an example of argument form

Q 15: State whether the following statements are true or false (True/False)

a) Deductive logic is also known as exact logic or formal logic. (True/False)

b) In inductive argument we proceed from general premise to particular Premise. (True/False)

c) The technical terms 'valid' and 'invalid' are used in inductive logic. (True/False)

d) The correctness and incorrectness of argument is associated with the subject matter of reasoning. (True/False)



ACTIVITY 2.1

- Do you find any relationship between truth and falsity of propositions and validity and invalidity of arguments? Discuss

Ans : .....

.....

.....

.....

.....



2.6 LET US SUM UP

From the above discussion it becomes clear that the concepts of Argument and Argument forms, truth and validity are the fundamental concepts of logic. These concepts of logic are interdependent. The concepts of argument and argument form without the concepts of truth and validity will be answerless. And again, the concepts of truth and Validity without the concepts of argument and argument form will be contentless discussion. That is why, the subject matter of logic starts with the discussion of argument and argument form and truth and validity. So, these two inter-related concepts are considered as fundamental concepts in logic.

The most important points discussed in this unit are:

- Logic is derived from the Greek word 'Logos'. It means 'thought' or 'reason' or 'law'.
- Logic is defined as the study of the methods and principles used in distinguishing correct from incorrect arguments.
- There are three laws of thought. They are respectively: The law of identity, the law of excluded middle and the law of non-contradiction.
- Argument consists of propositions. Conclusion is necessarily derived from the premises. Argument implies a relation between premise and conclusion.
- An argument form is a symbolic representation of propositions.
- Arguments are generally divided into kinds: Deductive and inductive argument
- Truth and falsity are dealt with propositions and validity and invalidity are concerned with arguments.



2.7 FURTHER READING

1) Copi, Irving M. (2006). *Symbolic Logic* (fifth edition). New Delhi: Prentice Hall of India Private Ltd.
2) Jain, Krishna.(1998). *A textbook of logic.* (fourth Revised Edition)
3) Singh, S. Shyam Kishore. *Modern Logic* (Vol.1) Lamyana Press
4) Copi, Irving M., Cohen, Carl., Jetli, Priyedarshi., & Prabhakar, Monica.(2006). *Introduction to Logic* (Twelfth edition).Delhi: Dorling Kindersley (India) Pvt. Ltd.



2.8 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: a) True b) False c) False d) True
e) False f) True g) True

Ans to Q No 2: An argument implies a logical relation between premise and conclusion.

Ans to Q No 3: All Philosophers are rationalists

Radhakrishnan is a philosopher

∴ Radhakrishnan is a rationalist

Ans to Q No 4: Logic is the study of the methods and principles used in distinguishing correct from incorrect arguments.

Ans to Q No 5: Conclusion is that proposition which is drawn from the premises.

Ans to Q No 6: The law of identity, the law of non-contradiction and the law of excluded middle

Ans to Q No 7: Premises are those propositions which provide the grounds to establish conclusion.

Ans to Q No 8: Traditional logic is defined as the study of laws of thought. These laws are: the law of identity, the law of non-contradiction, the law of excluded middle. These laws concern with the mental process. But logic is not concerned with the mental process. It deals with correctness or incorrectness of arguments. Therefore, the traditional definition of logic is not satisfactory.

Ans to Q No 9: Inductive and deductive argument

Ans to Q No 10: Inductive argument is that argument in which the conclusion is drawn with probability from particular premises

Ans to Q No 11: Deductive argument is that argument in which the conclusion is necessarily derived from premises.

Ans to Q No 12: All international cricketers are rich man

Sachin Tendulkar is an international cricketer

∴ Sachin Tendulkar is a rich man

Ans to Q No 13: An argument form is a symbolic formulation of propositions which contain variables.

Ans to Q No 14: If P then Q

P

∴ Q

Ans to Q No 15: a) True b) False c) False d) False



2.9 MODEL QUESTIONS

A. Objective Questions

Q 1: What is logic?

Q 2: Give an example of valid argument

Q 3: What are the three laws of thought?

Q 4: What are the kinds of arguments?

Q 5: Define inductive argument

Q 6: Give an example of deductive argument.

Q 7: What is the other name of deductive logic?

Q 8: What do you mean by argument form?

Q 9: What are three fundamental laws of thought?

B. Short Questions

Q 1: What is the nature of valid argument? Briefly explain

Q 2: Write short notes on Argument

Q 3: Write short notes on Argument form.

Q 4: Write short notes on Laws of thought

Q 3: Distinguish between truth and validity

Q 4: Distinguish between Deductive and inductive argument

C. Long Questions

Q 1: What is logic? Discuss the different definitions of logic and state which one you do accept more.

Q 2: What is argument? Explain the nature of argument with suitable examples.

Q 3: Do you find any relationship between truth and falsity and validity and invalidity of arguments? Illustrate with suitable examples.

Q 4: Are truth and validity the fundamental concepts in logic? What do you think?
Discuss with suitable examples.

Q 5: What do you mean by the nature of argument? Explain the distinction between argument and argument form with suitable examples.

Q 6: Is there any relation between logic and deductive argument? Discuss.

*** ***** ***

UNIT 3 : CONCEPT OF SYMBOL AND USE OF SYMBOL

UNIT STRUCTURE

3.1 Learning Objectives

3.2 Introduction

3.3 Symbolic Logic and Symbol

3.4 What is symbol?

3.5 The Difference Between Symbol and Sign

3.6 Different Types of Symbols

3.6.1 Symbols used in Propositional Logic

3.6.2 Symbols used in Predicate Logic

3.7 Different uses of symbols

3.8 Let Us Sum Up

3.9 Further Reading

3.10 Answers to Check Your Progress

3.11 Model Questions

3.1 LEARNING OBJECTIVES

After going through this unit you will be able to-:

- understand the relation between symbolic logic and symbol
- define symbol
- understand the difference between symbol and sign
- know different types of symbol
- understand utility of symbol in symbolic logic.

3.2 INTRODUCTION

In this unit we shall be dealing with the concept of symbol and some other issues connected with the concept of symbol. For a beginner in the subject of symbolic logic, a minimum preliminary knowledge of symbol seems to be necessary. Symbolic logic is a later development of traditional deductive logic. As the very name 'symbolic Logic' indicates, it involves the use of

symbols. The term 'symbol' in logic is used in a special technical sense. Symbolic logic, which is also known as 'formal logic' is deductive in nature. The use of symbol plays an important role in modern symbolic logic. In order to reveal its formal nature, modern logic makes use of a special technical language which is completely symbolic in nature.

3.3 SYMBOLIC LOGIC AND SYMBOL

It is well known that Symbolic logic is the result of intensive use of symbols upon traditional deductive logic. The exclusive use of deductive method in accordance with mathematical principles signifies the formal nature of symbolic logic. The formal nature of symbolic logic demands the use of symbols. If we go back to the history of symbolic logic, we can find that it is immensely influenced by the development of abstract reasoning that took place in mathematics. The triumph of deductive reasoning in mathematics inspired many great thinkers to apply a priori and deductive thinking in other branches of knowledge also. Symbolic logic is the result of this inspiration to extend mathematical method to other branches of knowledge. As a result, many able thinkers contributed the field of deductive logic in the later part of the nineteenth century and the early part of the twentieth century. The application of symbols became indispensible in this regard. Leibniz, the forerunner of mathematical logic planned to establish a universal scientific language in which scientific concepts could be represented by a combination of basic symbols. Thus it was realized by the thinkers like Leibniz and others that logic had a tremendous potential for a scientifically designed universal language which would be mathematical in nature. The attempt to give a mathematical interpretation of logical reasoning gave rise to a clear idea of a symbolic logical calculus. Great logicians like George Boole showed that algebraic formulae could be used to express logical relations with the special symbols, called connectives, such as conjunction, negation, disjunction and conditionals. German philosopher, mathematician, and logician Frege developed a rich formal language that involves the use of symbols. Bertrand Russell, a British philosopher and

mathematician along with A.N. Whitehead wrote an encyclopedic work entitled "Principia Mathematica" which established the point that pure mathematics can be reduced to logic without any residue. *Principia Mathematica* thus provided strong foundation to symbolic logic. Thus great mathematicians-logicians like Frege, George Boole, Russell, gave logic a solid formal nature. During the twentieth century much of the works in logic has focused on the formalization of logical systems. Formalization of logical systems made it incumbent upon logic to make use of symbols. Such developments in the path of formal deductive reasoning made it necessary for logic to be thoroughly symbolic in nature. Hence it came to be realized that symbols and symbolic logic are inseparable; It was John Venn (1834-1923) who first used the word "symbolic logic" to the emerging formal logic. In this way symbolic logic and the use of symbols came to be indispensably associated.



CHECK YOUR PROGRESS

Q 1: Fill up the blanks

a) Formalization of logical systems made it incumbent upon logic to make use of————

b) Symbols and ———— are inseparable.

c) It was ———— who first used the word "symbolic logic" to the emerging formal logic.

Q 2: Describe briefly how the use of symbols became indispensably associated with symbolic logic.

.....

3.4 WHAT IS SYMBOL?

It may generally be stated that symbols are signs or indicators representing some other thing. For example, the national flag of a country is the symbol of that country. By observing the flag one can say the country, which the flag symbolizes. Though symbol is a sign or indicator, indicating something, there is a difference between them. For example, the existence of smoke indicates the existence of fire. But smoke is not the symbol of fire.

Signs may be natural or artificial. Smoke is the natural sign of the presence of fire. But red light in a traffic point is an artificial signal to control the traffic. It is pre decided that running cars have to be stopped on the perception of red light in a traffic point, though there is no necessary connection between the two.

Thoughts operate through the signs or indicators. But operations of thought cannot achieve goals merely by depending on the natural signs because the number of natural signs or indicators is very limited as compared to the number of subjects to be expressed. Moreover, since abstract ideas cannot be presented to the senses, so the use of indicators which are pre decided becomes indispensable. Thus it can be said that artificial indicators which are pre decided may be said to be symbols.

In general use, signs either verbal or written, used to refer, understand, or express something are called symbols. For example, we use the sign '×' or '?' to evaluate answer. In such case, these signs may be said to be symbols.

3.5 THE DIFFERENCE BETWEEN SYMBOL AND SIGN

In symbolic logic, symbols are used in special sense. Symbol is a nonverbal artificial sign which is pre decided or consciously designated to represent or stand for something. Distinguished author of logic L.S. Stebbing defines a symbol as "a sign consciously designed to stand for something will be called a symbol". (Modern Introduction to Symbolic Logic, page 11)

From the above descriptions the features of symbol may be analyzed as follows-

1. Symbols are consciously used or decided beforehand to stand for or designate something. This feature differentiates symbol from signs. Sign are not always pre decided. For example, presence of smoke is a sign of the presence of fire. Here smoke is not consciously decided beforehand to represent fire.
2. Secondly, symbols are artificial. Natural signs are not sufficient to express the subject matter of the operations of thought, since their number is very limited. Hence in logic artificial symbols are used to

stand for abstract ideas or logical principles. This feature also differentiates signs from symbols. For example, the presence of smoke is a natural sign of the presence of fire. There is a necessary connection between smoke and fire, though the **reverse** is not true. On the other hand, alphabets like p, q, r, s, t etc. or mathematical symbols "×","±" "€" etc. are artificially designed symbols used in logic.

3. Thirdly, symbols used in logic are non-verbal. Symbols which are used to refer to substance, quality, action etc., are designated as verbal symbols. On the other hand, alphabets like p, q, r, s, t, etc., are non-verbal symbols. But signs are not necessarily non- verbal.



CHECK YOUR PROGRESS

Q 3: Fill up the blank

a) Symbols are ———— used beforehand to stand for or designate something.

b) Symbols are ————————

c) Symbols used in logic are————

Q 4: What is a symbol? Describe the three features of symbols.

.....

3.6 DIFFERENT TYPES OF SYMBOLS

We know that the extensive use of symbols is a special feature of symbolic logic. Before introducing different types of special symbols used in different major branches of symbolic logic, two types of symbols may roughly be distinguished at the initial stage. These are – ideograms and phonograms. In symbolic logic ideograms are used instead of phonograms.

Ideograms are those symbols which stand directly for concepts. For example, × ?÷, etc. are called ideograms. On the otherhand, phonograms are not symbols in the proper sense. They are signs which stand directly for sounds. For example, the written words "multiplication sign" or "question mark" are phonograms. Unlike the ideograms, phonograms represent the spoken English words which correspond to them. Thus while phonograms refer to the words in all languages which are written according to some sort

of phonetic rules, ideograms refer to the concept of the idea represented by the words.

Ideograms differ from the phonograms on the following three grounds—

1. Ideograms refer to the concepts directly. On the other hand, phonograms directly refer to the sounds and only indirectly to the concepts or the idea behind the word or the words. Thus the representing power of the ideograms is much more than the phonograms.
2. Secondly, ideograms are artificially constructed universal language while, phonograms represent the spoken or written words in any language which are constructed according to some sort of phonetic rules. Thus ideograms have universal appeal whereas phonograms don't have such universal appeal, as they are confined to specific languages.

Thus the symbols used in symbolic logic are called ideograms in a general sense. There are however different types of symbols within the general notion of what are called 'ideograms'.

We know that symbolic logic has three important branches. These three important branches are- propositional logic, predicate logic, and the logic of classes. Different specific symbols are used in different branches of logic.

3.6.1 Symbols used in Propositional Logic

Propositional logic is an important part of elementary symbolic logic. It is considered to be the starting point of symbolic logic. Propositional logic deals with the logical relationships holding between propositions without considering the interior structure of propositions. It analyses the terms, properties, relations and quantifications involved in the proposition. It is also known as the Sentential Logic or the Logic of Truth-functions.

In order to perform its specific functions, propositional logic makes use of mainly two types of symbols. This are-

- Propositional variable
- Logical constants.

Variables: —

In a very general sense, variables are symbols which can be replaced by any one of a definite range. In propositional logic, the small case letters of the English alphabet p, q, r, s, t are used as variables signifying simple statements. For example, let us consider the following argument:-

If p then q

P

Therefore, q

Here p, q are variables. They are symbols which can be replaced by any one of a definite range. They are called 'variables' because their meaning varies within a range and is not constant.

Logical constant:-Constants are those symbols which maintain the same meaning throughout every occurrence in the propositional formulae. A propositional formula means an expression or a propositional form which is a combination of symbols according to logical rules. In propositional logic, the following symbols are designated as logical constants———

1. Þ (negation sign read as 'not' or 'it is false that'....)
2. (conjunction sign read as 'and' or 'both….and…')
3. ∨ (disjunction sign read as 'or' , 'either/or')
4. (conditional sign or implicative, also known as horse shoe, read as 'if-then')
5. = (biconditional sign or equivalence, read as "if and only if")

The structure of propositional logic is explicated by these symbols. Thus while variables can assume any meaning within a definite range, the constants maintain the same meaning throughout every occurrence in propositional formula. Every logical constant with the exception of 4 (negation) is known as connective. They are called connectives because they form compound propositions from

simple or other compound propositions. Each connective connects two and only two propositions. Hence they are known as biennial constants.



CHECK YOUR PROGRESS

Q 5: Fill up the blank-

a) In symbolic logic ——— are used instead of phonograms.

b) Phonograms are not —— in the proper sense.

c) Propositional logic makes use of mainly ——— types of symbols.

Q 6: Give examples—

a) Propositional Variables-

b) Logical Constants-

Q 7: What are the logical constants and how many logical constants are there?

3.6.2 Symbols used in Predicate Logic

Predicate logic analyses the internal structure of the propositions (both simple and compound) in order to determine the validity of a given argument. It is also known as quantifier logic.

Predicate symbols: The fundamental component in predicate logic is the predicate. The letters which are used to symbolize these predicates are called predicate symbols. A predicate is symbolized by uppercase letters (A, B, C ….X,Y,Z)

Individual Constant: On the other hand, lower case letters from 'a' through'w' (the first twenty three letters of the alphabet) are used to denote individuals. These symbols are known as individual constants.

Example- The singular proposition 'India is great' is symbolized in predicate logic as- Gi.

Quantifiers: There are different symbols which are used to indicate quantity. These symbols are called quantifiers.

Words that indicate quantity such as 'all' 'every' 'most' are called quantifiers. Quantifiers indicate the quantity or how many of the individuals or objects have the property in question.

Predicate logic uses two types of quantifiers. They are 'universal quantifiers' and the 'existential quantifiers'. Universal quantifier is used to denote a universal proposition. On the other hand, existential quantifier is used to denote a particular or existential proposition.

The universal quantifier: The phrase 'given any x' is called a universal quantifier. The phrase, given any x' is symbolized as (x). The general proposition 'everything is temporary' is symbolized as: (x) Tx. Of course, various other alternative expressions may also be used.

The Existential Quantifier: The existential quantifier '(Žx)' is used to assert that some entities (at least one) have a given property. Thus to symbolize the proposition 'something is beautiful' or the proposition 'at least one thing is beautiful' we write- (Žx) Bx.



CHECK YOUR PROGRESS

Q 8: What is a quantifier? How many types of quantifiers are used in predicate logic?

Q 9: Define the following-

a) Universal quantifier

b) Existential quantifier.

Symbols used in Logic of Classes: It is one of the important branches of symbolic logic which is concerned with the classes and the relations among the classes. Special symbols are used in this branch of symbolic logic. Concept of set, Venn diagram technique and its different uses, modern interpretation of syllogistic arguments etc., are some of the important topics of this branch of symbolic logic. 'Membership' to the class' concept of 'empty class', 'operations on sets' etc.can also be symbolized accordingly. We can mention

just a few of the symbols as follows-

Symbol used to denote 'membership' (of a class) - €

Symbol used to denote 'not a member' (of a class) -

Symbol used to denote 'empty set' (empty class) - –", '"

Traditional A, E, I, O propositions are symbolized in the following way-

A = All S is P E =No S is P I= Some S is P. O= Some S is not P

SP= O SP = O SP = O SP =O

These are some of the symbols frequently used in this branch of symbolic logic.



CHECK YOUR PROGRESS

Q 10: State the symbols used to denote the following concepts respectively-

a) Membership of a class

b) Empty class.

3.7 DIFFERENT USES OF SYMBOLS

It has been seen that symbols are signs deliberately written used to signify something. In symbolic logic symbols are used to mean a special ideographic notation to express concepts peculiar to logic itself.

Modern or symbolic logic has made extensive use of symbols which helps the subject in the following direction.

1. The most significant use of symbols in symbolic logic lies in the fact that symbols exhibits the logical form of the argument and thereby helps to classify them according to their forms so that the validity of the argument can be determined.

The chief aim of logic is to determine the validity of the arguments. For this it is necessary to classify the arguments into some types according to their forms. This helps the application of the general rule into these types and their validity can be determined accordingly.

Example-: Let us consider the following arguments:

1. No arrogant student can be successful.
   Some intelligent students are arrogant.

Therefore, some intelligent students are not successful.

2. No good players are weak persons.
   Some Indians are good players.

Therefore, some Indians are not weak persons.

3. If the students are industrious then they can earn success.
   But they have not earned success.

Therefore, they are not industrious

4. If the rulers of the country are free of corruption, then the employees will also be free from corruption.

The employees are not free from corruption.

Therefore, the rulers are not free from corruption.

It can be observed that the arguments (1) and (2) and the arguments (3) and (4) are similar. This similarity is not because of subject matter but because of their forms. If we replace the three terms of the arguments no. 1 and 2, by using the symbols A, B and C respectively, this similarity of forms becomes clearer.

1. No A is B
   Some C is A.

Therefore, some C is not B

Similarly, if we use the variables p and q for the propositions of the arguments 3 and 4, we will get:

If p then q.

Not q

Therefore, not p.

When in this way the form of the argument is made clear through the use of symbols, it becomes easy to test the argument by applying the general rules.

2. The use of symbols helps to express the general nature of the logical rules. The rules of logic are applicable to all the arguments of a particular form. If a particular argument of a particular form is valid, any

other argument belonging to that form may be regarded as valid. Thus the use of symbol expresses the fact that a particular logical rule is applicable to all the arguments of a particular form. If a particular argument of a particular form is valid, any other argument belonging to that form may also be regarded as valid. Thus the use of symbols expresses the fact that a particular logical rule is applicable to all the arguments of a particular form.

3. The use of symbol gives conciseness, clarity and economy of expression to complicated statements and arguments which would be particularly unintelligible when expressed in ordinary language. The use of symbols minimizes the exercise of mental labor.

Example: Let us consider the following arguments:

1. The product of the sum and the difference of two numbers is equal to the difference of the squares of the two numbers.
1. (a+b) (a-b) = (a2-b2)

The example no. (2), which is expressed through the symbols is clearer than the example no. (1) which is expressed in ordinary language.

4. The use of symbols helps to express conveniently the concepts and technical terms which are peculiar to logic. Further, symbols like variables are also useful to represent different logical operations.

In every subject there are some special concepts and technical terms peculiar to the subject itself. Such terms in logic and mathematics are often most conveniently expressed through the symbols or ideographic notation. To give an example from mathematics- it is learned that Greek and Roman arithmetical notation lacked any symbol for “zero” and in consequence, simple arithmetical calculations like 7030×48 or 2323 ÷ 25 demanded considerable mathematical ability and enormous amount of mental labor.

5.The use of symbols helps to avoid the difficulties arising out of the use of ordinary language like-ambiguity, indefiniteness and the obscureness arising due to the use of metaphors, comparisons etc.

There are some inherent defects of ordinary language such as indefiniteness, ambiguity of the words etc. The fallacy of equivocation is an

example of such a defect which affects the syllogistic inferences. The use of symbols safeguards logic from such defects of ordinary language.

6. The use of symbols also helps to safeguard the argument from the influence of the material truth or falsity of the constituent propositions which creates difficulties in the determination of its validity.

Symbolic logic involves the use of deductive method. Accordingly, symbolic logic puts emphasis on formal truth rather than material truth. When an argument is expressed in ordinary language, the material truth or falsity of the constituent propositions and their meanings create difficulties in determination of the validity of the argument. This difficulty can be avoided by the use of symbols which brings the mind away from the material truth or falsity of the proposition and focuses the form of the argument.

Example: Let us consider the following arguments:

If I am a famous scientist, then I am a f

I am not a famous scientist.

therefore, I am not a famous person.

This argument seems to be valid superficially. Because premise and conclusion are materially true. But the invalidity of the argument can be easily recognized by comparing it with another argument of the same form.

If Mahatma Gandhi is a famous scientist, then he is a famous person. Mahatma Gandhi is not a famous scientist.

Therefore, Mahatma Gandhi is not a famous person.

Though the two premises of the argument no. (2) are true, but the conclusion is false. But this is not possible in a valid syllogism. If the premises of the above arguments are expressed through the symbols — p and q, then the form of the arguments will be:-

If p then

Not p

Therefore, q

Applying the rule of mixed hypothetical syllogism, it can be easily said that the hypothesis is invalid, because the consequent is denied in the conclusion by denying the antecedent of the premise. This violates the rule of mixed hypothetical syllogism. Thus the use of symbols helps to focus the

validity or invalidity of the argument by making the argument free from the influence of material truth or falsity of the constituent propositions.

Thus it can be seen that symbols are very useful in symbolic logic. Such advantages of the use of symbols have led the logicians to invent an artificial symbolic language which is free from the defects of ordinary language and through which the propositions and arguments can be conveniently expressed and the validity of the arguments can be determined.



CHECK YOUR PROGRESS

Q 11: Fill up the blank

a) The use of symbols exhibits the logical——— ——— of the argument.

b) The use of symbol gives————————, clarity and economy of expression to complicated statements and arguments.

Q 12: Explain briefly two advantages of using symbols in symbolic logic.



3.8 LET US SUM UP

- Symbolic logic is a modern development of traditional deductive logic. As the very name 'symbolic logic' indicates, it involves the extensive use of symbols. In symbolic logic, symbols are used in special sense.
- Symbol is a nonverbal artificial sign which is pre decided or consciously designated to represent or stand for something. Distinguished author of logic L.S. Stebbing defines a symbol as "a sign consciously designed to stand for something will be called a symbol". (Modern Introduction to Symbolic Logic, page)
- There are certain features of symbols of which distinguish them from signs. Thus symbols are *artificial non-verbal* signs used *consciously*.
- In symbolic logic different types of symbols are used. There are different branches of symbolic logic and to express the concepts different types

of symbols are needed.

- The use of symbols is indispensible in symbolic logic. The use of symbols helps the subject in the following directions——
- The use of symbols exhibits the logical form of the argument and thereby helps to classify them according to their forms. This helps to determine the validity of arguments by applying the general rules.
- The use of symbols helps to express the general nature of the logical rules. The use of symbols gives conciseness, clarity and economy of expression to complicated statements and arguments which would be practically unintelligible when expressed in ordinary language.
- Symbols help to express conveniently the concepts and technical terms peculiar to logic.
- The use of symbols also helps to safeguard the argument from the influence of the defects of ordinary language. In this way symbolic logic and the use of symbols came to be indispensably associated.



3.9 FURTHER READING

1) Copi, Irving M. & Cohen, Carl. 2007.*Introduction to Logic*. Prentice-Hall of India pvt.
2) Baronett, Stan & Sen, Madhuchanda. *2009*. *Logic.* Pearson:
3) Chakraborty,Chanda. 2007. *Logic Informal, Symbolic and Inductive* : Prentice-Hall India
4) Copi, I.M. 1973. *Symbolic Logic* :Macmillan Publishing Co., Inc.
5) Bason & O'Connor. 1977. *Introduction To Symbolic Logic*: Oxford University Press
6) Sharma, Banajit & Deka, Dr. Jahnabi. 2014. *A Text Book of Logic*. AANK- BAAK: Guwahati-1,

3.10 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: a) symbols b) symbolic logic c) John Venn.

Ans to Q No 2: The formal nature of symbolic logic demands the use of symbols. If we go back to the history of symbolic logic, we can find that it is immensely influenced by the development of abstract reasoning that took place in mathematics. The triumph of deductive reasoning in mathematics inspired many great thinkers to apply a priori and deductive thinking in other branches of knowledge also. Thus it was realized by the thinkers like Leibniz and others that logic had a tremendous potential for a scientifically designed universal language which would be mathematical in nature. During the twentieth century much of the works in logic has focused on the formalization of logical systems. Formalization of logical systems made it incumbent upon logic to make use of symbols. Such developments in the path of formal deductive reasoning made it necessary for logic to be thoroughly symbolic in nature.

Ans to Q No 3: a) Signs b) Artificial

Ans to Q No 4: Symbol is a non-verbal artificial sign which is pre decided or consciously designated to represent or stand for something. Distinguished author of logic L.S. Stebbing defines a symbol as “a sign consciously designed to stand for something will be called a symbol”. (Modern Introduction to Symbolic Logic, page 11)

From the above descriptions the features of symbol may be analyzed as follows-

Symbols are consciously used or decided beforehand to stand for or designate something. This feature differentiates symbol from signs. Sign are not always pre decided. For example, presence of smoke is a sign of the presence of fire. Here smoke is not consciously decided beforehand to represent fire.

b) Secondly, symbols are artificial. Natural signs are not sufficient to express the subject matter of the operations of thought, since their number is very limited. Hence in logic artificial symbols are used to stand for abstract ideas or logical principles. This feature also differentiates signs from symbols. For example, the presence of smoke is a natural sign of the presence of fire. There is a necessary connection between smoke and fire, though the **reverse** is not true. On the other hand, alphabets like p, q, r, s, t etc. or mathematical symbols "×","±" "€" etc. are artificially designed symbols used in logic.

c) Thirdly, symbols used in logic are non-verbal. Symbols used to refer to substance, quality, action etc. like 'table', 'chair', 'sincerity', 'honesty', etc.are verbal symbols. On the other hand, alphabets like p, q, r, s, t etc. or mathematical signs ÷,×, etc. are non-verbal symbols. This feature differentiates symbol from signs.

Ans to Q No 5: a) Ideograms b) Symbols c) Two

Ans to Q No 6: a) In propositional logic, the small case letters of the English alphabet p, q, r, s, t are used as variables signifying simple statements. For example, let us consider the following argument**:-**

If p then q

P

Therefore, q

Here p, q are variables.

b) In propositional logic, the following symbols are designated as logical constants—

÷ (Negation sign read as 'not' or 'it is false that)

. (Conjunction sign read as 'and' or 'both….and…')

Å (Disjunction sign read as 'or', 'either/or')

† (Conditional sign or implicative, also known as horse shoe, read as 'if-then')

= (Bi-conditional sign or equivalence, read as "if and only if")

Ans to Q No 7: Logical constants are important symbols of propositional logic. Constants are those symbols which maintain the same meaning

throughout every occurrence in the propositional formulae. There are five logical constants.

Ans to Q No 8: There are different symbols which are used to indicate quantity. These symbols are called quantifiers. Words that indicate quantity such as 'all' 'every' 'most' are called quantifiers. Quantifiers indicate the quantity or how many of the individuals or objects have the property in question.

Predicate logic uses two types of quantifiers; Universal quantifiers and the existential quantifiers.

Ans to Q No 9: a) The quantifier which is used to denote a universal proposition is called Universal quantifier. The phrase 'given any x' is called a universal quantifier. The phrase 'given any x' is symbolized as (x). The general proposition 'everything is temporary' is symbolized as: (x) Tx. Of course, various other alternative expressions may also be used.

b) The quantifier which is used to assert that some entities (at least one) have a given property is called an Existential quantifier. Thus to symbolize the proposition 'something is beautiful' or the proposition 'at least one thing is beautiful' we write- (Žx).

Ans to Q No 10: a) form

b) Conciseness

Ans to Q No 11: The most significant use of symbols in symbolic logic lies in the fact that symbols exhibits the logical form of the argument and thereby helps to classify them according to their forms so that the validity of the argument can be determined. Without the use of symbols, validity of the arguments cannot be easily determined.

The use of symbols helps to avoid the difficulties arising out of the use of ordinary language like-ambiguity, indefiniteness and the obscureness arising due to the use of metaphors, comparisons etc.

There are some inherent defects of ordinary language such as indefiniteness, ambiguity of the words etc. The fallacy of equivocation is an example of such a defect which affects the syllogistic inferences. The use of symbols safeguards logic from such defects of ordinary language



3.11 MODEL QUESTIONS

A) Very short questions

Q 1: What is symbolic logic?

Q 2: What is the other name of symbolic logic?

Q 3: Who is the author of *Principia Mathematica?*

Q 4: What is ideograms?

Q 5: What do you mean by phonograms?

Q 6: Write the definition of propositional variable and logical constants?

Q 7: What is universal quantifier?

Q 8: What is existential quantifier?

B) Short questions (Answer each question in about 150 words)

Q 1: Write a note on symbolic logic.

Q 2: What are two features of symbolic logic? Explain in brief.

Q 3: Do you find any difference between ideograms and phonograms? Briefly explain.

Q 4: What are the two types of symbols of propositional logic? Explain in brief.

Q 5: What are the different symbols used in predicate logic? Explain in brief.

Q 6: What are the different symbols used in logic of classes? Explain briefly

C) Long questions (Answer in about 300-500 words)

Q 1: What is symbol? State the difference between signs and symbols.

Q 2: Write a brief note on different types of symbols used in symbolic logic.

Q 3: Describe the utility of using symbols in symbolic logic.

Q 4: Define symbol and analyze the features of symbol.

*** ***** ***

UNIT 4 : LOGICAL FORM

UNIT STRUCTURE

4.1 Learning Objectives

4.2 Introduction

4.3 Concept of Logical Form

4.4 Distinction Between form and Matter

4.5 Recognizing the form of an Argument

4.6 Logical Form and the use of Symbols

4.7 Importance of Logical Form

4.8 Let Us Sum Up

4.9 Further Reading

4.10 Answers to Check Your Progress

4.11 Model Questions

4.1 LEARNING OBJECTIVES

After going through this unit you will be able to –

- explain the basic nature of modern symbolic logic
- define the meaning of the term 'Logical Form'
- describe the distinction between form and matter
- discuss the relation between logical form and the use of symbols
- state the Importance of logical form in modern logic
- explain the validity and invalidity of logical argument.

4.2 INTRODUCTION

Depending on its subject matter, method of study and specific aim, every science or arts has its nature peculiar to itself. We all know that symbolic logic is a modern form of traditional deductive logic where deductive method is applied along with extensive use of symbols and mathematical principles. Like traditional logic, the chief aim of symbolic logic is to determine the validity of arguments. In pursuing this aim, symbolic logic like mathematics makes use of deductive method.

It may be observed that the subject matter of symbolic logic is inference or argument; its aim is to determine the validity of the argument and the specific method is what is called deductive method. The subject-matter, aim and method specific to symbolic logic give the subject a special nature which can be termed as 'formal'. In this unit we shall discuss the concept of 'form' and other important issues related with logical form.

4.3 CONCEPT OF LOGICAL FORM

We can observe that in our day to day life we often use the term form and matter. Every material object e.g., a table has two aspects- one is its matter and the other is its form. When we speak of 'matter' of the table, we are talking of the elements out of which the table is made. It means for example, in the case of the wooden table, the 'wood' is the matter or the element which makes up the table. Again, supposing that the table is round in shape, then this "roundness" or round shape is the form of the table.

When the word 'form' is used in logic, it is used in connection with argument and constituent propositions. In other words, the word 'form' in logic is used as applied to arguments and propositions.

Like a material object, an argument and proposition also have particular forms and composed by certain elements which are called its 'matter'. For example, in the proposition, "All men are mortal", the constituent words are its matter and similarly, the matter of the argument is the constituent propositions or the premises and the conclusion.

On the other hand, the expression "logical form" indicates the way in which the matter or the elements of proposition or argument are expressed.

Logical form may be defined as the way in which the constituents or the elements of a proposition or argument are arranged to compose and express the proposition and argument. In other words, it is a type of structural co-ordination arising out of the mutual relations of the constituents or elements of a proposition or argument.

Thus it can be seen that the matter of a proposition or argument expresses the subject matter of a proposition or argument; on the other

hand, "form" of a proposition expresses the way in which the subject matter is expressed. Form relates to logical structures underlying the language content in any arguments. Content provides the matter of the argument. In other words, logical structure is the form or the mold to which the matter can be cast. Form relates to logical structure; it is the underlying logical relationship of an inference. The distinction between form and matter can be expressed with the help of two words namely, *syntax* and *semantics*. The 'form' relates to what is called, 'syntax'. On the other hand, 'matter' relates to what is called, *semantics* indicating the content or the matter or the subject which the argument is all about. Thus we can say that form is concerned with syntax (logical structure) of the argument but 'matter' is concerned with the semantics (meaning of the word) of the argument.



CHECK YOUR PROGRESS

Q 1: Fill in the blanks:

a) The word 'form' in logic is used as applied to arguments and ————

b) The constituent elements of an arguments are called its ———

c) Form of an argument relates to logical ————

d) Form is concerned with the syntax but matter is concerned with the ——— of the argument.

Q 2: Find out the correct answer

A. Logical form is related with the content of the argument / structural relations of the elements of the argument.

B. The nature of symbolic logic is formal /informal**.**

4.4 DISTINCTION BETWEEN FORM AND MATTER

Recognizing the form of a proposition and argument is very important. In order to discern the form of a proposition let us consider the following three propositions———

(1) All men are mortal

(2) All educational institutions are holy places.

(3) All pious individuals are honest.

If we observe above three propositions we can find that all the three propositions differ in respect to their content or matter but so far as their form is concerned, the propositions have the same form.

If we withdraw our attention from their subject matter, then we recognize their form to be of – "All p is q". Let us again observe the following propositions-

(1) All smoky objects are fiery

(2) If there is smoke, then there is fire.

What is striking about the above two propositions is that the two propositions have the same matter, but different forms. The form of the first proposition is as follows-All s is p. The second proposition has the following form- If p then q. Thus one need to withdraw one's attention from the subject matter of the proposition and concentrate on the way the subject matter is presented in order to recognize the form of the proposition. For this, the use of symbols is indispensible.

4.5 RECOGNIZING THE FORM OF AN ARGUMENT

We have learnt that in the case of an argument, the form is expressed in the way in which the constituent propositions are arranged to make the propositions. Let us explain it with the following examples-

EX.-1.-

I shall go if he comes.

He has come.

Therefore I shall go.

Ex. 2.-

If you study hard, then you will pass in the examination.

You have studied hard

Therefore you will pass in the examination.

Ex. 3.-

If it rains, then the crops will grow.

It rains.

Therefore the crops will grow.

All the three arguments have different matter but same form. The logical form of the above three arguments is one of hypothetical categorical syllogism. The form can be expressed as-

If p then q

p

Therefore, q

$p \supset q$

p

Therefore, q

This is modus ponens, a valid logical form. The use of symbols reveals the form of the argument.

4.6 LOGICAL FORM AND THE USE OF SYMBOLS

There is a close connection between logical form and the use of symbols. In order to express the form of proposition and argument, it is necessary to use symbols.

Symbolic logic is completely a formal science. In symbolic logic no importance is given at all to the matter or the elements of the proposition and the argument. As it has been observed in order to discern the form of the proposition or the argument one need to completely withdraw one's attention from the subject matter of the argument or proposition as the case may be and express the same through symbols. The use of symbols makes the logical form clear. Thus the use of symbols facilitates the discernment of logical form in the following ways-

*The use of symbols helps one to withdraw our attention from the subject matter and concentrate on the form of the proposition or the argument.

*The use of symbols helps one to perceive the arguments with same form but different subject matter .This further facilitates the classification of the arguments according to their form. Classification of the arguments

according to their form helps to determine the validity of the arguments.

In short, symbolization of the inference effectively defuses the truth content concern, thereby allowing us to concentrate entirely on logical questions at hand. The use of symbols to replace names reveals the logical structure of the inference because it elements any specific reference to the named objects. Since in modern logic, we are interested only in the logical component of the inference, the truth content of the statements is irrelevant.

Thus the use of symbols is indispensible in order to express the form of the argument.

Modern logic is a development of traditional deductive logic which is completely formal in nature just like mathematics. Like mathematics, it makes use of deductive method. The use of symbols makes the argument free from the ambiguities of ordinary language and the influence of material truth and falsity; it thereby reveals the formal nature clearly. This helps to determine the validity or invalidity of the argument.

Conventionally to express the argument form the lower case letters 'p', 'q', 'r'… are used as statement variables (symbols for propositions). Again these statement variables are replaced by arguments. Hence arises the distinction between argument and argument-form.

The words 'valid' and 'invalid' are applied to the form of the argument, and not to the argument, i.e., to the content of the argument. The word 'valid' implies the correctness of the structural arrangement i.e. the form of the argument which in its turn, depends upon the adherence to certain formal rules.

When the form of the argument is expressed by statement variables, it gives us what is called 'argument form' which then does not contain content or matter. In other words, whereas an argument can be expressed both through words and symbols, argument forms are expressed only through symbols. Like mathematics, modern or symbolic logic is purely a formal science emphasizing on 'form' of the argument. Consequently, it is concerned only with formal truth or logical consistency.



CHECK YOUR PROGRESS

Q 3: State in what way the use of symbols helps to reveal the form of an argument?

Ans: .....

Q 4: What is the basic nature of modern symbolic logic?

Ans: .....

Q 5: State whether the following statements are true or false

a) The words 'valid' and 'invalid' are applied to the contents or subject matter of the argument. (True/ False)

b) An argument form can be expressed both through words and symbols. (True/ False)

c) Modern logic is concerned with logical consistency. (True/ False)

4.7 IMPORTANCE OF LOGICAL FORM

The study of logical form is important because it is directly connected with the determination of the validity and invalidity of the argument. If the logical form of the argument is valid, then the deductive argument will be valid and if the form of the argument is incorrect, then the argument is invalid. Hence there is a close connection between the logical form and the validity and invalidity of the argument.

There can be a set of valid and invalid forms of reasoning such that any individual argument exhibiting any of these forms could be readily identified as valid or invalid.

Such argument forms become the standard of validation for syllogistic reasoning. Here comes the characteristic mark of the deductive argument that it is concerned with formal truth. This * * brings us to the distinction between formal logic and material logic. Formal logic is concerned merely with the forms of thought. It aims at the attainment of formal truth. It deals with thoughts which are consistent among themselves. In formal logic we do not question about the truth of the premises of an argument. The premises are assumed to be true. We only examine whether the rules of

the argument have been observed properly and whether the conclusion necessarily follows from the premises. Formal logic is also called the logic of consistency. On the other hand, material logic aims at the attainment material truth. It deals with thought which must agree with the actual facts of the world. Material logic is also known as applied logic or Logic of Truth.

- Concept of logical form has nothing to do with the question of truth and falsity of the constituent propositions of the argument.

  The following are some interesting points associated with the concept of logical form.

  a. A valid argument may have false premises.
  b. An invalid argument may have false premises.
  c. A valid argument may have false conclusion.
  d. An invalid argument may have true conclusion.
  e. In a valid argument, there cannot be true premises and false conclusion.
  f. In a valid argument, there cannot be true premises and false conclusion.

- Here come the two distinct concepts namely, “argument” and “argument form”. It is clear that argument deals with the actual ‘state of affairs’ whereas ‘argument form’ is the structural outline or the mere order of arrangement of the statements. Secondly, while an argument may be expressed both through words and symbols, an argument form must be expressed through symbols.

  In a valid argument, the form of the argument is correct. Correctness of the form depends upon the adherence to certain formal rules.

  Let us consider the following arguments to find out the distinction between ‘argument’ and ‘argument form’.

  Ex. (1)

  If you read hard, you will pass in the examination.

  You read hard.

  Therefore, you will pass in the examination.

  Ex. (2)

  If there is rain, the crops will grow.

There is rain.

Therefore, the crops will grow.

The above two arguments differ from each other so far as their content or subject matter is concerned. The form of the argument is however, the same,-i.e.,

P ⊃ q

P

Therefore, q.

We have observed earlier that this valid argument form is known as Modus Ponens (M.P). The above two arguments are valid since they have valid argument form. Thus I. M. Copi, the distinguished author of 'Symbolic Logic' has remarked –"any given argument can be proved valid if it can be shown that the specific form of the given argument is a valid argument."(p. 21).



CHECK YOUR PROGRESS

Q 6: State why the study of logical form is important?

.....

Q 7: What is the difference between formal logic and material logic?

.....

Q 8: State two points of difference between argument and argument-form.

Q 9: Fill up the blanks:

a) If the logical form of the deductive argument is valid, then the argument will be ——

b) Concept of logical form has nothing to do with the question of truth and falsity of the constituent ————of the argument.

c) A valid argument may have false ———



4.8 LET US SUM UP

- Logical form is the structural synthesis or coordination of the elements of the proposition or the argument as the case may be. Logicians put more emphasis on the concept of 'logical form' rather than on 'matter' or the elements of the proposition or the argument. It is because the validity of the argument depends on the form, and not on the matter. Such is the importance of logical form that modern logic now a day goes by the name 'Formal Logic.
- Because of the importance of the 'form', modern logic makes intensive use of symbols and goes by the name, 'symbolic logic'.
- Modern symbolic logic completely formal in nature. The use of symbols is indispensible because symbols make the form of the argument clear and help one to withdraw one's attention on the content or matter; the focus on the 'form' of the argument instantly helps to determine the validity of the argument.
- All form of inference can help one recognize the logical commitments or what is being claimed and to evaluate the inference and determine the validity or invalidity of the inference. After all, logical form is the underlying logical relationship, and inferences can be analyzed by classifying them according to their logical forms.



4.9 FURTHER READING

1) Copi, M Irving., & Cohen, Carl. *Introduction to Logic*: Prentice-Hall of India, New Delhi, 2007.

2) Copi, Irving. *Symbolic Logic*: Macmillan Publishing Co., Inc. New York and Collier Macmillan Publishers London. 1979.

3) Patrick J. Hurley. *Introduction to Logic:* Wadsworth Cengage Learning, 2007.

4) Baronett, Stan, & Sen, Madhucchanda. Pearson: 2009.

5) Chakraborty, Chhanda. *Logic Informal, Symbolic and Inductive:* Prentice-Hall of India, New Delhi, 2007.



4.10 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q No.1: a) Propositions b) Matter c) Structure d) Semantics

Ans.to Q No. 2: a) Structural relations of the elements of the argument

b) Formal

Ans. to Q No. 3: The use of symbols helps to withdraw our attention from the subject matter and thereby helps to have clear focus of the form of the argument. The use of symbols makes the argument free from the influence of the ambiguities of the ordinary language and the influence of material truth and falsity; it thereby reveals the formal nature of the argument clearly.

Ans. to Q No. 4: Modern symbolic logic is purely a formal science. It aims at the determination of the validity of the arguments. The validity of an argument depends only on its forms. Hence modern logic gives importance on 'form' of the argument rather than on its subject matter. The extensive use of symbols reveals the form of the argument. Consequently, it is concerned with formal truth and logical consistency.

Ans to Q No. 5: a) False b) False c) True

Ans to Q No. 6: The study of logical form is important because it is directly connected with the determination of the validity and invalidity of the argument. If the logical form of the argument is valid, then the deductive argument will be valid and if the form of the argument is incorrect, then the argument is invalid. Hence there is a close connection between the logical form and the validity and invalidity of the argument.

Ans to Q No. 7: Formal logic is completely formal and deductive in nature. It is concerned merely with the forms of thought. It aims at the attainment of formal truth and not with the material truth. It deals with

thoughts which are consistent among themselves. In formal logic we do not question about the truth of the premises of an argument. The premises are assumed to be true. We only examine whether the rules of the argument have been observed properly and whether the conclusion necessarily follows from the premises. Formal logic is also called the logic of consistency. On the other hand, material logic aims at the attainment of material truth. It deals with thought which must agree with the actual facts of the world. Material logic is also known as applied logic or Logic of Truth.

Ans. to Q No 8: There are certain differences between argument and argument. Argument deals with the actual 'state of affairs' whereas 'argument form' is the structural outline or the mere order of arrangement of the statements. Secondly, while an argument may be expressed both through words and symbols, an argument form must be expressed through symbols.

Ans. to Q No 9: a) Valid b) Propositions c) Premises



4.11 MODEL QUESTIONS

A. Very short Questions

Q 1: Define logical form.

Q 2: Give a symbolic expression of logical form.

Q 3: What is formal logic?

Q 4: What do you mean by logic of consistency?

Q 5: What is material logic?

B. Short Questions (Answer in about 150 words)

Q 1: Write a short note on logical form.

Q 2: What do you understand by logical form? Briefly explain.

Q 3: Why modern logic puts more importance on logical form? Briefly explain

Q 4: Distinguish between form and matter of an argument.

Q 5: Write briefly about the importance of logical form in modern logic

Q 6: Write a short note on formal logic.

Q 7: Write a short note on material logic.

Q 8: What do you mean by logical form? Briefly explain

Q 9: Briefly explain with suitable examples the relation between logical form and use of symbols.

Q 10: Distinguish between valid and invalid argument with suitable example.

Q 11: Do you find any distinction between argument and argument form? Explain in brief.

C. Long Questions (Answer in about 300 words)

Q 1: Explain how the use of symbols reveals the logical form of the argument. Explain in this connection the importance of the use of symbols.

Q 2: Explain the formal nature of modern symbolic logic.

Q 3: Explain the importance of logical form.

Q 4: Do you find any distinction between argument and argument form? Explain.

Q 5: Explain the distinction between argument and argument form.

*** ***** ***

UNIT 5 : INFERENCE AND IMPLICATION

UNIT STRUCTURE

5.1 Learning Objectives

5.2 Introduction

5.3 What is Implication?

5.4 Different meanings of Implication

5.5 Features of Implication

5.6 Material implication

5.7 Let Us Sum Up

5.8 Further reading

5.9 Answers to Check Your Progress

5.10 Model Questions

5.1 LEARNING OBJECTIVES

After going through this unit, you will be able to –

- understand the basic nature of formal logic
- examine the reasons or grounds behind a claim put forward in a deductive inference
- define Implication
- describe the features of implication
- know different meanings of implication
- define and differentiate inference and implication.

5.2 INTRODUCTION

Logic has been commonly defined as the science of reasoning. Reasoning is a special kind of thinking called inferring, in which conclusion is drawn from premises. We all know that inference is of two kinds, namely, deductive and inductive. The word 'implication' has its reference in connection with deductive inference only. In this unit we shall be discussing a very important aspect of deductive inference, i.e., implication. In other words, the word 'implication 'has its reference to deductive inference. As the

organized body of knowledge, or science, logic evaluates arguments or inferences and deals with the correctness of the reasoning process. We all infer in our day-to- day lives. Logic develops and provides a system of methods and principles that may be used as criteria for evaluating the inferences and guides in constructing correct inference and distinguishes them from incorrect ones.

An inference when it is expressed in language is called an argument. It is a group of statements, one or more of which (the premises) are claimed to provide support for, or reasons to believe one of the others (the conclusion). In some of the arguments, the premises do really support the conclusion and in some others, the premises do not support the conclusion, even though they are claimed to. The former are said to be good arguments and the latter, bad arguments. The questions at hands are, does the conclusion that is reached follow the premises which are used or assumed? What is it that provides reasons or grounds behind a claim of propositions or claims in a deductive inference? This unit makes an attempt to discern and define implication which is the most subtle element lying behind a deductive inference.

5.3 WHAT IS IMPLICATION?

Implication is the logical relation that holds between the antecedent and the consequent of a conditional or hypothetical statement. A conditional or hypothetical statement is a compound statement where two statements are joined by the word “if” before the first and inserting the word “then” between them. Such a hypothetical or conditional statement is also called an “implication”, or an “implicative statement”. The relation of implication is the basis of a conditional or hypothetical proposition. When two simple statements are joined together by the connective ‘if—then’, than the complex proposition obtained thereby is called a conditional or hypothetical proposition. In such a proposition the component statement that follows the “if” is called the “antecedent” or the “implican” and the component statement that follows the “then” is the consequent or the “implicate”.

Example: “if the rulers become corrupt, then the common people become rebellious”. In this conditional proposition the portion “the rulers become corrupt” is antecedent and the portion “common people become rebellious” is the consequent. Implication is the logical relation obtaining between the consequent and the antecedent by virtue of which the consequent follows from the antecedent. The meaning of a conditional proposition depends on this relation. To be more general, implication is the most important relation obtaining between the premises and the conclusion of a deductive argument. A deductive argument is in fact, a complex hypothetical statement which asserts that if the premises are true, the conclusion is also true.

Russell in his “Principia Mathematica” asserts that when a proposition is to be inferred from another proposition, there must be such a relation between the two propositions by virtue of which one is deduced from the other. Here Russell uses the word “inference” in the sense of ‘deductive inference’. For example, the relation between p and q is such that q is the result of p or deduced from p. In such case, the meaning of q is implied in p or deduced from p. A deductive inference is based on this relation of implication obtaining between the constituent propositions. In other words, it is because of this relation we can deduce one proposition from another proposition in a deductive inference.

In formal or deductive logic implication is very important. It is the basis of a conditional or hypothetical statement because of which consequent follows from the antecedent. Similarly, we can also say that the conclusion of a valid deductive argument follows necessarily because of the relation of implication. Such is the importance of implication in modern logic that modern logic is sometimes called the “Logic of Implication”.



CHECK YOUR PROGRESS

Q 1: Fill up the blank –

a. The word ‘implication’ has its reference only with ———— inference

b. The relation of implication is the basis of a ————————

proposition.

c. A deductive inference is based on the relation of ——— obtaining between the constituent propositions.

Q 2: What is implication? Explain with example.

.....

Q 3: Define implication. What is its importance in deductive inference?

.....

5.4 DIFFERENT MEANINGS OF IMPLICATION

There are different types of implication as there are different hypothetical or conditional statements. In other words, different hypothetical or conditional statements express different meanings of implications. Following are the different types of conditional statements each of which seems to assert different forms of implication:

1. If all the students are studious and Raja is also a student then Raja is also studious.
2. If ABC is a triangle, then the summation of its angles is equal to 180 degrees.
3. If this gold ring is placed in fire, then it will become brighter.
4. If you clean up your room, then I will buy you the toy you want.

About the examples cited above, the following observations may be made –

In the first example, the consequent is deduced logically from the antecedent. Thus example 1. expresses a logical relationship between the antecedent and the consequent. In the second case, the consequent is deduced from the definition of the word 'bachelor' in the antecedent part.Example 2. Conveys a definitional relationship. By definition of a triangle, the consequent follows. In the third case, the consequent follows from its antecedent empirically. It exhibits a causal connection based on experience. Finally, in the fourth case, the statement reports a decision of the speaker to behave in the specified way under the specified circumstance. It is a promise, the fulfillment of which depends upon a certain behavior. Thus all the four

conditional statements express different implications between the antecedent and the consequent. The word 'implication' therefore refers to four specific forms. These are—

1. Logical,
2. Definitional
3. Causal
4. Decisional or behavioral.

The above four types of implication are different in that each asserts a different type of implication between its antecedent and consequent. But they are not completely different; all assert types of implication. There is a common feature running through all these four forms. It is that all these conditional statements will be false in a particular circumstance; that is, when the antecedent is true and the consequent is false. A conditional statement is false when the antecedent is true and the consequent is false. This feature is the common partial meaning of implication. But it by itself does not express the complete or full meaning of implication. In order to express this common partial meaning of implication, the symbol '†' (horse shoe or hook sign) is used. This means, in order that any conditional statement like "if p then q" is to be true, it must be false that p is true and q is false and this is expressed symbolically in the way- ÷ (p. ÷q). Thus propositional logic maintains that the common core meaning of all conditionals or implications is that they are considered to be false when the antecedent is true, but the consequent is false. For instance, Example 3.will be considered as false when a gold ring is placed in fire and is left out without being brighter.



CHECK YOUR PROGRESS

Q 4: Fill up the blank

a) Different hypothetical or conditional statements express different ——of implications.

b) 'If India loses the cricket match with Pakistan, then I'll cut my nose' expresses —— implication between the antecedent and the consequent.

Q 5: State different meanings of implications as expressed by different conditional or hypothetical statements.

.....

5.5 FEATURES OF IMPLICATION DISTINGUISHING IT FROM INFERENCE

There are certain features of implication which distinguish it from an inference.

So far we have learnt that implication is a relation that holds between the antecedent and the consequent of a true conditional or hypothetical statement. In more general sense, it is the most important relation obtaining between the premises and the conclusion of a deductive argument which is in fact, a complex hypothetical statement. An analysis of the meaning of implication reveals two important characteristics. These are as follows

1. Implication is a necessary relation. When one proposition implies another proposition, the implied proposition is strictly determined by the first. In other words, if there is a relation of implication between p and q, then the relation is such that if p then only and only q.
2. Implication is a formal relation. This means it has no reference to material truth. This is the reason that implication holds in deductive inference only. The relation of implication is not concerned at all with the material truth of the proposition. These two features have distinguished implication from inference.

Example: (a) No philosophers are fit for the country and some philosophers are presidents. Therefore, some presidents are not fit for the country.

(b) If no philosophers are fit for the country and some philosophers are presidents, then some philosophers are not fit for the country.

Example no.(a) is an inference where constituent propositions are to be materially true besides being formally true in order to be valid. Example no (b) is an implication which expresses a logical statement on certain conditions. But here it is not important to see whether these conditions are really fulfilled or not. In an inference, a statement is established

(the one which is stated in the conclusion) on the basis of a statement or statements (expressed in the premises); where the question of material truth is a matter of consideration along with the question of formal truth.

On the basis of the above discussions, we can draw the differences between inference and implication as follows:

1. The statement, "if all humans are mortal and Socrates is a human, then Socrates is mortal" expresses implication. Implication is a statement of logic to the effect that if certain conditions are fulfilled, then certain consequences will result. It says nothing as to whether or not the conditions referred to in the if- clause are, in fact, fulfilled. On the other hand, in an inference the premise or the premises make certain assertions and as logical consequences of these assertions, a further assertion is made, namely, the fact stated in the conclusion.
2. The argument, "No man are perfect and some man are honest; therefore some honest man are not perfect" expresses inference. In order to be justified in asserting an inference, one has to know not only that the logical form of the argument guarantees that the conclusion follows from the premises but also that the premises are true. In other words, besides being formally true, the inference must also be materially true. On the other hand, an implication can be asserted without prejudice to the question of the truth or falsity of the premises. In other words, formal truth is the sole concern of an implication. It has nothing to do with the question of material truth. When we make an inference, we assume the truth of the premises; and as a consequence of the truth of the premises and of the logical validity of the argument, we are entitled to assert the conclusion. But when we commit ourselves to an implication, we do not thereby commit ourselves to the truth of the premises.



CHECK YOUR PROGRESS

Q 6: Explain the meaning of the statement, 'implication is a necessary relation'?

.....

Q 7: Explain the meaning of the statement, 'implication is a formal relation'?

.....

.....

Q 8: State two points of difference between inference and implication.

.....

Q 9: Fill up the blank

a) An inference must be ———— true besides being formally true.

b) ———————— is the sole concern of an implication.

5.6 MATERIAL IMPLICATION AND ITS TRUTH CONDITIONS

We have found that the word "implication" has more than one meaning. There are different senses of "implication" or "if—then". However the common core meaning all conditionals is that they are considered to be false when the antecedent holds true, but the consequent becomes false. For instance, the statement, "if a piece of wax is left in the hot sun, then it will melt" will be considered false when a piece of wax is left out in the hot sun but it does not melt. That is if we take p and q as any two antecedent and consequent, respectively, then pƒ"q is false when p becomes true but q becomes false; or when (p. ÷ q) is true. Material implication asserts that as a matter of fact, it is not the case that the antecedent is true when the consequent is false. "No real connection" is suggested between antecedent and consequent. It is to be noted that material implication symbol is a truth-functional connective like the symbols for conjunction and disjunction. The renowned authors of "The Introduction to Logic" I.M.Copi and Carl Cohen opine that material implication constitutes a fifth type that may be asserted

in ordinary discourse. As for example, we can take the statement, "If Mahatma Gandhi was a military genius, and then I am a monkey's uncle". No real connection whether logical, definitional, or causal obtains between the antecedent and the consequent here. A conditional of this type is often used as a humorous method of denying its antecedent. The full meaning of the present conditional seems to be the denial that "Mahatma Gandhi was a military genius's true when "I am a monkey's uncle" is false. And since the latter is so obviously false, the conditional must be understood to deny the former.

We can define material implication as a truth-functional relation (symbolized by the horseshoe) that may connect two statements. The statement "p materially implies q" is true when either p is false, or q is true. Material implication does not refer to the meaning of the statements connected, but merely asserts that it is not the case both that p is true and that q is false. The truth conditions of the material implication may be shown as follows——

The first and the third row of this table show that a material implication will be true whenever its consequent is true, irrespective of what value its antecedent has. Similarly, the third and fourth rows show a material implication is true whenever its antecedent is false, irrespective of what value the consequent has. In short, there is only one condition when a material implication is false; that is, when the antecedent is true and the consequent is false.



5.7 LET US SUM UP

- This unit has presented a very fundamental concept of modern symbolic logic. Implication is the most logical relation holding between the antecedent and the consequent of a conditional or hypothetical statement. In other words, implication is the logical basis of a conditional or hypothetical proposition by virtue of which the consequent logically follows from the antecedent. The same thing can also be expressed

by saying that implication is the most important relation obtaining between the premises and the conclusion of a deductive argument. A deductive argument is in fact, a complex hypothetical or conditional statement which asserts that if the premises are true, the conclusion is also true.

- Implication has different meanings; we can distinguish different senses of implication, i.e., logical, definitional, causal, and behavioural or decisional. Material implication is the type of implication which asserts that as a matter of fact, it is not the case that the antecedent is true when the consequent is false. The symbol for material implication is the '$\supset$' which is a truth-functional connective, like the symbols for conjunction and disjunction. As such it is defined by truth table, specific to it.
- There are some features of implication which distinguish it from inference. Implication is purely a necessary and formal relation. The statement, "if all humans are mortal and Socrates is a human, then Socrates is mortal" expresses implication.
- Implication is a statement of logic to the effect that if certain conditions are fulfilled, then certain consequences will result. It says nothing as to whether or not the conditions referred to in the if- clause are, in fact, fulfilled. On the other hand, in an inference the premise or the premises make certain assertions and as logical consequences of these assertions, a further assertion is made, namely, the fact stated in the conclusion.
- An inference in general, aims at both formal truth and material truth. On the other hand, formal truth is the sole concern of an implication. It has nothing to do with the question of material truth. When we make an inference, we assume the truth of the premises; and as a consequence of the truth of the premises and of the logical validity of the argument, we are entitled to assert the conclusion. But when we commit ourselves to an implication, we do not thereby commit ourselves to the truth of the premises.

- Material implication constitutes a fifth type that may be asserted in ordinary discourse. The statement, for example, "If Mahatma Gandhi was a military genius, then I am a monkey's uncle". No real connection whether logical, definitional, or causal obtains between the antecedent and the consequent here. A conditional of this type is often used as a humorous method of denying its antecedent. The full meaning of the present conditional seems to be the denial that "Mahatma Gandhi was a military genius" is true when "I am a monkey's uncle" is false. And since the latter is so obviously false, the conditional must be understood to deny the former.
- We can define material implication as a truth-functional relation (symbolized by the horseshoe) that may connect two statements. The statement "p materially implies q" is true when either p is false, or q is true. Material implication does not refer to the meaning of the statements connected, but merely asserts that it is not the case both that p is true and that q is false.



5.8 FURTHER READING

1) Copi, Irving M. and Cohen, Carl. 2007. *Introduction to Logic:* Prentice-Hall of India pvt.
2) Baronett, Stan and Sen, Madhuchanda. 2009. *Logic* : Pearson:
3) Chakraborty,Chanda. 2007. *Logic Informal, Symbolic and Inductive.* PrenticeHall India
4) Copi, I.M. 1973. *Symbolic Logic* . Macmillan Publishing Co., Inc..
5) Bason and O'Connor .1977. *Introduction To Symbolic Logic* : Oxford University Press.



5.9 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: a) Deductive b) Conditional or Hypothetical c) Implication.

Ans to Q No 2: Implication is the logical relation that holds between the antecedent and the consequent of a conditional or hypothetical statement. A conditional or hypothetical statement is a compound statement where two statements are joined by the word "if" before the first and inserting the word "then" between them. Such a hypothetical or conditional statement is also called an "implication", or an "implicative statement". The relation of implication is the basis of a conditional or hypothetical proposition. When two simple statements are joined together by the connective 'if—then', than the complex proposition obtained thereby is called a conditional or hypothetical proposition. In such a proposition the component statement that follows the "if" is called the "antecedent" or the "implicant" and the component statement that follows the "then" is the consequent or the "implicate".

Example: "if the rulers become corrupt, then the common people become rebellious". In this conditional proposition the portion "the rulers become corrupt" is antecedent and the portion "common people become rebellious" is the consequent. Implication is the logical relation obtaining between the consequent and the antecedent by virtue of which the consequent follows from the antecedent. The meaning of a conditional proposition depends on this relation. To be more general, implication is the most important relation obtaining between the premises and the conclusion of a deductive argument. A deductive argument is in fact, a complex hypothetical statement which asserts that if the premises are true, the conclusion is also true.

Ans to Q No 3: Implication is the logical relation that holds between the antecedent and the consequent of a true conditional or hypothetical statement. To be more general, implication is the most important relation obtaining between the premises and the conclusion of a deductive argument. A deductive argument is in fact, a complex hypothetical statement which asserts that if the premises are true, the conclusion is also true.

In formal or deductive logic implication is very important. It is the basis of a conditional or hypothetical statement because of which

consequent follows from the antecedent. Similarly, we can also say that the conclusion of a valid deductive argument follows necessarily because of the relation of implication. Such is the importance of implication in modern logic that modern logic is sometimes called the "Logic of Implication".

Ans to Q No 4: a) types b) Behavioral

Ans to Q No 5: Different conditional statements express different forms of implication between the antecedent and the consequent. The word 'implication' therefore refers to four specific forms. These ares-

a) Logical b) Definitional c) Causal d) Decisional or behavioral.

The above four types of implication are different in that each asserts a different type of implication between its antecedent and consequent.

Ans to Q No 6: Implication is a necessary relation. When one proposition implies another proposition, the implied proposition is strictly determined by the first. In other words, if there is a relation of implication between p and q, then the relation is such that if p then only and only q.

Ans to Q No 7: Implication is a formal relation. This means it has no reference to material truth. This is the reason that implication holds in deductive inference only. The relation of implication is not concerned at all with the material truth of the proposition. These two features have distinguished implication from inference.

Ans to Q No 8: Implication is a formal relation. This means it has no reference to material truth. This is the reason that implication holds in deductive inference only. The relation of implication is not concerned at all with the material truth of the proposition. Inference on the other hand, is not necessarily formal since it may be both deductive and inductive. Consequently, implication has no reference to material truth; it is concerned only with formal truth. These two features have distinguished implication from inference.

Ans to Q No 9: a) materially b) formal truth



5.10 MODEL QUESTIONS

A. Very short questions

Q 1: What is the logical basis of the relation between the premise and the conclusion of a deductive argument?

Q 2: What is the logical basis of the relation between the consequent and the antecedent of a conditional or hypothetical proposition?

Q 3: What is conditional proposition?

Q 4: State the kinds of inference.

Q 5: What is logic?

Q 6: Who is the author of *Principia Mathematiica*?

Q 7: What is conditional proposition?

Q 8: What do you mean by logic of implication?

B) Short questions (Answer in about 150 words)

Q 1: Write a short note on material implication.

Q 2: What are the different forms of implication?

Q 3: What are the two important characteristics of implication?

Q 4: Briefly explain the two senses of implication.

C) Long Questions (Answer in about 300-500)

Q 1: What is implication? What is its importance in deductive inference? Explain

Q 2: Describe the different meanings of implication with suitable examples.

Q 3: Explain the features of implication. In this connection explain the difference between inference and implication.

*** ***** ***

UNIT 6 : TRUTH FUNCTIONS

UNIT STRUCTURE

6.1 Learning Objectives

6.2 Introduction

6.3 Proposition: Simple and Compound

6.4 Variables

6.5 Logical Constants

6.6 The scope of Logical Constants and the use of Brackets

6.7 Truth Function

6.8 Basic Truth-Functions and their Tabular Representations

6.9 Truth Tables for Basic truth Functions

6.10 Inter-Definition of Basic Truth-Functions

6.11 Let Us Sum Up

6.12 Further Reading

6.13 Answers to Check Your Progress

6.14 Model Questions

6.1 LEARNING OBJECTIVES

After going through this unit you will be able to:

- explain the basic symbols used in propositional logic
- define the concepts of variable and logical constant
- discuss the features of logical constants
- describe the scope of logical constants and the use of brackets
- analyse the concept of truth-function
- describe the truth functions from the concept of function in mathematics
- explain the five basic truth-functions with the help of truth-table.

6.2 INTRODUCTION

This unit introduces you to the concepts of variables, logical constants, truth- function and inter-definition of truth-function. All these are

very important concepts of propositional logic. With this unit, we shall begin our formal approach to symbolic logic. As we go through this unit, we shall find that symbolic logic makes use of 'p' and 'q' and other symbols to express its well-formed concepts and formulas.

6.3 PROPOSITION: SIMPLE AND COMPOUND

An understanding of the concepts of variables and logical constants necessitates a preliminary understanding of the two kinds of propositions frequently referred to in propositional logic namely, simple and compound proposition. A simple proposition consists of a single statement. It has only one statement as its component part. A compound proposition on the other hand consists of two or more statements as component parts. For example, "you have studied hard" is a simple proposition and "you have studied hard and got 80% marks in Logic" is a compound proposition. In the first example, there is only one statement whereas in the second example, there are two statements.

6.4 VARIABLES

Variables are a type of symbol used in symbolic logic. The lower case letters of English alphabet e.g., p, q, r s, t, etc., are used as propositional variables to symbolize simple statements. They are thus place-holder for which a statement may be substituted and whose associated value may be changed. For example, in the proposition, "if p then q"; Here p, q are variables. They are symbols which can be replaced by any simple proposition within a definite range. Thus in the above example, the variable 'p' can be replaced by the proposition, "it rains" and 'q' can be replaced by the proposition "crops will grow". Any other propositions can replace the variables.



ACTIVITY

Is variable a symbol? If so, what does it symbolize? Give some examples.

Ans: .....

.....

.....



CHECK YOUR PROGRESS

Q 1: What is propositional logic?

.....

Q 2: What is the basic unit of propositional logic?

.....

Q 3: Fill in the blanks

a) A simple statement consists of a ———— statement.

b) Variables refer to ———— individual or object within a specified domain.

c) Compound propositions consist of more than ———— statements as its component parts.

6.5 LOGICAL CONSTANTS

In logic, a logical constant is a symbol that has the same value under every interpretation. Two important types of logical constants are-logical connectives and quantifiers. In propositional logic we are concerned only with the logical connectives which are as follows-

1. Negation sign ~ (interpreted as 'not' or 'it is false that'…)
2. Conjunction sign '" (interpreted as 'and, or 'both as ....')
3. Disjunction sign v (interpreted as 'or', 'either…or'…)
4. Conditional sign or the sign of implication $\supset$ (interpreted as 'if …then…)
5. Biconditional sign or the sign of equivalence $\equiv$ (interpreted as 'if and only if…)

Thus using the variables and constants, we can form following compound expressions or logical formulae-

~p (read as 'not p' or 'it is false that p')

p $\vee$ q (read as 'p or q')

p $\supset$ q (read as 'if p and q, or 'p implies q')

p $\equiv$ q (read as 'p if and only if q' or p is equivalent to q)

Not (read as 'it is not the case') or the negation is a monadic or unary constant or connective. For, at a time, it can connect only one statement. Some logicians do not accept it as a connective because that it does not serve to connect two statements. However, it serves to form compound propositions from simple statements by negating the simple statements. In this sense, some other logicians accept it as a connective. It is to be noted that the result of negating any simple proposition is a compound proposition. The other four logical constants or connectives are called dyadic connectives. They are called so because at a time they can connect two statements. The three connectives or constants viz., 'not', 'and' 'or' are also known as the Boolean operators or Boolean connectives. For, these were first used by George Boole, the English logician, as part of his system of logic.

The following table shows the logical constants along with the compound proposition which is formed, the English name of the symbol and the symbol itself:-

Logical Constants	The compound proposition	The English name	Symbol
'not', it is false that	Negative proposition	Tilde, curl	~
If-then	Conditional or implicative proposition	Hook, horse-shoe	⊃
and	Conjunctive	Dot	.
If and only if	Equivalent proposition	Equivalence (implies both ways)	↔, ≡
or	Disjunctive proposition	Vel, wedge	ν



LET US KNOW

- Propositional logic is an important part of elementary symbolic logic. It is considered to be the starting point of symbolic logic. Propositional logic deals with the logical relationships between propositions without considering the interior structure of the propositions. It analyses the terms, properties, relations and quantifications involved in the proposition. It is also known as the Sentential Logic or the Logic of Truth-functions.

- Propositional formula means an expression or a propositional form which is a combination of symbols according to logical rules.

6.6 THE SCOPE OF LOGICAL CONSTANTS AND THE USE OF BRACKETS

It has been observed that each of the connectives except the 'not' (~), connects two and only two propositions, whether these be simple or compound. So if we deal with a proposition in which two compound propositions are connected, or in which one simple proposition is connected to a simple one, it is essential to make clear which two propositions are connected. This can be known by using brackets.

It has already been stated that every constant except the negation connects two and only two propositions, i.e., the one which immediately precedes the constant and that which follows it, whether these be simple and compound. That is to say, the scope of the constant covers the expression which immediately precedes it, and that which immediately follows it. Thus in the following expression, —

1. $p \,.(q \supset r)$

   The scope of the conjunctive constant i.e., the ' '" sign is p (which immediately precedes it)and the whole form enclosed in the round brackets. The scope of the implicative constant i.e., the hook sign is q and r.

   Let us consider the following form—

2. $(p \supset q) \,.\, (r \supset s)$

   Here the scope of the first occurrence of the implicative constant is p and q, and the scope of the second hook sign or the implicative constant is r and s. The scope of the conjunctive constant or the dot (.) sign is the compound expression enclosed in the round brackets on either side of it.

In compound propositions, which have other compound propositions as their components, one of the connectives is the main connective. The main connective is that which has the widest scope. In the example

no.1, the dot sign is the main connective, as it is also the case in the example no.2. In the following example:-

[(p . q) ⊃ r] . ~ r

The second occurrence of the dot sign is the main connective.

- **The scope of the negation sign (~) -:**

  The scope of the negation sign is that expression which immediately follows it, whether this is simple or compound. Here are some examples:-

  1. ~p, here the scope of the negation sign is 'p'. There is no need for brackets here, because the scope of the negation sign covers a simple proposition only.
  2. ~p ⊃ q

     Here again, the scope of the negation sign is p. The scope of the hook sign is ~p (on the left hand side) and q on the right hand side.
  3. ~(p ⊃ q)

     Here the scope of the negation sign is the whole of the expression within the round brackets
  4. ~[p ⊃ (q . r)]

  In this example, the scope of the negation sign is (q.r). In example nos. 3 and 4, it is the main constant. In example no.1, it is the main constant. In example no. 2, it is not the main constant. Here the hook sign is the main constant.



CHECK YOUR PROGRESS

Q 4: What do you mean by logical constants? How many logical constants are there?

.....

Q 5: What is the difference between a monadic and a dyadic connective?

.....

Q 6: How many monadic constants or connectives are there in prepositional logic?

.....

Q 7: Fill in the blanks—

a) Every connective connects only..... propositions

b) The main connective is that which has the ..... scope.

c) The scope of the negation sign is that expression which immediately ……..it.

6.7 TRUTH FUNCTION

A truth function is a combination of propositions or sentences which has a definite truth-value. As for instance, a conjunction or negation is a truth function whose truth value is determined by the truth values of the components. 'p.q' is a truth function of p and q whose truth value is true when p is true and q is true, and false otherwise. '~p' is a truth function of p whose truth value is false when p is true and true when p is false.

Thus there is a correspondence between the truth-values of the truth-functional expression and its component variables. In other words, a compound expression is a function of the truth-values of its simple and atomic components. This is called the truth-functionality of the compound expression. In the case of the truth-functional compound, the value of the whole expression is called output value and values of the individual variables or the simple components are called the input values.

The concept of 'truth-function' is much close to the concept of 'function' in mathematics. To give an example from mathematics, the compound expression, '5+7' is a function because its value is determined by the values of the numbers signified by 5 and 7. Again, let us consider the expression –

X= 5y +3. Here x is a function of y because its value is determined as soon as a value is assigned to the variable y. Hence if the value of y is 5, then the value of x is 28(that is, x= 5×5+3=28). Again, if the value of y is 3, then the value of x= 5×3+3=18.

It may be observed that as in the case of logic where there is a functional correlation between the truth-value of a compound expression and the truth-values of its component variables, so in mathematics also there is a functional correlation between the number signified by a compound arithmetic expression and the numbers signified by its components. However, there are important differences between the concept of function in arithmetic and the concept of truth-function in logic. First, mathematics deals with numerical values such as 1,2,3 etc., whereas symbolic logic deals with truth-values. It is based upon only two truth-values, namely, truth (T) and falsity (F). Second, the range of numerical values in mathematics is infinitely large as there are infinitely many numbers; but the range of truth-values in logic is limited as there are only two truth-values namely (T) and false (F). In symbolic logic, all its statements will be either true or false and, similarly, its compounds will also be either true or false. Hence it is called a bi-valued system.



ACTIVITY

What is a bi-valued system? Can any statement of propositional logic be neither true nor false? Discuss.

Ans: .....

.....



CHECK YOUR PROGRESS

Q 8: What is truth-function? Give examples.

.....

.....

Q 9: Is there any difference between the concepts of logical truth-function and mathematical function?

.....

Q 10: Fill up the blanks-

a) The properties of "true" and "false" are known as ———— which characterize a proposition.

b) Each proposition has ———— and only ———— truth-value.

6.8 BASIC TRUTH-FUNCTIONS AND THEIR TABULAR REPRESENTATIONS

There are five basic truth-functions in propositional logic.

1. Negative (or contradictory) function- ~p
2. Conjunctive function— p.q
3. Disjunctive function- pvq
4. Conditional (or implicative) function- p⊃q
5. Biconditional (or equivalence) function- p≡q

These are called basic truth-functions because these are the foundation on the basis of which other forms of more complicated truth-functional expressions are formed

6.9 TRUTH TABLES FOR BASIC TRUTH FUNCTIONS

The correspondence between the truth-values of the truth-functional expression and its components can be presented conveniently in the form of tables, which are known as truth- tables. The capital letter 'T' or the numerical '1' is used for 'true' and the capital letter 'f' or the numerical '0' is used for 'false'. Thus, T, F, 1, O are the value signs and are written below the component variables of the function. The function is written above the horizontal line in the table. If there is only 1 variable there are just 2 cases of possible truth-values. With 2 variables there are 4 i.e., 2 x 2= 2 cases. With 3 variables there are 8 i.e., 2x2x2=2 cases. With 4 variables, there are 16 i.e., $2x2x2x2=2^4$ cases. In general, if there are n variables, there must be 2n cases. We shall now explain the basic truth-functions one by one with the help of truth-tables according to this rule.

- **Negative function: (~p):**

The compound statement which is obtained by prefixing a statement say, p with 'not' or 'it is false that' is called a negative function. T he negative expression of p is written as ~p.

Let 'p' be a true statement, then its negation (or denial) '~p' will be false. Again, if p is false, then '~p'will be false. Again, if p is false, then '~p' is true. This can be shown in the truth-table below-:

P	~p
T	F
F	T

'p' and 'q' are contradictions. So they have opposite truth-values. A proposition signified by 'p' is true if its contradiction is false, and false if its contradiction is true. We may take the truth-table of this function as a definition of the negative function which may be used as a rule for the logical constant occurring in the function.

- **Conjunctive function (p.q):**

The compound proposition obtained by connecting two simple propositions by the logical constant 'and' is called a conjunctive function. If p and q are two simple propositions then the conjunctive function obtained by connecting them with 'and' is written as p.q. Both p and q are conjuncts of the conjunctive function p.q.

A conjunctive function contains two variables 'p' and 'q'

A conjunctive function is true only when both the conjuncts are true. In other cases, it is always false. Here we have four possible combinations of truth-values – TT, TF, FT, and FF. The truth-table is as follows-:

p	Q	p.q
T	T	T
T	F	F
F	T	F
F	F	F

We may take the truth table of this function as a definition of the conjunctive function which may be used as a rule for the logical constant occurring in the function.

- **Disjunctive Function (pvq):**

The compound proposition obtained by connecting two simple propositions by the constant 'v' (either\or) is called a disjunctive function and is written as pvq. In logic, the disjunctive pvq is interpreted as "either p or q or both". This is inclusive sense of 'either\or', which is accepted, in propositional logic. Both p and q are called 'disjuncts'. The truth-table for the disjunctive function is as follows-:

p	**Q**	**p∨ q**
T	T	T
T	F	T
F	T	T
F	F	F

(There is another sense of 'either\or' in which 'or' means 'either this, or that, but not both'. This is the 'exclusive' sense of 'either\or'. For example, the proposition 'Sankaradeva was born in 1449 or 1514' expresses the exclusive sense of 'either\or' where both the disjuncts cannot be true. The corresponding function is known as 'alternative function'. In propositional logic, the exclusive sense of the 'either\or' is however not accepted).

A disjunctive function is true when at least one of the disjuncts is true and false when both the disjuncts are false. We may take the truth-table of this function as a definition of the disjunctive function which may be used as a rule for the logical constant occurring in the function.

- **Implicative or conditional function-(p ⊃ q):**

If p and q are two statements then the compound expression obtained by connecting them by the constant 'if-then' is called a conditional or implicative function. The conditional or implicative 'p ⊃q' is also known as Material Implication where 'p' is the implican (antecedent) and 'q' is the implicate (or consequent'). The function p ⊃q is false if and only if 'p' is true and 'q' is false. In other cases of truth-value combinations it is true. The truth-table for the implicative function is as follows—

p	**Q**	**p ⊃ q**
T	T	T
T	F	F
F	T	T
F	F	T

The truth-table of this function may be taken as a definition of the equivalent function which may be used as a rule for the logical constant occurring in the function.

- **The Biconditional (or equivalence) function (p a" q), (p↔ q):**

The functions of compound statements of the form "if p then q, and

if q then p" are called biconditional (equivalence or equivalent) functions. It is expressed variously in the following ways: 'p if and only if q', 'p is equivalent to q', 'p a" q' and q a" p' etc.

A biconditional function p a" q is true when its components have the same truth-value, and false when its components differ in truth-value. The following is its truth table:

p	Q	pa" q
T	T	T
T	F	F
F	T	F
F	F	T

The truth-table of this function may be taken as a definition of the equivalent function which may be used as a rule for the logical constant occurring in the function.



CHECK YOUR PROGRESS

Q 11: How many basic truth-functions are there in propositional logic? Why are they called basic truth-functions?

.....

Q 12: Write the name of five basic forms of truth-functions.

.....

Q 13: Give example-:

a) Negative function

b) Conjunctive function

c) Implicative function

d) Disjunctive function

e) Equivalent function

6.10 INTER-DEFINITION OF BASIC TRUTH-FUNCTIONS

It is to be noted that for the purposes of the propositional calculus, we may take the truth-table of each of the truth-functions set out above as a

definition of that function. Since the truth-table of a given function is a definition of that function, any other function having the same truth-table will be equivalent to it and may be inter-changed for all logical purposes. For example, the truth-table for '~p v q' is equivalent to the truth-table of 'p ⊃ q'. It can therefore be regarded as another way of defining the implicative function. Similarly, 'p ⊃ q' can be expressed in terms of '~' and '.'as ' ~(p .~q)')'.

The truth-table is as follows: (table-1)

p	Q	~q	~(p.~q)
T	T	F	T
T	F	T	F
F	T	F	T
F	F	T	T

Here the final column of the table is identical with the final column of the table for 'p ⊃ q.'Thus the expression 'p q', '~pvq' and '~(p. ~q)' are logically equivalent and may be for all logical purposes, be substituted one for another. The following tables show the inter definitions of logical functions:-

1. The truth-function 'p.q' may be defined in terms of ' ~' and 'v'. This means that they are equivalent. (Table-2)

p	q	~p	~q	~pv~q	p.q	~(~pv~q)
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F

It is seen that the truth-values of the truth-tables 'p.q' and ~(~pv~q) are the same. This means that two functions are equivalent.

2. The truth-function 'p ⊃ q' may be defined in terms of '~' and '.'. The following table shows that p ⊃ q is equivalent to ~(p. ~ q) by definition since they have same truth-value. (Table 3)

p	q	~q	p.~q	p ⊃ q	~(p.~q)
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

3. The truth-function p≡ q is equivalent to (p.q) v (~p . ~q); because, as the following table shows, they have the same truth-value. (Table-4)

p	q	~p	~q	p.q	~p.~q	p≡q	(p.q) v(~p.~q)
T	T	F	F	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	F	T	T	T

Thus, it can be seen that basic logical constants are interrelated. But ' ~' has to be taken as a primitive idea in the real sense of the term; because it is indefinable in terms of any other constants so far introduced.

The following equations sum up the inter definitions among the constants—:

1. p . q = Df. ~(~p v ~q)
   = Df. ~ (p ⊃ ~q)
2. p v q = Df. ~ (~p. ~q)
   = Df. ~p ⊃ q
3. p ⊃ q = Df. ~p v q
   = Df. ~ (p. ~ q)
4. p ≡ q = Df. (p ⊃ q). (q ≡ p)
   = Df. (p. q) v (~p. ~q)

In the above equations, 'Df' is the abbreviation for 'Definition'. Equation no. 1 shows that the conjunction 'p.q' can be defined first in terms of negation and disjunction, then secondly in terms of negation and implication.

Equation no. 2 shows that the disjunction 'pvq' can be defined first in terms of negation and conjunction, then in terms of negation and implication. Equation no. 3 shows that 'p ⊃ q' can be defined first in terms of negation

and disjunction, then in terms of negation and conjunction. Finally, equation no. 4 shows that p≡ q is equivalent of the conjunction of two conditionals; then, it is expressed in terms of negation, conjunction and disjunction.



CHECK YOUR PROGRESS

Q 14: Answer the following questions-

a) What will be the truth-value of an implicative function when the antecedent and the consequent are both false?

.....

b) What will be the truth-value of a conjunctive function when both the conjuncts are false.

.....

c) Under what condition an implicative function becomes false?

.....

d) Differentiate the inclusive sense and the exclusive sense of disjunction. Which of the two senses is accepted in . logic?

.....

e) Under what condition an equivalent function becomes true?

.....



6.11 LET US SUM UP

- Logical constants and variables are the basic symbols used in propositional logic. The constants are those symbols, which express the form of the compound propositions and maintain the same meaning throughout every occurrence in the propositional formulae. They have fixed meaning. Every logical constant is also known as a connective because they form compound propositions from simple or other compound propositions by connecting them. Each connective connects two and only two propositions. The constant '~' (negation) is

not a connective in this sense according to some logicians. At any rate, it forms a compound proposition simply by denying or negating a simple proposition. In compound propositions which have other compound propositions as their components, one of the connectives is the main connective. The main connective is that which has the widest scope.

- A variable on the other hand, is a symbol which can stand for any one of a given range of values within a domain. Lower case letters of English alphabet p, q, r, s, t are used as propositional variables which represent simple statements in expressions which contain several statements. They are called variables because like the symbols x and y in algebra, they appear in propositional forms of formulae and they could be replaced by any member of a certain range. Thus while variables may assume any meaning of a definite range, the constant maintain the same meaning throughout every occurrence in propositional formulae.
- Another very important notion of symbolic logic is the notion of truth-function. A truth-function is a compound expression where the value of the expression is determined by the truth-value of the constituent variables. In other words, the value of the compound expression is determined as soon as the truth- values of the constituent variables are known. For example, the statement 'pvq' is a truth-function, because the truth-value of the expression is determined by the truth-values of the constituent variables 'p' and 'q'.
- There are five basic truth-functions namely-(a) Negative function, (b) Conjunctive function,(c) Disjunctive function,(d) Implicative function, and (e) Biconditional function. These five basic truth-functional expressions may be defined with the help of truth-table which is a tabular order on which all possible truth-values of the compound statements are displayed, through the display of the truth-values of their simple components. The truth-table of each of the basic function is taken as a definition.
- It has also been observed that basic logical constants are inter-related. Since the truth-table of a given function is a definition of that function,

any other function having the same truth-table will be equivalent to it and may be inter-changed for all logical purposes.



6.12 FURTHER READING

1) Irving M. Copi and Carl, Cohen : Introduction to Logic, Prentice-Hall India, Eleventh Edition

2) A.H. Basson and D.J.O'Connor : Introduction to Symbolic Logic. Oxford University Press

3) Dr. Shyam kishor Sing : Modern Logic (vol. 1.) Jalukbari, Guwahati-14.



6.13 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: Propositional logic is an important part of elementary symbolic logic, which deals with the logical relationships holding between the propositions without considering the interior structure of propositions. It is also known as the Sentential Logic or the logic of Truth-functions.

Ans to Q No 2: A simple statement is the basic unit of propositional logic.

Ans to Q No 3: a) Single b) unspecified c) one

Ans to Q No 4: The logical constants are those symbols, which express the form of the compound propositions and maintain the same meaning throughout every occurrence in the prepositional formulae. There are five logical constants: -' '(implication), 'v'(disjunction), ' .'(conjunction), ' ≡' (equivalence), ' ~` (negation).

Ans to Q No 5: A monadic connective connects only one statement at a time whereas a dyadic connective connects two statements at a time. ' ~` (negation) is an example of monadic constant. On the other hand, '⊃ '(implication), 'v'(disjunction), ' .' (conjunction), '≡'(equivalence) are called the dyadic constants.

Ans to Q No 6: There is one monadic constant or connective in prepositional logic.

Ans to Q No 7: a) two, b) widest c) follows.

Ans to Q No 8: A truth-function is a compound expression where the value of the expression is determined by the truth- value of the constituent variables; the statement or the compound expression is determined as soon as the truth-values of the constituent variables are known. For example, the statement, 'pvq' is a truth-function, because the truth-value of the expression is determined by the truth-values of the two constituent variables 'p' and 'q'. Similarly, 'p.q', [(p.q) . (p v~q)], (pvq) . (qvp) etc. are some examples of truth-functions.

Ans to Q No 9: Though the concept of truth-function in logic is derived from the notion of function in mathematics, yet there are differences between the two. First, mathematics deals with numerical values such as 1,2,3 etc., whereas symbolic logic deals with truth-values. It is based upon only two truth-values, namely, truth (T) and falsity (F). Second, the range of numerical values in mathematics is infinitely large as there are infinitely many numbers; but the range of truth-values in logic is limited as there are only two truth-values namely (T) and false (F). In symbolic logic, all its statements will be either true or false and, similarly, its compounds will also be either true or false. Hence it is called a bi-valued system.

Ans to Q No 10: a) truth-value b) one and only one.

Ans to Q No 11: There are five basic truth-functions in propositional logic They are called basic truth-functions because on the basis of these truth-functional expressions, other more compound forms of truth-functional expressions are formed.

Ans to Q No 12: The name of five basic truth-functional expressions are – (1) Negative function, (2) Conjunctive function, (3) Disjunctive function, (4) Implicative function, (5) Equivalent function.

Ans to Q No 13 : ~, . , ⊃,V, ≡ .

Ans to Q No 14: a) Ans: true (b) Ans: false

c) Ans: When the antecedent is true and the consequent is false.

d) Ans: In the inclusive sense of 'either \ or' 'or' means "either this or that or both" On the other hand, in the exclusive sense, 'or' means 'either this or that but not both'. In propositional logic, the inclusive sense is accepted.

(e) Ans: An equivalent function becomes true only when both the components have the Same truth-value.



6.14 MODEL QUESTIONS

A. Very short questions

Q 1: Define truth function.

Q 2: What is variable?

Q 3: What is implicative function?

Q 4: What do you mean by logical constant?

Q 5: Give an example of simple proposition.

Q 6: What is compound proposition?

Q 7: What is a bi-valued system?

Q 8: What is truth-table?

Q 9: How many kinds of truth-values are there in symbolic logic?

Q 10: What are the five basic truth functions?

B. Short questions (Answer each question in about 150 words)

Q 1: Write a short note on truth function.

Q 2: Write is truth table? Explain briefly

Q 3: What is truth function? Explain in brief.

C. Long questions (Answer in about 300-500 words)

Q 1: What is truth-function? Differentiate the concept of truth-function in logic from the concept of function in mathematics.

Q 2: What are the basic truth-functions? Explain each of them with the help of truth-tables.

Q 3: Define truth-function. Explain implicative and the equivalent truth-function by constructing truth-tables for them.

Q 4: Explain the interdefinition of basic truth functions with examples.

*** ***** ***

UNIT 7 : TRUTH TABLE METHOD

UNIT STRUCTURE

7.1 Learning Objectives

7.2 Introduction

7.3 Truth Table Method

7.4 Determining the Truth-value of Compound Expressions by Using Truth Table Method

7.5 Basic Rules for Determining the Validity of Arguments by Truth Table Method

7.6 Indirect Truth table Method or the Method of Reductio Ad Absurdum

7.7 Let Us Sum Up

7.8 Further Reading

7.9 Answers to Check Your Progress

7.10 Model Questions

7.1 LEARNING OBJECTIVES

After going through this unit you will be able to:

- define truth table method
- examine the truth-value of different compound expressions by using the truth table method
- explain the basic rules for determining the validity of arguments by truth table method
- discuss the indirect truth table method or the method of Reductio ad Absurdum
- evaluate the truth table method.

7.2 INTRODUCTION

The foregoing unit presented the fundamentals of constructing and using truth tables for analysis of basic truth-functional statements. In the present unit, you will focus on the understanding of the concept of truth

tables, a very important part of propositional logic, along with the construction of the truth tables to analyse the truth-content of truth-functional statements. Determining the validity and invalidity of truth-functional inferences is an important feature of truth table method.

7.3 TRUTH TABLE METHOD

Truth table is a tabular order on which all possible truth-values of compound statements are displayed, through the display of all possible combinations of the truth-values of their simple components. It is a two dimensional array formed by rows and columns by which one can find out the truth-values of compound statements or sets of compound statements mechanically.

A truth table may be used to define truth-functional expressions and it may also be used to test the validity of many deductive arguments.

As has been mentioned above, a truth table will have rows and columns of truth-values. It has also been mentioned that the capital letter 'T' or numerical '1' is used for 'true' and the capital letter 'F' or the numerical 'o' is used for 'false'. Thus, T, F, 1, 0 are the value signs indicating the truth-values and are written below the component variables of the function. The function is written above the horizontal line in the table. If there is only one variable, there are just two cases of possible truth-values. With two variables, there are 4 i.e., 2x2= 2 cases. With three variables, there are 8 i.e., 2x2x2= 2 cases. With 4 variables, there are 16 i.e., 2x2x2x2=16 cases of possible truth-values.

The rows of a truth table are supposed to provide all possible truth-value situations for a given set of statements. Basically, truth table is a method by which one can mechanically find out the truth- values of compound statements or sets of compound statements. You shall now find out how to construct a truth table.



LET US KNOW

Truth- content: - The truth or falsity of a statement and the method of its determination.

Truth-functional statements: - A complex statement whose truth-value is determined by an analysis of the individual components (the simple statements) together with the logical operators or constants.



ACTIVITY

- What is truth table? Is it a method? Discuss

Ans: ……

……

……

……

……



CHECK YOUR PROGRESS

Q 1: In what sense truth table is a two dimensional array?

……

Q 2: What are the uses of truth table?

……

Q 3: If there are two variables, how many cases of possible truth-values will be there in a truth table?

……

……

7. 4 DETERMINING THE TRUTH-VALUE OF COMPOUND EXPRESSIONS

From the preceding unit, we have learned about the truth tables for the basic truth-functional expressions and also about the scope of logical constants and the use of brackets. Now we are in a position to explain how

to determine the truth-value of a compound expression in which brackets are used. Truth table is an ordered arrangement, in which all possible truth-values of compound statements are displayed, through the display of all possible combinations of the truth-values of their simple components. A truth table may be used to define truth-functional expressions and it may also be used to test the validity of many deductive arguments. Let us start with the following example in order to explain the way in which its truth-value is to be determined**:** -

Example -1.

(p ⊃ q) . (q ⊃ p)

The procedure shall be as follows:-

First we are to see how many propositional variables are there and accordingly arrange the truth possibilities. We assign the truth-values to the variables in the following way-

p	q	(p ⊃ q)	.	(q ⊃ p)
1	1	1 1 1	1	1 1 1
1	0	1 0 0	0	0 1 1
0	1	0 1 1	0	1 0 0
0	0	0 1 0	1	0 1 0

(a) We assign truth-values to the logical constants, beginning with that which has the least scope. To find out the constant, which has the least scope, we look for the main connective. The main connective is one which has the widest scope, which in this case, is the dot (.) sign. Here we have a conjunction, the conjuncts of which are both compound propositions. The compound proposition on the left hand side consists of two simple propositions connected by the hook (⊃) sign and we already have the truth-values of the simple component propositions. So begin from here. We consult the definition of the truth table for the basic truth-function of the implication and fill in the column just below the hook sign in our example. The compound proposition on the right hand side of the conjunction also consists of two simple propositions connected by the hook sign, so we can assign truth-

values to this proposition. On the first line, we have the hook sign connecting two propositions, both of which have the truth-value 'true' (1); we consult the definition to find out what is the truth-value of a material implication with true antecedent and a true consequent. Accordingly, we assign the value 1(true) just below the hook sign. Assigning value in this way, we have the following table so far-:

p	q	(p ⊃ q) . (q ⊃ p)
1	1	1 1 1 1 1 1 1
1	0	1 0 0 0 0 1 1
0	1	0 1 1 0 1 0 0
0	0	0 1 0 1 0 1 0

Now we are left with the truth-values for the main constant which is the dot (.) sign in this case. Consulting the truth table for the basic conjunctive function, we fill in the column just below that sign in our example. Accordingly, we assign truth-values under the main constant as follows to get the following final table-

p	q	(p ⊃ q) . (q ⊃ p)
1	1	1 1 1 1 1 1 1
1	0	1 0 0 0 0 1 1
0	1	0 1 1 0 1 0 0
0	0	0 1 0 1 0 1 0

Here are some rather more complex examples fully worked out following the procedure as mentioned above, proceeding from the left hand side to the right hand side.

Example 2. (p . ~ q) ⊃ ~ (p ⊃ q)

p	q	(p . ~ q) ⊃ ~ (p ⊃ q)
1	1	1 0 0 1 1 0 1 1 1
1	0	1 1 1 0 1 1 1 0 0
0	1	0 0 0 1 1 0 0 1 1
0	0	0 0 1 0 1 1 0 0 0

Example 3. ~ [~ (p ⊃ q) ⊃ (~ p ⊃ q)]

p	q	~ [~ (p ⊃ q) ⊃ (~ p ⊃ q)]
1	**1**	**0 0 1 1 1 1 0 1 1 1**
1	0	0 1 1 0 0 1 0 1 1 0
0	1	0 0 1 1 1 1 1 0 1 1
0	0	0 0 1 1 1 1 1 0 1 1

Example -4 [p⊃ (p ⊃ q)] q

p	q	[p⊃ (p ⊃ q)] ⊃ q
1	**1**	**1 1 1 1 1 1 1**
1	**0**	**1 0 1 0 0 1 0**
0	**1**	**0 1 0 1 1 1 1**
0	**0**	**0 1 0 1 0 0 0**

Example -5- [(p ⊃ q) . (p ⊃ r)] ⊃ (p ⊃r)]

p	q	r	[(P ⊃ Q) . (P ⊃ R)] ⊃ (P ⊃R)]
1	**1**	**1**	**1 1 1 1 1 1 1 1 1 1 1**
1	**1**	**0**	**1 1 1 0 1 0 0 1 1 0 0**
1	**0**	**1**	**1 0 0 0 0 1 1 1 1 1 1**
1	**0**	**0**	**1 0 0 0 0 1 0 1 1 0 0**
0	**1**	**1**	**0 1 1 1 1 1 1 1 0 1 1**
0	**1**	**0**	**0 1 1 0 1 0 0 1 0 1 0**
0	**0**	**1**	**0 1 0 1 0 1 0 1 0 1 1**
0	**0**	**0**	**0 1 0 1 0 1 0 1 0 1 0**

In the above examples, we may observe that in some of the truth tables the main constant contains the truth-values of '1' throughout the column, in some truth tables the main constant contains the truth-values of 1 and o mixed up and in some other truth tables, the column under the main

constant contains the truth-values of 'o' throughout. When the column under the main constant contains in this way 1,1,1,1 as its truth-values, it means that the statement is a logical truth, tautology. When the column under the main constant contains o, o, o, o as its truth-value, then the statement is a contradictory one and when on the other hand, the column under the main constant contains 1 and o mixed up, then the expression is to be described as a contingent statement or expression.



LET US KNOW

- LOGICAL EQUIVALENCE

When two truth-functional statements apparently different from each other but have identical truth-values, then these two statements are called logically equivalent statements. By constructing the truth tables for two such statements, we compare the tables by looking at the result of the column under the main constant. When they are found to have identical truth-values, they are called, 'logically equivalent' statements

Examples : Statement – 1. ~(p.q),

Statement- 2. ~p v ~q

Truth table for the statement –1.

~ (P . Q)
F T T T
T T F F
T F F T
T F F F

Truth table for the statement-2

~P V ~ Q
F T F F T
F T T T F
T F T F T
T F T T F

The truth table shows that the statement 1, and the statement 2. are logically equivalent statements since the column under the main constant contains the same truth-values.

- While constructing a truth table for a complex truth-functional statement one has to proceed step by step. Since the main operator controls the final determination of the statement's truth-value, it is the last step.

- Truth tables of basic truth-functional statements for conjunction, disjunction, negation, implicative and equivalent statements are used to analyze and construct complex statements.
- A contingent statement is possibly true or possibly false.
- A tautology is a non-contingent statement that is true.
- A self-contradictory statement is a non-contingent statement that is always false.



CHECK YOUR PROGRESS

Q 4: a) What will be the truth-values under the column of the main constant in the truth table of a tautological statement?

b) What will be the truth-values under the column of the main constant of a Contingent statement?

c) What will be the truth-values under the column of the main constant of a contradictory statement?

Q 5: What do you mean by logically equivalent statement?

.....

.....

7. 5 BASIC RULES FOR DETERMINING THE VALIDITY OF ARGUMENTS BY TRUTH TABLE METHOD

- So far we have got some elementary knowledge about the application of truth table method in determining the truth-value of compound expressions. Now we are in a position to test the validity of arguments through truth table method. In fact, determining or testing the validity of arguments by truth table method is an important feature of truth table method. The necessary procedure to test the validity of arguments by truth table method may be laid down as follows-

(a) The given argument is to be symbolised in the form of a material implication, with the premises as the antecedent and the conclusion as the consequent.

(b) The argument is to be symbolized using abbreviator letters or prepositional variables.

(c) Truth-values are to be assigned to the statement form, following the procedure given before.

(d) It would then have been shown that there is no possibility of assigning truth-values in such a way that there is a true antecedent and a false conclusion. The argument, which has been written as a material implication, with the premises as the antecedents and the conclusion as the consequent may be taken in this way to be valid.

(e) If the completed truth table in this way does not have a column of '1's under the main connective as truth-value, then the statement is not to be taken as a logical truth. The argument, which was written as a material implication, is not valid. There is the possibility of its having true premises and a false conclusion.

Example 1.

If you want to achieve your target, you must be able to manage your time.

You have achieved your target. Therefore, you are able to manage your time.

First it must be symbolized in the form of an argument which may be presented in the form of a material implication as follows: -

$[(T \supset M) . T] \supset M$

(Using T for 'you want to achieve your target' and M for 'you must be able to manage your time')

Assigning truth-values for the propositions we construct the following truth table:-

[(T ⊃ M) . T] ⊃ M
1 1 1 1 1 1 1
1 0 0 0 1 1 0
0 1 1 0 1 1 1
0 1 0 0 0 1 0

Assigning truth-values in this way to the simple propositions (T,M), To the constants respectively according to their scope. The main constant contains all the '1's as its truth-value, hence the argument is valid.

Example 2.

If the chief witness is telling the truth, then A is guilty. But A is not guilty; so the chief witness is not telling the truth. The argument is expressed in the form of material implication in the following way**:-**

[(W ⊃ G). ~G] ⊃ ~W

Now we can construct the following truth table by assigning truth-values to test the argument**:-**

[(W ⊃ G) . ~ G] ⊃ ~W
1 1 1 0 0 1 1 0 1
1 0 0 0 1 0 1 0 1
0 1 1 0 0 1 1 1 0
0 0 0 1 1 0 1 1 0

The argument is valid because the main constant, that is, the second occurrence of the hook sign contains '1's throughout as its truth-value**.**

Example 3.

If K. K. Handique was an Assamese, then he was an Indian. K.K.Handique was an Assamese. Therefore K.K.Handique was an Indian**.**

The logical form of the argument**: - [(A ⊃ I). A] ⊃ I**

Now we construct the following truth table by assigning truth-values as per procedure: -

[(A ⊃ I) . A] ⊃ I
1 1 1 1 1 1 1
1 0 0 0 1 1 0
0 1 1 0 0 1 1
0 1 0 0 0 1 0

The argument is valid because the main constant, that is, the second occurrence of the hook sign contains all the 1s as its truth value.



LET US KNOW

Modus ponens- the inference structure in the above examples is called modus ponens. Modus, means 'method' and ponens, means 'affirming'. In this particular case, it stands for the 'method of affirming the consequent'. Any argument or inference with the logical structure of modus ponens is valid.



CHECK YOUR PROGRESS

Q 6: Examine the validity of the following arguments by using truth table method.

a) If it rains, then crops will grow. It rains. Therefore the crops will grow.

b) If he is pious, then he is honest. But he is not honest. Therefore he is not pious.

Q 7: Fill in the blanks:-

a) All the truth-values under the main constant of valid argument shall be———

b) All the truth-values under the main constant of an invalid argument shall be either ———— or ————————.

7. 6 INDIRECT TRUTH TABLE METHOD OR THE METHOD OF REDUCTIO AD ABSURDUM

To test the validity of arguments involving more than three simple statements will involve a great deal of time and space. For an argument involving four statements we should need 16 lines. For one, involving five, we should need thirty-two lines. However there is a shorter method, which is known as the indirect method or the method of *Reductio ad absurdum*. This phrase was originally used to refer to a method of argument first explained in Euclid's *Elements of Geometry* a book which dates from the fourth century B.C.

In the indirect truth table or reductio ad absurdum method we begin by assuming that the given argument is false. Hence we put a '0' under the

main constant of the truth table. Accordingly, we go on assigning values. Then the consequences are carried out till the end to find out whether this involves in any contradiction or not. If it is found to result in any logical contradiction, than it is understood that the given argument, which is the opposite of our assumption, is valid. If on the other hand, the consequence of our assumption does not involve us in any contradiction, then it is understood that our assumption is valid; in other words, the given argument which is the opposite of our assumption, is invalid.

Let us apply the shorter or indirect method of truth table in determining the validity of the following argument—

Example 1. Either A is guilty or B and C. If A is guilty, then C is guilty. Therefore C is guilty.

Assuming that A, B and C to be constituent propositions, let us symbolize the argument in the following form and arrange the argument horizontally to make it convenient to construct the truth table**:-**

1. **(A V (B. C)**
2. **A ⊃ C**

∴ **C**

[{A V (B. C)}. (A ⊃ C)] ⊃ C

We shall construct the truth table in the following steps: -

Step 1. :- We write out the form with a box underneath for each variable and each constant. (the making of 'boxes' one for each variable and constant is helpful; in this way, we can be sure that we have assigned a truth-value to each variable and each constant)**.**

[A V (B . C)} . (A ⊃ C)] ⊃ C

Step 2. :- Assuming the statement form to be false, we put the value sign '0' under the main connective i.e., the last occurrence of the hook sign in this example**.**

[A V (B . C)} . (A ⊃ C)] ⊃ C

[A	V	(B	.	C)}	.	(A	⊃	C) }	⊃	C
									0	

Step 3. : - When a material implication is false, the antecedent is true and the consequent false. We assign the value sign '1' below the antecedent and 'o' below the consequent**.**

[A V (B . C)} . (A ⊃ C) } ⊃ C

					1				0	0
					1					2

Step 4:- When a material implication is false, the antecedent is true and the consequent is false. We assign the value '1' below the antecedent and '0' below the consequent.

[A V (B . C)} . (A ⊃ C) } ⊃ C

	1				1		1		0	0
	3				1		4			2

Step 5. : - The antecedent is a conjunction; we have assigned the value '1' to it. If the conjunction is true both the conjuncts must be true. So we assign the value '1' to both the conjuncts.

[A V (B . C)} . (A ⊃ C) } ⊃ C

	1		0		1		1		0	0
	3		5		1		4			

Here we assign the value 'o' to the conjunction B.C as we have already found that one of its conjuncts (c) is false.

Step 6. :-

[A V (B . C)} . (A ⊃ C) } ⊃ C

	1		0	0	1	1	1		0	0
	3		5		1	6	4			

In this final step we assign the value '1' to A because, it must be true since the other components of the disjuncts i.e., (5), is false. After assigning

the values in this way we find that the conditional 'A C', which is shown to have the value '1' has the value 'o' for the consequent. But this goes against the definition of the table for implicative function. Hence our assumption that the given argument is invalid must be wrong. The opposite assumption i.e., the argument to is valid, must be true. We write 'valid' by the side of the table to indicate this.

Example 2.

[(P	⊃	Q)	.	P]	⊃	Q
1	1	0	1	1	0	0

The above is an example fully worked out in a single step which we normally have to do. It is to be noted that shorter or indirect truth table method can be applied to test the validity of any argument. We have to symbolize the argument, using abbreviatory letters, and present the argument in the form of a material implication, and then follow the procedure given above. The indirect truth table method is much quicker and less cumbersome than the ordinary truth table method. But while using this method, it is necessary to know -(a) the definitions and to keep a sharp eye out for any violations ;(b) that we cannot assign truth-value in any arbitrary fashion. But by and large, it is more convenient to apply this method in determining the truth-values of expressions involving a large number of variables, which are found to be cumbersome when tested with the help of the ordinary (direct) method of truth table decision.



CHECK YOUR PROGRESS

Q 8: What is the other name for the indirect truth table method?

.....

Q 9: Name the book in which this term or the phrase was first used to refer to a method of argument.

.....

Q 10: State the principle which we adopt to proceed in the indirect truth table method?

.....



7.7 LET US SUM UP

- Truth table is a tabular order on which all possible truth-values of compound statements are displayed, through the display of all possible combinations of the truth-values of their simple components. A truth table may be used to define truth-functional expressions and it may also be used to test the validity of many deductive arguments.
- When the truth-value under the main constant of the table contains the values '1' throughout the column, it means that the statement is a logical truth. If the statement happens to be an argument, then it is said to be valid. When the main constant contains the values 'o' throughout the column, it means that the statement is not a logical truth; in the case of an argument, it is described as invalid. Again, when the main constant contains 1 and o mixed up, then the expression is to be described as contingent; in the case of an argument, it is described as invalid. It is to be noted that the constant which has the widest scope is called the main constant.
- Truth table test of validity has several advantages. First, the test is capable of proving both validity and invalidity. Second, when applied to truth-functional argument forms, the test is guaranteed (if executed properly) to yield the correct result in finite time. Third, the test is essentially 'mechanical' and does not require insight, creativity, genius, or even the process of trial and error.
- The truth-table test also has severe drawbacks. First, the method is effective only for truth-functional argument forms containing nothing but statement variables and truth-functional connectives like ., v, ~ , etc. If an argument contains a premise or conclusion that is not truth-

functional, we will generally be unable to evaluate it in all of the rows. For the truth-value of that or the conclusion will not be completely determined by the truth-values of its simple components.

- Second, while the test does not require genius, it does require careful, concentrated and accurate work.
- Third, the truth table test is dull and tedious precisely because it is so mechanical.



7.8 FURTHER READING

1) Stan Baronett and Madhucchanda Sen: Logic : Pearson 2008

2) Chanda Chakraborty: Logic: Informal, Symbolic and Inductive: Prentice -Hall of India,New Delhi, 2007

3) I.M. Copi and Cohen Nagen : Introduction to Logic:Prentice -Hall India, 2007

4) David Harel, Dexter Kozen, Jerzy Tiuryn: Dynamic Logic:Prentice - Hall India, 2007



7.9 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q No 1: Truth table is a two- dimensional arrangement because it is formed by rows and columns of truth-values by which one can find out the truth-values of compound statements or sets of compound statements.

Ans to Q No 2: A truth table may be used to define truth-functional expressions and it may also be used to test the validity of many deductive arguments. Besides, truth tables may be used to determine whether different truth-functional statements are logical equivalence, contradictory, consistent and inconsistent statements.

Ans to Q No 3: With 2 variables there are 4 i.e., 2x2=2 cases of possible truth-values.

Ans to Q No 4: a) 1 ,1,1,1 (T, T, T ,T)

b) 1 and 0 or T and F mixed up.

c) 0 or F throughout the column.

Ans to Q No 5: Logically equivalent statements are truth-functional statements that have

identical truth- values.

Ans to Q No 6: a) Ans: 1(T)

b) O(F) or 1(T) mixed up

Ans to Q No 7: a) 1 b) 0

Ans to Q No 8: Reductio ad Absurdum.

Ans to Q No 9: Elements of Geometry (a book written by Euclid which dates from the fourth century B.C.)

Ans to Q No 10: As a principle, in this method we assume the contradictory of what we have to prove. If this assumption results in a contradiction, it is known that what we are trying to prove does indeed follow from our premises, i.e., valid. If this assumption does not result in contradiction, then the argument is not valid.



7.10 MODEL QUESTIONS

A) Very short questions

Q 1: Define truth table method.

Q 2: What is indirect truth table method?

B) Short questions

Q 2: Construct truth tables for the following expressions-

a) P (P.Q)

b) (P. Q) . (PVQ)

c) [(PVQ) . ~P] Q

d) ~(P . Q)

e) (P VQ) V P

f) (P . ~ Q)

g) P V (Q V ~Q)

h) (P Q) (~ Q ~P)

i)~(P. Q) (~P V ~Q)

j) (P V Q) . (~P . ~Q)

Q 3: Write short notes on the following-

(a) Truth table method

(b) Indirect truth table method.

C) Long Questions (Answer in about 300-500 words)

Q 1: Explain the indirect truth table method with suitable example.

Q 2: What are the basic rules for determining the validity of arguments by truth table method? Explain.

*** ***** ***

UNIT 8 : TRADITIONAL ANALYSIS OF PROPOSITION

UNIT STRUCTURE

8.1 Learning Objective

8.2 Introduction

8.3 Proposition

8.4 Structure of Proposition

8.5 Nature of Copula

8.6 Proposition and Judgment

8.7 Proposition and Sentence

8.8 Classification of Proposition

8.9 Categorical Propositions and Classes

8.10 Doctrine of Distribution of Terms

8.11 Reducing Sentences to Logical Forms

8.12 Opposition of Propositions

8.13 Drawbacks of Traditional Analysis and Classification of Proposition

8.14 Let Us Sum Up

8.15 Further Reading

8.16 Answers to Check Your Progress

8.17 Model Questions

8.1 LEARNING OBJECTIVES

After going through this unit, you will be able to

- discuss the meaning of proposition as discussed by traditional logicians
- distinguish proposition from ordinary speech, writing and grammatical sentence
- understand the traditional analysis and classification of proposition
- learn the logical form of proposition.

8.2 INTRODUCTION

This unit introduces you to the traditional analysis and classification of proposition. Here we will find that a proposition is the statement of a certain relation between two terms. Every proposition either asserts or denies. Aristotle defines proposition as a statement in which something is said regarding something else, either affirmatively or negatively. When we say that the proposition is either affirmative or negative we point out the 'quality' of the proposition. When we affirm or deny the whole subject or a part of the subject we point out the 'quantity' of the proposition. The fundamental kinds of proposition are *categorical* and *conditional*. In short every categorical proposition is affirmative or negative in quality and universal or particular in quantity. Hence we have four kinds of categorical propositions viz. universal affirmative, universal negative, particular affirmative and particular negative that have been symbolized by the alphabets A, E, I and O respectively. According to the traditional logicians, every proposition consists of a subject, a predicate and the copula which connects the subject and the predicate term. Copula is usually expressed by the word 'is' 'are' or 'is not' and 'are not'. Every logical proposition must be expressed in strict logical order of subject- copula –predicate. In addition, every proposition must clearly state the quantity. The subject term has the sign of quantity. If any expression is not in this order, it is to be reduced in one of these four forms for use in logic. This process is called reduction or simplification of proposition. There are some general rules for reducing sentences into strict logical form. Traditional logicians recognize singular proposition as universal proposition.

8.3 PROPOSITION

Proposition is the basic unit of logical thinking. Logic is defined as the science of thought as expressed in language. The word reasoning may be substituted for thought in logic. The reasoning which is expressed in language is called an argument. An argument is constituted by more than one proposition with its definite structure. Hence proposition is the unit of logical thinking.

Traditionally a proposition is meant the statement of a certain relation between two terms**.** For example: “All crows are black” - is a proposition. Here ‘crows’ and ‘black’ are two terms and a certain relation between these two terms are stated. The relation between these two terms is expressed by the word ‘are’.

Every proposition either asserts or denies. It is the affirmation or denial of a certain relation between two terms. Aristotle defines proposition as a statement in which something is said regarding something else, either affirmatively or negatively.

8.4 STRUCTURE OF PROPOSITION

The structure of proposition means the constitution of proposition. The analysis of above definition of proposition directly refers to the structure of a proposition. A proposition states a relation between two terms. So a proposition must be constituted by three parts- two terms and the sign of relation between them. Of these two terms, one is the subject and the other is the predicate. The word ‘are’ that holds the relation between them is known as copula. The subject term is that about which something is spoken. The predicate term is that term which represents what is being spoken of the subject term. The copula is the word which relates the subject term and the predicate term. For example: (1) all men are mortal (2) some men are not learned. In the first proposition, ‘men’ is the subject as something is said or affirmed about ‘all men’. And ‘mortal’ is the predicate as ‘mortality’ is affirmed of ‘all men’. And the word ‘are’ is the copula as it connects the subject and the predicate. In the second proposition, something is denied about ‘some men’. So ‘men’ is the subject. ‘Learned’ is the predicate and ‘are not’ is the copula.

8.5 NATURE OF COPULA

The copula stands as the sign of relation between the subject and the predicate**.** The copula indicates the quality of the proposition. In Aristotelian logic the nature of copula can be described by the following points.

- The copula is not a term**.** It is a connecting word which stands as the sign of relation between the subject and the predicate.
- The verb 'to be' can only be used as the copula. Because only the verb 'to be' can clearly and distinctly express the relation between the subject and the predicate. Further the copula should be expressed separately which is made possible only by the verb 'to be'.
- The copula should always be expressed in the present tense of the verb 'to be'. But the verb 'to be' has three different forms in three different tenses. The different forms in past tense –was, were; in present tense-am, is, are; in future tense-shall be will be. So question arises, whether the copula can be expressed in the present tense of the verb 'to be' or any other tense of the verb 'to be'.

  There are some controversies among logicians regarding this question. Logicians like **Hamilton, Mansell and Fowler** maintain the view that the copula should always be used only in present tense of the verb 'to be'. But **Mill** holds the view that the copula may be used in any tense of the verb 'to be'. This controversy finally ends with the view that the relation between the subject and the predicate is maintained by the copula. And this relation does not admit of variation in time. So copula should be free from the element of time. And it is the predicate that refers the element of time. Therefore copula should always be expressed in the present sense of the verb 'to be'. For example, Socrates lived in Athens- is a sentence and in the logical form it should be expressed-'Socrates is a person who lived in Athens'.
- **The copula may be affirmative or negative.** Logicians differ among themselves regarding this issue. Some logicians like **Hobbes** maintain that the copula should always be affirmative. For them if the negative sign is attached to the copula then it shows that there is no relation between the subject and the predicate. So they attempt to attach the negative sign to the predicate. For example, the proposition- 'some people are not happy' should be expressed as 'some people are not-happy'. But this view is not accepted. Because by mere attaching the negative sign to the predicate we can not avoid the distinction between

affirmation and negation. Though the proposition is affirmative in form, it is negative in its content. Hence the copula is affirmative when it expresses the relation of affirmation and negative when it expresses the relation of denial.

8.6 PROPOSITION AND JUDGMENT

The two words 'judgment' and 'proposition' are often used synonymously, but there is an important difference between them. A judgment is regarded as a unit of thought. It is a mental process of comparing two concepts with each other. Sometimes judgment is used in the sense of mental act of judging and sometimes in the sense of what is judged. However logic is mainly concerned with the product of this mental process. We arrive at an affirmative or negative judgment according to the character of the relation between them. Judgments as mental products become definite and precise when clothed in language. And proposition is a judgment expressed in language.



LET US KNOW

A concept means a general idea. It is formed by abstracting the common and essential attributes of different individuals or things and then a name is given to that generalized group. For example, 'man.

8.7 PROPOSITION AND SENTENCE

A logical proposition roughly corresponds to what is known as a sentence in grammar. But a proposition must not be confused with a sentence. A proposition is not identical with a sentence. There is a clear difference between a proposition and sentence. For example,

- A sentence is a group of words put together in a grammatically correct method carrying a complete meaning. But a proposition is the explicit statement of the relation between two terms. For example, 'Ram defeats Ravana'-is a sentence while 'Ram is a man who defeats

Ravana' is a proposition.

- Every sentence is not a proposition but every logical proposition is a grammatical sentence.
- A logical proposition consists of three parts- the subject, the predicate and 'the copula'. For example, 'All cows are ruminant'- in this proposition 'cows' is the subject, 'are' is the copula and 'ruminant' is the predicate. But a sentence consists of two parts- the subject and the predicate.
- In a logical proposition the copula shows the relation between the subject and the predicate and it is expressed separately while in a sentence the copula is included in the predicate.
- A sentence may be in past, present and future. But a logical proposition is stated in the present tense only.
- A sentence may express thoughts, feelings, desires, commands, likes and dislikes. So there are various kinds of sentences- Interrogative, Imperative, Optative, Exclamatory and Indicative. But a logical proposition expresses thoughts alone. The logical proposition has its nearest approach only in an indicative sentence. If any sentence is not expressed in indicative sentence, they have to be reduced to the forms of the indicative sentences. Sometimes even an indicative sentence requires to be re-written in logical form to be used in logic. For example; the indicative sentence 'John runs' is not a logical proposition. It is to be expressed in strict logical form as- 'John is a person who runs'
- The quality and quantity are expressed clearly in a logical proposition. But not always so expressed in a grammatical sentence.



LET US KNOW

Modern logicians define proposition as a statement which is either true or false but cannot be both true and false at the same time.

8.8 CLASSIFICATION OF PROPOSITION

Traditional logicians have classified proposition under different heads according to different principles.

According to Structure or Composition: According to structure proposition are divided into simple and compound.

A simple proposition expresses a single statement. For example: It is raining now.

A compound proposition, on the other hand, consists of more than one simple proposition. As for instance, 'Radhakrishnan is both philosopher and politician'.

A combination of two or more affirmative propositions is called Copulative proposition, such as 'Rabindranath Tagore is both a philosopher and a poet- This proposition combines two simple and affirmative propositions-(1) Rabindranath Tagore is a philosopher and (2) Rabindranath Tagore is a poet.

A combination of two or more negative proposition is called Remotive proposition, such as 'Neither cunning nor hypocrisy is good.' This proposition consists of two simple and negative propositions- (1) Cunning is not good and (2) hypocrisy is not good.

According to quality: According to quality, propositions are divided into Affirmative and Negative.

An affirmative proposition is one in which the predicate is affirmed of the subject. For example: 'All men are mortal'. 'In this proposition the predicate 'mortal' is affirmed of the subject 'men'.

A negative proposition is one in which the predicate is denied of subject. For example: Some Indians are not lazy. In this proposition the predicate 'lazy' is denied of the subject 'Indians'.

The quality of a proposition is determined by the copula. But in case of universal negative proposition, the negative sign is detached from the copula and associated with the subject, as for example, 'No men are perfect'.

According to quantity: According to quantity, propositions are divided into Universal and Particular.

A universal proposition is one in which the predicate is affirmed or denied of the whole subject. For example, 'All men are mortal', 'No men are perfect'. Here in the first proposition the predicate 'mortal' is affirmed of all men and in the second proposition, the predicate 'perfect' is denied of 'all men'.



LET US KNOW

Traditional logicians consider singular proposition as universal proposition. In a singular proposition the subject term is singular and it is used in the whole of its extent. Hence it is considered as universal proposition. For example,

Socrates was a philosopher.

India is a member of U.N.O.

These singular propositions are regarded as universal proposition by the traditional logicians.

A particular proposition is one in which the predicate is affirmed of a part of the subject. For example, 'Some people are happy', 'Some girls are not bold'. In the first proposition, the predicate 'happy' is affirmed of some men, and in the second proposition, the predicate 'bold' is denied of some men.

The quantity of a proposition is determined by the quantity of the subject. A universal affirmative proposition is signified by the words like- 'all', 'every', 'each' and these words are attached to the subject. Similarly a universal negative proposition is signified by the words, 'no', 'none' and they are attached to the subject.

According to relation: According to relation propositions are divided into categorical and conditional.

A categorical proposition is one which expresses a relation between the subject and the predicate categorically or unconditionally. In other words in a categorical proposition the predicate is either affirmed or denied of the subject unconditionally. For example, 'All man are mortal', 'Some men are not learned'. These propositions express a relation between the subject and the predicate without any condition. Here the predicate 'mortal' is affirmed

of the subject 'all men' and the quality 'learned' is denied of the subject 'men' unconditionally.

On the other hand in a conditional proposition the relation between the subject and the predicate is made under certain condition. For example, (a) 'If there is sun, there is light'. (b) 'If he comes, I shall go out, (3) 'Shyam is either foolish or wicked'. Conditional propositions are of two kinds (a) Hypothetical and (b) Disjunctive. In a hypothetical proposition the condition is introduced by 'if-then' or such words. A hypothetical proposition consists of two parts-the antecedent and the consequent. The antecedent part contains the condition and the consequent part contains the statement of a hypothetical proposition. In the above illustrations 'If there is sun', 'If he comes' are the antecedents and 'there is light', 'I shall go out' are the consequents. The quality of a hypothetical proposition is determined by the quality of its consequent and the quantity of a hypothetical proposition is determined by the quantity of its antecedent.

In a disjunctive proposition alternatives are mentioned, of which at least one true and its two alternatives being joined by 'Either-or'. For example: Either Shyam is foolish or he is wicked'.

According to modality: According to modality proposition are divided into Necessary, Assertory and Problematic.

In a necessary proposition the relation between the subject and the predicate is based on their very nature. A necessary proposition is universally and necessarily true. For example: 'Two and two must be four'.

In an assertory proposition the relation between the subject and the predicate is based on experience. An assertory proposition is true so far as our experience goes. For example: 'All swans are white'.

In a problematic proposition the relation between the subject and the predicate is only probable. For example: 'He may come', 'It may rain'.

According to import: According to import Proposition are divided into Verbal and Real.

In a verbal proposition the predicate merely states the connotation or a part of the connotation of the subject. For example: 'All men are rational'. Here the predicate 'rational' states only a part of the connotation of subject

'men'. It does not give any new information but merely analyses the connotation or a part of the connotation of the subject. A verbal proposition is called an analytic proposition. Such a proposition is also known as Essential or Explicative proposition.

On the other hand in a real proposition the predicate asserts an additional fact about the subject that is not contained in the connotation of the subject. For example: 'All men are mortal', 'Man is a laughing animal'. A real proposition is called synthetic proposition. Such a proposition is also known as Accidental or Ampliative.

8.9 CATEGORICAL PROPOSITIONS AND CLASSES

The fundamental kinds of proposition are *categorical* and *conditional*. Every proposition either asserts of denies. A categorical proposition is one that can be analyzed as being about classes, or categories, affirming or denying that one class, S, is included in some other class, P, in whole or in part. In other words every categorical proposition either *affirms* that the subject term is related partially or completely to the predicate term or *denies* that the subject term is related partially or completely to the predicate term. So various relationships between classes are affirmed or denied by categorical propositions. There are four standard-forms categorical propositions viz**. Universal Affirmative, Universal Negative, Particular Affirmative and Particular Negative.**

A **universal affirmative** proposition says that all members of the subject term (first class) are members of the predicate term (second class). In other words in a universal affirmative proposition, the predicate is affirmed of the whole subject. For example 'All residents are voters'. This proposition is named by the vowel –A

A **Universal Negative** proposition asserts that no members of the subject term are members of the predicate term. In other words in a universal negative proposition the predicate is denied of the whole subject. For example: 'No reptiles are warm-blooded animals'. This proposition is symbolized by the vowel –E

A **Particular Affirmative** proposition asserts that at least one member of the subject term is a member of the predicate term. In other words in a Particular Affirmative proposition the predicate is affirmed of a part of the subject. For example 'Some politicians are liars'. This proposition is symbolized by the vowel –I

A **Particular Negative** proposition asserts that at least one member of the subject term is not member of the predicate term. In other words in a Particular negative proposition the predicate is denied of a part of the subject. For example: 'Some judges are not lawyers'. This proposition is symbolized by the vowel –O This kind of classification is called **four-fold scheme of proposition.**

We can represent the four kinds of proposition in the following table.

Kinds of proposition	Description	Forms of proposition
A	Universal affirmative	All S are P
E	Universal negative	No S are P
I	Particular affirmative	Some S are P
O	Particular negative	Some S are not P

So every standard-form categorical propositions is said to have a quality-affirmative and negative. Similarly every standard-form categorical propositions is said to have a quantity also-universal or particular. The words 'All' and 'No' indicate that the proposition is universal and the word 'some' indicate that the quantity is particular. But the quantifier 'no' indicate the negative quality of the proposition.



LET US KNOW

The letters 'A' & 'I' and the letters 'E' and 'O' have been used for the affirmative and negative propositions respectively because the first two vowels of the Latin word ***Affi****rmo* (meaning, 'I affirm') and *n****E****g****O*** (meaning 'I deny') signify the quality of the proposition.

8.10 DOCTRINE OF DISTRIBUTION OF TERMS

The distribution of terms is concerned with the denotation of the terms. Distribution does not mean division. If a categorical proposition says something definite about every member of the class designated by a term, then the term is said to be ***distributed***. On the other hand, if the statement does not say something definite about every member of the class designated by a term, then the term is said to *be* ***undistributed***. In other words a term is said to be distributed when it is taken in its entire denotation and a term is said to be undistributed when only a part of its denotation is taken into account. For example, 'All swans are white'–in this proposition, the subject term 'swans' is taken in its entire denotation i.e. 'all swans' hence the term 'swans' is distributed here. On the other hand 'Some scientists are philosophers' - in this proposition only a part of the denotation of the subject term 'scientists' is taken into account. Hence the term 'scientists' is undistributed here.

Let us see which terms are distributed in the four categorical propositions-A, E, I, O.

A-Universal Affirmative Proposition

Let us take one example of universal affirmative proposition- 'All men mortal'. Here the subject term 'men' is distributed as it is taken here in its entire denotation and it is clearly expressed by the word 'all'.

But in case of the predicate term there is no such word to explain its entire denotation. So let us see whether the predicate term is distributed or not. The denotation of the predicate term 'mortal' is 'all mortal organisms'. And man is a part of all mortal organisms. So the predicate term is undistributed here.

So, A proposition distributes subject only, not the predicate**.**

E-Universal Negative Proposition

Let us take one example of Universal Negative Proposition-'No men are perfect'. Here both the subject and the predicate terms are taken in their entire denotation; hence both the terms are distributed. The two classes 'man' and 'perfect' are mutually exclusive. It means no part of the demotion of the subject term 'man' is included in the denotation of the predicate term 'perfect'. Similarly no part of the denotation of the term 'perfect' is included

in the denotation of the term 'man'. Hence in E proposition both subject and the predicate terms are distributed.

I-Particular Affirmative Proposition

Let us take one example of I proposition-'Some mangoes are sweet'. Here the subject term 'mangoes' is undistributed because it is taken in its partial denotation and it is clearly expressed by the word 'some'. The denotation of the predicate term 'sweet' is 'all sweet things'. So some mangoes can not cover the entire denotation of the term 'sweet'. There are other sweet things apart from mangoes also. Hence here the predicate term 'sweet' is undistributed.

O-PARTICULAR NEGATIVE PROPOSITION

Let us take one example of O proposition-'Some students are not honest'. Here the subject term is taken in its partial extent and it is clearly expressed by the word 'some' Hence in this proposition the subject term is undistributed.

So far as the predicate term is concerned, it is denied to some students. In a negative proposition, the predicate must be denied in its entire extent, otherwise negation does not possess any meaning. So the subject term 'some students' is excluded from the entire denotation of the term 'honest' i.e. 'All honest beings'. Thus the predicate term is taken here in its entire denotation, hence it is distributed.

Thus O proposition distributes predicate only.

From the above discussion we can find out the following principles.

(1) Universal proposition distributes their subject

(2) Negative propositions distribute their predicates.



LET US KNOW

To remember the doctrine of the distribution of term we can use the word-'**ASEBINOP'**. It implies A proposition distributes subject, E distributes both the subject and the predicate, I distributes neither the subject nor the predicate and O distributes predicate only.

8.11 REDUCING SENTENCES TO LOGICAL FORMS

In the traditional logic every proposition must clearly state the quantity, the subject, the copula and the predicate. Normally this strict logical order is not maintained in the ordinary speech and writing. So in order to use in logic, the ordinary and grammatical sentences must be reduced into the strict logical forms. This process is called reduction or simplification of proposition. In deductive logic A, E, I, and O are recognized as the four standard form categorical propositions. In simplification of proposition the ordinary and grammatical sentences are reduced into one of these four categorical propositions. While we reduce a sentence into logical form we should pay attention to the meaning of the sentence, not to its mere form.

General rules for reducing sentences into logical form

1. (A) Affirmative sentences containing words like 'All', "Every', 'Each', "Any' 'Always', 'Ever', 'Everywhere', 'Absolutely' 'Necessarily', 'Universally', 'Invariably' are to be reduced to 'A', i.e. universal affirmative and

   (B) Negative sentences containing are to be reduced to 'O', i.e. particular negative.

The following tables will clarify it.

i)

words	sentence	Ordinary sentences	logical forms
'All',.. "Every', 'Each' etc	affirmative	Any man can do this Every man is mortal	A - All men are those who can do this- A - All men are mortal

ii)

words	sentence	Ordinary sentences	logical forms
'All', "Every', 'Each', "Any' etc.	negative	Every man is not honest All that glitters are not gold	Some men are not honest.-O Some things that glitter are not gold.-O

2.(A) Affirmative sentences containing words like 'Most', 'Many', 'A few', 'Certain', 'All most all', 'All but one', 'Several', 'Mostly' 'Generally',

'Frequently', 'Often', 'Perhaps', 'Nearly always', 'Sometimes' and 'Occasionally', are to be reduced to 'I' particular affirmative proposition and

(B) Negative sentences containing these words are to be reduced to 'O' i.e. particular negative proposition.

i)

words	sign	ordinary sentences	logical forms
Most, Many, A few, Certain, All most all, All but one, S'	affirmative	Most of the teachers are hard-working. Many students are clever	Some teachers are hard-working-I Some students are clever -I

ii)

words	Sign	Ordinary sentence	logical forms
Above words	negative	A few boys are not bold. Children generally do not like bitter things.	Some boys are not bold.-O Some children are not those who like bitter things.-O

3). (A) The affirmative sentences containing the word 'few' are to be reduced to 'O' i.e. particular negative proposition. The word 'few' in its ordinary use means, ''Almost none'. But it must be noted that the word 'A few' means 'some are'.

(B) On the other hand negative sentences containing the word 'few' are to be reduced to 'I' i.e. particular affirmative proposition.

Words	sentence	Ordinary sentences	logical forms
'Few' A few	Affirmative Negative	Few boys are not bold. Few scientists are not rich. A few students study Hindi	Some boys are not bold.-O Some scientists are rich-I Some students are those who study Hindi.-I

4. The word 'Seldom', 'Hardly', Scarcely' and 'Rarely' are used in negative sense. So the affirmative sentences containing these words are to be reduced to 'O' proposition. Similarly, if they are used in negative sentences they are to be reduced to 'I' proposition.

i)

Words	sentence	Ordinary sentences	logical forms
'Seldom', 'Hardly', Scarcely' and 'Rarely'	Affirmative	People seldom openly condemn such acts.	Some people are not those who openly condemn such acts-O
	Negative	Hardly some stories are not interesting.	Some stories are interesting.-I

5) Sentences containing the words like 'No', 'Never', 'None', 'Not at all', 'Not a single', 'Nobody', 'Nothing', 'Not even one' etc. are to be reduced to E proposition.

Words	sentence	Ordinary sentences	logical forms
'No', 'Never', 'None', 'Not at all', etc.		None is dishonest	No men is dishonest-E
		The enemies are not at all reliable	No enemies are reliable.-E

6) Sentences containing the words like 'Only', 'Alone', 'None but', 'o one else but' are to be reduced to 'A' 'E' and 'I' propositions according to three different ways. Such sentences are called exclusive proposition.

A) Sentences are to be reduced to 'A' proposition by transposing the subject and the predicate. For example: Only commerce graduates are eligible for the post. Logical form: All those who are eligible for the post are commerce graduates.-A

B) Sentences are to be reduced to 'E' proposition by using the contradictory of the given subject as the subject in the proposition. For example: Only commerce graduates are eligible for the post. Logical form: No non- commerce graduates are eligible.

C) The another method is to reduce to 'I' proposition without any change of the terms.

For example: Only commerce graduates are eligible for the post. Logical form: Some commerce graduates are eligible for the post.

7) The sentences containing the words such as 'except', 'but', other

than make exception about the subject. If the exception is definitely stated the sentences are to be reduced to universal proposition and if it is indefinitely stated the sentences are to be reduced to particular proposition. For example: All candidates except one have been selected. Logical form: Some candidates are selected.-I

8) In singular proposition, where the subject is definite singular term is treated as universal and where it is indefinite singular term is treated as particular. For example:

Russell was a mathematician. Logical form: Russell is a man who was a mathematician-A. One player wins the game. Logical form: Some players are those who win the game. - I

8.12 OPPOSITION OF PROPOSITIONS

There is a particular type of logical relation between categorical propositions. The opposition of proposition states this relation. Thus 'opposition of proposition' may be defined as the logical relation between two categorical propositions having the same subject and predicate, but differing in quantity only, or in quality, or both in quality or in quantity. There are four forms of opposition of propositions:

1. Sub-alternation or Sub-altern opposition
2. Contrary opposition or Contrariety
3. Sub-contrary opposition or Sub-contrariety
4. Contradictory Opposition.

Sub-alternation or Sub-altern opposition: This kind of relation holds between two propositions having the same subject and predicate, and having the same quality but differing in quantity. For example;

All scholars are wise.-A

Some scholars are wise.-I

Contrary opposition: This kind of relation holds between two universal propositions having the same subject and predicate but differing in quality. Two propositions are contraries if they can not both be true, although they may both be false at the same time. In the Aristotelian square of opposition **A** and **E** statements are contraries. For example:

All philosophers are politician.-A

No philosophers are politician.-E

Sub-contrary opposition: This kind of relation holds between two particular propositions having the same subject and predicate but differing in quality. In this relation two propositions are related in such a way that they can not both be false at the same time, but they both can be true at the same time. In addition, if one is false then the other must be true. For example:

Some Americans are Indians-I

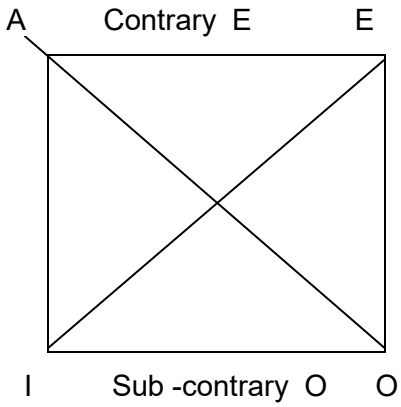
Some Americans are not Indians-O

Contradictory Opposition: This kind of relation holds between two propositions having the same subject and predicate but differing both in quantity and in quality. In the contradictory relation one proposition is the denial or negation of the other. Therefore, in this relation it is impossible for both the propositions to be true or both to be false at the same time. The categorical A and O proposition and E and I propositions are contradictories. For example: All logicians are mathematicians-A

Some logicians are not mathematicians.-O

The Square of Opposition

The square of opposition is a diagram that represents the four kinds of opposition of proposition. The diagram is as follows:



- Immediate inferences based on the Square of Opposition

This square of opposition has shown us that we obtain different combinations of truth and falsity between pairs of categorical propositions. This traditional Square of Opposition clearly provides a basis for validating

a number of immediate inferences. Thus if an A proposition is taken as premise then according to the Square of Opposition, we can validly infer that the corresponding O proposition having the same subject and predicate terms is false. Similarly from the same premise one can immediately infer that the corresponding I proposition is true. Thus the immediate inferences based on the Square of Opposition may be listed as follows:

A being given as true: E is false, I is true, O is false.

E being given as true: A is false, I is false, O is true.

I being given as true: E is false, while A and O are undetermined.

O being given as true: A is false, while E and I are undetermined

A being given as false: O is true, while E and I are undetermined

E being given as false: I is true, while A and O are undetermined

I being given as false: A is false, E is true, O is true.

O being given as false: A is true, E is false, I is true.

8.13 DRAWBACKS OF TRADITIONAL ANALYSIS AND CLASSIFICATION OF PROPOSITION

Modern logicians find some drawbacks of the traditional analysis and classification of proposition. For example:

- Modern logicians do not accept that all propositions are of the same logical form i.e. subject-copula-predicate. For example, subjectless proposition e.g. exclamatory and impersonal propositions. Moreover according to them this subject-predicate type of proposition is one of the forms of propositions and this form is distinguished from various kinds of other propositions. Further the subject and the predicate of a proposition are recognized as constituents by the modern logicians.
- According to modern logicians the simplification of proposition into strict logical form makes the proposition awkward, logically futile and practically absurd. Moreover, it makes obscure the fundamental logical distinction among propositions. For them every proposition cannot be meaningfully analyzed to strict logical form.
- According to the traditional logicians every proposition asserts a predication. But according to modern logicians, this is not true. For

example, 'Ram defeats Ravana'. This proposition asserts only relation between subjects; they do not attribute a characteristic to a subject. So the simplification of proposition into strict logical form makes it difficult to distinguish between relations and attribute.

- In the modern classification of proposition we find subjectless, class-membership, relational, singular, simple general etc. which are not thoroughly analyzed and separately discussed by traditional logicians. In traditional classification the four forms of proposition include all these types of proposition.
- The traditional notion of the distribution of terms derives from the insufficient analysis of propositions and that is why they consider singular and general propositions as equivalent and these propositions are categorized as 'A'. But modern logicians recognize the existential import singular proposition.



CHECK YOUR PROGRESS

Q 1: Describe the nature of copula.

.....

.....

Q 2: Define real proposition.

.....

.....

Q 3: Distinguish between categorical and conditional proposition.

.....

.....

Q 4: What is four-fold scheme of proposition?

.....

.....

Q 5: What do you mean by simplification of proposition?

.....

.....

Q 6: Find out the correct answer:

a) The copula of a proposition is a term/an attribute// none/a sign of relation

b) Universal and particular are the divisions of proposition under quality/quantity.

Q 7: Fill in the blanks

a) The subject term is distributed in___ proposition.

Q 8: Reduce the following sentences into proper logical form'

a) One should never drive rashly.



8.14 LET US SUM UP

- A proposition is the statement of a certain relation between two terms. Traditional logicians analyze proposition into constituent terms and the sign of relation connecting one term with another term.
- There are two criteria, on the basis of which proposition can be classified. One is the criteria of quality and the other is the criteria of quantity. It is possible to combine these two criteria. By combining them we can classify proposition as - universal affirmative (A), universal negative (E), particular affirmative (I) and particular negative (O).
- In traditional logic ordinary and grammatical sentences are reduced into one of these four categorical propositions. 'A' proposition distributes the subject, 'E' proposition distributes both the subject and the predicate, 'I' proposition distributes neither the subject nor the predicate and 'O' proposition distributes the predicate only.



8.15 FURTHER READING

1) Chakraborti, Chhanda: Logic. 2007 Informal, Symbolic & Inductive. Prentice Hall of India Pvt. Ltd., New Delhi,

2) Copi, I. M. & Cohen, Carl. 1997. Introduction to Logic. Prentice Hall of India Pvt. Ltd, New Delhi,

3) Stebbing, L. Susan . 1969 .A Modern Elementary Logic, Methuen & Co. Ltd., London,

4) Korde, Ashok & Ankush Sawant: 1983. Logi.c. Himalaya Publishing House, Bombay,



8.16 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No. 1: The copula is connecting word that stands as the sign of relation between the subject and the predicate term. The copula is not a term. Only the verb 'to be' is used as the copula. And it should be expressed in the present tense of the verb 'to be'. And it may be affirmative or negative.

Ans to Q No. 2: In a real proposition the predicate asserts an additional fact about the subject that is not contained in the connotation of the subject. For example: 'All men are mortal',

Ans to Q No. 3: A categorical proposition expresses a relation between the subject and the predicate categorically or unconditionally. For example, 'All men are mortal'. On the other hand in a conditional proposition the relation between the subject and the predicate is made under certain condition. For example, (a) 'If there is sun, there is light'.

Ans to Q No 4: According to the mixed principle of quantity and quality we have four kinds of categorical propositions viz. Universal Affirmative (A), Universal Negative (E), Particular Affirmative (I) and Particular Negative (O). This kind of classification is called four-fold scheme of proposition.

Ans to Q No. 5: The process of reducing the ordinary and grammatical sentences into the strict logical forms is called reduction or simplification of proposition.

Ans to Q No. 6. a) a sign of relation b) quantity.

Ans to Q No 7:. A

Ans to Q No.8: No person is one who should drive rashly.



8.17 MODEL QUESTIONS

A) Short questions

Q 1: Define proposition with example.

Q 2: What do you mean by copula?

Q 3: What do you mean by opposition of proposition?

Q 4: Give an example of universal affirmative proposition.

Q 5: Give an example of universal negative proposition.

B) Short questions (Answer in about 150 words)

Q 1: What do you mean by proposition? Explain briefly the different parts of a proposition.

Q 2: Classify proposition according to import. Explain each of them.

Q 3: How does a proposition differ from a judgment? Explain in brief

Q 4: What do you mean by the distribution of terms? Briefly explain

Q 5: What do you mean by simplification of proposition? Briefly explain the process of simplification of Exclusive Proposition.

Q 6: Write short notes on:

a) Square of proposition

b) Doctrine of Distribution of terms

c) Classification of proposition

d) Structure of proposition

Q 7: Briefly explain the structure of a proposition.

Q 8: Explain briefly the nature of copula.

Q 9: Do you find any distinction between proposition and sentence? Briefly explain

Q 10: How do the categorical propositions differ from classes? Briefly explain.

C) Long questions (Answer in about 300-500 words)

Q 1: Describe the traditional analysis of proposition

Q 2: Explain the doctrine of distribution of terms.

Q 3: What do you mean by opposition of propositions? Explain with suitable example.

Q 4: Explain the drawbacks of traditional analysis of propositions.

*** ***** ***

UNIT 9 : MODERN CLASSIFICATION OF PROPOSITION: SIMPLE AND COMPOUND, SINGULAR AND GENERAL

UNIT STRUCTURE

9.1 Leaning Objective

9.2 Introduction

9.3 Modern Analysis of Proposition.

9.4 Proposition and Sentence

9.5 Modern Classification of Proposition

9.6 Singular and General Proposition

9.7 Comparison Between Modern and Traditional Classification of Proposition

9.8 Let Us Sum Up

9.9 Further Reading

9.10 Answer to Check Your Progress

9.11 Model Questions

9.1 LEARNING OBJECTIVES

After going through this unit you will be able to

- explain the meaning of proposition as defined by modern logicians
- understand the modern analysis of proposition
- describe the traditional and modern classification of proposition
- acquaint with symbols used in modern logic.

9.2 INTRODUCTION

This unit introduces you to the modern analysis and classification of proposition. Modern logicians do not agree with the traditional definition of proposition. They define proposition as a statement which is either true or false. But it can not be both true and false. A proposition is not identical with sentence. Similarly a proposition may express a fact but it is not a fact.

Only indicative sentences may be judged to be either true or false. A proposition may have any number of constituents and these constituents are combined in various ways. The mode of combination expresses the logical form of proposition. Modern logicians have classified proposition into simple, compound and general.

9.3 MODERN ANALYSIS OF PROPOSITION

In modern logic proposition means a statement which is said to be true or false. For example, 'Asoka was the king of Pataliputra', 3+6=9, 'Socrates was a philosopher' 'No man is immortal'. Truth or falsity can be ascribed to every one of them. Every one of them can be either true or false. But 'May God help us' –is not a proposition; because the property's truth or falsity can not be ascribed to it. In modern logic the components of a proposition i.e. the subject and the predicate are known as 'constituents' of a proposition. **The truth or falsity of a proposition is called its truth value.** The truth-value of a proposition is determined by the fact. If a proposition represents a fact as it is, then the proposition is true; otherwise it is false. But a proposition is different from a fact.

9.4 PROPOSITION AND SENTENCE

There is a clear difference between a proposition and a sentence. Though a proposition is expressed in the form of a sentence, it is not identical with a sentence. We can point out following points as their differences,

- A proposition may be understood as that to which truth or falsity can be ascribed. But truth or falsity can not be ascribed to all kinds of sentences.
- All grammatical sentences are not propositions. Only indicative sentence can be judged to be either true or false. Interrogative sentences, Imperative sentences, Optative sentences can not be judged to be either true or false. For example, 'Have you ever gone to Calcutta'? 'May God help us!' Truth or falsity can not be ascribed to it. So they are not propositions.

- The same proposition may have different verbal expressions. Therefore the same proposition may be expressed by different sentences. But various sentences have a single underlying meaning which is called a proposition.

9.5 MODERN CLASSIFICATION OF PROPOSITION

The modern logicians have broadly classified the propositions into elementary proposition and non-elementary proposition. Elementary propositions are of two kinds viz. A) Simple and B) Compound propositions. Non-elementary propositions include all general propositions. Let us discuss all types of proposition.

A) Simple proposition: A simple proposition expresses a simple fact and it can not be analyzed into further propositions. Moreover a simple proposition makes an assertion about an individual, a person, a place, a thing, a country and so on. For example: Seema is doing a course in logic. Simple proposition have four forms: (1) Subjectless proposition (2) Subject-predicate proposition (3) Relational proposition (4) Class-membership proposition.

1. Subjectless proposition: The simplest kind of proposition is the subjectless proposition e.g. Fire! Thieves! These propositions are also known as the exclamatory propositions. The proposition such as: It rains, it thunders. etc. are known as impersonal propositions. Such propositions have no logical subject. In these propositions the thinker asserts something but the statement is not fully expressed. These propositions give information and therefore they are regarded as propositions.

2. Subject-predicate proposition: A proposition which asserts that a quality or an attribute belongs to something is called a subject-predicate proposition. e.g. 1. Shyam is intelligent. 2. This paper is white. Subject of this type proposition is a singular term. The subject-predicate type of proposition is represented simply as 'S-P'. In predicate logic small letters a, b, c, — are used to symbolize individuals, and capital letters A, B, C, — and are used to symbolize attributes. For example: Shyam

is intelligent. The symbolic expression of this proposition is –I

Here the symbols 'I' stands for intelligent and's' stands for Shyam.

3. **Relational proposition:** Relational proposition asserts a relation between two or more constituents. There are various ways to express relations. We may use the verbs –drink, love, enemy, hurt, or words greater than, smaller than etc. to express various relations. For example: the lecturer teaches a course in English, he owes me fifty rupees for a bag.

   A relational proposition may contain any number of constituents. According to the number of constituents the relations are called dyadic, triadic, tetradic, pentadic. Relations involving more than five terms are called polyadic relations.

   The constituents of a relational proposition are not the subjects and predicates but they are the terms of relations or subjects of relations. In a relational proposition the relation proceeds from something to something else. This is called the direction or sense of relation. The term from which the relation proceeds is called the *referent* and the term to which the relation proceeds is called the *relatum.* In the above example the term 'The lecturer' is a referent and the term, a course in English is relatum.

4. **Class-membership proposition:** Class-membership proposition asserts that an individual is a member of a class. For example: Rabindra Nath Tagore is a poet. This class-membership proposition is symbolically expressed as r ° P. Here 'r' stands for Rabindra Nath Tagore and 'P' stands for the class of poet. In symbolic logic capital letters A, B, C—are used to symbolize the class and small letters a, b, c, —are used to symbolize the individual.

B) **Compound proposition:** A compound proposition contains two or more simple propositions as its components. For example: Socrates was a philosopher and Russell was a mathematician. This proposition is a single proposition. Simple propositions are combined by various ways to form a compound proposition. According to the modes of combination we have four kinds of compound proposition. They are-

1. Conjunctive proposition
2. Disjunctive proposition
3. Alternative proposition
4. Implicative proposition

1. **Conjunctive proposition:** The compound proposition in which two or more simple propositions are combined by the word 'and' is known as conjunctive proposition. For example: Monmohan Singh is the Prime Minister of India and Barak Obama is the President of the U.S.A. This conjunctive proposition is symbolically expressed as p .q
   In this expression 'p' and 'q' stand for two simple propositions; and the symbol dot (.) stands for 'and'.
   The components of a conjunctive proposition are called 'conjuncts'.
   A conjunctive proposition is true if and only if both (or all) the conjuncts are true. The words such as 'but', 'although', 'while', 'yet' etc. are also used to combine the simple proposition.
2. **Disjunctive proposition:** The compound proposition in which two propositions are combined into one by the word 'or' (either-or) used in inclusive sense is called disjunctive proposition. For example: Either Logic is interesting subject or it is a scoring subject. The components of a disjunctive proposition are presented in the form of 'disjuncts'. This disjunctive proposition may be symbolically stated as p v q
   Here p and q stand for two disjuncts and the symbol 'v' (the Latin word 'vel' or 'wedge') stands for 'either-or' in its inclusive sense.
   A disjunctive proposition is true if and only if at least one of the disjuncts is true. In other words a disjunctive proposition is true in case of one or other or both disjuncts are true.
3. **Alternative proposition:** The compound proposition in which two propositions are combined into one by the word 'or' (either-or) used in exclusive sense is called alternative proposition. As for example, the man is either literate or illiterate. The components of an alternative proposition are presented in the form of Alternatives.
   This proposition may be symbolically stated as p Λ q

Here, p and q stand for two alternatives and the symbol Λ (the Latin word 'aut') stands for 'or' in its exclusive sense. The meaning of 'or' in its exclusive sense can be expressed as (p Λ q).- (p. q)

An alternative proposition is true only if one of the alternatives is false.

4. Implicative proposition: The compound proposition in which two simple propositions are combined by the word 'if-then' is called an implicative proposition. For example: 'If there is drought then the price of food grains will rise'.

The component proposition which follows 'if' is called the antecedent or the '*implicans*' and the component proposition which follows 'then' is called the 'consequent' or the '*implicate'*. In an implicative proposition one component proposition implies the other. The proposition which implies the other is 'implicans' and the proposition which is implied is 'implicate'.

In an implicate proposition the implicate logically follows from the implicant.

In other words, if the antecedent is true, then consequent is also true. It means (1) If antecedent is true then consequent is also true or it is not the case that the antecedent is true and the consequent is false.

C) General proposition

General propositions are non-compound propositions. They are about classes. Such propositions either affirm or deny (1) the existence of something or (2) a property of the whole universe or 3) the relation between two classes. So there are three kinds of general propositions. They are

1) Existential General Proposition

2) One-Predicate General Proposition.

3) General Propositions asserting relations between two classes.

1) Existential General Proposition

An existential proposition directly affirms or denies the existence of something. For example: a) Tiger exists.

b) There are no unicorns.

In predicate logic the first propositions can be symbolized by using existential quantifier as (3x) (Tx)

This proposition states that 'there is something which is a tiger'. The second proposition can be symbolized as ? (3x) (Ux) This proposition states that, 'it is not the case that there is something which is a unicorn'. The general form of affirmative and negative existential propositions may be obtained by using ô for the property. Thus we can express existential affirmative proposition as (Ýx) (ôx) and negative existential proposition as ?(Ýx) (ôx)

2) **One-Predicate General Proposition:** One-Predicate General Propositions either affirm or deny a property or an attribute about the whole universe. For example: a) everything is mortal

b) Nothing is permanent.

The first proposition is symbolized as (x) (Mx) and the second proposition is symbolized as (x) (?Px)

3) **General Propositions asserting relations between two classes:** This proposition states that one class is wholly or partly included in or excluded from another class. For example: a) All lions are animals.

b) No man is perfect

c) Some students are cleaver

d) Some philosophers are not scientists

The traditional categorical propositions i.e. A, E, I, O belong to this category. According to modern logicians both universal and particular propositions belong to general propositions. Modern logicians have tried to bring out the formal structure of general proposition by using quantifies. Thus we can symbolize the above examples as follows.

a) (x) (Lx ; ûAx)

It means 'Given any x, if x is lion, then x is mortal'.

b) (x) (Mx ; û ?Px)

It means if 'Given any x, if x is man, then x is not perfect'.

c) (Ýx) (Sx. Cx)

It means 'There is atleast one x such that x is student and x is cleaver'.

d (Ýx) (Px . ? Sx)

It means 'There is atleast one x such that x is philosopher and x is not student'.



LET US KNOW

Quantifiers are symbols used in predicate logic to signify the quantity of the General propositions. A quantifier is a specific collection of number, objects or persons. Two types of quantifiers are– (1) Universal Quantifier and (2) Existential Quantifier. Universal Quantifier is used to symbolize the universal proposition and Existential Quantifier is used to symbolize the particular proposition. The symbols for the universal quantifier are (x), (y), (z) which means, 'Given any x'. The symbol for the Existential quantifier is '(Ýx)' which means 'there is at least one x'.

A general proposition is different from a class membership proposition. A general proposition is a proposition about classes. It asserts a relation of inclusion or exclusion between two classes. But a class-membership proposition asserts an individual is a member of a class. A general proposition can be distinguished from a simple proposition. A simple proposition refers to a particular individual. A general proposition can also be distinguished from a compound proposition. A general proposition consists of a single statement but a compound proposition consists of two or more simple propositions.

9.6 SINGULAR AND GENERAL PROPOSITION

There is a clear difference between a singular proposition and a general proposition.

- A singular proposition asserts that the individual possesses a particular property. For example: 'Lata Mangeshkar is a singer', 'Sakuntala is beautiful'. But a general proposition asserts that some individuals or all individuals possess a certain attribute. For example: all men are mortal', 'Some flowers are beautiful'.
- A singular proposition is obtained from a propositional function by the process of instantiation. A singular proposition is a substitution instance of a propositional function. But a general proposition is obtained from

a propositional by the process of quantification or generalization.

- The process of symbolization of a singular proposition is different from that of a general proposition. In the symbolization of a singular proposition we use capital letters to symbolize the attributes or properties and small letters to symbolize the individual. For example; 'Socrates is wise' -this singular proposition can be expressed symbolically as 'Ws'

But in the symbolization of a general proposition quantifies are used. For example; 'Everything is perishable'- this proposition can be symbolically expressed by the use of universal quantifier as (x) Px



LET US KNOW

A propositional function is an expression which contains an individual variable and which becomes a proposition when the individual variable is replaced by an individual constant. Propositional functions are neither true nor false. For example; Px (x is a philosopher) is a propositional function. Here 'x' is an individual variable. When this individual variable is substituted by an individual constant 's' we get a singular proposition 'Ps' means 'Socrates is a philosopher'. The process of obtaining singular proposition from a propositional function is called instantiation. Quantification or generalization states that a propositional function is true for all or some of the values of the variables.

9.7 COMPARISON BETWEEN MODERN AND TRADITIONAL CLASSIFICATION OF PROPOSITION

In the light of above discussion we can bring out the following points on the comparison between modern and traditional classification of proposition.

- According to modern logicians there are innumerable kinds of propositions whereas traditional schedule is mainly confined to the four forms of the categorical propositions.

- According to modern logicians the subject and the predicate of a proposition are the constituents of proposition but according to the traditional logicians they are the constituents only.
- According to modern logicians there may be any number of constituents and they may be combined in various ways. But traditional analysis of proposition is limited to two constituents and one mode of combination.

The traditional logicians have completely ignored some kinds of propositions and others are confused. So traditional classification of proposition is based on imperfect analysis and therefore it is unscientific. But in comparison to it modern classification of proposition may be regarded as scientific.



ACTIVITY

- Do you find any relationship between the modern and the traditional classification of proposition? Discuss

Ans. .....

.....

.....

.....

.....



CHECK YOUR PROGRESS

Q 1: How do modern logicians define proposition?

.....

.....

Q 2: Analse the modern analysis of proposition

.....

.....

Q 3: What are the different kinds of proposition according to modern logic?

.....

.....

Q 4: Distinguish between Class-membership proposition and General proposition.

.....

.....

Q 5: Give example: a) Conjunctive proposition and b) Implicative proposition

.....

Q 6: Do you think the modern classification of proposition is more scientific than traditional classification of proposition?

.....

.....

Q 7: Symbolize the following sentences-

a) All existentialists are philosophers b) Socrates is a philosopher



9.8 LET US SUM UP

- Proposition is the basic unit of logical thinking. By 'Proposition' we mean any statement which must be either true or false, but which can not be both true and false. A proposition is expressed in the form of a sentence; but it is not identical with a sentence. Similarly the truth or falsity of a proposition is determined by the fact, but a proposition is different from a fact. Only indicative sentences can be regarded as propositions because they can be judged to be either true or false.
- Unlike traditional logicians modern logicians do not accept that every proposition must be expressed in the same logical form i.e. subject-copula -predicate. According to modern logicians, there are innumerable kinds of propositions. And a proposition may have any number of constituents and these constituents may be combined in various ways.
- According to modern logicians there are some drawbacks of the traditional classification of proposition. Though traditional logicians try to express all types of proposition in the same logical form,

fundamentally they are different in their logical structures. Therefore, there seems to arise some confusion in their classification of proposition. In order to get rid of the defects and limitations of traditional classification of proposition modern logicians attempt to present their classification scientifically.

- According to modern logicians there are three kinds of proposition- Simple, Compound and General. A simple proposition contains only one statement. A compound Proposition contains two or more simple proposition and General propositions are about classes.



9.9 FURTHER READING

1) Chakrabort, Chhanda: 2007. Logic- Informal, Symbolic & Inductive. Prentice Hall of India Pvt. Ltd., New Delhi,

2) Copi, I. M. & Cohen, Carl. 1997.Introduction to Logic, Prentice Hall of India Pvt. Ltd, New Delhi,

3) Stebbing, Susan L. 1969.A Modern Elementary Logic, Methuen & Co. Ltd., London

4) Korde, Ashok & Sawant, Ankush. 1983. Logic. Himalaya Publishing House, Bombay,



9.10 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: According to modern logicians a proposition is a statement which is either true or false. For example; Abdul Kalam is a scientist.

Ans to Q No 2: By the term proposition modern logicians meant a statement which is said to be true or false. The truth or falsity of a proposition is called its truth value. The truth-value of a proposition is determined by the fact. If a proposition represents a fact as it is, then the proposition is true; otherwise it is false. But a proposition is different from a fact. A

proposition is expressed in the form of a sentence; it is not identical with a sentence.

Ans to Q No 3: In modern logic, propositions are classified into: Simple, Compound and General. A simple proposition contains only one single statement. It can not be analyzed into other propositions. There are four forms of simple proposition. (1) Subject less proposition (2) Subject-predicate proposition (3) Relational proposition (4) Class-membership proposition.

A compound proposition contains two or more simple statements. There are four forms of compound proposition: Conjunctive proposition, Implicative proposition, Disjunctive proposition and Alternative proposition.

General propositions are non-compound propositions. They are about classes. Such propositions either affirm or deny (1) the existence of something or (2) a property of the whole universe or the relation between two classes. So there are three kinds of propositions. They are

1) Existential General Proposition

2) One-Predicate General Proposition.

3) General Propositions asserting relations between two classes.

Ans to Q No 4: A Class-membership proposition differs from a General proposition. A class-membership proposition asserts that an individual is a member of a class. For example; Russell is a mathematician. But a General proposition states about classes. It asserts a relation of inclusion and exclusion between two classes. For example; Some philosophers are scientists.

The process of symbolization of a Class-membership proposition is different from that of a general proposition. The above Class-membership proposition can be symbolically expressed as r € P But the above General proposition can be symbolically expressed as (?x) (Px. Sx)

Ans to Q No 5: a) Bertrand Russell was a mathematician and a philosopher

b) If there is tremendous rise in the population then the housing problem will become difficult.

Ans to Q No 6: It can be said that the modern classification of proposition is more scientific than traditional classification of proposition. In support of it we may put forward some reasons. In modern logic we find innumerable kinds of proposition with symbolic expression. By this they have clearly shown the differences of the logical structures of the propositions. But traditional schedule is mainly confined to the four forms of categorical proposition and they should have same logical form for which it becomes difficult to distinguish between quality and relation. Besides it the fundamental difference between various simple propositions and general propositions are not clearly shown in traditional logic.

Ans to Q No 6: a) (x)(Ex ? Px) b)s ? P



9.11 MODEL QUESTIONS

A) Very short questions

Q 1: What is simple proposition?

Q 2: What are the kinds of simple proposition?

Q 3: Define subject less proposition.

Q 4: Give an example of subject-predicate proposition.

Q 5: Define implicative proposition.

B) Short questions (Answer in about 150 words)

Q 1: Distinguish between simple and compound proposition

Q 2: Distinguish between sentence and proposition

Q 3: Distinguish between subject-predicate proposition and General proposition

Q 4: Distinguish between disjunctive proposition and alternative proposition

Q 5: Distinguish between singular and general proposition.

Q 6: Write a note on General proposition

Q 7: What is general proposition? Briefly explain

Q 8: Do you find any difference between singular and general proposition.

C) Long questions (Answer in about 300-500 words)

Q 1: Describe modern analysis of proposition.

Q 2: Explain modern classification of proposition with examples.

Q 3: Describe the different types of general proposition. Give examples with symbolic expression.

Q 4: Make a comparison between modern and traditional classification of proposition.

*** ***** ***

UNIT 10 : SQUARE OF OPPOSITION OF PROPOSITIONS

UNIT STRUCTURE

10.1 Learning Objectives

10.2 Introduction

10.3 Existential Import of A, E, I, and O Propositions

10.4 Traditional Analysis of Square of Opposition

10.5 Modern Analysis of Square of Opposition

10.6 Let Us Sum Up

10.7 Further Reading

10.8 Answers to Check Your Progress

10.9 Model Questions

10.1 LEARNING OBJECTIVES

After going through this unit you will be able to:

- explain the concept of existential import of A, E, I, and O propositions
- analyse the traditional analysis of square of opposition
- discuss modern analysis of square of opposition.

10.2 INTRODUCTION

This unit introduces to you the concept of square of opposition of propositions. It includes the concepts of existential import of A, E, I and O proposition, traditional analysis of square of opposition, and modern analysis of square of opposition.

10.3 EXISTENTIAL IMPORT OF A, E, I, AND O PROPOSITIONS

It indicates the existence of a certain object or thing that is implied by the subject of a given proposition. If the class designated by the subject of the given proposition has members, the proposition is said to have

existential import. George Boole, a distinguished logician, mentions that the universal propositions 'A' and 'E' do not have existential import. And he at the same time writes that the particular propositions 'I' and 'O' have existential import.

Universal propositions 'A' and 'E' are existentially negative in the sense that they cannot make existential claim of objects. In contrast to this, the particular propositions 'I' and 'O' can make existential claim of objects or entities designated by the subject term. Therefore, in modern logic the particular propositions are considered existential propositions. 'A' and 'E'propositions differ from 'I' and 'O' propositions in cases of generality and existential import.

'A' proposition that is "All men are mortal" holds the relation between 'men' and 'mortality'. It also can be paraphrased that if anything that is man, than it is mortal. Here, the meaning of the proposition does not depend on the assumption or presupposition that man exists. Rather, the meaning of the proposition depends on the relation between the subject and the predicate. Besides, the existence of the object is neither necessary nor important for the meaningfulness of a universal proposition. As for instance, the proposition "The present king of France is bald" is meaningful although there is no present king of France. It holds that if there is a present King of France, he would be bald.

The Universal negative proposition also indicates the mere existence of the object or entity which is not accepted. As for instance, "No man is immortal". This can be stated that if there is a man, he must be mortal. Again, this proposition can be logically represented that "whatever X may be, if X is man, then he is not immortal." The universal propositions "A" and "E" state the internal and inseparable relation between subject and predicate terms. The meaningfulness of "A" and "E" propositions do not depend on the existence of the objects indicated by the subject of the propositions. Rather the meaningfulness of the propositions like "A" and "E" depends on the inseparable relation between subject and predicate terms.

But, the nature of universal proposition is different from particular proposition. The proposition, "Some students are honest" is an example of

particular proposition. This proposition is meaningful only when we find honest students. This indicates that the meaningfulness of particular propositions. 'I' and 'O' propositions depends on the existence of the objects indicated by the subject of the propositions. The proposition "some students are intelligent" can be written as "there exists at least one student who is honest". Again, the proposition "some students are not honest" is an example of "O" proposition. It can be analyzed that there exists at least one student who is not honest.

From the above discussion, we can make a distinction between traditional account of A, E, I, and O proposition and existential import formulated by modern logic. The traditional fourfold propositions- A, E, I, and O propositions hold the view that a proposition is true when it describes the facts (make a relation between the subject and the object) and false when it fails to describe the fact (does not make a relation between the subject and the object). On the other hand in case of existential import, a proposition "All S is P" is true when there are no members in the class of subject term "S". Again, a proposition "No S is P" is true when the subject term "S" is empty and false when there are members in the subject term "S".

Particular propositions are considered true when there are members in the class of subject term. For instance, "Some S is P" is true when the class "S" has members, and false when the class "S" has no members. Similarly, particular negative propositions are also true when there are members in the class of subject term. And false when there are no members in the class of subject term. For instance, "Some S is not P" is true when the class 'S' has members and false when the class 'S' has no members.

The above discussion shows how the existential import of the categorical propositions influenced George Boole and John Venn's formulations of categorical propositions. Boole had formulated A, E, I, and O propositions in terms of empty class, on the other hand Venn had represented the four-fold propositions through diagrams. Through the discussion of John Venn and George Boole, it is clear that both the logicians agree with the point that universal propositions do not imply the existence of

members in the subject term class. On the other hand, particular propositions imply the existence of members in the subject term class. But, Aristotelian formulation of propositions presupposed that the classes represented by the terms of every categorical proposition have members.



CHECK YOUR PROGRESS

Q 1: State whether the following statements are True or False

a) Boole mentions that the universal proposition 'A' and 'E' do not have existential import. (True/False)

b) Particular propositions 'I' and 'O' have existential import. (True/ False)

c) 'No man is immortal' is an example of 'E' proposition. (True/ False)

d) The particular propositions are considered existential propositions. (True/ False)

Q 2: What is existential import? (Answer in about 80 words)

.....

.....

Q 3: Why are 'A' and 'E' propositions existentially negative? (Answer in about 40 words)

.....

Q 4: How can we make a distinction between traditional account of A, E, I, O proposition and existential import? (Answer in about 80 words)

.....

10.4 SQUARE OF OPPOSITION OF PROPOSITIONS: A TRADITIONAL ANALYSIS OF PROPOSITION

The four-fold categorical propositions- A, E, I and O are related to each other by various truth-relations. The various relations are of oppositions. The traditional logicians have explained the relationship of categorical propositions (A, E, I and O) through a diagram of square. Therefore, the

diagrammatic representation of the relationships of A, E, I and O propositions by a square are known as square of opposition.

Each side of the square and also the diagonals assert the various kinds of relationships among A, E, I and O propositions.

'A' proposition is known as Universal affirmative proposition. It is symbolized as "All S is P". And 'E' proposition is Universal negative proposition (E). It is symbolized as "No S is P". 'A' and 'E' propositions are related to each other by the contrary relation which we can understand through examples. The proposition "All writers are rich" is contrary to "No writers are rich". Similarly, "No men are im-mortal" is contrary to "All men are mortal".

'I' proposition is particular affirmative proposition. "Some students are genius". It is symbolically represented as "Some S is P". And O proposition is particular negative proposition. It can be symbolically presented as "Some S is not P". 'I' and 'O' propositions are related to each other by sub-contrary relation. This subcontrary relation of 'I' and 'O' proposition can be understood through examples. As for instance, the proposition- "Some men are honest" is contrary to the proposition of "Some men are not honest". Similarly, the proposition "Some men are not honest" is also subcontrary to "Some men are honest" and vice-versa.

The universal affirmative proposition, "All S is P" is related to particular negative proposition, "Some S is not P" by contradictory relation. Again, the proposition, "Some S is not P" is contradictory to the proposition "All S is P" and vice-versa. Similarly, the universal negative proposition(O), "No S is P" is related to I proposition, "Some S is P" by the contradictory relation.

The universal affirmative proposition, "All S is P" is related to Particular affirmative proposition, "Some S is P" by superaltern relation. Similarly, particular affirmative proposition, 'Some S is P' is related to universal affirmative proposition by superaltern relation.

The different relationships among A, E, I, and O propositions which can be shown through a diagrammatic relation by a square can be in such a way:

1. "All S is P" is contrary to "No S is P" and vice-versa

2. “Some S is P” is subcontrary to “Some S is not P” and vice-versa
3. (i) “All S is P” is contradictory to “Some S is not P” and vice-versa
   (ii) “No S is P” is contradictory to “Some S is P”and vice-versa
4. (i) “All S is P” is superaltern to “Some S is P”
   (ii) “No S is P” is superaltern to “Some S is not P”
   (iii) “Some S is P” is superaltern to “All S is P”
   (iv) “Some S is not P” is subaltern to “No S is P”

The categorical propositions are said to be opposites. They differ either in quality or quantity, or both in quantity and quality.

Contrary propositions “A” and “E” have same quantity but they differ in qualities. Similarly, subcontrary propositions “I” and “O” have same quantity but they differ in qualities. And propositions related to subaltern relationship like “A” and “I”, and also “E” and “O” have the same quality but they differ in quantities. Contradictory propositions “A” and “O”, and also “E” and “I” differ both in quantities and qualities.

Contrary propositions “A” and “E” cannot be both true together though they both can be false at the same time. If one of the contrary propositions is true, the other contrary proposition is necessarily true. While if one of the contrary propositions is false, the other contrary proposition cannot be determined to be true or false. It can be either true or false.

Subcontrary relationship shows that “I” and “O” propositions cannot both be false together though they both can be true together. If “I” is true, “O” is undetermined; where as if “I” proposition is false, “O” is necessarily true. Similarly, if “O” is true, “I” is undetermined, but if “O” is true, “I” is undetermined but if “O” is false, “I” is definitely true.

Subaltern relationship shows that if “A” is true, then “I” is necessarily true, but if “I” is true, “A” remains undetermined. The same case also happens with “E” and “O”. If “E” is true, “O” is true. Similarly, If “O” is true, “E” is undetermined.

Contradictory relation between “A” and “O” Propositions, and “E” and “I” is strictly opposite. If “A” is true, “O” is false; if “O” is true, “A” is false. Similarly, if “E” is true, “I” is false and if “I” is true, “E” is false.

These different relationships among A, E, I, and O propositions provide the foundation for testing the validity of arguments in logic.



CHECK YOUR PROGRESS

Q 5: What is square of opposition? (Answer in about 80 words)

.....

Q 6: Mention the different relationships among A,E,I, and O propositions

.....

Q 7: What are the different kinds of valid relationship among categorical propositions?

.....

Q 8: Give an example of A,E,I and O proposition.

.....

10.5 SQUARE OF OPPOSITION OF PROPOSITIONS: MODERN LOGICIANS'POINT OF VIEW

Modern logicians criticised the traditional account of square of proposition given by traditional logicians. They criticised the traditional square of opposition in the context of existential import. We have already understood from the previous section that (traditional account of proposition) universal propositions, "A" and "E" do not provide existential import, on the other hand particular propositions, "I" and "O" can provide existential import. This kind of view of traditional square of opposition has been criticised by the modern logic. Modern logicians view that only one kind of relationship of categorical propositions, A, E, I, and O is valid, that is contradictory relation of propositions. And other relations of propositions like contrary, sub-contrary, subaltern are invalid.

The existential import of the categorical propositions shows that the universal proposition, "All S is P" is true when the subject term "S" has no member. Though "S" is an empty term, still the Universal propositions, A and E are true. However, if the subject term has member, the universal propositions, "All S is P" (A) as well as "No S is P" (E) would have been false propositions.

The existential import also shows that particular proposition, “Some S is P” is true when the subject term has member. On the other hand, if the subject term “S” has no member, then the proposition “Some S is P” would be false. The same occurrence also happens in “O” proposition. If the subject term has members, the proposition “Some S is not P” would be true. While if the subject term has no members, the proposition “Some S is not P” would be false.

Now, we can examine the different relations of A, E, I, and O propositions in order to find out the differences between traditional and modern analysis of Square of Opposition in the context of existential import.

The contrary relation states that propositions “All S is P” and “No S is P” cannot be both true together though they both can be false together. But the propositions, “All houses made of wax are long lasting” and “No houses made of wax are long lasting” are true together although the subject term of these propositions is an empty class. (because there are no houses made of wax). If the subject term of A and E propositions are true together in spite of the empty class indicated by the subject term, the question naturally involves, how can they be contrary?

The subcontrary relation between “I” and “O” propositions also becomes invalid. The existential import of “I” and “O” propositions becomes true when the subject term of both the propositions has members. And the propositions “I” (Some S is P) and “O” (Some S is not P) are false when the subject term of both the propositions has no member. The sub-contrary relation holds that both “I” and “O”propositions can be true together, but they cannot be false together. But, if there are no members in the subject term of “I” and “O” propositions, both these propositions are false together. As for instance, “Some intelligent men are famous” and “Some intelligent men are not happy” are false together because the subject term “Some intelligent men” is an empty class. Therefore, the question involved- if both “I” and “O” are false together, how can there be subcontrary relation involved between “I” and “O” propositions?

The subaltern relation between “A” and “I” propositions, and between “E” and “O” propositions also have become invalid. The subaltern relation

shows that the truth of "A" proposition implies the truth of "I"proposition, and also the truth of "E" proposition implies the truth of "O" proposition. From the existential import, we cannot validly infer the truth of "I" proposition from the truth of "O" proposition. Again, we cannot infer the truth of "O" proposition from the truth of "O" proposition.

"A" and "E" propositions are existentially negative propositions, so, they cannot validly infer "I" and "O" propositions, which are existentially loaded. The deductive reasoning also shows that if the premises do not have existential import, they cannot imply the propositions. For instance, "A" and "E" propositions cannot imply the propositions like "I" and "O" propositions.



CHECK YOUR PROGRESS

Q 9: Write the name of the categorical propositions which indicate contradictory relation.

.....

Q 10: How do the modern logicians criticise the traditional square of opposition? (Answer in about 80 words)

.....



10.6 LET US SUM UP

- Existential import indicates the existence of a certain object or thing that is implied by the subject of a given proposition. If the class designated by the subject of the given proposition has members, the proposition is said to have existential import. George Boole, a distinguished logician, mentions that the universal propositions 'A' and 'E' do not have existential import. At the same time he writes that the particular propositions 'I' and 'O' have existential import.
- 'A' and 'E' propositions are existentially negative in the sense that they cannot make existential claim of objects. In contrast to this, the particular propositions 'I' and 'O' can make existential claim of objects

or entities designated by the subject term. Therefore, in modern logic the particular propositions are considered existential propositions. 'A' and 'E' propositions differ from 'I' and 'O' propositions in cases of generality and existential import.

- "Some students are honest" is an example of particular proposition. This proposition is meaningful only when we find honest students. This indicates the meaningfulness of particular propositions. 'I' and 'O' propositions depends on the existence of the objects indicated by the subject of the propositions.
- The traditional logicians have explained the relationship of categorical propositions (A, E, I and O) through a diagram of square. The relationship among A, E, I and O propositions was diagrammed by a square, therefore, these diagrammatic relationships by a square are known as square of opposition.
- Modern logicians criticized the traditional square of opposition in the context of existential import. According to the traditional account of proposition, universal propositions, "A" and "E" do not provide existential import, on the other hand, particular propositions, "I" and "O" can provide existential import. This kind of viewpoint of traditional square of opposition has been criticised by the modern logic. Modern logicians show that only one kind of relationship of categorical propositions (A, E, I, and O) is valid, that is contradictory relation of propositions. And other relations of propositions like contrary, sub-contrary, subaltern have become invalid.



10.7 FURTHER READING

1) Jain, Krishna. *A Textbook of Logic*. D.K. Printworld (P) New Delhi.

2) Hurley, Patrick J. *Introduction to Logic*. Wadsworth Cengage Learning.

10.8 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: a) True b) True c) True d) True

Ans to Q No 2: It indicates the existence of a certain object or thing that is implied by the subject of a given proposition. If the class designated by the subject of the given proposition has members, the proposition is said to have existential import. George Boole, a famous logician holds the view that the universal propositions 'A' and 'E' do not have existential import. He also writes that the particular propositions 'I' and 'O' have existential import.

Ans to Q No 3: 'A' and 'E' propositions are existentially negative in the sense that they cannot make existential claim of objects. These propositions do not indicate the existence of a certain object or thing that is implied by the subject of a given proposition.

Ans to Q No 4: According to the traditional four-fold propositions (A, E, I and O), a proposition is true when it describes the facts and false when it fails to describe the fact. On the other hand in case of existential import, a proposition is true when there are no members in the subject term 'S'. Again, a proposition is true when the subject term 'S' is true and false when there are members in the subject term 'S'.

Ans to Q No 5: The traditional logicians have explained the relationship of categorical propositions (A, E, I, and O) through a diagram of square. Therefore, these diagrammatic relationship of A, E, I, and O propositions represented by a square are known as square of opposition.

Ans to Q No 6: Contradictory relation

Ans to Q No 7: A: All men are mortal, E: No men are immortal, I: Some men are rich, O: Some men are not happy

Ans to Q No 8: 'A' and 'E' propositions, and 'E' and 'I' proposition

Ans to Q No 9: The modern logicians criticise the traditional square of opposition in the context of existential import. As for instance, according

to the modern logicians, 'A' and 'E' propositions do not provide existential import. On the other hand, particular propositions, 'I' and 'O' can provide existential import. Modern logicians view that only one kind of relationship of categorical propositions (A, E, I, and O) is valid, that is contradictory relation. And other relations like contrary, sub-contrary, subaltern relation are invalid.



10.9 MODEL QUESTIONS

A. Very short Questions

Q 1: What do you mean by existential import?

Q 2: Give an example of A, E, I, and O proposition.

Q 3: Define square of opposition.

Q 4: What is sub-contrary relation?

Q 5: Identify the two categorical propositions which indicate contrary relation.

Q 6: Identify the two categorical propositions which indicate sub-contrary relation.

Q 7: Identify the two categorical propositions which indicate contradictory relation.

Q 8: Identify the two categorical propositions which indicate sub-altern relation.

B. Short Questions (Answer each question in about 150 words)

Q 1: Does the universal proposition make claim of existential import? Write briefly.

Q 2: Does the particular proposition make claim of existential import? Answer briefly.

Q 3: Write a short note on existential import.

Q 4: Do you find any difference between universal and particular proposition from the point of nature? Answer briefly

Q 5: Write a brief note on Square of opposition.

Q 6: Distinguish between traditional and modern analysis of opposition

D. Long Questions (Answer each question in about 300-500 words)

Q 1: Explain the concept of existential import.

Q 2: What is square of opposition? Explain the traditional point of view.

Q 3: How do the modern logicians criticize the traditional square of opposition? Explain

Q 4: What are the valid kinds of relationship among categorical propositions, according to Modern logic? Explain with examples.

Q 5: Critically explain the different relationships of A, E, I, and O propositions.

Q 6: Explain the different relationships among A, E, I, and O proposition from the traditional point of view.

Q 5: How do the modern logicians criticize the contrary, sub-contrary, and subaltern relation formulated by traditional logicians? Explain

*** ***** ***

UNIT 11 : PROPOSITIONAL LOGIC AND PREDICAE LOGIC

UNIT STRUCTURE

11.1 Learning Objectives

11.2 Introduction

11.3 Branches of Symbolic Logic

11.4 Development of Propositional Logic and Predicate Logic

11.5 Propositional Logic

11.6 Forms of Propositional Logic

11.7 Sentence, Proposition and Statement

11.8 Simple and Compound Proposition

11.9 Logical Constant, Propositional Variable and Propositional Constant

11.10 Truth Function

11.11 Decision Procedure

11.12 Formal Proof of Validity

11.13 Predicate Logic

11.14 Need of Predicate Logic

11.15 Singular and General Proposition

11.16 Propositional Function

11.17 Quantifiers

11.18 Rules of Inference

11.19 Difference Between Propositional Logic and Predicate Logic

11.20 Let Us Sum Up

11.21 Further Reading

11.22 Answers to Check Your Progress

11.23 Model Questions

11.1 LEARNING OBJECTIVES

After going through this unit you will be able to:

- define propositional logic

- define predicate logic
- list symbols used in modern logic
- analyse 'what decision procedure is?'
- explain the methods for proving the validity of arguments adopted both in propositional logic and predicate logic.

11.2 INTRODUCTION

This unit introduces to you the two important branches of symbolic logic viz propositional logic and predicate logic. Propositional logic is the logic of propositions. It is that section of symbolic logic which considers simple statements or propositions as its atomic or the most basic units. This logic does not look into the sub-structures of simple statements. The Simple statements form the foundation of the language of propositional logic. There are different forms of propositional logic. This unit will primarily deal with **truth-functional propositional logic** that limits itself to the study of truth-functional operators.

Predicate logic on the other hand deals with the sub-structures of both simple and compound propositions. This logic has its own grammar to formulate statements which are considered well-formed in this logic. Predicate logic is also known as **quantifier logic.**

The three basic components of symbolic logic are: **syntax, semantics and proof system**. This unit introduces to you only the elementary concepts that fall in the syntax of propositional logic and predicate logic.

11.3 BRANCHES OF SYMBOLIC LOGIC

In the history of western logic, Symbolic logic is a relatively recent development of Aristotelian logic which has strong connection with mathematics. It has been developing till today. Symbolic logic is known as 'mathematical logic', 'formal logic', 'modern logic', 'algebra of logic', 'logistic' etc. Symbolic logic has three distinct branches. These are propositional logic, predicate logic and logic of classes.

Propositional logic is the logic of propositions. It considers simple statements as it's atomic or the most basic or elementary units. This logic assumes simple propositions as indivisible units.

Predicate logic is the logic of predicates or properties, and things or objects to which the predicates may be ascribed. This kind of logic deals with the internal structure of both the simple and compound propositions.

Logic of classes deals with the classes and their relationship which form the content of the categorical propositions or statements. Categorical statements are statements which assert something about the classes. Aristotelian formal logic is exclusively concerned with these kind of statements and with arguments viz. syllogistic arguments that are formed out of these statements. But the modern approach of syllogistic reasoning is somewhat different from Aristotelian syllogism. John Venn gave a modern approach to syllogistic reasoning representing them in a diagram what is called Venn diagram. The modern logicians interpret syllogistic reasoning in terms of the inter-relationship among three classes.

11.4 DEVELOPMENT OF PROPOSITIONAL LOGIC AND PREDICATE LOGIC

The ancient Greek philosopher Aristotle is regarded as the father of logic in the western tradition. The early Megarians and the Stoic philosophers in the late 3rd century B.C. had developed some basic principles of propositional logic. They attempted to study the connectives such as negation, conjunction, disjunction and conditionals. Greek philosopher Chrysippus (280-206 B.C.), who was one of the founders of the Stoic logic developed a logic in which the fundamental elements were whole propositions. He advanced a kind of propositional logic, by marking out a number of different ways of forming complex premises for arguments. This system was also studied by medieval logicians. However propositional logic really came to fruition only in the mid-19th Century, with the advent of Symbolic Logic in the works of logicians such as Augustus DeMorgan, George Boole and Gottlob Frege. George Boole rediscovered this logic and formulated it

like a calculus. Boole showed that the algebraic formulae can be used to express logical relations with the connectives, such as conjunction, negation, disjunction and conditionals. From Boolean Algebra, modern logic has received the theory of truth functions, the disjunctive normal forms and conjunctive normal forms.

Modal propositional logics are the most widely studied form of non-truth-functional propositional logic. The first systematic inquiry into this modal propositional logic can be found in the work of C. I. Lewis in 1912 and 1913. Among other well-known forms of non-truth-functional propositional logic, **deontic logic** began with the work of Ernst Mally in 1926, **and epistemic logic** was first treated systematically by Jaakko Hintikka in the early 1960s. The modern study of three-valued propositional logic began in the work of Jan Łukasiewicz in 1917**. Relevance propositional logic** is relatively more recent; dating from the mid-1970s in the work of A. R. Anderson and N. D. Belnap. **Paraconsistent logic**, while having its roots in the work of Łukasiewicz and others, has blossomed into an independent area of research only recently, mainly due to work undertaken by N. C. A. da Costa, Graham Priest and others in the 1970s and 1980s.

Predicate Logic was initially developed by Gottlob Frege and Charles Peirce in the late 19th Century. Charles Sanders Peirce, an American philosopher, devised a very precise, almost mathematical notation system for relational arguments. But it reached full fruition in the Logical Atomism of Whitehead and Russell in the 20th Century.



LET US KNOW

Propositional logic considers simple statements as its atomic or the most basic units. The part of language which includes the symbols and the formal rules by which well-formed sequences of symbols are to be formed is known as the **syntax** of the language.The part of language which tells us how to interpret or assign meaning to the well-formed sequences of symbols is known as the **semantics** of the language.

11.5 PROPOSITIONAL LOGIC

Propositional logic is that branch of symbolic logic which considers simple statements as its atomic or the most basic units. It studies ways of combining or altering statements or propositions to form more complicated statements or propositions. This logic is also known as ***sentential logic*** or ***sentential calculus*** or ***statement logic***. Sometimes, it has been called propositional calculus by those who are keen to emphasise the analogy to mathematical calculus. We can bring out the following points that may mark propositional logic in some way.

- Propositional logic is about statements. It considers simple statements as its atomic or the most basic or elementary units. This logic assumes that simple propositions as indivisible units. That is, in this kind of logic simple propositions are not further analysable. Thus, propositional logic does not deal with the internal structure or the sub-structure of simple propositions.
- The simple propositions form the **foundation of the language** of propositional logic. Thus simple propositions are supposed to be the starting point of propositional logic. Compound propositions are formed by combing the simple propositions with the help of connectives.
- Propositional logic can be thought of as primarily the study of **sentential connectives or logical operators**. A logical operator is any word or phrase used either to modify one statement to make a different statement, or join multiple statements together to form a more complicated statement. In English, words such as “and”, “or”, “not”, “if ... then...”, “because”, and “necessarily”, are all used as operators. It is therefore worth mentioning that in propositional logic two kinds of symbols namely, (1) **logical constant** or logical operator and (2) **propositional variable** are used to form compound statements.
- Propositional logic also studies **way of modifying** statements, such as the addition of the word “not” that is used to change an affirmative statement into a negative statement.
- Propositional Logic studies the **ways of joining** propositions to form

more complex propositions or statements as well as the **logical relationships and properties** that are derived from these methods of combining or altering statements**.**

- Propositional logic has its own symbolization key or procedure. In this procedure each simple statement is to be replaced by the chosen capital letter and hence a unique capital letter for each distinct simple statement has to be selected. Secondly, logical connective is to be replaced by its respective symbol. For example, the conjunctive proposition- 'Radhakrishnan is a philosopher and Rabindranath Tagore is a poet' can be symbolically expressed as:

  R • T.

- Instead of numerical values, propositional logic deals with truth values. It is based upon only **two truth values**, namely, truth and falsity. Hence, propositional logic is a **two-valued logic** or a **bi-valued system.** All its statements are either true or false.
- Propositional logic is based on the notion of **truth function**. There are five basic truth functions in the propositional logic. They are:
  1) Negative or Contradictory Function
  2) Conjunctive Function
  3) Disjunctive Function
  4) Implicative or Conditional Function
  5. Equivalent or Biconditional Function

Apart from these basic truth functions propositional logic deals with other truth functions. One of them is **Stroke function.**

- Propositional logic is concerned exclusively with **truth-functional compounds** and thus non-truth-functional compounds do not fall within its scope.
- Propositional logic is mainly concerned with the different methods called **decision procedures** to test the validity of arguments and argument forms and also to examine whether statement or statement forms are tautologous or not. Some of the decision procedures are: Truth table method, Shorter truth table method, Conjunctive Normal Form (CNF), Disjunctive Normal Form (DNF) and Truth tree method.

- Propositional logic adopts a method of proof which is known as **Formal proof of validity**. It is also known as **Formal Derivation** with an aim to establish the validity of arguments.



CHECK YOUR PROGRESS

Q 1: What is propositional logic?

.....

Q 2: State one mark of propositional logic

.....

Q 3: What are the symbols used in propositional logic to form a compound statement?

Q 4: State whether statements are True or Falsea) Propositional logic is about statements / argumentsb) Logical constant/ propositional variable connects simple propositions.

11.6 FORMS OF PROPOSITIONAL LOGIC

Classical truth-functional propositional logic is the most widely studied branch of truth-functional propositional logic which assumes that a statement can have two possible truth-values viz. (1) truth, and (2) falsity that is, every statement is either true or false but not both. Truth-functional propositional logic limits itself to the study of truth-functional operators.

The most well known form of non-truth-functional propositional logic is **modal propositional logic**. Modal propositional logic involves introducing operators into the logic involving necessity and possibility, usually along with truth-functional operators. Typically, the sign '&' is used in place of the English operator, "it is necessary that...", and the sign 'Ç%' is used in place of the English operator "it is possible that...". In modal propositional logic, the truth-value of a statement does not depend entirely on the truth-values of the parts.

Deontic propositional logic and **epistemic propositional logic** are two other forms of non-truth-functional propositional logic. The former involves introduction of operators similar to the English operators "it is morally obligatory that..." and "it is morally permissible that...". Again, the truth-value

of a statement in deontic logic does not depend wholly on the truth-value of the parts. Epistemic logic involves the addition of operators similar to the English operators “it is known that...” and “it is believed that ...”.

Yet another widely studied form of non-truth-functional propositional logic is **relevance propositional logic**, which involves the addition of an operator ‘Rel’ used to connect two statements α and β to form a statement ‘Rel(α, β)’, which is interpreted to mean that α is related to β in theme or subject matter. There are other forms of propositional logic such as **Intuitionist propositional** logic, and Paraconsistent propositional logic.

11.7 SENTENCE, PROPOSITION AND STATEMENT

A sentence is a set of words arranged in a grammatically correct manner to form a meaningful expression. In other words, a sentence is a word or a group of words conveying some meaning. For example, ‘Sita is pure’ ‘Come here Ã’, ‘Are you alright?’

‘Sentence’ represents a broad category which includes many different types of sentences. For example, it covers indicative sentences, interrogative sentences, optative sentences, imperative sentences and exclamatory sentences.

A proposition is an expression which can be either true or false, but cannot be both true and false. Cohen and Nagel defined proposition as anything which can be said to be true or false. For example, ‘Lachit Barphukan was brave’, ‘Earth is a planet’. A proposition is the basic unit of logical thinking. The logical proposition has its nearest approach in a sentence indicative. So, **it is not true that every sentence is a proposition.**

A statement is understood as what is said in a declarative statement. In Encyclopaedia of Philosophy, a statement is defined as a declarative sentence, or part of a **sentence**, that is capable of having a truth-value, such as being true or false. So, **every kind of sentence cannot be proposition or a statement**. It is worth mentioning that **statements are close to the propositions.**

11.8 SIMPLE AND COMPOUND PROPOSITION

The fundamental elements of propositional logic are whole statements. Statements are represented by letters, and these letters are combined by means of the operators to form more complex symbolic representation. To understand the symbolic representation used in propositional logic, we must distinguish simple statements from compound statements.

A simple statement is one that does not contain any other statement as a component. Simple statements are structurally simple. For example, 'Some Indians are scientists', Fast foods tend to be unhealthy', 'Descartes is a mathematician' etc. Any convenient uppercase letter may be selected to represent each statement. Thus*, S* might be selected to represent the first*, F* the second and *D* the third.

A compound statement is one which contains more than one simple proposition as its component. For example, (i) 'Mita is rich and smart', (ii) 'Either Radhakrishnan is a philosopher or a politician'. Using letters (M, S, R, and P) for the simple statements, these compound expressions may be represented as follows:

(i) 'Mita is rich and smart' *M* and *S*.

(ii) Either Radhakrishnan is a philosopher or a politician' Either *R* or *P.*

Compound statements can be of two types, namely truth-functional compound and non-truth-functional compound. A compound statement is truth functional if and only if its truth value is entirely determined by the truth values of its simple or atomic components. Truth functional compounds are formed by truth functional connectives, such as conjunction, negation, disjunction and implication. For example, 'Earth is a planet and it is round' – is a truth-functional compound statement because the truth value of this compound statement depends on its simple components 'Earth is a planet' and 'it is round'. On the other hand a compound statement is a non-truth-functional compound if and only if its truth value depends on further information than the truth values of its components. For example, 'Some

people believe that the pyramids of Egypt are built by aliens'. Its truth value further depends on a psychological fact of some people.

It is worth mentioning that all truth functional statements are compound statements but all compound statements are not truth functional. Propositional logic is exclusively concerned with truth functional compounds not with the non -truth functional compounds.



LET US KNOW

- It is not true that every sentence is a proposition or a statement.
- Statements are close to the propositions.

11.9 LOGICAL CONSTANT, PROPOSITIONAL VARIABLE AND PROPOSITIONAL CONSTANT

Logical constants are those words which are used to connect simple propositions to produce compound propositions and which express the form or structure of the compound propositions. They connect statements. Hence, they are also called the connectives. Propositional logic uses five basic logical constants or connectives. They are- not, and, either-or, if-then, if and only if. The logical constants are represented by their respective symbols. The three connectives 'not', 'and' and 'or' are also known as Boolean operators or Boolean connectives, because these constants were first used by George Boole as a part his system of logic.

Propositional variables or ***statement variables*** are those letters which can stand for any simple proposition as a whole. Lower case letters of the English alphabet p, q, r, s, t etc. stand for simple or atomic proposition. These letters are referred to as propositional variables. For example:

If p then q

P

Therefore, q

This argument can be symbolically represented in the following way:

P f! q

P

. . q

Here, p and q are variables because they can be replaced by any simple proposition within a definite range. In the above argument form the variable 'p' can be replaced by the proposition, 'If Sun rises' and 'q' can be replaced by the proposition 'there will be light'. Again the same propositional variable, 'p' and 'q' may be replaced by 'x is a triangle', and 'x has three sides' respectively.

Propositional variables are thus place-holder symbols. They do not have any fixed reference. Their reference can vary. They are called variables as they can be replaced by various actual statements.

Propositional constants or ***Statement constants*** are the appropriate capital letters of the English alphabet which are used to refer to specific simple statements in a given context. For example, we may use the capital letter E to represent 'Earth is a planet'. If used in the same context E should not also represent 'Earth is the third planet in the solar system'. Some other capital letter such as M may be used to represent the statement 'Earth is the third planet in the solar system'. So, the capital letters such as A, B, C etc. …are referred to as statement constants. Statement constants have a fixed reference in a given context. They will represent exactly one sentence and no more. Statement constants have therefore a unique reference to a specific simple statement in a given context while statement variable has non-specific reference to statement.

STATEMENT FORMS: A statement form is a sequence of statement variables such that, when the statement variables are substituted by actual statement- the same statement variable being replaced by the same actual statement throughout - the result is an actual statement. The statements, "Either the papers are difficult or the students are not intelligent", "Either the train is late or the watch is fast" can be symbolised as 'D V S' and 'T V W' Both of these expressions are of the form p v q. In statement forms all its content is set aside. Several actual statements may share the same statement form. Actual statement is therefore obtained by properly substituting the variables by constants, what are called the substitution instance of the statement form.



ACTIVITY

- Do you find any distinction between simple proposition and compound proposition? Discuss

Ans: ……..……..……..……..

- Do you find any distinction between logical constant and propositional variable? discuss

Ans: ……..……

- Distinguish between statement variable and statement constant.

Ans: ……..……

11.10 TRUTH FUNCTION

Truth function is a basic concept in propositional logic. It is mathematical in origin. In propositional logic a truth function can be defined as any compound expression whose truth value is completely determined by the truth value of its constituent simple propositions. For example, 'Rina is rich and smart'. This expression may be symbolically expressed as follows:

'p • q'.

It is a truth function because its truth value is determined by the truth value of its variable 'p' and 'q'. The following are basic truth functions in propositional logic.

	Truth function	**Symbolic representation**
a)	Negative or Contradictory function	4p
b)	Conjunctive function	p • q
c)	Disjunctive function	p v q
d)	Implicative or Conditional function	p f! q
e)	Equivalence or Biconditional function	p a" q

Propositional logic deals with other type of truth functions apart from these basic truth functions. One of these truth functions is **stroke function.** In propositional logic truth functions are interdefinable, that means, logical constants or truth functions can be defined and reduced into each other. For example, a conjunction can be defined in terms of other constants such as '4', '•', 'v', 'f!', and 'a"'. Similarly, the conjunctive function can be defined

in terms of the contradictory and the implicative functions, such as

i) (p • q)= df 4 (4p v 4 q)

ii) (p • q)= df 4 (p ƒ! 4 q)

Defining and reducing the truth functions or constants into one another is called '**interdefinability of truth function'** or **'interdefinability of constants'**.

11.11 DECISION PROCEDURE

Decision procedures are those methods used in propositional logic to determine the validity or invalidity of arguments and argument forms and also examine whether statements or statement forms are tautologous or not. The following are the fundamental decision procedures that propositional logic mainly deals with.

A) Truth table method: A truth table can be defined as tabular representation in which all possible truth values of a compound statement are calculated through the calculation of the truth values of their simple constituent propositions. In propositional logic truth table method is considered as the most basic and important decision procedure. This method is also known as matrix method.

Truth table method is useful in various ways. By this method we can determine the nature and logical status of the compound statements. That is a truth table can conclusively establish whether a given statement is tautology or contradictory or contingent. Secondly, it can show that one function is equivalent to another function. That means the inter definitions of logical constants or functions can be shown through this method. Thirdly, truth tables are useful for determining the validity or invalidity of argument and argument forms.

B) Shorter truth table method: A shorter truth table, as its name suggests, is a truth table in its short form. In propositional logic shorter truth table method is constructed to establish directly the invalidity or indirectly the validity of an argument or argument form. In this method such a technique is used to establish the invalidity of an argument that there exists at least one possible truth-value assignment on which the premises are all true

while the conclusion is false.

Indirect truth tables provide a shorter and faster method for testing the validity or invalidity of arguments than the ordinary truth table method. This method is convenient in case of arguments that contain a large number of simple statements. For example, an argument containing five different simple propositions would require thirty two lines in truth table method. On the other hand, instead of drawing thirty two lines which is a time and labour consuming process, the indirect truth table method can test the validity or invalidity of an argument effectively only by a single row. Shorter truth table method is also known as 'Indirect Truth Table Method' because in this method we start with an assumption that the given argument is invalid. If this assumption results in a contradiction then the given argument is proved to be valid. That means our assumption is wrong and thus the given argument is valid. On the other hand if this assumption does not result in contradiction, then the given argument is proved to be invalid. As such, shorter truth table method is a 'direct proof of invalidity' and an 'indirect proof of validity'. This method is also known as the method of 'Reductio ad Absurdum'.

C) Conjunctive Normal Form (CNF): The Method of Reduction to the Conjunctive Normal Form is an important method for determining the truth value of complex logical expressions.

A statement form in which we find a conjunction of two or more disjunctions each of which consists of a variable followed by its negation is known as Conjunctive normal Form. In other words, given a propositional formula A, we say that it is in Conjunctive Normal Form (CNF) if it is a conjunction of disjunctions of literals. **A literal** is a sentence letter or the negation of a sentence letter. That is, a literal is an atomic or simple statement or its negation, such as A or 4A.

In logic, the disjunction of a variable with its negation is logically true. Thus, 'p v ~ p' is logically true. It is tautology. Again the conjunction of two true statements is logically true. It follows that the conjunction (p v~p). (q v ~ q) is logically true. Further, the addition of a variable or a function to a logically true expression is logically true. Thus, the disjunction of 'q' to (p v~ p), that is ' (p v ~p) v q is a tautology. In the Process of Reduction to

the Conjunctive Normal Form a given expression is shown to be a tautology by reducing it to a conjunction of tautologies by employing a few reference formulae and also interdefinitions of functions.

D) Disjunctive Normal Form (DNF) : A formula is in disjunctive normal form if it is a disjunctive of conjunctions of literals.

E) Truth tree method: Like the truth tables and unlike the method of proofs, the truth tree method provides a mechanical decision procedure for the validity and invalidity of any sentential argument. Truth trees provide a kind of picture or representation of truth conditions.

The truth trees can be applied to both propositional logic and predicate logic. But in predicate logic, truth trees lose their effectiveness somewhat while in the domain of propositional logic, the effectiveness of truth trees is unquestionable.

11.12 FORMAL PROOF OF VALIDITY

Propositional logic adopts a deductive proof procedure that establishes the validity of an argument by deducing the logical conclusion from given premises by means of a small number of valid argument forms and logical equivalences which are considered as rules. In this process conclusion is formally derived from the premises to prove that the given argument is valid. This proof procedure is also known as 'Formal Derivation' or 'Natural Deduction' or 'Formal Proof of Validity' or Method of Deduction' or 'Derivation by substitution'.



CHECK YOUR PROGRESS

Q 5: How do you define a proposition in modern logic?

.....

Q 6: What is simple proposition?

.....

Q 7: What is compound proposition?

.....

Q 8: What is truth functional compound proposition?

.....

Q 9: What are the basic logical constants or connectives?

.....

Q 10: What are the basic truth functions?

.....

Q 11: What do you mean by decision procedure?

.....

Q 12: State whether the following statements true (T) or false (F)

a) Truth- functions are interdefinable.

b) Truth table method is a decision procedure.

c) The symbolic expression of conjunctive function is pvq

d) The symbolic expression of disjunctive function is p • q

Q 13: Choose the correct answer

a) The symbol 'a"' is a logical constant /propositional variable.

b) The small letters such as p, q, r, s, t etc. stand for propositional variable / propositional constant.

c) The capital letters such as A, B, C etc. stand for statement constants / logical constants

d) There are five /four basic truth functions.

11.13 PREDICATE LOGIC

Predicate logic is that branch of symbolic logic which deals with the internal structure of both the simple and compound propositions. It is concerned with predicates or properties, and things or objects to which the predicates may be ascribed. This logic is also known as **Predicate calculus.** In this chapter we are mainly concerned with the **First Order Predicate logic.** We can bring out some important points about predicate logic in the following way.

- Predicate logic is the logic of properties and things to which the predicates may be ascribed.
- Predicate logic deals with the internal structure of both the simple and compound propositions.

- Predicate logic contains the entire propositional logic. Hence, it concerns with the symbols used in propositional logic.
- This logic has a set of new vocabulary and a language of its own. It has its own grammar to formulate statements which are considered well-formed in this logic.

11.14 NEED OF PREDICATE LOGIC

Propositional logic sometimes fails to determine the validity of most valid arguments. In such cases the necessity of predicate logic arises as it provides us with the method for proving the validity of a number of valid arguments which are invalid when symbolized in the notation of propositional logic. Predicate logic thus deals with those arguments which contain propositions, whose structures are different from that of the compound or truth functional propositions. For example;

All scientists are atheists.

Russell is a scientist

Therefore Russell is atheist.

This argument is obviously valid. Using statement constant we can symbolise the argument in the notation of propositional logic as follows:

1. P

2. Q / $R

This argument form is obviously invalid. Propositional logic is unable to prove the validity of this argument form. These statements have different inner structures. In order to prove the validity of this argument form we have to depend on predicate logic that provides a symbolic system which can bring out the internal structure of the proposition.

11.15 SINGULAR AND GENERAL PROPOSITION

A **singular proposition is** one which states that a particular individual possesses or does not possess a particular property or attribute. Singular proposition is the very simplest kind of proposition. For example: 'Sita is pure', 'Kalidasa is a poet'. On the other hand a **general proposition**

asserts that some individuals (it may be even one individual but not a particular individual) or all individuals possess or do not possess a certain property. For example: 'All men are honest', 'Some flowers are beautiful', 'No man is perfect', 'A teacher is learned'.

INDIVIDUAL CONSTANT, INDIVIDUAL VARIABLE AND PREDICATES: The small or lowercase letters from 'a' through 'w' which we use to denote particular individuals are known as **individual constants**. Their function is to denote one, unique individual or object from the domain of discourse. Since their reference remains fixed or constant within a given context, they are called individual constants. The singular terms are represented in predicate logic by the individual constants. Usually, an individual or entity is referred to by the first letter of the name. For example; we may use r to denote Radhakrishnan's***, s*** to denote Socrates, ***d*** to denote Delhi and so on.

On the other hand small letters towards the end of the alphabets such as x, y, z which we use to denote any individual are known as **individual variables.** They stand for objects or individuals in the domain. Individual variables are by nature quite different from the constant. A variable is a place maker; it is not like a name. They refer to some unspecified individuals. For example: 'x' is an individual variable in the expression 'Hx' because 'x' refers to an arbitrary entity within a specified class.

Predicate logic as its name suggests, is concerned with predicates. **Predicates** are words or expressions that we use to refer to properties or attributes that things have. For example, 'Socrates is wise' –in this statement 'wise' is a predicate term. In predicate logic capital letters such as 'A', 'B' are used to symbolise properties. These letters are known as **predicate letters or predicate symbols.**

Symbolization key: In order to symbolise a singular proposition we need two symbols- one for the individual and the other for the attribute. Then we write the predicate symbol before the individual constant. For example: 'Socrates is wise' can be symbolised as 'Ws' where 'W' stands for 'wise' and 's' stands for 'Socrates'.



LET US KNOW

- **Predicate** logic **is** also known as first-order logic, First –order predicate calculus, lower predicate calculus and quantification theory.
- **First order predicate logic:** First Order Predicate logic is the most elementary kind of Predicate logic. In it, only simpler predications such as properties of individuals or objects are considered.
- **Second Order Predicate Logic**: In the Second Order Predicate Logic, there can be complicated predications where we consider the properties of properties and quantity of properties.
- **In higher order logic** we can have variables standing for properties such as 'F' or 'G' etc. and property constants.

11.16 PROPOSITIONAL FUNCTION

A propositional function is an expression which contains individual variables, such that, when the individual variable is replaced by individual constant, the expression becomes a proposition. For example, the expression 'x is human' (Hx) is a propositional function. If the individual variable, 'x' is replaced by the individual constant 's' (Socrates) then the expression becomes a singular proposition -'Socrates is human' (Hs). Thus, a singular proposition is a **substitution instance** of a proposition function. Propositional function is neither true nor false, but a proposition is either true or false.

A Propositional function that contains only one variable is known as **monadic** or **one-place** propositional function, but which contains more than one name is known as **polyadic** or many-place functions.

There are two ways of obtaining propositions from propositional functions. One is called ***Instantiation or Substitution*** and other is called ***Quantification*** or ***Generalization***.

Instantiation: Instantiation is the process of obtaining a singular proposition from a propositional function by replacing the individual variable

by an individual constant. For example: The singular proposition, 'Socrates is human' (Hs), is obtained from the propositional function 'Hx' by replacing the individual variable 'x' by the individual constant 's' (Socrates).

***Quantification*:** Quantification is the process of obtaining a general proposition from a propositional function by prefixing the quantifier to the propositional function. In other words, asserting or denying a propositional function for 'all' or 'some' of the values of the individual variable is called quantification. That is, quantification is the process of adding appropriate quantifier to a propositional function to bring out the formal structure of general proposition.

11.17 QUANTIFIERS

Quantifiers are expressions that state how many of the individuals or objects have the property in question. Thus the terms signifying quantity such as 'all', 'some', 'every', 'none' are known as quantifiers. In predicate logic only two kinds of quantifier symbols are used to signify quantity: Universal Quantifier and Existential Quantifier.

The expression "given any x", or "for all x" or "for any x" or "for every x" is known as **universal quantifier** and is symbolised as (x). It is used to assert that all entities have a certain property or properties. Thus it yields a universal proposition when prefixed to a propositional function. Hence, to symbolise the statements containing the words 'all', 'every', 'each', 'any', their equivalent expression universal quantifier is to be used. For example: The general proposition 'Everything is movable' can be symbolised by using universal quantifier as: (x) Mx

It can be expressed as:

Given any individual thing whatever, it is movable or Given any x, x is movable.

The phrase 'given any x' or 'for all x' or 'for any x' or 'for every x' is known universal quantifier.

The expression "there is at least one x" is known as **existential quantifier** and is symbolised as (Žx). It is used to assert that some entities or at least one entity have a given property. Thus, the statement 'Something

is solid' or the statement 'at least one thing is solid' is to be symbolised by as using existential quantifier as: (Žx) Sx

Symbolisation of traditional categorical proposition: The four standard form categorical propositions i.e. A, E, I, and O can be symbolised in predicate logic as follows:

A-	All S is P	(x) (Sx f! Px)
E-	No S is P	(x) (Sx f! 4 Px)
	I-Some S is P	(Žx) (Sx •Px)
O-	Some S is not P	(Žx) (Sx • 4 Px)



ACTIVITY

- Do you find any utility of predicate logic over propositional logic?

  Ans : ……

- How do you distinguish a proposition from a propositional function?

Ans : ……..

11.18 RULES OF INFERENCE

In addition to all the rules of inference and replacement, adopted in propositional logic four rules namely Universal Instantiation (UI), Universal Generalization (UG), Existential Instantiation (EI) and Existential Generalization (EG) are required to determine the validity of arguments in predicate logic. These four rules are for introducing and eliminating quantifiers. The rule of universal instantiation allows us to eliminate the universal quantifier while universal generalization allows us to introduce universal quantifier. Similarly, the rule of existential instantiation allows us to eliminate the existential quantifier while existential generalization allows us to introduce the existential quantifier.



CHECK YOUR PROGRESS

Q 14: What is predicate logic?

.....

Q 15: Write a short note on propositional function.

.....

Q 16: Define a) Singular proposition b) General proposition c) Quantifier d) Instantiation e) Quantification

Q 17: State whether the following statements are true (T) or false (F)

a) Predicate logic deals with the internal structure of both the simple and compound propositions.

b) Predicate logic contains the entire propositional logic.

c) 'Some flowers are beautiful'- It is a singular proposition.

d) 'Anil is sincere'- It is a general proposition.

11.19 DIFFERENCE BETWEEN PROPOSITIONAL LOGIC AND PREDICATE LOGIC

There is no sharp distinction between propositional logic and predicate logic; predicate logic is a transition from the logic of unanalysed proposition to the logic of predicates. Predicate logic contains the entire propositional logic but it goes beyond it. Predicate logic includes propositional logic in the sense that, predicate calculus is based on propositional calculus. Secondly, it uses the same proof procedure namely Formal Derivation and all the inference and replacement rules employed in this method to demonstrate the validity of arguments. But their difference lies on the following points.

- Propositional logic considers simple proposition as its atomic or most basic unit. This logic assumes that simple propositions are not further analysable. On the other hand predicate logic deals with the sub-structure of both simple and compound propositions. That is, in predicate logic simple propositions are to be further analysed.

- While propositional logic deals with simple declarative statements, predicate logic additionally covers predicate and quantification.
- To demonstrate the validity of arguments, the proof procedure namely the Method of Deduction and all the derivation rules are used both in propositional logic and predicate logic. But predicate logic requires additional four rules of inference to introduce and eliminate the quantifiers while conducting derivation. These four additional inference rules are: Universal Instantiation (UI), Universal Generalization (UG), Existential Instantiation (EI) and Existential Generalization (EG).
- The symbols which stand for logical connectives are used both in propositional logic and predicate logic. But predicate logic additionally uses quantifier symbols to signify the quantity of the propositions.
- Both propositional logic and predicate have their own symbolization key. For example, the statements:
  'Anita is good' can be symbolised in propositional logic as 'P' whereas in predicate logic as it would be- 'Ga'.
  Similarly, 'Anita is good and Bipul is honest' can be symbolically put in propositional logic as: 'P •Q' whereas in predicate logic as it would be- 'Ga •Hb'.
  Again the statement 'All philosophers are scientists' can be symbolised in propositional logic as: 'P' whereas in predicate logic as:
  (x) (Px f! Sx)
- Predicate logic deals with those forms of argument which are beyond the scope of the propositional calculus.



LET US KNOW

- The word 'quantification' was introduced by Charles S. Peirce.
- Variables are of two types: Free variable and Bound variable. An unquantified variable is a free variable and a quantified variable is a bound variable.



11.20 LET US SUM UP

- Propositional logic considers simple statements as its atomic or the most basic units.
- Simple propositions are supposed to be the starting point of propositional logic.
- Compound propositions are formed by combining the simple propositions with the help of connectives.
- In propositional logic two kinds of symbols namely, (1) logical constant or logical operator and (2) propositional variable are used to form compound statement.
- Propositional logic determines the validity of arguments and argument forms and also examines whether statements or statement forms are tautologous or not by means of Decision procedures.
- Predicate logic deals with the internal structure of both the simple and compound propositions. It concerns with predicates or properties, and things or objects to which the predicates may be ascribed. Predicate logic additionally uses quantifier symbols viz. universal quantifier and existential quantifier and four inference rules namely Universal Instantiation (UI), Universal Generalization (UG), Existential Instantiation (EI) and Existential Generalization (EG).
- There is no sharp distinction between propositional logic and predicate logic; Predicate logic is a transition from the logic of unanalysed proposition to the logic of predicates. Predicate logic contains the entire propositional logic but it goes beyond it.



11.21 FURTHER READING

1) Chakraborti. C., Logic-Informal, Symbolic & Inductive

2) Basson. A. H. and O'Corner, D.J., Introduction to symbolic logic

3) Hurley. J. P., Introduction to Logic
4) Baronett. S. and Sen. M., Logic
5) Copi. I.M. & Cohen. C., Introduction to Logic
6) Sharma. B. and Deka. J., A text Book of logic
7) http://www.iep.utm.edu/prop-log/;InternetEncyclopaedia of Philosophy



11.22 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: Propositional logic is the logic of propositions. It considers simple statements as its atomic or the most basic or elementary units. This logic assumes simple propositions as indivisible units.

Ans to Q No 2: One of the significant marks of propositional logic is that it primarily studies sentential connectives or logical operators.

Ans to Q No 3: In propositional logic two kinds of symbols namely, (1) logical constant or logical operator such as "and", "or", "not", "if ... then...", and (2) propositional variable are used to form compound statement.

Ans to Q No 4: a) Propositional logic is about statements.
b) Logical constant connects simple propositions.

Ans to Q No 5: A proposition is an expression which can be either true or false, but cannot be both true and false. For example: 'Kolkata is the capital of West Bengal'.

Ans to Q No 6: A simple statement is one that does not contain any other statement as a component. Simple statements are structurally simple statements. For example, 'Some Indians are scientists', 'New Delhi is the capital of India'.

Ans to Q No 7: A compound statement is one which contains more than one simple proposition as its component. For example, 'Mita is rich and smart', 'Either Radhakrishnan is a philosopher or a politician'.

Ans to Q No 8: A compound statement is truth functional if and only if its truth value is entirely determined by the truth values of its simple or

atomic components. Truth functional compounds are formed by truth functional connectives, such as conjunction, negation, disjunction and implication. For example, 'Earth is a planet and it is round' - is a truth-functional compound statement because the truth value of this compound statement depends on its simple components 'Earth is a planet' and 'it is round'.

Ans to Q No 9: Propositional logic uses five basic logical constants or connectives. They are -not, and, either-or, if-then, if and only if.

Ans to Q No 10: The basic truth functions are: Negative or Contradictory function (?p), Conjunctive function (p o q), Disjunctive function (p v q), Implicative or Conditional function (p ? q),Equivalence or Biconditional function (p ? q)

Ans to Q No 11: Decision procedures are those methods used in propositional logic to determine the validity or invalidity of arguments and argument forms and also examine whether statements or statements forms are tautologous or not.

Ans to Q No 12: a) True b) True c) False d) False.

Ans to Q No 13: a) The symbol '?' is a logical constant.

b) The small letters such as p, q, r, s, t etc. stand for propositional variable.

c) The capital letters such as A, B, C etc. stand for statement constants.

d) There are five basic truth functions.

Ans to Q No 14: Predicate logic is that branch of symbolic logic which deals with the internal structure of both the simple and compound propositions. It is concerned with predicates or properties, and things or objects to which the predicates may be ascribed.

Ans to Q No 15: A propositional function is an expression which contains one or more individual variables, such that, when the individual variable is replaced by individual constant, the expression becomes a proposition. For example, the expression 'x is human' (Hx) is a propositional functions. If the individual variable, 'x' is replaced by the individual constant 's' (Socrates) then the expression becomes singular proposition 'Socrates is human' (Hs).

Ans to Q No 16: a) A singular proposition is one which states that a particular individual possesses or does not possess a particular property or attribute. For example: 'Sita is pure'.

b) A general proposition asserts that some individuals or all individuals possess a certain property. For example: 'All men are honest'.

c) Quantifiers are expressions that state how many of the individuals or objects have the property.

d) Instantiation is the process of obtaining a singular proposition from a propositional function by replacing the individual variable by an individual constant.

e) Quantification: Quantification is the process of obtaining a general proposition from a propositional function by prefixing the quantifier to the propositional function.

Ans to Q No 17: a) True b) True c) False d) false



11.23 MODEL QUESTIONS

A) Very Short questions

Q 1: What is propositional logic?

Q 2: What are the basic components of symbolic logic?

Q 3: Mention the three branches of symbolic logic.

Q 4: Define predicate logic.

Q 5: Who is considered the father of symbolic logic?

Q 6: What is Venn diagram?

Q 7: Define compound proposition with example.

B) Short questions

Q 1: Write a note on propositional logic.

Q 2: What are the five basic truth functions in the propositional logic?

Q 3: What is truth function? Briefly explain.

Q 4: What do you mean by quantifiers? Explain in brief.

Q 5: Distinguish between propositional logic and predicate logic.

Q 6: Write a short note on truth function.

C) Long questions (Answer each question in about 300-500 words)

Q 1: What is propositional logic? Explain.

Q 2: What is simple proposition? Explain with example.

Q 3: What is decision procedure? Describe.

Q 4: What is predicate logic? Explain.

Q 5: Do you find any difference between propositional logic and predicate logic? Explain.

*** ***** ***

UNIT 12 : IDEA OF QUANTIFICATION

UNIT STRUCTURE

12.1 Learning Objectives

12.2 Introduction

12.3 Propositional Logic and Predicate Logic

12.4 Singular and general Propositions

12.5 Propositional Function

12.6 Quantification: What it is

12.6.1 Types of Quantifiers

12.7 Quantification of Four-Fold Classification of Propositions

12.7.1 Square of Opposition

12.8 Proving Validity and Invalidity: Rules of Quantification

12.9 Let Us Sum Up

12.10 Further Reading

12.11 Answers to Check Your Progress

12.12 Model Questions

12.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- explain 'what is quantification'
- discuss simple and compound proposition
- define propositional function
- identify quantifiers and kinds of quantifiers
- describe four-fold classification of proposition by quantification
- discuss the rules of quantification.

12.2 INTRODUCTION

This unit introduces to you to the quantification theory of predicate logic. Predicate logic is dealt with the symbolization of both simple and compound propositions. This logic is also concerned with the internal logical structure of both simple and compound propositions. Predicate logic

introduces special symbols for quantifying. That is why this logic is known as predicate calculus or predicate logic or quantifier logic. Predicate logic provides the technique of symbolization of the logical structure of simple propositions which is different from the symbolic structure of compound propositions. It also gives us the technique or method for determining the validity or invalidity of arguments.

This unit also provides us the informations about quantification, kinds of quantifiers, Symbolizations of A, E, I, and 0 through quantifiers and rules of quantification theory from which we determine the validity of invalidity of arguments respectively.

12.3 PROPOSITIONAL LOGIC AND PREDICATE LOGIC

The two important branches of symbolic logic are- propositional logic and predicate logic. In Propositional logic simple or atomic proposition is considered as the unit of logical thinking. This kind of logic does not analyse the internal structure of the simple proposition. It deals only with the internal structure of compound propositions. So, propositional logic is also known as the truth-functional logic. But predicate logic deals with the internal structure of both simple and compound propositions. This logic is also known as 'quantifier logic' as it introduces special symbolic for quantifying.

Predicate logic provides us the technique for proving the valid arguments which are invalid when symbolized in the notation of propositional logic. For example, the following argument is a valid argument. It is the BARBARA mood of first figure.

All men are mortal

All poets are man

All poets are mortal

This argument has to be symbolized in propositional logic as-

1. A
2. B / C

In this notation it appears to be invalid. This symbolic expression shows that the conclusion 'C' does not follow from the premises as there is nothing common between the premises. Premises together do not imply

the conclusion. But its validity can be shown by using the technique of symbolization adopted by predicate logic.

However, like propositional logic predicate logic follows the rules of inference and the rules of replacement provided by the method of deduction.

12.4 SINGULAR AND GENERAL PROPOSITIONS

There is a clear difference between singular and general proposition.

A singular proposition asserts that a particular individual or object has a particular property or an attribute. For example,

Socrates is a human,

Bombay is a big city,

Sita is beautiful etc.

Singular proposition is not identical with a simple proposition. Of course a singular proposition is non-compound and being a non-compound it is included within simple propositions. On the other hand a general proposition states that not a particular but some or all individuals possess or do not possess a particular property. For example:

For example: Everything is movable

Something is solid

All men are animals

Some students are not honest etc.

General propositions are concerned with the relation of agreement and disagreement of classes. The traditional categorical proposition namely, universal and particular come under the category of general proposition.

The technique for symbolizing singular proposition is different from that of general proposition. To symbolize a singular proposition two symbols are needed. They are (a) individual and (b) attribute. We use the first letter of the name of the individual to denote it. Thus we denote Socrates by ‘s’, Bombay by ‘b’ and ‘Sita’ by ‘s’ and so on. These symbols are called individual constants.

We use capital letters to denote attributes. We generally use first letter of the attribute to denote it. Thus, we denote human by ‘H’, port by ‘P’, beautiful by ‘B’, and so on.

Having symbolized the individual and attribute in this way we write the predicate or attribute symbol before the individual symbol. As for instance, the proposition "Sita is beautiful' can be symbolized as 'Bs'. Similarly,

1. Amit is honest (Ha)
2. Bipul is good (Gb)
3. Either Amit is honest or Bipul is Good (Ha V Gb)

On the other hand general propositions are to be symbolized by the employment of appropriate quantifiers.

Now it becomes clear that the subject of singular proposition is singular while the subjects of general propositions are class names. Moreover the quantifying word 'All' and 'Some' are present at the beginning of the general propositions.

Further, both singular and general propositions are obtained from 'propositional function' but the process is different in both the cases.

12.5 PROPOSITIONAL FUNCTION

In order to understand the nature of a singular and a general proposition we are to know what a propositional function is. Quantification theory is directly related with the notion of propositional function.

A propositional function is a symbolic expression which contains individual variable. For example, Hx, Px are propositional functions. Here 'X' is an individual variable. This individual variable can be replaced by an individual constant which results a singular proposition. A propositional function is neither true nor false, but a proposition is either true or false.

Any singular proposition is a substitution instance of a propositional function. As for example, 'Hx' (x is human) is propositional function. Here the individual variable 'X' may be substituted by the individual constant 's' (Socrates). In that case the proposition will be 'Socrates is human'. The symbolic expression of this proposition is 'Hs'. It is a singular proposition. Singular proposition is obtained from propositional function. Thus the process or technique of obtaining a singular proposition from a propositional function by replacing an individual variable is known as instantiation.

But the process of obtaining a general proposition from propositional function is somewhat different. In case of general proposition it is quantification or generalization.



CHECK YOUR PROGRESS

Q 1: State whether the following statements are true or false

a) Propositional logic is known as the truth functional logic.

b) Propositional logic deals with the internal structure of compound propositions.

c) 'Hx' is an example of propositional function.

Q 2: What is propositional function? (Answer in about 15 words)

Q 3: What is instantiation? (Answer in about 40 words)

12.6 QUANTIFICATION: WHAT IT IS

Instantiation and quantification are two processes of converting propositional function into proposition. Through the process of instantiation propositional function is converted to singular proposition. While through the process of quantification propositional function is converted to general proposition.

Thus quantification is a process or method or technique for obtaining general proposition from propositional function through the employment of quantifiers. As for example, 'Mx' (X is moving) is a propositional function. By prefixing a quantifier we may have the following general proposition.

(X) Mx (Everything is moving). It means for all values of X, or given any X, X is moving. Thus, asserting or denying a propositional function for all or some of the values of the individual variable is quantification.

Quantification is a method or technique for symbolizing general proposition by considering their inner logical structure. By this technique appropriate quantifiers are added to bring out the formal nature of general propositions.

12.6.1 Types of Quantifiers

Quantifier is a specific collection of number, objects and persons. So, quantifiers are the symbols used in predicate logic to signify the quantity (i.e., how many) of the general propositions.

There are two types of quantifiers:

- Universal quantifier
- Existential quantifier

Universal quantifier: Universal quantifier is the symbol which is used to assert that all entities have a certain property or properties. The symbols for universal quantifier are (X), (Y), (Z). They are used to signify the quantity of 'All' or 'Every'. As for example, the general proposition, 'Everything is moving ' can be symbolized by prefixing universal quantifiers as - (X) Mx. It is to be read as "For all x, Mx", or "Given any x, Mx" or "Given any x, x is moving". Similarly,

Everything id perfect-(X) Px

Everything is illusive —(X) Ix

All Humans are mortal ——(X) (Hx ƒ" Mx)

Existential quantifier: Another quantifier is existential quantifier. The symbol for the existential quantifier is (x). It is used to assert that 'some entities' or 'at least one entity' have a certain property. As for instance - 'something is solid' can be symbolized by using existential quantifier as- (x) Sx. It is to be read as 'there is atleast one x such that, x is solid' or for some x, x is solid. Similarly,

'Something is moving' — (x) Sx

Ghost exists ——(x) Gx (It means that there is something which is Ghost.) Some men are honest- (x) (Mx.Hx). The phrase 'there is at least one x such that 'is called an existential quantifier and is symbolized as (x). In predicate logic brackets are used to avoid ambiguities. The brackets indicate the scope of the quantifier.

Variables are of two types. They are free and bound variable. An unquantified variable is free variable e.g., in the sentence form 'Mx', x is free variable. On the other hand quantified variable is bound variable. For example, in (X) Mx, 'X' is bound variable.



CHECK YOUR PROGRESS

Q 4: State whether the following statements are true or false:

a) (X) is used to asset that 'some entities' or 'at least one entity' have a certain property.

b) (x) is used to assert that all entities have a certain property.

c) Variables are two types, free and bound variable.

d) There are three types of quantifiers.

Q 5: What is quantification? (Answer in about 20 words)

Q 6: What are the two types of quantifiers?



ACTIVITY: 12.1

Is quantification theory the replacement of the formal proof of validity?

Discuss

Ans: .....

.....

.....

.....

12.7 QUANTIFICATION OF FOUR-FOLD CLASSIFICATION OF PROPOSITIONS

We can symbolize the traditional four-fold types of propositions by using the existential and universal quantifiers in predicate logic. The four fold types of traditional subject-predicate propositions are:

- Universal affirmative (A)
- Universal negative (E)
- Particular affirmative (I)
- Particular negative (O)

These four-fold propositions can be shown by examples and can be symbolized by quantifiers:

All humans are mortal (X) (Hx ∫" Mx)

No humans are mortal (X) (Hx ∫" ~Mx)

Some humans are mortal (x) (Hx. Mx)

Some humans are not mortal (x) (Hx. ~Mx)

Now, we can analyse A, E, I and O propositions by the quantification theory.

- Quantification of A Proposition

  A. All humans are mortal. This proposition can be analysed that- Given any individual thing, whatever, if it is human, then it is mortal. This expression can be symbolically formulated as, given any X, If X is human then X is mortal. Replacing the horse-shoe symbol for 'if-then' we get: Given any X, X is human X is mortal. This symbolic expression can be finally formulated as: (X) (Hx ∫" Px)

- Quantification of E proposition

  E. No humans are mortal. This proposition can be analysed that-Given any individual thing, whatever, if it is human, then it is not mortal. This expression can be symbolically formulated as, Given any x, if x is human, then x is not mortal.

  Substituting the horseshoe symbol ' ' for 'if-then', we can express the symbolic form of sentence as such- Given any x, x is human x is not mortal. Finally, this symbolic expression can be symbolized as (X) (Hx ∫" ~Mx)

- Quantification of I proposition

  I. Some humans are mortal. This proposition can be asserted that there is atleast one thing that is human and mortal. This expression can be analysed as there is atleast one x such that x is human and x is mortal. Substituting the dot symbol '.' for 'and' we can symbolize the proposition as: there is atleast one x such that x is human. X is mortal. Using the existential quantifier (x) we finally symbolize the proposition as: (x) (Hx. Mx)

- Quantification of O proposition

  Some humans are not mortal. This proposition can be paraphrased as there is atleast one thing that is a human and not mortal. Substituting 'x' for humans in the sentence form we can analyse the propositions

as - there is atleast one x such that 'x' is a human . 'x' is not mortal. Using quantifiers and propositional function we finally symbolize the proposition as (x) (Hx . ~Mx)

12.7.1 Square of Opposition

The general connection between universal and existential quantification can be shown in terms of square array which is known as square of opposition.

We use the Greek letters Phi, Psi etc. to stand for any predicate symbol whatsoever. The four fold traditional forms of propositions can be symbolized as:

Contraries

(X) (ϕ x) (X) (~ϕ x)

Sub-alternation Sub-alternation

($\square$ x) ($\phi \in$x) ($\square$ x) (~ϕx)

Sub-contraries

We can explain the square of opposition in such a way:

A. The two top propositions are contraries. Because they might both be false but both cannot be true.
B. The two bottom propositions are sub-contraries, because they might both be true but cannot both be false.
C. Propositions that are at opposite ends of the diagonals are contradictories.
D. On each side of the square, the truth of the lower proposition is implied by the truth of the proposition directly above it.

12.8 PROVING VALIDITY AND INVALIDITY: RULES OF QUANTIFICATION

To test the validity of arguments in predicate logic, rules of inference and rules of replacement of the method of deduction are applied. In addition to the two sets of rules of propositional logic, four rules of quantification are

also used to test the validity of arguments in predicate logic.

They are respectively: i) Universal instantiation

ii) Universal generalization

iii) Existential instantiation

iv) Existential generalizations

Now, we can apply these rules to test the validity of arguments in predicate logic.

Rules of universal quantifiers:

Universal instantiation: This rule states that, if a proposition is true for all individuals or everything is true for an individual or any arbitrarily specified thing. This rule permits us to infer validly any substitution instance from a true universal quantification. Symbolically it can be expressed as— If (X) Mx then Ma, where a is individual instance or as

(X) Mx

Ma where a is any individual. As for instance true. This rule is denoted as UI. By using this rule we can prove the validity of the following argument.

All men are rational

Hari is a man

Hari is rational

This argument can be symbolized and proved as follows.

1. (X) (Mx Rx)
2. Mh

Rh

We can now apply the rules of inference and the rules of replacement to test the validity of argument.

3. Mh Rh 1 U.I
4. Rh 2, 3 M.P

Universal generalization: The second rule of universal quantifier is universal generalization. This rule states that if a proposition is true for any arbitrarily selected individual then it is true for all individuals. This rule is denoted as (U.G). This rule permits us to infer

Fy ……

(X) Fx

Here Y is any arbitrarily selected individual. By this rule of propositional logic we can make a formal proof of validly of the following argument.

As for instance:

All human beings are mortal

All rational beings are human

All rational beings are mortal

We can symbolize this argument using quantifiers and variables.

1. (X) (Hx Mx)
2. (X) (Rx Hx)/ (X) (Rx Mx)
3. Ha Ma 1. U.I
4. Ra Ma 2. U.I
5. Ra Ma 4,3 H.S
6. (X) (Rx Mx) U.G

Rules of Existential Quantifiers:

Existential instantiation: This rule states from the existential quantification of a propositional function we may infer the truth of its substitution instance. This substitution contains individual constant that occurs nowhere earlier in that context. For example, according to this principle, if 'some businessman are honest' is true then we can infer from the premise that there is atleast one individual who is a businessman is also honest.

This rule allows us to infer— Fx ___

Fa

Where a is an individual constant. This rule is denoted as E.I.

Existential generalization: This rule states that from any true substitution instance of a propositional function we may validly infer the existential instantiation of that propositional function. Thus this rule permits us to infer

Fa ___

(X) Fx Where a is any individual. In other words this rule states that if a proposition is true for one individual then there exists at least one individual for which the proposition is true. This rule is denoted as E.G. Thus we can prove the validity of the following argument.

No honest men are perfect

Some industrialists are honest man

Some industrialists are not perfect

We symbolize this argument using existential and universal quantifiers.

1. (X) (Hx ~Px)
2. (X) (Ix. Hx) /

∴ (x) (Ix. ~Px)

3. Ia. Ha	2. E.I
4. Ha ~Pa	1. U.I
5. Ha	3 Simp
6. ~Pa	4, 5 M.P
7. Ga	3 Simp
8. Ga. ~Pa	6, 7 Conjunction
9. (x) (Gx. ~ Px)	8 E.G



CHECK YOUR PROGRESS

Q 7: What are the traditional fourfold types of propositions?

Q 8: Symbolize '0'proposition using the quantification technique.

Q 9: What are the four rules of quantification theory?

Q 10: What is universal generalization? (Answer in about 60 words)

Q 11: What do you mean by existential instantiation? (Answer in about 60 words)

Q 12: What is 'Square of opposition'? (Answer in about 60 words)

Q 13: Fill in the blanks:

a)..... is the symbol of existential quantifier.

b) Predicate logic is also known as .....

c) (X) (Gx) means that……which is ghost.



12.9 LET US SUM UP

- The quantification theory is the technique to symbolize the inner logical structure of general propositions with the help of quantifiers.
- The process of symbolizing simple propositions by replacing individual constant for individual variable from propositional functions in predicate logic is known as instantiation.
- There are two types of quantifiers: existential and universal quantifier.
- (x) is the symbol for existential quantifier.
- (X) is the symbol for universal quantifier.
- The four fold types of traditional subject-predicate propositions are: Universal affirmative, Universal negative, Particular affirmative, Particular negative
- There are four rules of quantification: Universal instantiation, Universal generalization, Existential instantiation, and Existential generalization
- Quantification theory helps us to symbolize not only A, E, I and O propositions but also provides us the technique to symbolize singular and general proposition.
- Quantification is the technique of bringing out the inner logical structure of general propositions with the help of quantifiers.



12.10 FURTHER READING

1) Copi, Irving M. Symbolic Logic (fifth edition). New Delhi. Prentice Hall of India (Private Ltd.).
2) Jain, Krishna(1998). A textbook of logic (fourth Revised Edition).
3) Singh, S. Shyam Kishore. Modern Logic (Vol.1) Lamyana Press.
4) Copi, Irving M., Cohen, Carl, Cohen., Priyedarshi, Jetli.,& Monica, Prabhakar. Introduction to Logic (Twelfth edition). Dorling Kindersley (India) Pvt. Ltd.

12.11 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: (a) True b) True c) True

Ans to Q No 2: A propositional function is a symbolic expression which contain individual variable.

Ans to Q No 3: Instantiation is the method of obtaining propositions replacing individual constant for individual variable from propositional functions. As for instance, Ha, Hd, Hr etc. are the examples of instantiation of getting singular propositions out of propositional function, 'Hx'.

Ans to Q No 4: a) False b) True c) True d) True

Ans to Q No 5: Quantification is the technique of symbolizing general propositions with the help of quantifiers.

Ans to Q No 6: Universal Quantifier and Existential quantifier

Ans to Q No 7: A, E, I and O are four fold types of propositions.

Ans to Q No 8: (x) (Hx. ~Mx) is the example of 'O' proposition

Ans to Q No 9: Two rules of universal quantifier:

i) Universal instantiation

ii) Universal generalization

Two rules of existential quantifier:

i) Existential instantiation

ii) Existential generalization

Ans to Q No 10: This rule states that if a proposition is true for any arbitrarily selected individual then it is true for all individuals. This rule is denoted as (U.G) As for instance, if Ram is mortal is true then all men are mortal is also true. Similarly, If 'Ram is four footed' is a false proposition, 'all men are four footed' is also a false proposition.

Ans to Q No 11: This rule states that from the existential quantification of a propositional function we may infer the truth of its substitution instance. As for instance, if "Some businessmen are honest" is true then we can infer from the premise that 'the businessman is honest' or 'there is at least one individual who is a Businessman' is also true.

Ans to Q No 12: The logical connection between universal and existential quantification can be shown in terms of square array which is known as square of opposition.

Ans to Q No 13: a) (x) b) Quantifier logic

c) There is atleast something



12.12 MODEL QUESTIONS

A) Very Short Questions

Q 1: What is predicate logic?

Q 2: What is quantification?

Q 3: What is propositional function?

B) Short Questions (Answer each question in about 150 words)

Q 1: Do you find any difference between instantiation and propositional function? Discuss

Q 2: Explain the quantifiers with suitable examples.

C) Short notes (Answer each question in about 150 words)

Q 1: Square of opposition

Q 2: Existential quantifier

Q 3: Universal quantifier

Q 4: Rules of quantification

Q 5: Propositional function

D) Distinguish between (Answer each question in about 150 words)

Q 1: Universal quantifier and existential quantifier

Q 2: Singular and general proposition

Q 3: Propositional logic and predicate logic

Q 4: Universal instantiation and existential instantiation

Q 5: Universal generalization and existential generalization

E) Long Questions (Answer each question in about 300-500 words)

Q 1: What is predicate logic? Do you find any difference between instantiation and propositional function? Discuss

Q 2: What is quantification? Explain the quantifiers with suitable examples.

Q 3: Discuss the symbolic formulations of A, E, I and O by quantification

Q 4: What are the rules of quantification? Discuss with suitable examples.

*** ***** ***

UNIT 13 : EXISTENTIAL AND UNIVERSAL QUANTIFIERS

UNIT STRUCTURE

13.1 Learning Objectives

13.2 Introduction

13.3 Predicate Logic

13.4 Symbols used in Predicate Logic

13.5 Quantifiers and Types of Quantifiers

13.6 Difference Between Universal Quantifier and Existential Quantifier.

13.7 Symbolisation of Categorical Statements with Quantifiers

13.8 Scope of Quantifier

13.9 Symbolisation

13.10 Let Us Sum Up

13.11 Further Reading

13.12 Answers to Check Your Progress

13.13 Model Questions

13.1 LEARNING OBJECTIVES

After going through this unit you will be able to:

- define predicate logic
- explain the symbols used in predicate logic
- describe the distinction between universal quantifier and existential quantifier
- discuss the statements symbolised in predicate logic.

13.2 INTRODUCTION

The three distinct branches of symbolic logic are propositional logic, predicate logic and logic of classes. Predicate logic deals with the internal structure of both the simple and compound propositions. It is the logic of predicates or properties, and things or objects to which the predicates may

be ascribed. Now question arises, how many of the individuals have the property in question? In predicate logic, the expressions which state that, how many of the individuals have the property in question are known as quantifiers. This logic uses only two kinds of quantifier symbols: Universal quantifier and Existential quantifier. Universal quantifier stands for "for all x" or "for any x" . The symbol used for universal quantifier is –(x). The existential quantifier stands for 'there is at least one x' and it has existential commitment. The symbol used for existential quantifier is –(Žx). These quantifiers give us the way to symbolise quantity terms such as 'all', 'some', 'every', 'none' which occur in statements about predications and thereby bring out the formal structure of general proposition.

13.3 PREDICATE LOGIC

Predicate logic is that branch of symbolic logic which deals with the internal structure of both the simple and compound propositions. It is concerned with predicates or properties, and things or objects to which the predicates may be ascribed. Predicate logic on the other hand deals with the sub-structures of both simple and compound propositions. In more formal terms, it is also known Quantification Theory or the Predicate calculus.

Predicate logic contains the entire propositional logic. In predicate logic, a distinction between singular proposition and general proposition is made. A singular proposition is one which states that a particular individual possesses or does not possess a particular property or attribute. 'kalidasa is a poet'. On the other hand a general proposition asserts that some individuals or all individuals possess or do not possess a certain property. For example: 'All men are wise', 'some flowers are red', etc.

13.4 SYMBOLS USED IN PREDICATE LOGIC

Symbols play a pivotal role in predicate logic. A symbol is an ideogram which represents logical concepts. Predicate logic contains entire propositional logic. So, it uses same symbols adopted in propositional logic. For example, logical constants and propositional variable are used both in

propositional logic and predicate logic. In addition to it predicate logic uses a kind of symbol to signify the quantity term occur in statements that is called quantifier.

Now, it is necessary to deal with the symbols used in propositional logic. Propositional logic uses two types of symbols namely, (1) logical constant or logical operator and (2) propositional variable.

Logical constants: Logical constants are those words which are used to connect simple propositions to produce compound propositions and which express the form or structure of the compound propositions. They connect statements. Hence, they are also called the connectives. Propositional logic uses five basic logical constants or connectives. They are- not, and, either-or, if-then, if and only if. These logical constants are represented by their respective symbols. The logical constants and their respective symbols are the following:

Symbol	Name	Meaning
~	(Tilde)	Not, It is not the case that
•	(Dot)	and
”	(Wedge)	or
f!	(Horseshoe)	if-then
a™	(triple bar)	if and only if

Propositional variables: The lower case letters of the English alphabet p, q, r, s, t etc. which stand for simple propositions are known as propositional variables. For example:

If p then q

P

Therefore q

This argument can be symbolically represented in the following way:

P f! q

P

. . q

Here, p and q are variables because they can be replaced by any simple proposition within a definite range.

In addition to these symbols, predicate logic uses a kind of symbol to signify the quantity term occurring in statements what is called quantifier. Predicate logic uses only two kinds of quantifiers: Universal quantifier and Existential quantifier.

13.5 QUANTIFIERS AND TYPES OF QUANTIFIERS

Quantifiers are expressions that state how many of the individuals or objects have the property in question. They do not state which one of the individuals have the property. Prof. Chhanda Chakraborti states that a quantifier consists of:

- A left parenthesis or '('
- A quantifier symbol
- One of the individual variable symbol
- A right parenthesis or ')'

In predicate logic, we have two quantifier symbols:

- The inverted 'A' or V
- And the inverted 'E' or 'Ž'

The quantifiers provide us a way to symbolise quantity terms such as 'all', 'some', 'every', 'none' which occur in the propositions. Predicate logic uses two types of quantifiers: Universal Quantifier and Existential Quantifier.

Universal quantifier: The expression "given any x", or "for all x" or "for any x" or "for every x" is known as universal quantifier. Universal quantifier is used to assert that all entities have a certain property or properties. Thus it yields a universal proposition when prefixed to a propositional function. (We have already discussed about propositional function in the chapter: 11)

The symbol used for universal quantifier is: '(Vx)'. The symbol '(Vx)' consists of all the components of a quantifier. First it is enclosed within a pair of parentheses. Secondly, it is expressed with an individual variable 'x'. Thirdly, it stands for 'for all x', or 'for any x', or 'for every x', or 'given any x'. But conventionally the symbols '(x)', '(y)', '(z)' are used for this purpose. Hence, universal quantifier is used to symbolise the statements containing the words 'all', 'every', 'each', 'any', and their equivalent expressions.

Let us symbolise the proposition 'Everything is movable' in predicate logic step by step. The proposition can be paraphrased as:

In the first step, we express it as: For all x, x has the property of being temporary

or

for all x, x is temporary or 'given any x, x is temporary'.

In the second step, we use a predicate symbol to symbolise the property 'being movable'. Here, we can use 'Mx' to symbolise the property 'being movable'.

In the third step, the expression 'for all x', which is called universal quantifier is expressed by the quantifier symbol '(x)'. Thus the general proposition 'everything is movable' can be symbolically expressed in predicate logic as follows:

(x) Mx

Existential quantifier: The expression "there is at least one x" is known as existential quantifier. It is used to assert that some entities or at least one entity have a given property. Thus it yields an existential proposition when prefixed to a propositional function.

The symbol 'Ž' stands for particular quantity terms such as 'some', 'a few', etc. and their equivalent terms in English. As a quantifier, it is used as '(Žx). The symbol '(Žx)' stands for the expression "there is at least one x" which is known as the existential quantifier. Let us symbolise the proposition 'Something is solid' by as using the existential quantifier.

The proposition may be paraphrased as –There is at least one x such that x is solid. Using the same process as shown above we may symbolise the statement as follows:

(Žx) Sx

Similarly, 'Some things are not solid' may be rephrased as 'There is at least one x such that x is not temporary. We put negation sign (~) in the predication part and symbolise the proposition as follows:

(Žx) ~ Sx

We have already discussed in the preceding chapter (11.15) that general propositions are different from singular propositions because general

propositions contain words signifying quantity. So quantifiers are used to bring out the formal structure of general proposition.



CHECK YOUR PROGRESS

Q 1: What is quantifier?

.....

.....

Q 2: What are the types of quantifier?

.....

Q 3: What is universal quantifier?

.....

.....

Q 4: What is existential quantifier?

.....

.....

Q 5: Choose the correct answer.

a) The symbol (x) / (Žx) stands for universal quantifier.

b) The symbol (x) /(Žx) stands for existential quantifier.

Q 6: Fill in the blanks

a) Predicate logic contains …….logic.

b) Quantifiers are used to signify the quantity of …….proposition.

13.6 DIFFERENCE BETWEEN UNIVERSAL QUANTIFIER AND EXISTENTIAL QUANTIFIER

Propositional logic uses two types of quantifier. viz. universal quantifier and existential quantifier. There is a clear difference between them. We can bring out their differences in the following way.

- The expression “given any x”, or “for all x” or “for any x” or “for every x” is known as universal quantifier. On the other hand, the expression “there is at least one x” is known as existential quantifier.
- Universal quantifier is used to assert that all entities have a certain property or properties. On the other hand, existential quantifier is used

to assert that some entities or at least one entity have a given property. Hence, the statement which contains words like 'all', 'every', 'each', 'any', 'everything', etc. and their equivalent terms in English are symbolised by the aid of universal quantifier. On the other hand, the statement which contains particular quantity terms such as 'some', 'a few', at least one' etc. and their equivalent terms in English are symbolised by the aid of existential quantifier.

- The symbol (x), (y), (z) stand for universal quantifier i.e. for the expression "There is at least one x". But the symbol (Žx) stands for existential quantifier i.e. for the expression 'for all x' of 'for any x'.
- When universal quantifier is placed before a propositional function, we obtain a universal proposition. For example, 'Mx' or 'x is movable' is a propositional function. If we prefix the universal quantifier to the propositional function 'Mx' we obtain a universal proposition: (x) Mx which is to be read as "For all x, x is movable". On the other hand, when universal quantifier is placed before a propositional function, we obtain a universal proposition. For example, If we prefix the existential quantifier to the propositional function 'Mx' we obtain an existential proposition i.e. a particular proposition: (Žx) Mx which is to be read as "There is at least one x, such that x is movable".
- According to Boolean interpretation, Universal statement does not carry existential import. For example, universal propositions such as "All S is P' is logically translated as "for all x, such that if x is a S the x is a P'. In the expression 'for all x' no existential import is acknowledged i.e, it has no existential commitment. On the other hand, particular statements carry an existential import. The particular proposition "Some S is P'- is logically translated as "there exists at least one x such that x is a S and x is a P'. Existential import is acknowledged in the expression 'there is at least one x'. Therefore, existential quantifier (There is at least one x) not only refers to the quantity (at least one) but also commits that at least one of the things exists.

In this way we can show the difference between universal quantifier and existential quantifier.

13.7 SYMBOLISATION OF CATEGORICAL STATEMENTS WITH QUANTIFIERS

Let us now see how the four standard forms of categorical propositions: A ,E, I, and O can be translated in predicate logic and symbolised with the help of quantifiers.

"All cats are mammals"- it is an example of universal affirmative proposition. In this proposition there are two properties, 'being a cat' and 'being a mammal'. So we have to use two property letters with a variable. In Boolean interpretation, universal propositions are conditional proposition with no existential commitment. Accordingly, we may paraphrase the proposition as:

For every x, if x is a cat then x is a mammal. This expression can be symbolised by using universal quantifier as:

(x) (Cx f! Mx)

The 'E' proposition "No cat is a mammal" can be paraphrased as:

For every x, if x is a cat then x is a mammal. Using the same symbolisation key we may symbolise the proposition as:

(x) (Cx f! ~ Mx)

Here the proposition does not deny the predication predication of the property 'being a cat' but it denies the property of 'being a mammal' to every cat. Therefore negation sign is put before Mx.

'I' and 'O' propositions have import. We symbolise them by using existential quantifier. Let us take the 'I' proposition first.

"Some cats are mammals". It can be paraphrased as:

There is at least one x such that x is a cat and x is a mammal. We symbolise it as: (Žx) (Cx • Mx)

Similarly, we can paraphrase the 'O' proposition

"Some cats are not mammals"

as: There is at least one x such that x is a cat and x is not a mammal. It can be symbolised as: (Žx) (Cx • 4 Mx)

We may use Greek letter phi (ô) and psi (ψ) to represent any property symbol whatever. Thus, the four traditional propositions may be

symbolised as:

A – (x) (ôx f! ψx)

E– (x) (ô f! ~ψx)

I– (x) (ôx • ψx)

O– (ôx • ~ψx)



ACTIVITY

- How can you symbolise traditional A, E, I and O propositions by means of quantifiers?

.....

.....

13.8 SCOPE OF QUANTIFIER

A quantifier is used to signify the quantity i.e. 'how many'. So in order to do that it must be within its scope. That means a quantifier falls within its own scope. The scope of a quantifier indicates the extent of its interpretive power. For example, the universal quantifier (x) is interpreted as 'all x' which is its scope. Similarly, the existential quantifier ((Žx) is interpreted as 'at least one x' which is its scope. The scope of a quantifier is shown by the parenthesis or bracket. Parentheses are used to avoid ambiguities. If a quantifier is followed by a left parenthesis '('or a left square bracket '[' its scope continues to the next matching parenthesis or bracket. It can be clearly explained with the following examples:

1. (x) (Mx f! Hx)
2. (Žx) (Dx • Bx)
3. (x) Dx f! Da
4. (Žx) Mx •Da

In the examples 1 and 2 the scope of the quantifier includes firstly the variable in the quantifier itself i.e. (x) and (Žx) and secondly it extends from the left-hand bracket to the end of the matching bracket.

On the other hand, in the examples 3, and 4 none of the statements has a left bracket after the quantifier. In the example 3, the scope of the universal quantifier starts with itself and ends with 'Dx' i.e., before the logical

connective 'f!'. The 'a' of 'Da' is not within its scope. Moreover 'a' is a constant. Similarly, in the example 4, the scope of the existential quantifier starts with itself and ends with 'Mx' i.e. before the logical connective '•' If negation sign is used before brackets, it signifies that the scope of the negation sign covers the expression within the brackets. For example, the statement "It is not the case that some books are expensive" is symbolised as: 4 [(Ž x) (Bx •Ex)] Here negation sign covers the expression within the bracket. On the other hand, in the statement- "Somethings are not solid" the negation sign appears before the predicate, not in the quantity part. This statement can be translated as: (Ž x) 4Sx

It can be written as "there is at least one x, such that x is not solid.

Thus, in predicate logic parentheses (brackets) indicate the scope of an expression. Of course, we may omit the brackets in some cases. We may symbolise the sentence "Everything is movable" as (x) Mx without enclosing the expression Mx.

Bound variable and free variable: A quantified variable is known as bound variable. In other words if the variable is either part of the quantifier or lies within the scope of quantifier is known as bound variable. For example, in (x) Mx , 'x' is a bound variable. Similarly, in example 1. "(x) (Mx f! Hx)", and 2. "(Ž x) (Dx • Bx)" all three occurrences of the variable 'x' is bound.

On the other hand, an unquantified variable is free. For example in the sentence form Px, 'x' is a free variable.



ACTIVITY

- Distinguish between quantified statement and truth-functional compound statement.

Ans: ……

13.9 SYMBOLIZATION OF SENTENCES INVOLVING QUANTIFIERS

Symbolising with universal quantifier:

We use universal quantifier for universal statements. For example,

'Every man is selfish'. This proposition is to be reduced to 'A' proposition and thus its logical form is 'All men are selfish'. This proposition is to be symbolised as:

(x) (Mx f! Sx)

Similarly, consider the following example:

'No men are selfish'. Its meaning is 'Men are not selfish'. Using the same translation key it is to be symbolised as:

(x) (Cx f! ~ Mx)

Symbolising with existential quantifier: The quantity terms 'someone', 'something', 'some' are symbolised by the existential quantifier '(Žx)'. For example, the statement, 'some men are selfish' is paraphrased as: There is at least one x such that x is a man and x is selfish'.

Similarly, the statement 'Every man is not selfish' is to be reduced to 'O' proposition i.e. particular negative proposition. Thus its logical form is 'Some men are not selfish'. This proposition is to be paraphrased in predicate logic as "There are men that are not selfish" and it is to be symbolised as:

(Žx) (Cx • 4 Mx)

The proposition 'Lion exists' indicates that there is or there exists at least one thing which is a lion". In predicate logic it is translated as: "Something is a lion". The symbolic expression of this statement is:

(Žx) Lx

Symbolising 'only'. The statement 'only virtuous are happy' is properly paraphrased as: 'For any x, if x is happy, then x is virtuous.' Therefore its proper translation is:

(x) (Hx f! Mx).

So the sentences containing such words as 'Alone', 'None but', 'No one else but' are translated in the same way.

Multiply quantified statements: Statements containing more than one quantifier are referred to as multiply quantified statements and are alternatively known as multiply general statements. In other words, in a multiply quantified statement, there are multiple quantifiers. To symbolise such kind of statements two or more variables are required. Let us consider the following examples:

'Everyone loves everyone.' The symbolic expression of the statement is:

(x) (y) Lxy

It means 'given any x, and given any y, x loves y. Similarly the symbolic expression of the statement 'someone loves someone' is:

(Žx) (Žy) Lxy

It means 'there is an x, and there is a y, such that x loves y'.

The statement 'someone loves everybody' is symbolised as:

(Žx) Px (y) (Py f! Lxy)

It means 'there is an x, such that x is a person, and given any y, if y is person, then x loves y'.



CHECK YOUR PROGRESS

Q 7: Symbolise 'A' proposition in predicate logic.

.....

Q 8: Symbolise 'O' proposition in predicate logic.

.....

Q 9: Symbolise the following statements by using quantifiers.

a. Tiger exists.

b. Everything is temporary.

c. All scientists are philosophers.

d. Some people are not reliable.

Q 10: Fill in the blanks.

a. The scope of a quantifier is shown by the …….

b. quantified variable is known as …… variable.

Q 11: State whether the following statements are true (T) or (F)

a) Particular statements carry an existential import. (True/False)

b) The expression "there is at least one x" is known as existential quantifier.



13.10 LET US SUM UP

- Predicate logic deals with the internal structure of both the simple and compound propositions.
- Logic uses only two kinds of quantifier symbols: Universal quantifier and Existential quantifier.
- The expression “given any x”, or “for all x” or “for any x” or “for every x” is known as universal quantifier. The symbol used for universal quantifier is: (x).
- The expression “there is at least one x” is known as existential quantifier. The symbol used for existential quantifier is –(Žx). The four standard form categorical propositions i.e. A ,E, I, and O can be symbolised with the help of quantifiers.
- The scope of a quantifier indicates the extent of its interpretive power. The scope of a quantifier is shown by the parenthesis or bracket.
- A quantified variable is known as bound variable. On the other hand, an unquantified variable is free.



13.11 FURTHER READING

1) Chakraborti. C., *Logic-Informal, Symbolic & Inductive*
2) Hurley. J. P., *Introduction to Logic*
3) Baronett. S. and Sen. M., *Logic*
4) Copi. I.M. & Cohen. C., *Introduction to Logic*
5) Sharma. B. and Deka J., *A text Book of logic*



13.12 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: Quantifiers are expressions that state how many of the individuals or objects have the property in question.

Ans to Q No 2: There are two types of quantifier.

Ans to Q No 3: The expression "given any x", or "for all x" or "for any x" or "for every x" is known as universal quantifier.

Ans to Q No 4: The expression "there is at least one x" is known as existential quantifier.

Ans to Q No 5: a) The symbol '(x)' stands for universal quantifier.

b) The symbol '(Žx)' stands for existential quantifier.

Ans to Q No 6: Fill in the blanks

a) propositional b) General

Ans to Q No 7: (x) (Sx f! Px)

Ans to Q No 8: (Žx) (Sx • 4 Px)

Ans to Q No 9: a) (Žx) Tx b. (x) Tx c. (x) (Sx f! Px)

d. (Žx) (Sx • 4Rx)

Ans to Q No 10: a) Bracket b) Bound

Ans to Q No 11: a) (T) b) (T)



13.13 MODEL QUESTIONS

A) Very short questions

Q 1: What is quantifier?

Q 2: What are the types of quantifier?

Q 3: What is existential quantifier?

Q 4: What is universal quantifier?

Q 5: What is bound variable?

Q 6: Symbolise the following statements in terms of predicate logic.

a) Tiger exists

b) Only teachers are honest

c) Everything is temporary

d) Some people are not reliable.

Q 7: What is predicate logic?

B) Short questions (Answer each question in about 100-150 words)

Q 1: What are the types of quantifier? Briefly explain

Q 2: Distinguish between universal and existential quantifier.

Q 3: Write a note on existential quantifier.

Q 4: Write a note on universal quantifier.

Q 5: What is predicate logic? What are the symbols used in predicate logic?

C) Long questions (Answer each question in about 300-500 words)

Q 1: What do you mean by the scope of quantifier? Explain

Q 2: Explain the traditional A, E, I and O proposition by using quantifiers.

Q 3: Explain the types of quantifiers with suitable examples.

*** ***** ***

UNIT 14: CONCEPT OF SET

UNIT STRUCURE

14.1 Learning Objectives

14.2 Introduction

14.3 Concept of Set

14.4 Set Membership

14.5 Null Set

14.6 Finite and Infinite Set

14.7 Subset and Superset

14.8 Universal Set

14.9 Power Set

14.10 Let Us Sum Up

14.11 Further Reading

14.12 Answers to Check Your Progress

14.13 Model Questions

14.1 LEARNING OBJECTIVES

After going through this unit, you will be able to

- learn the concept, definition and examples of set;
- learn the concepts and notations of members and membership in a set;
- learn the methods of writing sets;
- learn the concept and examples of null set;
- learn the concepts and examples of finite and infinite sets;
- learn the concepts and examples of subset and superset;
- learn the concept and examples of universal set;
- learn the concept and examples of power set.

14.2 INTRODUCTION

The language of set theory is required in all branches of mathematics. This unit will first introduce to you the concept and the definition of a set.

The concept of set membership, null set, finite set, infinite set, subset, superset, universal set and power set will be introduced afterwards. In each case, enough examples are given to illustrate the concepts.

14.3 CONCEPT OF SET

In our daily life, we use bags, boxes, containers etc. to keep different items properly and systematically. In mathematics, we deal with certain mathematical objects, mostly numbers at the initial stages. So, we need something to keep those mathematical objects systematically. Similar to bags, boxes or containers, we use the term 'set' to refer such an entity inside which we keep our mathematical objects to be studied and we use a pair of braces '{ }' inside which we list the objects.

The theory of sets is the language of modern mathematics. The concept of sets plays an integral part in all branches of modern mathematics. The concept of sets is basically to group different objects of a particular mathematical discussion and view the whole as a single entity. Formally, a **set** is a collection of some **well-defined** objects. The objects which will be in the set should be well-defined in the sense that there should not be any ambiguity to decide whether a particular object belongs to a given set or not. Thus, if a set and an object is given to us, then there will be only two possibilities, namely, either the object is in the set or not. So, while referring to a set, either the objects it contains should be visible to us or there should be a description which will determine the objects in the set. Let us take the following examples to clarify this idea.

1. The collection of all the students in your class.
2. The collection of all the **tall** students in your class.
3. The collection of all the students in your class who can speak English **fluently**.
4. The collection of the small letters in the English alphabet.
5. The collection of the first five natural numbers.
6. The collection of the natural numbers which are **near** to 10.
7. The collection of all the natural numbers.
8. The collection of all the **large** natural numbers.

9. The collection of all the **even** natural numbers.
10. The collection of books that are inside your school bag.
11. The collection of the names of the days in a week.
12. The collection of the names of the **cold** days in a particular week.

Notice that given any student from your school, college or university, it is clear whether he/she is in your class or not. Clearly, the letters a to z are all small letters in the English alphabet, and there are no others. Similarly, the numbers 1, 2, 3, 4 and 5 are the first five natural numbers and so, given any number, we can determine whether the number is in the list 1, 2, 3, 4, 5 or not. Given any number, from the knowledge of numbers it is clear whether it is a natural number or not. Indeed, given any number, it is clear whether it is an even natural number or not, as the natural numbers which are divisible by 2 is precisely the even numbers. If one says the name of any book of their choice, one can precisely check and tell whether it is in your school bag or not. Similarly, the names Monday, Tuesday, Wednesday, Thursday, Friday and Saturday are all the names of the days in a week, and so whatever name is taken by someone, we can determine whether that is a name of one of the days in a week or not. Thus the descriptions in Example 1,4,5,7,9,10 and 11 are well-defined and eligible for being a set.

However, the term tallness is quite vague unless we fix a certain height to be the minimum for being tall. So, a particular student may be tall enough for someone and may be quite short for others. So, it is difficult to decide whether a given student in your class is tall or not. In the same way, it is ambiguous to decide whether a person speaks fluent English or not. Similarly, for example, the number 5 may be very near to 10 for someone and may be far enough from 10 for others. So, given a number, it is not possible to decide whether it is near to 10 or not. In the same manner, the idea of large numbers is also vague, as a number like 1,000,000,000,000,000,000 may also be small enough for some people depending on the context. So, a given number cannot be classified as large or small. It is clear with the same logic that a particular day of a week may be colder for someone and may not be so cold for others. Thus the descriptions in Example 2, 3, 6, 8 and 12 are **not** fit for being a set.



CHECK YOUR PROGRESS

Q 1: Write a short note on the concept of set.
Give the formal definition of set.

Q 2: Check whether the following descriptions determine a set or not:

(i) The collection of all the professors of your university;

(ii) The collection of all the books in your university library;

(iii) The collection of all the costly books in your university library;

(iv) The collection of all the chairs in your class room;

(v) The collection of all the alphabets in the word 'UNIVERSITY';

(vi) The collection of all the negative integers;

(vii) The collection of all the natural numbers which are divisible by 10;

(viii) The collection of all the brilliant students of your class;

(ix) The collection of the numbers 2, 3 and the alphabets a, b, c;

(x) The collection of the numbers 2, 3 and the word 'NAME';

(xi) The collection of all the large natural numbers;

(xii) The collection of the words 'NAME', 'MONDAY', 'SET', 'PRIME', 'HOME';

(xiii) The collection of the alphabets in the words 'NAME', 'MONDAY', 'SET', 'PRIME' and 'HOME';

(xiv) The collection of all the stars in the universe.



LET US KNOW

The modern theory of sets was first given by German Mathematician Georg Cantor, who lived during 1845 – 1918, towards the end of the nineteenth century. His papers on set theory appeared during 1874 to 1897. The theory of sets given by him is known as '*naïve set theory*'. Cantor's work was further developed by several famous mathematicians like

Richard Dedekind (1831-1916), Kronecker (1823-1891), GottlobFrege (1848-1925) etc. Till then the theory of sets was based on the assumption that there is a set of all sets. However in 1902, famous English Philosopher Bertrand Russell showed that there does not exist a set of all sets. This is linked with the famous Russell's Paradox. Due to the assumption of existence of all sets, mathematicians found many more paradoxes after Russell's Paradox was discovered. To overcome these difficulties, Ernst Zermelo in 1908 first gave axiomatic description of set theory. Abraham Fraenkel proposed another axiomatisation of set theory in 1922. Paul Bernays gave a more satisfactory axiomatisation of set theory in 1937. Famous Austrian mathematician Kurt Gödel modified these axioms in 1940. This axiomatic set theory is known as Godel-Bernays set theory.

Though there are logical difficulties in Cantor's theory of sets, it is still used in present day mathematics to express most of the concepts and results.

14.4 SET MEMBERSHIP

Whenever we have a collection which defines a set, we use the capital letters of English alphabets to denote it. And we normally use the small letters of English alphabets to denote the objects available in the set, though any other symbols can be used. The objects inside a given sets are formally called ***members*** or ***elements*** of the set. Suppose A is a set and a is an element of. Then we write $a \in A$ to say that 'a is an element of A'. If a is **not** an element of the set A, then we write that $a \notin A$. The symbols '$a \in A$' and '$a \notin A$' are read as 'a belongs to A' and 'a does not belong to A', respectively.

For example, let A be the set of all even natural numbers. Then we can write $4 \in A$ or $1 \notin A$. However, $3 \in A$ or $6 \notin A$ are wrong as 3 is not an even number while 6 is an even number.

Though sets are determined by certain descriptions, such mere descriptions are often not enough to deal with them further mathematically.

So, we need some systematic method of writing sets. There are two methods of writing a set.

14.4.1 TABULAR METHOD

In this method, all the elements of the set are listed inside the braces { }, a comma is used between each element, an element is not written more than once and there is no importance given on the order of the elements written inside the braces. For example, let be the set of the first three natural numbers. Then we can write A = {1, 2, 3}. Notice that writing A= {1, 2, 3}, A = {1, 2, 3, 1), A = (1, 2, 3) or A = [1, 2, 3] are all wrong, as the comma is used twice between 1 and 2 in the first example, 1 is listed twice in the second example, and other brackets are used instead of braces in the last two examples. However, A = (1, 3, 2) or A = (2, 1, 3) are both correct as the order among the elements in a set does not matter. This method of writing sets is also known as ***roster method***.

A major drawback of this method is that large sets are difficult to write in this method. For example, a set with 10000 elements will take a lot of space and time in writing all the elements. However, if the elements of the set have a certain pattern, then instead of writing all the elements, we sometimes write first few elements followed by three dots, which is read as “and so on”. If possible, we also write the last element. For example, the set of first one thousands natural numbers can be written as {1, 2, 3, . . . ,1000}

Similarly, the set of all even natural numbers can be written as {2, 4, 6 . . .}. Even with this convention, there are many sets which cannot be written in this method. For example, consider the set of all real numbers. In an attempt to write th e setin tabular method, whatever real number we choose to write in the first place, there is no pattern to write the second element. Thus the set R cannot be written in tabular method.

14.4.2 SET BUILDER METHOD

In most of the times, we consider sets where all the elements of the set possess a common property and elements which are outside the set do not possess the property. In such situations, the elements of the set can be characterized by that common property. For example, consider the set A = {1, 2, 3}. Notice that the elements of are the natural numbers less than or equal to 3. In set builder method, we write this set as.

A = { x : x is a natural number less than or equal to 3} or A = {x : x $\in$ $\mathbb{N}$, x $\leq$ 3}

Note that the symbol $\mathbb{N}$ stands for the set of all natural numbers. In general, when the elements of a set can be characterized by a common property, we can write the set in set builder method. In this method, we describe the elements of the set by using a symbol like or any other letters from small English or small Greek alphabets. Then the symbol representing the elements is followed by a colon ":". After the colon sign, we write the characteristic property of the elements of the set and finally the whole thing is enclosed within braces. Sometimes, a vertical bar '|' is also used instead of a colon ':'.

The set described above should be read as "the set of all such that is a naturalnumber less than or equal to 3". Notice that the braces stands for "the set of all" and the colon stands for "such that". Also notice that the characteristic property of the elements of the set may be written in more than one ways, depending on the situations. As another example, consider the set of all even natural numbers. We can write this set as.

B = {x : x is an even naural number} or B = {x : x = 2k, for some k$\in\mathbb{N}$}

It is worthwhile to remark here that all the elements of a given set may not have a common characteristic property, and in that case it may be difficult to write the set in this method. For example, the

set {1, c, α, red, book, 5} is not suitable to be written in set builder method.

The set builder method is also known as ***rule method*** or ***property method***.



CHECK YOUR PROGRESS

Q 3: Give two examples of sets which can be written in both tabular method and rule method.

Q 4: Give two examples of sets which cannot be written in tabular method.

Q 5: Give two examples of sets which are not suitable to be written in rule method.

Q 6: Write the following sets in both tabular method and rule method:

(i) The collection of all the negative integers;

(ii) The collection of all the days in a week;

(iii) The collection of all the natural numbers which are divisible by 10;

(iv) The collection of all the letters in the word 'AMERICA'.

Q 7: Write the following sets in tabular method:

(i) {x : x is an integer, $3x \le 13$ };

(ii) {x : x is a prime number, $x \le 13$ }

(iii) {x : x is a month of the year having exactly 30 days};

(iv) {x : x is prime number which divides 70}.

Q 8: Write the following sets in rule method:

(i) {1, 4, 9, 16, 25}, (ii) {1, 4, 9, 16, ...}

(iii) {1, 3, 5, 7, ...} (iv) {–3. –1, 1, 3, 5, 7, ...}.

14.5 NULL SET

Many times, we need to deal with sets where the description of the set is given in such a way that we find no elements in the set. For example, consider the set $A = \{x : x \in N, x < 1\}$. Since there is no natural number

which is less than 1, we see that there are no elements available in this set. This set is just like an empty bag or an empty box. Formally, ***a set which does not contain any elements is called a null set***. Though it is surprising, it is to be noted that there is only one null set. To justify it, suppose that and are two null sets. If they are not same, they are not equal. So, one of them should contain at least one element that does not belong to the other. Accordingly, one of the sets is not a null set. This contradicts our initial assumption that both the sets are null sets. Hence there is only one null set.

As an example, let us take the sets $A = \{x : x \in N, x < 1\}$ and $B = \{x : x \in N, x^2 = -1\}$. From the description of these two sets, it seems that these two sets are different. However, it turns out that both of them are null sets, and hence are equal. Thus, the description of a null set can be given in many different ways.

Since there is only one null set, a single notation to denote a null set is enough. A null set is usually denoted by the symbol ϕ, which is a letter from the Norwegian and Danish alphabet. This symbol is basically a small circle cut by a forward slash. Note that the symbol ϕ of null set is not the same with Greek letter φ or Φ and so the symbol ϕ of null set should never be read as 'phi'. Null sets also have the name ***void set*** or ***empty set***. Therefore the set ϕ should be read as 'null set', 'void set' or 'empty set'. Sometimes, the notation { }, a pair of braces with nothing inside, is also used to denote a null set.



CHECK YOUR PROGRESS

Q 9: Give two (different descriptions) examples of null sets.

Q 10: Determine which of the following sets are empty:

(i) {x : x is an integer, $x^2 = 3$} ;

(ii) {x : x is a prime number, $3x \leq 13$ } ;

(iii) the set of prime numbers dividing 20 and greater than 6;

(iv) the set of positive real numbers whose square is 0.

14.6 FINITE AND INFINITE SET

Consider the following sets:

A = {a, b, c, d, 3}, B = {x : x is a natural number, x < 1000}, C = the set of all points on a piece of paper, D = the set of all men living presently on earth, and E = the set of all natural numbers.

We notice that the set A contains five elements. The set B contains 999 elements. The sets C and E contain infinitely many elements. However, even though we do not know the precise number of elements in the set D, it must be a natural number which may be a quite big number. We say that the sets A, B andare finite sets and the sets andare infinite sets. An empty set is also regarded as finite set. Thus, a set which is either empty or contains a definite number of elements is called a ***finite set***. Otherwise, the set is called an ***infinite set***.

We use the notation $n(S)$ to denote the number of elements of a set S. The number $n(S)$ is also called the ***size*** or ***order*** of the set S. If S is not a finite set, then we write $n(S) = \infty$ .Thus, $n(\phi) = 0,\ n(A) = 5$ while $n(C) = \infty$. It is clear that the order of a non-empty finite set is a natural number. Sometimes the notation $|S|$ is also used to denote the number of elements in a set S.

A set which contains exactly one element is called a ***singleton set***. For example, the sets {1} and $\{\phi\}$ are singleton sets.

14.7 SUBSET AND SUPERSET

Consider the sets A ={1, 2, 3} and B = {a, 1, 2, b, 3, 5}. We notice that every element of *A* is also an element of *B*. In such case, we say that *A* is a subset of *B* and *B* is a superset of *A*. Formally, if every element of a set *A* is also an element of another set *B*, then we say that *A* is a ***subset*** of *B*. We write this in symbols as $A \subseteq B$, which is read as '*A* is a subset of *B*'. Equivalently, we also say that *B* is a ***superset*** of *A*, written in symbols as $B \supseteq A$, which is read as '*B* is a superset of *A*'. Notice that '*A* is a subset of *B*' and '*B* is a superset of *A*' expresses the same meaning in different phrases.

Using symbols, we write the definition of subset as follows:

$A \subseteq B$ if $x \in A \Rightarrow x \in B$

Recall that the symbol '$\Rightarrow$' stands for 'implies'. Thus the above statement is read as "*A* is a subset of *B* if *x* is an element of *A* implies that *x* is an element of *B*".

If *A* is not a subset of *B*, then we write that $A \not\subset B$. From the definition of subset, it is clear that if $A \not\subset B$ then there is at least one element in *A* which is not in *B*. Conversely, if there is at least one element in *A* which is not in *B*, then $A \not\subset B$. In symbols, we write this as $A \not\subset B \Rightarrow$ there is some $x \in A$ such that $x \notin B$.

Note that the symbol '$\Rightarrow$' stands for both way implications, and is read as 'if and only if'.

Let us now consider an arbitrary set *A*. Notice that the statement "every element of *A* is also an element of *A*" is clearly true. Therefore *A* is a subset of itself, that is, $A \subseteq B$. Now consider the empty set ϕ. Since there is no $x \in \phi$ such that $x \notin A$, the condition for $\phi \not\subset A$ is not satisfied. Therefore $\phi \subseteq A$ is true for any set *A*. Thus we find that

$\phi \subseteq \phi,\ \phi \subseteq A,\ A \subseteq A$ for every set *A*.

Whenever we write $A \subseteq B$, it covers the possibility of *A* = *B*. In case, $A \subseteq B$ and $A \neq B$ then we say that *A* is a ***proper subset*** of *B*, written symbolically as $A \subset B$. Thus if $A \subset B$, then every element of *A* is also an element of *B* and there is some element in *B* which is not an element in *A*. Symbolically,

$A \subset B$ if $x \in A \Rightarrow x \in B$ and there is some $y \in B$ but $y \notin A$.

Notice that $A \subset B$ means either *A* = *B* or $A \subset B$. Therefore if *A* is known to be a proper subset of *B*, then both $A \subseteq B$ and $A \subset B$ are correct, although writing $A \subset B$ gives stronger information.

Two sets *A* and *B* are said to be equal if they contain the same elements. Equality of two sets can be equivalently described with the help of subset relation. We are stating and proving this fact as a theorem below.

Theorem14.1: Let *A* and *B* be two sets. Then *A* = *B* if and only if $A \subseteq B$ and $B \subseteq A$.

Proof: Suppose first that *A* = *B*. Then according to the definition of equality

of sets, *A* and *B* contain the same elements. Therefore both the statements $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$ are true. Accordingly, we have $A \subseteq B$ and $B \subseteq A$.

Conversely, suppose that $A \subseteq B$ and $B \subseteq A$ If $A \neq B$, then at least one of the following two alternatives must be true:

a) there is $x \in A$ somesuch that $x \notin B$;

b) there is $x \in B$ somesuch that $x \notin A$.

But (a) implies that $A \not\subset B$, which is against our assumption $A \subseteq B$. Similarly, (b) implies that $B \subseteq A$, which is against our assumption $B \subseteq A$. Thereforeis not possible, and hence we must have *A* = *B*. //

Let us take the following examples for illustrations:

(a) Suppose

N = the set of all natural numbers, Z = the set of all integers,

Q = the set of all rational numbers, T = the set of all irrational numbers,

R = the set of all real numbers, C = the set of all complex numbers.

We know that every natural number is an integer, but there are integers (for example–1) which are not natural numbers. Every integer is a rational number, but there are rational numbers (for example $\frac{1}{2}$) which are not integers. Every rational number is a real number, but there are real numbers (for example $\sqrt{2}$) which are not rational numbers. Every irrational number is a real number, but there are real numbers (for example 1) which are not irrational numbers. Every real number is a complex number, but there are complex numbers (for example 1+2i) which are not real numbers. From these explanations, we have that

$\mathrm{N} \subset Z, Z \subset Q. Q \subset R, \mathrm{T} \subset R$ and $R \subset C$.

(b) Let *A*={2, 4, 6} and *B* = {x : x is an even natural number less than 100}. Notice that $2 \in A$ and 2 is an even natural number less than 100. Therefore $2 \in B$. Similarly, we have $4 \in B$ and $6 \in B$. Thus we see that $x \in A \Rightarrow x \in B$ and so $A \subseteq B$. However, $8 \in B$ but $8 \in A$. Therefore, *A* is a proper subset of *B*, that is, $A \subset B$.

(c) Let *A* = {x : x is a letter in 'later'} and *B* ={x : x is a letter in 'lateral}. We

see that *A* = {l, a, t, e, r} and *B* = {l, a, t, e, r} Clearly, *A* and *B* contains the same elements, and so *A* = *B*.

(d) Let *A* = {x : x $\in$ N and x is divisible by 3} and *B* = {x : x = 3k, for some k $\in$ N} Suppose that $x \in A$. Then x is a natural number and it is divisible by 3. Therefore $\frac{x}{3}$ is a natural number, say, $\frac{x}{3} = r$ and x $\in$ N. This implies that x = 3r for some r $\in$ N. Thus we have x $\in$ B. We found that $x \in A \Rightarrow x \in B$ and so $A \subseteq B$. Again, suppose that $x \in B$. Then we see that x = 3k for some natural number *k*. Since the natural number 3*k* is divisible by 3, we find $x \in A$. We found that $x \in B \Rightarrow x \in A$ and so $B \subseteq A$. Now from $A \subseteq B$ and $B \subseteq A$ we can conclude that *A* = *B* (according to **Theorem 14.1**).

(e) Let *A* = {1, 2, 3, b} and *B* = {1, 2, b, c, 5} We see that $3 \in A$ but $3 \in B$ Therefore $A \not\subset B$. Similarly, we have $B \not\subset A$.

From the examples (c) and (d), it is clear that if the elements of two given sets are clearly seen to be the same, then we can directly conclude that they are equal. However, if their elements are not visible precisely then we must use **Theorem 14.1** in order to show them to be equal.



CHECK YOUR PROGRESS

Q 11: Give five examples of singleton set.

Q 12: What are the orders of the sets ϕ and $\{\phi\}$? Are ϕ and $\{\phi\}$ equal?

Q 13: Give five examples of finite sets of different orders.

Q 14: Determine which of the following statements are true:

(i) subset of a finite set is finite;

(ii) subset of an infinite set is infinite.

Q 15: Determine whether the following sets are finite or infinite:

(i) the set of all prime numbers;

(ii) the set of all real numbers;

(iii) the set of all cells in a human body;

(iv) the set of all animals in the world;

(v) the set of all triangles that can be drawn in the plane;

(vi) the set of all natural numbers that are not divisible by 2.

Q 16: Let $A = \{0, 1, a, b\}$. Determine which of the following statements are true:

(i) $\{0\} \in A$, (ii) $\{0\} \subseteq A$ (iii) $0 \in A$ (iv) $0 \subset A$

(v) $\phi \in A$ (vi) $\{\phi\} \in A$ (vii) $\phi \subseteq A$

Q 17: Show that if $A \subseteq B$ and $B \subseteq A$ then $A \subseteq C$.

14.8 UNIVERSAL SET

In some situations, if our discussion is limited to a particular topic or context, then it is possible to get all sets of that discussion as a subset of a fixed basic set. Such a basic set is called a ***universal set*** of that particular context. A universal set is usually denoted by *U*. For example, if our discussion is limited to counting, then our universal set can be the set N of natural numbers. It is to be noted that, if N is a universal set of a particular context then any set that contains N as a subset, can also be a universal set of that context. The reason being, any subset of N will also be a subset of the set that contains N. For example, if N is a universal set of a particular context, then Z can also be a universal set of that context.

The situation is something like this: suppose you are interested to know the name of some particular student of your class and you have contacted your principal for this purpose. If you are provided the list of all the students of the class, then surely you can find the name you wished to know. Instead, if you are provided the list of all the students of the University, then also all the name you wished to know can be found in the list (though your work may be a bit tedious now).

Thus it is clear that a universal set of a particular context is never unique. It depends on the choice and conveniences of persons involved in the discussion. The only condition is that every set of the discussion should be a subset of the chosen universal set.

14.9 POWER SET

Consider the set $A = \{a, b\}$. We see that the subsets of *A* are ϕ, {a}, {b} and *A*. If we list these four subsets inside a pair of braces, then we find

a new set in which these four subsets will be elements. This new set {ϕ, {a}, {b}, *A*} is called the power set of *A*. Formally, the set of all subsets of a given set *A* is called the ***power set*** of *A*. The power set of *A* is denoted by P (*A*) or 2^A.

As ϕ is the only subset of the null set ϕ, we find P (ϕ) = {ϕ}. Similarly, if *A* = {1}, then P(*A*) = {ϕ, *A*}. If *A* = {1, 2}, then P (*A*) = {ϕ, (1), (2), *A*}. If *A* = {1, 2, 3}, then P(*A*) = {ϕ, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3} *A*}. From these examples, we observe the following:

- if $n(A)$ =0 (that is, where $A = \phi$), then $n\{P(A)\} = 1 = 2^0$;
- if $n(A)$ =1 (for example,), then $n\{P(A)\} = 2 = 2^1$;
- if $n(A)$ =2 (for example,), then $n\{P(A)\} = 4 = 2^2$;
- if $n(A)$ =3 (for example,), then $n\{P(A)\} = 8 = 2^3$.

From this observation, we see a pattern that if $n(A)$ = k,, then most possibly $n(P(A) = 2^k$,. Indeed, this is always true, which we mention now as a theorem below. The proof of this theorem is avoided as it is beyond the scope of the present discussion.

Theorem 14.2: Let *A* be a set containing *k* elements. Then the power set *A* contains 2^k elements.

The idea of power set is something like putting several small boxes inside a big box. Elements of a power set are itself sets. It is also clear that a subset of a power set contains some sets. In the definition of a set in the beginning, it was said that any well-defined objects can be in a set. As we have seen now, we can put sets also inside a set.



CHECK YOUR PROGRESS

Q 18: In human population studies, some groups of humans are taken at a time to study. What universal set may be considered in this case?

Q 19: Consider the sets *A* = {0, 1}, *B* = {1, 2, 5} and *C* = {0, 2, 4, 6, 8}. Which of the following may be considered as universal set(s) for all the three sets *A, B* and *C* ?

(i) N

(ii) Z

(iii) {0, 1, 2, 3, 4, 5, 6, 7, 8}

(iv) {0, 1, 2, 3, 4, 5, 6, 8}

Q 20: Write the power set for the following sets:

(i) $\{\phi\}$ (ii) $\{\phi, a\}$ (iii) $\{\phi, \{a\}\}$ (iv) {1, 2, 3, 4}

(v) $\{\phi, 1, \{1,2\}, b\}$ (vi) $\{\{a\}\}$



14.10 LET US SUM UP

- A **set** is a collection of some **well-defined** objects.
- Capital letters of English alphabets are usually used to denote a set. The small letters of English alphabets are usually used to denote the members in a set.
- If *A* is a set and *a* is a member of *A*, then we write $a \in A$, which is read as '*a* does not belongs to *A*'.
- If *A* is a set and *a* is **not** a member of *A*, then we write $a \notin A$, which is read as '*a* does not belong to *A*'.
- There are two methods of writing sets. They are tabular (roster) method and set builder (rule) method.
- In tabular method, all the elements of a set are listed, separated each by comma, inside a pair of braces. For example, {1, 2, red, *a*, Man}.
- If the elements in a set are characterized by a common property, then the set can be written in set builder method. For example,

  B = {x : x is an even natural number} of B = {x : x = 2k, for some k $\in$ N}
- There are sets which cannot be written in tabular method. There are sets which are not suitable to be written in set builder method. There are sets which can be written in both the methods.
- A set which does not contain any element is called a null set. Null set is unique, denoted by ϕ.

- A set which contains a definite number of elements is called a finite set. Otherwise, it is called an infinite set.
- The number of elements in a set is denoted by $n(S)$.
- $n(\phi) = 0$ while $n(S) = \infty$ if S is an infinite set.
- If every element of a set A is also an element of set B, then we say that A is a ***subset*** of B. We write this in symbols as $A \subseteq B$, which is read as 'A is a subset of B'.
- If $A \subseteq B$, we also say that B is a ***superset*** of A, written in symbols as $B \subseteq A$, which is read as 'B is a superset of A'.
- $A \subseteq B$ if $x \in A \Rightarrow x \in B$
- If A is not a subset of B, then we write that $A \not\subset B$.
- If $A \not\subset B$ then there is at least one element in A which is not in B. Indeed, $A \not\subset B \Rightarrow$ there is some $x \in A$ such that $x \notin B$.
- $\phi \subseteq \phi$, $\phi \subseteq A$, $A \subseteq A$ for every set A.
- If we write $A \subseteq B$, it covers the possibility of $A = B$.
- If $A \subseteq B$ and $A \neq B$ then we say that A is a ***proper subset*** of B, written symbolically as $A \subset B$.
- $A \subset B$ if $x \in A \Rightarrow x \in B$ and there is some $y \in B$ but $y \notin A$.
- Let A and B be two sets. Then $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
- In a particular topic or context, if every sets of that context are subsets of a fixed basic set, then that basic set is called a ***universal set*** of that particular context.
- A universal set is usually denoted by U.
- A universal set of a particular context is never unique. It depends on the context, personal choice and conveniences.
- The set of all subsets of a given set A is called the ***power set*** of A.
- The power set of A is denoted by P (A) or 2^A.
- If $n(A) = k$ then the power set P (A) contains 2^k elements.
- Power sets are never empty.



14.11 FURTHER READING

1) Seymour Lipschutz, 2005, *Set Theory and Related Topics*, Tata McGraw-Hill.

2) V. K. Balakrishnan, 1996, *Introductory Discrete Mathematics*, Dover Publications, Inc.

3) 2006, *Mathematics, Text book for Class XI*, NCERT, New Delhi.



14.12 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: In our daily life, we use bags, boxes, containers etc. to keep different items properly and systematically. In mathematics, we deal with certain mathematical objects, mostly numbers at the initial stages. So, we need something to keep those mathematical objects systematically. Similar to bags, boxes or containers, we use the term 'set' to refer such an entity inside which we keep our mathematical objects to be studied and we use a pair of braces '{ }' inside which we list the objects.

The theory of sets is the language of modern mathematics. The concept of sets plays an integral part in all branches of modern mathematics. The concept of sets is basically to group different objects of a particular mathematical discussion and view the whole as a single entity. This idea of set was first given by German Mathematician Georg Cantor, who lived during 1845 – 1918, towards the end of the nineteenth century. The theory of sets given by him is known as '*naïve set theory*'. Formally, a **set** is a collection of some **well-defined** objects. The objects which will be in the set should be well-defined in the sense that there should not be any ambiguity to decide whether a particular object belongs to a given set or not.

Ans to Q No 2: Descriptions that determines a set: (i), (ii), (iv), (v), (vi), (vii), (ix), (x), (xii), (xiii), (xiv).

Descriptions that does not determine a set: (iii), (viii),(xi).

Ans to Q No 3: *A* = set of natural numbers less than 5, *B* = set of roots of roots of $x^2 - 3x + 2 = 0$.

Ans to Q No 4: *A* = set of all circles that can be drawn in a plane, *B* = set of all complex numbers.

Ans to Q No 5: *A* = {1, *b,* rice}, *B* = {Ram, Monday, 5, x}.

Ans to Q No 6: (i) {–1, –2, –3, –4, . . .} = {x : x = –n for some $n \in \mathrm{N}$}.

(ii) {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday} = {x : x is a day in a week}.

(iii) {10, 20, 30, 40, . . .} = {x : x = 10k for some $k \in \mathrm{N}$}.

(iv) {A, M, E, R, I, C} = {x : x is a letter in the word 'AMERICA'}.

Ans to Q No 7: (i) {4, 3, 2, 1, 0, –1, –2, –3, . . .}.

(ii) {2, 3, 5, 7, 11, 13}.

(iii) {April, June, September, November}.

(iv) {2, 5, 7}

Ans to Q No 8: (i) {x : x = n^2 for $n \in$ (1, 2, 3, 4, 5)}.

(ii) {x : x = n^2 for some $n \in \mathrm{N}$}.

(iii) {x : x is an odd integer} or {x : x = 2k – 1 for some $k \in \mathrm{N}$}

(iv) {x : x is an odd integer, $x \geq -3$}

Ans to Q No 9: *A* = {x : $x^2 < 0$, $x \in \mathrm{N}$}, *B* = set of the names of the deserts located in Assam.

Ans to Q No 10: The empty sets are (i), (iii) and (iv).

Ans to Q No 11: {1}, {SET}, $\{\{\phi, 1\}\}$, {x : $x^2 = 4$, $x \in \mathrm{N}$}, {x : $x^2 = 4$, x < 0}.

Ans to Q No 12: The orders of ϕ and $\{\phi\}$ are 0 and 1, respectively. Also, $\phi \neq \{\phi\}$.

Ans to Q No 13: {1}, {1, 2}, {1, 2, 3}, {1, 2, 3, 4}, {1, 2, 3, 4, 5}.

Ans to Q No 14: (i) True. If a set *A* has *k* elements then every subset of *A* has at most *k* elements.

(ii) False. The set N of natural numbers is infinite. The set of even natural numbers is infinite and also a subset of N.

Ans to Q No 15: Finite sets: (iii), (iv).**Infinite sets**: (i), (ii), (v), (vi).

Ans to Q No 16: True: (ii), (iii), (vii).**False**: (i), (iv), (v), (vi).

Ans to Q No 17: Given that $A \subseteq B$ and $B \subseteq C$. Let $x \in A$. Since $A \subseteq B$, we have $x \subseteq B$. Now $x \subseteq B$ and $B \subseteq C$ implies that $x \subseteq C$. We have started with $x \in A$ and found that $x \subseteq C$ .Therefore we find $A \subseteq C$.

Ans to Q No 18: The set of all living men and women in the world.

Ans to Q No 19: (ii), (iii) and (iv).

Ans to Q No 20: (i) $\{\phi, \{\phi\}\}$

(ii) $\{\phi, \{\phi\}, \{a\}, \{\phi, a\}\}$

(iii) $\{\phi, \{\phi\}, \{\{a\}\}, \{\phi, \{a\}\}\}$

(iv) $\left\{\begin{matrix}\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\} \\ , \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\end{matrix}\right\}$

(v) $\left\{\begin{matrix}\phi, \{\phi\}, \{1\}, \{1,2\}, \{b\}, \{\phi,1\}, \{\phi, \{1,2\}\}, \{\phi, b\}, \{1, \{1,2\}\}, \{1, b\} \\ \{\{1,2,\}, b\}, \{\phi,1, \{1,2\}\}, \{\phi,1,b\}, \{\phi, \{1,2\}, b\}, \{1, \{1,2\}, b\}, \{\phi,1, \{1,2\}, b\}\end{matrix}\right\}$

(vi) $\{\phi, \{\{a\}\}\}$.



14.13 MODEL QUESTIONS

Q 1: Determine which of the following descriptions represent a set. In case the description represents a set, write the set in tabular or set builder method.

a) Collection of all natural numbers that are greater than –2.

b) Collection of good musician of our country.

c) Collection of the number 3, your surname, the word 'Assam' and the set {a, b}.

d) Collection of the positive real numbers which are less than –2.

e) Collection of all the finite sets whose order is 5.

Q 2: Let the set of all natural numbers, N = the set of all integers, Z = set of all integers that are divisible by 3. Express the following sets in language:

a) B = {x : x$\in$N, x$\in$Z };

b) C = {x : $x \notin N$, $x \in Z$};

c) D = {x : $x \in N$, $x \in A$ };

d) E = {x : $x \in N$, $x \notin A$ };

e) F = {x : $x \notin N$, $x \in A$ };

Q 3: Represent the sets in Q 2 in tabular method.

Q 4: Give five (different descriptions) examples of null sets and five examples of singleton sets.

Q 5: Let A = {a, b, c}. How many subsets of P (P (A)) can have?

Q 6: Write all the subsets of $\{0, \{\phi, 1, 2\}\}$.

Q 7: Determine, with justification, which of the following sets are finite or infinite:

a) the set of all prime numbers;

b) the set of all integers that are less than 10;

c) the set of all bicycles in the world;

d) $\left\{x : x = \frac{m}{n}\right.$, m and n are natural numbers $\left.\right\}$;

e) $\left\{x : x = \frac{m}{n}\right.$, m and n are natural numbers $m \le 5, n \le 6$ $\left.\right\}$;

f) {x : x is an integer, $-2 \le x \le 5, 9$ }

Q 8: Let *A* = {ϕ , 2, 3} and *B* = {2, 3, *A*, 5}. Determine, with justification, which of the following statements are true or false.

(i) $A \subseteq B$, (ii) $\varnothing \in B$, (iii) $\varnothing \subset A$, (iv) $A \in B$, (v) $\{\varnothing\} \subset A$, (vi) $\{\varnothing\} \subset B$
(vii) $\{\varnothing, A\} \subseteq A$, (viii) $\{\varnothing, A\} \subseteq B$, (ix) $\varnothing \in A$, (x) $\{A\} \subset B$.

Q 9: For each of the following, two sets *A* and *B* are given. Determine whether

$A = B, A \subset B, B \subset A, A \not\subset B$ or $B \not\subset A$

a) $A = \{1, 2\}$, $B = \{x : x \in \square, x^2 - 3x + 2 = 0\}$

b) *A* = { x : x is a letter in the word FOLLOWER}, *B* = {x : x is a letter in the word WOLF}

c) *A* = {–2, 2, 4, 6}, *B* = {x : x is an even positive integer and $x \le 9$ }

d) *A* = { x : x is an even natural number}, *B* = {x : x is a natural number divisible by 6}.

e) $A = \{\varnothing\}$, $B = \{x : x \in \square, x^2 = 1\}$.

Q 10: Write all the subsets of {a, b, c, d, (a)}.

Q 11: Show that {2, 3, 4, 5} is a proper subset of {1, 2, 3, 4, 5, 6, 7, 8, 9}.

Q 12: Show that {2, 3, 4, 5} is not a subset of {x : x is an even number, $x \in \mathrm{N}$}

Q 13: If the power set of *A* is equal to the power set of *B*, does it follow that *A* and *B* are equal? Justify.

Q 14: Determine whether each of the following statements is true or false in case of three arbitrary sets *P, Q* and *R*.

(i) If is an element of *Q* and if *Q* is a subset of *R*, then *P* is an element of *R*.

(ii) If is an element of *Q* and if *Q* is a subset of *R*, then *P* is also a subset of *R*.

(iii) If is a subset of *Q* and if *Q* is an element of *R*, then *P* is an element of R.

(iv) If is a subset of *Q* and if *Q* is an element of *R*, then *P* is a subset of *R*.

Q 15: Let *A* and *B* be two sets such that P (*A*) = P (*B*). Prove that *A* = *B.*

*** ***** ***

UNIT 15: SET OPERATION

UNIT STRUCURE

15.1 Learning objectives

15.2 Introduction

15.3 Union of sets

15.4 Intersection of sets

15.5 Difference of Sets

15.6 Complement of a set

15.7 Let us sum up

15.8 Further reading

15.9 Answers to check your progress

15.10 Model questions

15.1 LEARNING OBJECTIVES

After going through this unit, you will be able to

- learn the concept, examples and properties of union of sets;
- learn the concept, examples and properties of intersection of sets;
- learn the concept, examples and properties of difference of sets;
- learn the concept, examples and properties of relative complement of a set;
- learn several properties involving union, intersection and difference of sets.

15.2 INTRODUCTION

In our daily life, sometimes we need to place objects of two bags into a new bag. In our primary school, we learnt additions, subtraction, multiplication and division, called operations on numbers, to produce a new number out of a few given numbers. In the same spirit, we have some operations on sets through which we can produce new sets out of a few given sets.

In this unit, we shall discuss three basic operations of sets, that is,

the operation of union, intersection and difference. After that we shall discuss complement of a set, which is indeed a difference of the set from a universal set. We shall also discuss several examples and properties of these set operations.

15.3 UNION OF SETS

The idea of union of two sets is similar to keeping objects of two bags into one bag. This operation of sets is also similar to the addition of two numbers. Let us take an example. Consider the sets $A = \{1, 2, 3, a\}$ and $B = \{2, 4, 5, a, b\}$.

If we include all the elements of these two sets in a new set, since repetition of elements is not allowed in a set, we find the new set $\{1, 2, 3, a, 4, 5, b\}$. Formally, the union of two sets A and B is the new set which only contains all the elements of A and all the elements of B , the common elements being taken only once to avoid their repetition. The symbol $A \cup B$ is used to denote the union of the sets, which is read as 'A union B'. More precisely, $A \cup B$ ={x : x $\in$ A or x$\in$B
}

It is to be noted that the meaning of 'or' here is inclusive, in the sense that '$x \in A$' means either x belongs to A or x belongs to B or x belongs to both *A* and *B*. In other words, $x \in A \cup B$ implies exactly one of the following three possibilities are true:

- $x \in A$ but $x \notin B$;
- $x \notin A$ but $x \in B$;
- $x \in A$ but $x \in B$.

Let us consider the sets A = {1, 2}, B = {1, 2, 3}, C = {1, 3, 5} and U = {1, 2, 3, 4, 5, 6, 7} as universal set. Let us do the following unions and observe certain facts:

a) We have $A \cup C = \{1, 2\} \cup \{1, 3, 5\} = \{1, 3, 5\} = \{1, 2, 3, 5\}$ and $C \cup A = \{1, 3, 5\} \cup \{1, 2\} = \{1, 2, 3, 5\}$. We see that $A \cup C = C \cup A$. This is something like commutative property of addition of two numbers, for example, 2+3=3+2.

b) We have

$(A \cup B) \cup C = \big(\{1,2\} \cup \{1,2,3\}\big) \cup \{1,3,5\} = \{1,2,3\} \cup \{1,3,5\} = \{1,2,3,5\}$ and

$A \cup (B \cup C) = \{1,2\} \cup \big(\{1,2,3\} \cup \{1,3,5\}\big) = \{1,2\} \cup \{1,2,3,5\} = \{1,2,3,5\}$.

We see that $(A \cup B) \cup C = A \cup (B \cup C)$. This is something like associative property of addition of three numbers, for example, (2+3)+4 = 2+(3+4). Note that in a union of three sets like $(A \cup B) \cup C$, first we need to take the union of the sets inside the bracket and then union of the new set with the third set is taken.

c) We have $A \cup \varnothing = \{1,2\} \cup \varnothing = \{1,2\} = A$. This is something like law of identity of addition of numbers, for example, 2+0=2.

d) We have $A \cup A = \{1,2\} \cup \{1,2\} = \{1,2\} = A$.

e) $U \cup A = \{1,2,3,4,5,6,7\} \cup \{1,2\} = \{1,2,3,4,5,6,7\} = U$.

f) We see that $A \subseteq B$ and that $A \cup B = \{1,2\} \bigcup \{1,2,3\} = \{1,2,3\} = B$.

g) As $B \cup C = \{1,2,3\} \cup \{1,3,5\} = \{1,2,3,5\}$, we see that $B \subseteq B \cup C$ and $C \subseteq B \cup C$.

All the observations made in the previous examples are indeed true in case of any sets. Now we state these properties for any arbitrary sets and provide their proof.

Theorem 15.1: Let *A, B* and *C* be three arbitrary sets and let *U* be their universal set. Then the following properties hold good:

(i) $A \subseteq A \cup B$ and $B \subseteq A \cup B$;

(ii) $A \cup B = B \cup A$ [Commutative Law];

(iii) $(A \cup B) \cup C = A \cup (B \cup C)$ [Associative Law];

(iv) $A \cup \varnothing = A$ [Law of Identity Element];

(v) $A \cup A = A$ [Idempotent Law];

(vi) $U \cup A = U$ [Law of Universal Set];

(vii) $A \subseteq B \Leftrightarrow A \cup B = B$.

Proof: Except the first property, notice that all other properties are about to show equality of sets. Since these sets are arbitrary, we do not have idea about the exact elements of these sets. So, we must use Theorem 14.1 of Unit 14 to establish the equality of these sets. Recall Theorem 14.1 that two sets *X* and *Y* are equal if and only if $X \subseteq Y$ and $Y \subseteq X$.

(i) We shall prove $A \subseteq A \cup B$. The other part is left as an exercise for 'CHECK YOUR PROGRESS'. We have

$x \in A \Rightarrow x \in A$ or $x \in B \Rightarrow x \in A \cup B$

Therefore. $A \subseteq A \cup B$

(ii) Applying the definition of union, we have

$x \in A \cup B \Rightarrow x \in A$ or $x \in B \Rightarrow x \in B$ or $x \in A \Rightarrow x \in B \cup A$.

Thus every element of $A \cup B$ is also an element of $B \cup A$, and therefore $A \cup B \subseteq B \cup A$. Again

$x \in B \cup A \Rightarrow x \in B$ or $x \in A \Rightarrow x \in A$ or $x \in B \Rightarrow x \in A \cup B$

Therefore $B \cup A \subseteq A \cup B$. Now from $A \cup B \subseteq B \cup A$ and $B \cup A \subseteq A \cup B$, we get $A \cup B = B \cup A$.

(iii) Applying the definition of union, we have

$$\begin{aligned} x \in (A \cup B) \cup C &\Rightarrow x \in A \cup B \text{ or } x \in C \\ &\Rightarrow (x \in A \text{ or } x \in B) \text{ or } x \in C \\ &\Rightarrow x \in A \text{ or } x \in B \text{ or } x \in C \\ &\Rightarrow x \in A \text{ or } (x \in B \text{ or } x \in C) \\ &\Rightarrow x \in A \text{ or } x \in B \cup C \\ &\Rightarrow x \in A \cup (B \cup C). \end{aligned}$$

Thus every element of $(A \cup B) \cup C$ is also an element of $A \cup (B \cup C)$, and therefore $(A \cup B) \cup C \subseteq A \cup (B \cup C)$. Again

$$\begin{aligned} x \in A \cup (B \cup C) &\Rightarrow x \in A \text{ or } x \in B \cup C \\ &\Rightarrow x \in A \text{ or } (x \in B \text{ or } x \in C) \\ &\Rightarrow x \in A \text{ or } x \in B \text{ or } x \in C \\ &\Rightarrow (x \in A \text{ or } x \in B) \text{ or } x \in C) \\ &\Rightarrow x \in A \cup B \text{ or } x \in C \\ &\Rightarrow x \in (A \cup B) \cup C. \end{aligned}$$

Therefore $A \cup (B \cup C) \subseteq (A \cup B) \cup C$. Now fromand $(A \cup B) \cup C \subseteq A \cup (B \cup C)$ and $A \cup (B \cup C) \subseteq (A \cup B) \cup C$, we get $(A \cup B) \cup C = A \cup (B \cup C)$.

(iv) Recall from the definition of union that if $x \in A \cup B$, then exactly one of the three possibilities, namely, ($x \in A$ and $x \notin B$) or ($x \notin A$ and $x \in B$) or ($x \in A$ and $x \in B$) are true. If we know that $x \notin B$, then surely

the last two possibilities are false, and so the first possibility must be true. On the other hand, if we know that $x \in A$ is true, then irrespective of the truth value of $x \in B$ the statement '$x \in A$ or $x \in B$' is true. We are going to use this logic in proving the Law of Identity. We have

$$x \in A \cup \varnothing \Rightarrow x \in A \text{ or } x \in \varnothing$$
$$\Rightarrow x \in A, \text{ as } x \in \varnothing \text{ is false.}$$

Thus every element of $A \cup \varnothing$ is also an element of *A*, and therefore $A \cup \varnothing \subseteq A$. Again $x \in A \Rightarrow x \in A \text{ or } x \in \varnothing$
$\Rightarrow x \in A \cup \varnothing$.

Therefore $A \subseteq A \cup \varnothing$. Now fromand $A \cup \varnothing \subseteq A$ and $A \subseteq A \cup \varnothing$, we get $A \cup \varnothing = A$.

(v) We have $x \in A \cup A \Rightarrow x \in A \text{ or } x \in A \Rightarrow x \in A$

Thus every element of $A \cup A$ is also an element of *A*, and therefore $A \cup A \subseteq A$. Again

$x \in A \Rightarrow x \in A \text{ or } x \in A \Rightarrow x \in A \cup A.$

Therefore $A \subseteq A \cup A$. Now fromand $A \cup A \subseteq A$ and $A \subseteq A \cup A$, we get $A \cup A = A$.

(vi) Since *U* is the universal set and $U \cup A$ is a set in this universal set, we have $U \cup A \subseteq U$. Again

$x \in U \Rightarrow x \in U \text{ or } x \in A \Rightarrow x \in U \cup A.$

Therefore $U \subseteq U \cup A$. Now fromand $U \cup A \subseteq U$ and $U \subseteq U \cup A$, we get $U = U \cup A$.

(vii) Recall that the meaning of the symbol '$\Leftrightarrow$' is 'if and only if' that is, both way implications. So to prove $A \subseteq B \Leftrightarrow A \cup B = B$, we basically need to prove two facts. First, if $A \subseteq B$ then to show $A \cup B = B$. Conversely, if then to show. We will prove the first fact, and the second is left as an exercise for 'CHECK YOUR PROGRESS'.

Assume that $A \subseteq B$. Now we have

$$x \in A \cup B \Rightarrow x \in A \text{ or } x \in B \Rightarrow x \in B \text{ or } x \in B, \text{ as } A \subseteq B$$
$$\Rightarrow x \in B.$$

Thus every element of $A \cup B$ is also an element of *B*, and therefore $A \cup B \subseteq B$. Again

$$x \in B \Rightarrow x \in A \text{ or } x \in B \Rightarrow x \in A \cup B.$$

Therefore $B \subseteq A \cup B$. Now from $A \cup B \subseteq B$ and $B \subseteq A \cup B$, we get $A \cup B = B$. //

Let us do the following analysis based on the commutative and associative laws of union. According to associative law, we have $(A \cup B) \cup C = A \cup (B \cup C)$. This means that while taking the union of three sets, it does not matter whether we calculate the union of *A* and *B* first or the union of *B* abd *C* first. Further, using the commutative and associative laws, we find that $A \cup (B \cup C) = A \cup (C \cup B) = (A \cup C) \cup B$.

Thus in the union of three sets *A, B* and *C*, taking the union of *B* and *C* first or *A* and *C* first will give us the same result. It turns out that, we can take the union of any two sets first in a union of three sets *A, B* and *C*. Due to this fact, sometimes the union $(A \cup B) \cup C$ is simply written as $A \cup B \cup C$. In the same manner, we can take union of four sets; five sets or rather, any number of sets. Indeed, we can take union of infinitely many sets. However, such unions are avoided here as it is beyond the present scope of discussion.

Note: The inclusions $A \subseteq A \cup \varnothing, A \subseteq A \cup A, U \subseteq U \cup A$ and $B \subseteq A \cup B$ in parts (iv), (v), (vi) and (vii) of **Theorem 15.1**, respectively, can also be obtained by using part (i) of **Theorem 15.1**. However, for the sake of offering a direct independent proof, usage of part (i) has been avoided. Otherwise, you are free to use part (i) for the proof of these inclusions.



CHECK YOUR PROGRESS

Q 1: For each of the questions below, find the union of the pair of sets given.

(i) $A = \{1, 2, a, f, x, 4, 0\}$, $B = \{1, 2, \varnothing, \theta, f, 3, \pi\}$.

(ii) $A = \{1, 2, a, f, 0\}$, $B = \{\varnothing\}$.

(iii) *A* = {x : x is an odd natural number}, B = □.

(iv) *A* = {x : x is a multiple of 2 and $x \in$ □}, *B* = {x : x is a multiple of 3 and $x \in$ □}

(v) *A* = {x ; x = –n for some $n \in$ □}, B = □.

Q 2: Show that $B \subseteq A \cup B$.

Q 3: If $A \cup B = B$ then show that $A \subseteq B$.

Q 4: Let *A* and *B* be sets such that $A \cup X = B \cup X$ for every set *X*. Does it imply that *A* = *B* ? Justify.

Q5. If $A \cup B = \varnothing$ then prove that $A = \phi$ and $B = \phi$.

15.4 INTERSECTION OF SETS

The idea of intersection of two sets is similar to keeping the common objects of two bags into a new bag. Though this operation of sets is not similar to the multiplication of two numbers, we will see that this operation has some properties that are similar to some properties of multiplication of two numbers. Let us take an example. Consider the sets *A* = {1, 2, 3, a} and *B* = {2, 4, 5, a, b}.

If we choose all the common elements of these two sets and form a new set out of these elements, we find a new set {2, a}. Formally, the intersection of two sets *A* and *B* is the new set which contains all the common elements of *A* and *B*. The symbol $A \cap B$ is used to denote the intersection of the sets *A* and *B*, which is read as '*A* intersection *B*'. More precisely,

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Thus $x \in A \cap B$ implies that $x \in A$ as well as $x \in B$. Note that if $x \notin A \cap B$ then either $x \in A$ or $x \in B$. It is so natural that while taking two bags, there can surely be the possibility that they do not have any objects in common. Similarly, there are sets also having no elements common. Such sets are called disjoint sets. Formally, two sets *A* and *B* are said to be ***disjoint*** if they do not have any common elements, and symbolically we write $A \cap B = \varnothing$. For example, if *A*= {a, b} and *B* = {c, d, e} then *A* and *B* are disjoint and so we can write $A \cap B = \varnothing$.

Let us consider the sets

A = {1, 2}, *B* = {1, 2, 3}, *C* = {1, 3, 5} and U = {1, 2, 3, 4, 5, 6, 7} as universal set.

Let us do the following intersections and observe certain facts:

a) We have $A \cap C = \{1,2\} \cap \{1,3,5\} = \{1\}$ and $C \cap A = \{1,3,5\} \cap \{1,2\} = \{1\}$

We see that $A \cap C = C \cap A$. This is something like commutative

property of multiplication of two numbers, for example, 2 x 3 = 3 x 2..

b) We have

$(A \cap B) \cap C = \big(\{1,2\} \cap \{1,2,3\}\big) \cap \{1,3,5\} = \{1,2\} \cap \{1,3,5\} = \{1\}$ and

$A \cap (B \cap C) = \{1,2\} \cap \big(\{1,2,3\} \cap \{1,3,5\}\big) = \{1,2\} \cap \{1,3\} = \{1\}$.

We see that $(A \cap B) \cap C = A \cap (B \cap C)$. This is something like associative property of multiplication of three numbers, for example, (2 x 3) x 4 = 2 x (3 x 4). Note that in an intersection of three sets like $(A \cap B) \cap C$, first we need to take the intersection of the sets inside the bracket and then the intersection of the new set with the third set is taken.

c) We have $A \cap \varnothing = \{1,2\} \cap \varnothing = \varnothing$. This is something like the multiplication of any number with 0 produces always 0, for example, 2 x 0 = 0.

d) We have $A \cap A = \{1,2\} \cap \{1,2\} = \{1,2\} = A$.

e) $U \cap A = \{1,2,3,4,5,6,7\} \cap \{1,2\} = \{1,2\} = A$.

f) We see that $A \subseteq B$ and that $A \cap B = \{1,2\} \cap \{1,2,3\} = \{1,2\} = A$.

g) As $B \cap C = \{1,2,3\} \cap \{1,3,5\} = \{1,3\}$, we see that $B \cap C \subseteq B$ and $B \cap C \subseteq C$.

h) We have $B \cup (B \cap C) = \{1,2,3\} \cup \{1,3\} = \{1,2,3\} = B$. Similarly, $B \cap (B \cup C) = B$.

i) We have $A \cup (B \cap C) = \{1,2,3\} \cup \{1,3\} = \{1,2,3\}$ and

$(A \cup B) \cap (A \cup C) = \big(\{1,2\} \cup \{1,2,3\}\big) \cap \big(\{1,2\} \cup \{1,3,5\}\big) = \{1,2,3\} \cap \{1,2,3,5\} = \{1,2,3\}$

Thus $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. Similarly, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. This is something like the distributive property of multiplication of numbers over addition. For example,

2 x (3+5)=(2x3)+(2x5) and (2+3) x 5 = (2x5)+(3x5).

All the observations made in the previous examples are indeed true in case of any sets. Now we state these properties for any arbitrary sets and provide their proof.

Theorem 15.2: Let *A, B* and *C* be three arbitrary sets and let *U* be their universal set. Then the following properties hold good:

(i) $A \cap B \subseteq A$ and $A \cap B \subseteq B$

(ii) $A \cap B = B \cap A$ [Commutative Law];

(iii) $(A \cap B) \cap C = A \cap (B \cap C)$ [Associative Law];

(iv) $A \cap \varnothing = \varnothing$ [Law of Identity Element];

(v) $A \cap A = A$ [Idempotent Law];

(vi) $U \cap A = A$ [Law of Universal Set];

(vii) $A \subseteq B \Leftrightarrow A \cap B = A$;

(viii) $A \cap (A \cup B) = A$ and $A \cup (A \cap B) = A$ [Absorption Law];

(ix) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [Distributive Law (of union over intersection)];

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [Distributive Law (of intersection over union)].

Proof: Recall Theorem 14.1 that two sets *X* and *Y* are equal if and only if $X \subseteq Y$ and $Y \subseteq X$.

(i) We shall prove $A \cap B \subseteq A$. The other part is left as an exercise for 'CHECK YOUR PROGRESS'. We have

$x \in A \cap B \Rightarrow x \in A$ and $x \in B \Rightarrow x \in A.$

Therefore $A \cap B \subseteq A$.

(ii) Applying the definition of intersection, we have

$x \in A \cap B \Rightarrow x \in A$ and $x \in B \Rightarrow x \in B$ and $x \in A \Rightarrow x \in B \cap A.$

Thus every element of $A \cap B$ is also an element of $B \cap A$, and therefore $A \cap B \subseteq B \cap A$. Again

$x \in B \cap A \Rightarrow x \in B$ and $x \in A \Rightarrow x \in A$ and $x \in B \Rightarrow x \in A \cap B.$

Therefore $B \cap A \subseteq A \cap B$. Now from $A \cap B \subseteq B \cap A$ and $B \cap A \subseteq A \cap B$, we get $A \cap B = B \cap A$.

(iii) Applying the definition of intersection, we have

$$\begin{aligned} x \in (A \cap B) \cap C &\Rightarrow x \in A \cap B \text{ and } x \in C \\ &\Rightarrow (x \in A \text{ and } x \in B) \text{ and } x \in C \\ &\Rightarrow x \in A \text{ and } x \in B \text{ and } x \in C \\ &\Rightarrow x \in A \text{ and } (x \in B \text{ and } x \in C) \\ &\Rightarrow x \in A \text{ and } x \in B \cap C \\ &\Rightarrow x \in A \cap (B \cap C). \end{aligned}$$

Thus every element of $(A \cap B) \cap C$ is also an element of $A \cap (B \cap C)$, and therefore $(A \cap B) \cap C \subseteq A \cap (B \cap C)$. Again

$$x \in A \cap (B \cap C) \Rightarrow x \in A \text{ and } x \in B \cap C$$
$$\Rightarrow x \in A \text{ and } (x \in B \text{ or } x \in C)$$
$$\Rightarrow x \in A \text{ and } x \in B \text{ and } x \in C$$
$$\Rightarrow (x \in A \text{ and } x \in B) \text{ and } x \in C)$$
$$\Rightarrow x \in A \cap B \text{ and } x \in C$$
$$\Rightarrow x \in (A \cap B) \cap C.$$

Therefore $A \cap (B \cap C) \subseteq (A \cap B) \cap C$. Now from $(A \cap B) \cap C \subseteq A \cap (B \cap C)$ and $A \cap (B \cap C) \subseteq (A \cap B) \cap C$, we get $(A \cap B) \cap C = A \cap (B \cap C)$.

(iv) Let *x'* be any possible element of $A \cap \varnothing$. We have

$$x \in A \cap \varnothing \Rightarrow x \in A \text{ and } x \in \varnothing$$
$$\Rightarrow x \in \varnothing, \text{ which is impossible.}$$

Thus $x \in A \cap \varnothing$ is false for any element *x*, and therefore $A \cap \varnothing = \varnothing$.

(v) We have

$$x \in A \cap A \Rightarrow x \in A \text{ and } x \in A \Rightarrow x \in A$$

Thus every element of $A \cap A$ is also an element of*A*, and therefore $A \cap A \subseteq A$. Again

$$x \in A \Rightarrow x \in A \text{ and } x \in A \Rightarrow x \in A \cap A.$$

Therefore $A \subseteq A \cap A$. Now from $A \cap A \subseteq A$ and $A \subseteq A \cap A$, we get $A \cap A = A$.

(vi) We have

$$x \in U \cap A \Rightarrow x \in U \text{ and } x \in A$$
$$\Rightarrow x \in A.$$

Thus every element of $U \cap A$ is also an element of *A*, and therefore $U \cap A \subseteq A$. Again

$$x \in A \Rightarrow x \in A \text{ and } x \in A$$
$$\Rightarrow x \in U \text{ and } x \in A, \text{ as } A \subseteq U$$
$$\Rightarrow x \in U \cap A.$$

Therefore $A \subseteq U \cap A$. Now from $U \cap A \subseteq A$ and $A \subseteq U \cap A$, we get $U \cap A = A$.

(vii) Recall that the meaning of the symbol '$\Leftrightarrow$' is 'if and only if' that is, both way implications. So to prove $A \subseteq B \Leftrightarrow A \cap B = A$, we basically need

to prove two facts. First, if $A \subseteq B$ then to show $A \cap B = A$. Conversely, if $A \cap B = A$ then to show $A \subseteq B$. We will prove the first fact, and the second is left as an exercise for 'CHECK YOUR PROGRESS'.

Assume that $A \subseteq B$. Now we have

$x \in A \cap B \Rightarrow x \in A$ and $x \in B \Rightarrow x \in A.$

Thus every element of $A \cap B$ is also an element of *A*, and therefore $A \cap B \subseteq A$. Again

$$\begin{aligned} x \in A &\Rightarrow x \in A \text{ and } x \in A \\ &\Rightarrow x \in A \text{ and } x \in B, \text{ as } A \subseteq B \\ &\Rightarrow x \in A \cap B. \end{aligned}$$

Therefore $A \subseteq A \cap B$. Now from $A \cap B \subseteq A$ and $A \subseteq A \cap B$, we get $A \cap B = A$.

(viii) We will prove $A \cap (A \cup B) = A$. The proof of $A \cup (A \cap B) = A$ is left as an exercise for 'CHECK YOUR PROGRESS'. We have now

$x \in A \cap (A \cup B) \Rightarrow x \in A$ and $x \in A \cup B \Rightarrow x \in A$

Thus every element of $A \cap (A \cup B)$ is an element of *A*, and therefore $A \cap (A \cup B) \subseteq A$,. Again

$$\begin{aligned} x \in A &\Rightarrow x \in A \text{ and } x \in A \\ &\Rightarrow x \in A \text{ and } (x \in A \text{ or } x \in B) \\ &\Rightarrow x \in A \text{ and } x \in A \cup B \\ &\Rightarrow x \in A \cap (A \cup B). \end{aligned}$$

Therefore $A \subseteq A \cap (A \cup B)$. Now from $A \cap (A \cup B) \subseteq A$ and $A \subseteq A \cap (A \cup B)$, we conclude that $A \cap (A \cup B) = A$.

(ix) We shall prove the first law. The other one is left as an exercise for 'MODEL QUESTIONS'. Applying the definitions of union and intersection, we have

$$\begin{aligned} x \in A \cup (B \cap C) &\Rightarrow x \in A \text{ or } x \in B \cap C \\ &\Rightarrow x \in A \text{ or } (x \in B \text{ and } x \in C) \\ &\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C) \\ &\Rightarrow x \in A \cup B \text{ and } x \in A \cup C \\ &\Rightarrow x \in (A \cup B) \cap (A \cup C). \end{aligned}$$

Thus every element of $A \cup (B \cap C)$ is also an element of $(A \cup B) \cap (A \cup C)$, and therefore $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$. Again

$$\begin{aligned} x \in (A \cup B) \cap (A \cup C) &\Rightarrow x \in A \cup B \text{ and } x \in A \cup C \\ &\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C) \\ &\Rightarrow x \in A \text{ or } (x \in B \text{ and } x \in C) \\ &\Rightarrow x \in A \text{ or } x \in B \cap C \\ &\Rightarrow x \in A \cup (B \cap C). \end{aligned}$$

Therefore $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$. Now from $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ and $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$, we get. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ //

Let us do the following analysis based on the commutative and associative laws of intersection. According to associative law, we have. This means that while taking the intersection of three sets $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, it does not matter whether we calculate the intersection of *A* and *B* first or the intersection of *B* and *C* first. Further, using the commutative and associative laws, we find that

$$A \cap (B \cap C) = A \cap (C \cap B) = (A \cap C) \cap B.$$

Thus in the intersection of three sets *A, B* and *C*, taking the intersection of *B* and *C* first or *A* and *C* first will give us the same result. It turns out that, we can take the intersection of any two sets first in an intersection of three sets. Due to this fact, sometimes the intersection $(A \cap B) \cap C$ is simply written as $A \cap B \cap C$. In the same manner, we can take intersection of four sets; five sets or rather, any number of sets. Indeed, we can take intersection of infinitely many sets. However, such intersections are avoided here as it is beyond the present scope of discussion.

Note: The inclusions $A \cap A \subseteq A, U \cap A \subseteq A, A \cap B \subseteq A$ and $A \cap (A \cup B) \subseteq A$ in parts (v), (vi), (vii) and (viii) of **Theorem 15.2**, respectively, can also be obtained by using part (i) of **Theorem 15.2**. However, for the sake of offering a direct independent proof, usage of part (i) has been avoided. Otherwise, you are free to use part (i) for the proof of these inclusions.



CHECK YOUR PROGRESS

Q 6: For each of the questions below, find the intersection of the pair of sets given.

(i) $A=\{1,2,a,f,x,4,0\}, \quad B=\{1,2,\varnothing,\theta,f,3,\pi\}$

(ii) $A=\{1,2,a,\varnothing,f,0\}, \quad B=\{\varnothing\}$

(iii) $A=\{x:x$ is an odd natural number$\}, \ B=N$

(iv) $A=\{x:x$ is a multiple of 2 and $x=N\}$,

$A=\{x:x$ is a multiple of 3 and $x=N\}$

(v) $A=\{x:x$ is an odd natural number$\}, \ B=N$

Q 7: Show that $A\cap B\subseteq B$.

Q 8: If $A\cap B=A$ then show that $A\subseteq B$.

Q 9: Let *A* be *B* two sets such that $A\cap X=B\cap X$ for every set x. Does it imply that $A=B$? Justify.

Q 10: If $A\cap B=U$ then prove that $A=U$ and $B=U$ where U is the associated universal set.

Q 11: Prove the absorption law $A\cup(A\cap B)=A$.

15.5 DIFFERENCE OF SETS

Given two bags of books, sometimes we may wish to select those books in one bag which are not available in the other bag. In the same way, suppose the sets $A=\{1,2,3,a\}$ and $B=\{2,4,5,a,b\}$ are given. Then $\{1,3\}$ is the set of those elements of A which are not present in B. The set $\{1,3\}$ is called the difference of A and B. Notice that, the set of those elements of B which are not present in A is {4, 5, b}. Formally, if A and B are two given sets, then the set of those elements of A which are not in B is called the ***difference*** of A and B. Symbolically, we write this set as $A-B$, which is read as 'A minus B'. More precisely,

$$A-B=\{x:x\in A \text{ and } x\notin B\}$$

In the same way, the set of those elements of B which are not in A is called the ***difference*** of B and A. Symbolically, we write this set as $B-$

A. More precisely,

$$B - A = \{x : x \in B \text{ and } x \notin A\}$$

Sometimes, we may wish to find the set of those elements which are either belonging to A or belong to B, but do not belong to both. Formally, such a set is called the ***symmetric difference*** of A and B, and is denoted by the symbol $A \oplus B$. That is $A \oplus B = (A - B) \cup (B - A)$.

Note that is $A \oplus B$ obtained by removing the common part $A \cap B$ from the union $A \cup B$. Indeed, the identity $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$ is true, which is left as an exercise for 'CHECK YOUR PROGRESS'.

Set difference has some properties that are similar to some properties of difference of two numbers. Let us consider the sets $A = \{1,2,3,4\}, B = \{1,3,5\}$ and $C = \{5,7,8\}$ and do the following set differences to observe certain facts:

a) We have $A - B = \{1,2,3,4\} - \{1,3,5\} = \{2,4\}$ and $B - A = \{1,3,5\} - \{1,2,3,4\} = \{5\}$. We see that $A - B \neq B - A$. This is something like the property of difference of two numbers, for example, unless both the numbers are equal, $a - b \neq b - a$.

b) We have $A - B = \{2,4\} \subseteq A$ and $B - A = \{5\} \subseteq B$.

c) We have $A \cap C = \varnothing$ and that $A - C = \{1,2,3,4\} - \{5,7,8\} = \{1,2,3,4\} = A$. Similarly, also we find $C - A = \{5,7,8\} = C$.

d) We have $A - \varnothing = \{1,2,3,4\} - \varnothing = \{1,2,3,4\} = A$ and $A - A = \{1,2,3,4\} - \{1,2,3,4\} = \varnothing$. This is something like $a - 0 = a$ and $a - a = 0$ for any number a.

e) As $A \cap B = \{1,2,3,4\} \cap \{1,3,5\} = \{1,3\}$, $A - B = \{2,4\}$ and $B - A = \{5\}$, we see that $(A - B) \cap (A \cap B) = \varnothing$, $(B - A) \cap (A \cap B) = \varnothing$ and $(A - B) \cap (B - A) = \varnothing$. Also we have $(A - B) \cup (A \cap B) \cup (B - A) = \{2,4\} \cup \{1,3\} \cup \{5\} = \{1,2,3,4,5\} = A \cup B$. Thus the three sets $A - B, A \cap B$ and $B - A$ are pair-wise disjoint and their union is $A \cup B$.

All the observations made in the previous examples are indeed true in case of any sets. Now we state these properties for any arbitrary sets and provide their proof.

Theorem 15.3: Let *A* and *B* be two arbitrary sets. Then the following properties hold good:

(i) $A - B \neq B - A$, unless both the setsare equal;

(ii) $A - B \subseteq A$ and $B - A \subseteq B$;

(iii) If $A \cap B = \varnothing$, then $A - B = A$ and $B - A = B$;

(iv) $A - \varnothing = A$;

(v) The sets $A - B, A \cap B$ and $B - A$ are pair-wise disjoint, and $(A - B) \cup (A \cap B) \cup (B - A) = A \cup B$.

Proof:

(i) If $A = B$, then we see that $x \in A - B \Rightarrow x \in A - A \Rightarrow x \in A$ and $x \notin A$. Notice that $x \in A$ and $x \notin A$ cannot be true at the same time. Therefore $x \in A - B$ is **not** possible, which gives $A - B = \varnothing$, that is, $A - A = \varnothing$. Now assume that $A \neq B$. Then at least one of the following two possibilities must be true:

- there is some $x \in A$ but $x \notin B$;
- there is some $x \in B$ but $x \notin A$;.

In the first case, $x \in A - B$ but $x \notin B - A$, and accordingly $A - B \neq B - A$. In the second case, $x \in B - A$ but $x \notin A - B$, and accordingly $A - B \neq B - A$.

Thus we see that unless both the sets *A* and *B* are equal, we have $A - B \neq B - A$.

(ii) We shall prove $A - B \subseteq A$. The other part is left as an exercise for 'CHECK YOUR PROGRESS'. We have $x \in A - B \Rightarrow x \in A$ and $x \notin B \Rightarrow x \in A$. Thus every element of *A – B* is also an element of *A*. Hence $A - B \subseteq A$.

(iii) Suppose $A \cap B = \varnothing$. We shall prove $A - B = A$. The other part is left as an exercise for '**MODEL QUESTIONS**'. We already have in Part (ii) that $A - B \subseteq A$. Now as $A \cap B = \varnothing$, we see that if $x \in A$ then cannot belong to B, as otherwise will be an element common to both *A* and *B*, contradicting $A \cap B = \varnothing$. Therefore

.Thus every element of *A* is also an element of, $A \cap B = \varnothing$ and therefore. Now from $A - B \subseteq A$ and $A \subseteq A - B$, we get $A = A - B$.

(iv) Since $A \cap \varnothing = \varnothing$, applying part (iii), we obtain that $A - \varnothing = A$.

(v) We shall prove that $(A-B)\cap(A\cap B)=\varnothing$. The proof of $(B-A)\cap(A\cap B)=\varnothing$ is left as an exercise for 'CHECK YOUR PROGRESS' and the proof of $(A-B)\cap(B-A)=\varnothing$ is left as an exercise for 'MODEL QUESTIONS'. We have

$$
\begin{aligned}
& x \in (A-B)\cap(A\cap B) \\
\Rightarrow\ & x \in A-B \text{ and } x \in A\cap B \\
\Rightarrow\ & (x \in A \text{ and } x \notin B) \text{ and } (x \in A \text{ and } x \in B) \\
\Rightarrow\ & x \in A \text{ and } x \notin B \text{ and } x \in B \\
\Rightarrow\ & x \notin B \text{ and } x \in B, \text{ which is impossible.}
\end{aligned}
$$

Thus $x \in (A-B)\cap(A\cap B)$ is **not** possible, which gives that $(A-B)\cap(A\cap B)=\varnothing$.

Now we will prove that $(A-B)\cup(A\cap B)\cup(B-A)=A\cup B$. Recall that $x \in A\cup B$. implies exactly one of the following three possibilities is true and vice versa:

- $x \in A$ and $x \notin B$;
- $x \notin A$ and $x \in B$;
- $x \in A$ and $x \in B$.

We have now

$$
\begin{aligned}
& x \in (A-B)\cup(A\cap B)\cup(B-A) \\
\Rightarrow\ & x \in A-B \text{ or } x \in A\cap B \text{ or } x \in B-A \\
\Rightarrow\ & (x \in A \text{ and } x \notin B) \text{ or } (x \in A \text{ and } x \in B) \text{ or } (x \in B \text{ and } x \notin A) \\
\Rightarrow\ & x \in A \text{ or } x \in B \\
\Rightarrow\ & x \in A\cup B.
\end{aligned}
$$

Thus $(A-B)\cup(A\cap B)\cup(B-A)\subseteq A\cup B$. Again we have

$$
\begin{aligned}
& x \in A\cup B \\
\Rightarrow\ & x \in A \text{ or } x \in B \\
\Rightarrow\ & (x \in A \text{ and } x \notin B) \text{ or } (x \in A \text{ and } x \in B) \text{ or } (x \in B \text{ and } x \notin A) \\
\Rightarrow\ & x \in A-B \text{ or } x \in A\cap B \text{ or } x \in B-A \\
\Rightarrow\ & x \in (A-B)\cup(A\cap B)\cup(B-A).
\end{aligned}
$$

Therefore $A\cup B\subseteq(A-B)\cup(A\cap B)\cup(B-A)$. Now $(A-B)\cup(A\cap B)\cup(B-A)\subseteq A\cup B$ and

$A \cup B \subseteq (A - B) \cup (A \cap B) \cup (B - A)$ altogether gives

$A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$. //



CHECK YOUR PROGRESS

Q 12: For each of the questions below, two sets are given. Find $A - B, B - A$ and $A \oplus B$.

(i) $A = \{1, 2, a, f, x, 4, 0\}$, $B = \{1, 2, \varnothing, \theta, f, 3, \pi\}$.

(ii) $A = \{1, 2, a, \varnothing, f, 0\}$, $B = \{\varnothing\}$.

(iii) *A* = { x : x is an odd natural number} *B* = *N*.

(iv) *A* = { x : x is a multiple of 2 and }, *B* = { x : x is a multiple of 3 and x = *N*}.

(v) *A* = { x : x = –*N* for some n}, *B* = *N*.

Q 13: Prove that $B - A \subseteq B$.

Q 14: Prove that $(B - A) \cap (A \cap B) = \varnothing$.

Q 15: Prove that $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$.

15.6 COMPLEMENT OF A SET

Let *U* = the set of all natural numbers. Let *U* be the universal set of a particular discussion, and let $A = \{x : x \geq 5, x \in N\}$. We observe that $1 \in U$ but $1 \notin A$, $2 \in U$ but $2 \notin U$, $3 \in U$ but $3 \notin U$ and $4 \in U$ but $4 \notin A$. Apart from these four elements, all other elements of *U* are also elements of *A*. So, the set of those elements of *U* that do not belong to *A* is {1, 2, 3, 4}. This set is nothing but the set *U*–*A*, and it is called the complement of *A*. In case, our universal set is *U* = the set of all even natural numbers, then we find *U* – *A* = {2, 4}. Thus we notice that if our universal set is changed, then its complement may also get changed. Due to this reason, the complement of a set is also referred as relative complement. This leads to the following definition:

Let A be a subset of a universal set U . Then the ***complement*** of A relative to U is the set of all elements of U that do not belong to A . In symbols, we write A^c to denote the complement of A with respect to U . Thus, $A^c = U - A$ and indeed $A^c = \{x : x \in U$ and $x \notin A\}$

Note again that we can talk about the complement of a set provided we know the associated universal set. The complement of a set is the difference between the universal set and the given set.

Let us consider the sets A = {1, 2, 3, 4} B = {1, 3, 5} *and* U = {1, 2, 3, 4, 5, 6, 7, 8} and do the following set operations to observe certain facts:

a) We have $U^c = \{1,2,3,4,5,6,7,8\} - \{1,2,3,4,5,6,7,8\} = \varnothing$ and $\varnothing^c = U - \varnothing = U.$

b) $A^c = \{1,2,3,4,5,6,7,8\} - \{1,2,3,4\} = \{5,6,7,8\}$ and $A \cup A^c = \{1,2,3,4\} \cup \{5,6,7,8\} = U.$

c) $A^c = \{5,6,7,8\}$ and so $A \cap A^c = \{1,2,3,4\} \cap \{5,6,7,8\} = \varnothing.$

d) $A^c = \{5,6,7,8\}$ and so $\left(A^c\right)^c = \{1,2,3,4,5,6,7,8\} - \{5,6,7,8\} = \{1,2,3,4\} = A.$

e) $A^c = \{5,6,7,8\}, B^c = \{1,2,3,4,5,6,7,8\} - \{1,3,5\} = \{2,4,6,7,8\},$
$A - B = \{1,2,3,4\} - \{1,3,5\} = \{2,4\}, A \cap B^c = \{1,2,3,4\} \cap \{2,4,6,7,8\} = \{2,4\}$ and
$B^c - A^c = \{2,4,6,7,8\} - \{5,6,7,8\} = \{2,4\}$. We see that
$A - B = A \cap B^c = B^c - A^c.$

f) $A \cup B = \{1,2,3,4\} \cup \{1,3,5\} = \{1,2,3,4,5\}$ and $\left(A \cup B\right)^c = U - \{1,2,3,4,5\} = \{6,7,8\};$
$A^c \cap B^c = \{5,6,7,8\} \cap \{2,4,6,7,8\} = \{6,7,8\}$. We see that
$\left(A \cup B\right)^c = A^c \cap B^c$.

g) $A \cap B = \{1,2,3,4\} \cap \{1,3,5\} = \{1,3\}$ and $\left(A \cap B\right)^c = U - \{1,3\} = \{2,4,5,6,7,8\};$
$A^c \cup B^c = \{5,6,7,8\} \cap \{2,4,6,7,8\} = \{2,4,5,6,7,8\}$. We see that
$\left(A \cap B\right)^c = A^c \cup B^c$.

All the observations made in the previous examples are indeed true in case of any sets. Now we state these properties for any arbitrary sets and provide their proof.

Theorem 15.4: Let A and B be two arbitrary sets and let U be the associated universal set. Then the following properties hold good:

(i) $U^c = \varnothing$ and $\varnothing^c = U$;

(ii) $A \cup A^c = U$;

(iii) $A \cap A^c = \varnothing$;

(iv) $\left(A^c\right)^c = A$;

(v) $A - B = A \cap B^c = B^c - A^c$;

(vi) $\left(A \cup B\right)^c = A^c \cap B^c$ [***De Morgan's Law***];

(vii) $\left(A \cap B\right)^c = A^c \cup B^c$ [***De Morgan's Law***].

Proof:

(i) Applying parts (i) and (iv) of Theorem 15.3, we have

$U^c = U - U = \varnothing$ and $\varnothing^c = U - \varnothing = U$.

(ii) As U is the universal set, we clearly have $A \cup A^c \subseteq U$. Again we have

$x \in U \Rightarrow x \in A$ or $x \in A^c \Rightarrow x \in A \cup A^c$.

Therefore we have $U \subseteq A \cup A^c$. Now from $A \cup A^c \subseteq U$ and $U \subseteq A \cup A^c$, we get $A \cup A^c = U$.

(iii) We have $x \in A \cap A^c \Rightarrow x \in A$ and $x \in A^c$, which cannot hold altogether. Therefore $x \notin A \cap A^c$, and hence $A \cap A^c = \varnothing$.

(iv) We have $x \in \left(A^c\right)^c \Rightarrow x \notin A^c \Rightarrow x \in A$. Therefore $\left(A^c\right)^c \subseteq A$. Again we have $x \in A \Rightarrow x \notin A^c \Rightarrow x \in \left(A^c\right)^c$.

Therefore $A \subseteq \left(A^c\right)^c$. Now $\left(A^c\right)^c \subseteq A$ and $A \subseteq \left(A^c\right)^c$ altogether gives $\left(A^c\right)^c = A$.

(v) We shall prove $A - B = A \cap B^c$. The other part is left as an exercise for 'CHECK YOUR PROGRESS'. We have

$$\begin{aligned} x \in A - B &\Rightarrow x \in A \text{ and } x \notin B \\ &\Rightarrow x \in A \text{ and } x \in B^c \\ &\Rightarrow x \in A \cap B^c. \end{aligned}$$

Therefore $A - B \subseteq A \cap B^c$. Again

$$\begin{aligned} & x \in A \cap B^c \\ \Rightarrow\ & x \in A \text{ and } x \in B^c \\ \Rightarrow\ & x \in A \text{ and } x \notin B \\ \Rightarrow\ & x \in A - B. \end{aligned}$$

Thus $A \cap B^c \subseteq A - B$. Now from $A - B \subseteq A \cap B^c$ and $A \cap B^c \subseteq A - B$, we get $A - B = A \cap B^c$.

(vi) We have

$$\begin{aligned} x \in (A \cup B)^c &\Rightarrow x \notin A \cup B \\ &\Rightarrow x \notin A \text{ and } x \notin B \\ &\Rightarrow x \in A^c \text{ and } x \in B^c \\ &\Rightarrow x \in A^c \cap B^c. \end{aligned}$$

Therefore we have $(A \cup B)^c \subseteq A^c \cap B^c$. Again

$$\begin{aligned} x \in A^c \cap B^c &\Rightarrow x \in A^c \text{ and } x \in B^c \\ &\Rightarrow x \notin A \text{ and } x \notin B \\ &\Rightarrow x \notin A \cup B \\ &\Rightarrow x \in (A \cup B)^c. \end{aligned}$$

Therefore $A^c \cap B^c \subseteq (A \cup B)^c$. Now from $(A \cup B)^c \subseteq A^c \cap B^c$ and $A^c \cap B^c \subseteq (A \cup B)^c$, we get $(A \cup B)^c = A^c \cap B^c$.

(vii) The proof of this part is left as an exercise for 'CHECK YOUR PROGRESS'.//

We have so far discussed several properties of union, intersection and difference. All these properties are very basic properties of these set operations. While proving equality of any two sets, either some of these properties can be applied as formula or else, we can try directly to use Theorem 14.1, that is, to show one of the set is a subset of the other and vice versa. Now we give some examples, where some of these properties are used as formula, and some examples where Theorem 14.1 is used.

Example 1: If $A \subseteq B$, then prove that $A \cup (B - A) = B$.

Solution: Given that $A \subseteq B$. Now we have

$$\begin{aligned} A \cup (B - A) &= A \cup (B \cap A^c), \text{ as } B - A = B \cap A^c \\ &= (A \cup B) \cap (A \cup A^c), \text{ by distributive law} \\ &= (A \cup B) \cap U, \text{ as } A \cup A^c = U \text{ the universal set} \\ &= A \cup B, \text{ as } U \text{ the universal set} \\ &= B, \text{ as } A \subseteq B. // \end{aligned}$$

Example 2: Prove that $A = B$ if and only if $A \cup B = A \cap B$.

Solution: First suppose that $A = B$. Clearly, we have $A \cup B = A = A \cap B$. Conversely, suppose that $A \cup B = A \cap B$. To prove that $A = B$. We know that $A \subseteq A \cup B$ and $A \cap B \subseteq B$. So

$$A \subseteq A \cup B, A \cup B = A \cap B, A \cap B \subseteq B \Rightarrow A \subseteq B \quad .$$

Similarly, we have

$$B \subseteq A \cup B, A \cup B = A \cap B, A \cap B \subseteq A \Rightarrow B \subseteq A$$

Now from $A \subseteq B$ and $B \subseteq A$, we conclude that $A = B$ .//

Example 3: For any three sets *A, B* and *C*, show that

$$A - (B \cup C) = (A - B) \cap (A - C).$$

Solution: We have

$$\begin{aligned} A - (B \cup C) &= A \cap (B \cup C)^c, \text{ by a property of set difference} \\ &= A \cap (B^c \cap C^c), \text{ by De Morgan's Law} \\ &= (A \cap A) \cap (B^c \cap C^c), \text{ as } A \cap A = A \\ &= (A \cap B^c) \cap (A \cap C^c), \text{ by associative law} \\ &= (A - B) \cap (A - C). // \end{aligned}$$

Example 4: For any three sets *A, B* and *C*, show that

$$A - (B \cap C) = (A - B) \cup (A - C).$$

Solution: This identity can surely be proved by method similar to Example 3. However, we wish to prove it by using Theorem 14.1 for the purpose of illustration. Indeed, the identity in Example 3 can also be proved using Theorem 14.1. We have now

$$\begin{aligned} x \in A - (B \cap C) &\Rightarrow x \in A \text{ and } x \notin B \cap C \\ &\Rightarrow x \in A \text{ and } (x \notin B \text{ or } x \notin C) \\ &\Rightarrow (x \in A \text{ and } x \notin B) \text{ or } (x \in A \text{ and } x \notin C) \\ &\Rightarrow x \in A - B \text{ or } x \in A - C \\ &\Rightarrow x \in (A - B) \cup (A - C). \end{aligned}$$

Thus every element of $A - (B \cap C)$ is also an element of $(A - B) \cup (A - C)$. Therefore we have $A - (B \cap C) \subseteq (A - B) \cup (A - C)$. Again we have

$$x \in (A-B)\cup(A-C) \Rightarrow x \in A-B \text{ or } x \in A-C$$
$$\Rightarrow (x \in A \text{ and } x \notin B) \text{ or } (x \in A \text{ and } x \notin C)$$
$$\Rightarrow x \in A \text{ and } (x \notin B \text{ or } x \notin C)$$
$$\Rightarrow x \in A \text{ and } x \notin B \cap C$$
$$\Rightarrow x \in A-(B \cap C).$$

Therefore $(A-B)\cup(A-C) \subseteq A-(B\cap C)$. Now from $A-(B\cap C) \subseteq (A-B)\cup(A-C)$ and $(A-B)\cup(A-C) \subseteq A-(B\cap C)$, we get $A-(B\cap C) = (A-B)\cup(A-C)$ .//

To observe another important fact, let us take the sets $A=\{a,b,c,d,e,f\}$ and $B=\{a,c,1,2,3\}$. We have $A\cup B=\{a,b,c,d,e,f,1,2,3\}, A\cap B=\{a,c\}$ and $A-B=\{b,d,e,f\}=A-(A\cap B).$ Recall that $A-B=A-(A\cap B)$. We observe now that $n(A\cup B)=9$ and that $n(A)+n(B)-n(A\cap B)=6+5-2=9$. Thus $n(A\cup B)=n(A)+n(B)-n(A\cap B)$. However, though we expect $n(A-B)=n(A)-n(B)$, unfortunately $n(A)-n(B)=6-5=1, n(A-B)=4$, and so $n(A-B)\neq n(A)-n(B)$. Indeed, $n(A-B)=n(A-(A\cap B))=5=n(A)-n(A\cap B)$. These facts are indeed true for any two finite sets. In general, similar kind of formulae is available for unions and intersections of any finite number of finite sets. However, as the discussion goes beyond the scope of present context, we only give the statements (without proof) of these results in case of two sets.

Theorem 15.5: Let be two finite sets. Then the following holds:

(i) $n(A\cup B)=n(A)+n(B)-n(A\cap B)$;

(ii) $n(A-B)=n(A)-n(A\cap B)$.

Theorem 15.5(i) is about the number of elements of finite sets. $A, B, A\cup B$ and $A\cap B$. Therefore if the numbers of elements of any three of these sets are given, then the number of elements of the fourth sets can be determined. Similarly, if the numbers of elements of any two of the sets $A-B, A$ and $A\cap B$ are known, then Theorem 15.5(ii) can be applied to determine the number of elements of the third set. Using this theorem, we can solve several practical problems easily. We now give two such examples.

Example 5: Suppose that every student of your class can speak at least one of the languages of Hindi and English. If 70 percent can speak Hindi and 80 percent can speak English, find the percentage of students speaking both the languages.

Solution: Let $A =$ set of students who speak Hindi and $B =$ set of students who speak English. As the question is about finding percentage of students speaking both the languages, we may assume that the total number of students is 100. Accordingly, we haveand. Therefore we have $n(A) = 70, n(B) = 80$ and $n(A \cup B) = 100$. Therefore we have

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$\Rightarrow 100 = 70 + 80 - n(A \cap B)$$
$$\Rightarrow n(A \cap B) = 70 + 80 - 100 = 50.$$

Thus the percentage of students speaking both the languages is 50. //

Example 6: While surveying your class of 50 students, it is found that 25 students studied French, 20 students studied German and 5 students studied both. Find the number of students who

(i) studied only French (ii) did not study German

(iii) studied French or German (iv) studied neither language.

Solution: Let. It is clear that of all students in the class, is the universal set. According to the question, we have

$$n(U) = 50,\ n(A) = 25,\ \ n(B) = 20 \ \text{ and } \ n(A \cap B) = 5$$

(i) The set of students who studied only French $= A - B$. We have

$$n(A - B) = n(A) - n(A \cap B) = 25 - 5 = 20.$$

(ii) The set of students who did not study German $= B^c$. We have

$$n(B^c) = n(U - B) = n(U) - n(U \cap B) = n(U) - n(B) = 50 - 20 = 30$$

(iii) The set of students who studied French or German $= A \cup B$. We have

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 25 + 20 - 5 = 40$$

(iv) The set of students who studied neither language $= (A \cup B)^c$. We have $n((A \cup B))^c = n(U - (A \cup B)) = n(U) - n(U(A \cup B))$

$$= n(U) - n(A \cup B) = 50 - 40 = 10.//$$



CHECK YOUR PROGRESS

Q 16: Consider the $A=$ set of even natural numbers and $B=\{x:x\in n$ and $x\geq 5\}$ let be the associated universal set. Find the following:

(i) A^c (ii) B^c (iii) A^c-B^c (iv) $(A\cup B)^c$ (v) $A\cap B^c$
(vi) $A^c\cap B$ (vii) $A^c\cap B^c$ (viii) $(A\cap B)^c$ (ix) $A^c\cup B^c$ (x) B^c-A^c.

Q 17: For any two sets A and B prove that $A-B=B^c-A^c$

Q 18: Prove the De Morgan's law $\left(A\cap B\right)^c=A^c\cup B^c$.

Q 19: Fill the blanks using the symbols $\subseteq$ and $\supseteq$:

(i) $A\ldots A-B$ (ii) $A\cap B\ldots A\cup(B-A)$ (iii) $A^c\ldots B-A$ (iv) $A-B\ldots(B-A)^c$.

Q 20: For any two sets A and B, prove that $A=(A\cup B)\cap(A\cup B^c)$

Q 21: Suppose $n(U)=100,n(A)=50,n(B)=40,n(A\cap B)=20$ that. Find the following:

$(i)\ n(A\cup B)$ $(ii)\ n(A^c)$ $(iii)\ n(B^c)$ $(iv)\ n(A^c\cap B^c)$ $(v)\ n(A\oplus B)$.



15.7 LET US SUM UP

- $A\cup B=\left\{x:x\in A\ \text{ or }\ x\in B\right\}$, called the union of A and B.
- If $x\in A\cup B$ then exactly one of the following three possibilities is true: "$x\in A$ but $x\notin B$" or "$x\notin A$ but $x\in B$" or "$x\in A$ but $x\in B$".
- $x\notin A\cup B\Rightarrow x\notin A$ and $x\notin B$.
- $A\subseteq A\cup B$ and $B\subseteq A\cup B$.
- $A\cup B=B\cup A$ [Commutative Law].
- $(A\cup B)\cup C=A\cup(B\cup C)$ [Associative Law].
- $A\cup\varnothing=A$ [Law of Identity Element].
- $A\cup A=A$ [Idempotent Law].
- $U\cup A=U$ [Law of Universal Set].
- $A\subseteq B\Leftrightarrow A\cup B=B$.

- $A \cap B = \{x : x \in A \text{ and } x \in B\}$, called the intersection of A and B
- If $x \in A \cap B$ then x must belong to both A and B.
- If $x \notin A \cap B$ then either $x \notin A$ or $x \notin B$.
- If $A \cap B = \varnothing$ then A and B are said to be disjoint.
- $A \cap B \subseteq A$ and $A \cap B \subseteq B$.
- $A \cap B = B \cap A$ [Commutative Law].
- $(A \cap B) \cap C = A \cap (B \cap C)$ [Associative Law].
- $A \cap \varnothing = \varnothing$ [Law of Identity Element].
- $A \cap A = A$ [Idempotent Law].
- $U \cap A = A$ [Law of Universal Set].
- $A \subseteq B \Leftrightarrow A \cap B = A$.
- $A \cap (A \cup B) = A$ and $A \cup (A \cap B) = A$ [Absorption Law].
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [Distributive Law (of union over intersection)].
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [Distributive Law (of intersection over union)].
- $A - B = \{x : x \in A \text{ and } x \notin B\}$.
- $A - B \neq B - A$, unless both the sets A and B are equal.
- $A - B \subseteq A$ and $B - A \subseteq B$.
- If $A \cap B = \varnothing$, then $A - B = A$ and $B - A = B$.
- $A - \varnothing = A$.
- The sets $A - B, A \cap B$ and $B - A$ are pair-wise disjoint, and $(A - B) \cup (A \cap B) \cup (B - A) = A \cup B$.
- $A \oplus B = (A - B) \cup (B - A)$.
- $A^c = \{x : x \in U \text{ and } x \notin A\}$, is called the (relative) complement of A and is dependent on the choice of the universal set.
- $U^c = \varnothing$ and $\varnothing^c = U$.
- $A \cup A^c = U$, $A \cap A^c = \varnothing$ and $\left(A^c\right)^c = A$.
- $A - B = A \cap B^c = B^c - A^c$.
- $\left(A \cup B\right)^c = A^c \cap B^c$ and $\left(A \cap B\right)^c = A^c \cup B^c$ [***De Morgan's Laws***].



15.8 FURTHER READING

1) Seymour Lipschutz, 2005, *Set Theory and Related Topics*, Tata McGraw-Hill.

2) V. K. Balakrishnan, 1996, *Introductory Discrete Mathematics*, Dover Publications, Inc.

3) 2006, *Mathematics, Text book for Class XI*, NCERT, New Delhi.



15.9 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1:

(i) $A \cup B = \{1, 2, a, f, x, 4, 0, \varnothing, \theta, 3, \pi\}$.

(ii) $A \cup B = \{1, 2, a, f, 0, \varnothing\}$.

(iii) $A \cup B = n$.

(iv) $A \cup B = \{x : x$ is a multiple of 2 or 3, $x \in n\}$

(v) $A \cup B = \{x : x \in n,\ x \neq 0\}$

Ans to Q No 2: We have $x \in B \Rightarrow x \in A$ or $x \in B \Rightarrow x \in A \cup B$. Therefore $A \subseteq A \cup B$.

Ans to Q No 3: Assume that $A \cup B = B$. To show that $A \subseteq B$. We have

$x \in A \Rightarrow x \in A$ or $x \in B \Rightarrow x \in A \cup B \Rightarrow x \in B$, as $A \cup B = B$.

Thus every element of A is also an element of B, and therefore $A \subseteq B$.

Ans to Q No 4: Given that $A \cup X = B \cup X$ for every set X. Therefore putting $X = \varnothing$, in particular, we have $A \cup \varnothing = B \cup \varnothing \Rightarrow A = B$.

Ans to Q No 5: Given that $A \cup B = \varnothing$. If $A \neq \varnothing$, then $A \cup B$ will contain the elements of A, and so $A \cup B$ cannot be empty. Similarly, if $B \neq \varnothing$, then also $A \cup B$ cannot be empty. Thus the only possibility for $A \cup B = \varnothing$ remains is that $A = \varnothing$ and $B = \varnothing$.

Ans to Q No 6:

(i) $A \cap B = \{1, 2, f\}$

(ii) $A \cap B = \{\varnothing\}$

(iii) $A \cap B = A$

(iv) $A \cup B = \{x : x$ is a multiple of 6, $x \in n\}$

(v) $A \cap B = \varnothing$

Ans to Q No 7: We have $x \in A \cap B \Rightarrow x \in A$ and $x \in B \Rightarrow x \in B.$ Therefore $A \cap B \subseteq B$.

Ans to Q No 8: Assume that $A \cap B = A$. To show that $A \subseteq B$. We have

$$\begin{aligned} x \in A &\Rightarrow x \in A \cap B, \text{ as } A = A \cap B \\ &\Rightarrow x \in A \text{ and } x \in B \\ &\Rightarrow x \in B. \end{aligned}$$

Thus every element of A is also an element of B, and therefore $A \subseteq B$.

Ans to Q No 9: Given that $A \cap X = B \cap X$ for every set X. Therefore putting $X = U$, an associated universal set, we have $A \cap U = B \cap U \Rightarrow A = B$.

Ans to Q No 10: Given that $A \cap B = U$, where U is the associated universal set. If A is a proper subset of U, then from $A \cap B \subseteq A$ we have that $A \cap B$ is a proper subset of U, contradicting the equality $A \cap B = U$ Therefore we must have $A = U$. Applying the same argument for B, we find that $B = U$ as well.

Ans to Q No 11: We have

$$\begin{aligned} x \in A \cup (A \cap B) &\Rightarrow x \in A \text{ or } x \in A \cap B \\ &\Rightarrow x \in A \text{ or } (x \in A \text{ and } x \in B) \\ &\Rightarrow x \in A \text{ or } (x \in A) \\ &\Rightarrow x \in A. \end{aligned}$$

Thus every element of $A \cup (A \cap B)$ is also an element of A, and therefore $A \cup (A \cap B) \subseteq A$. Again

$$\begin{aligned} x \in A &\Rightarrow x \in A \text{ or } x \in B, \text{ as } A \subseteq A \cup B \\ &\Rightarrow x \in A \text{ or } (x \in A \text{ and } x \in B) \\ &\Rightarrow x \in A \text{ or } x \in A \cap B \\ &\Rightarrow x \in A \cup (A \cap B). \end{aligned}$$

Therefore $A \subseteq A \cup (A \cap B)$. Now from $A \cup (A \cap B) \subseteq A$ and $A \subseteq A \cup (A \cap B)$, we conclude that $A \cup (A \cap B) = A$.

Ans to Q No 12:

(i) $A - B = \{a, x, 4, 0\}$, $B - A = \{\varnothing, \theta, 3, \pi\}$, $A \oplus B = \{a, x, 4, 0, \varnothing, \theta, 3, \pi\}$.

(ii) $A - B = \{1, 2, a, f, 0\}$, $B - A = \varnothing$, $A \oplus B = \{1, 2, a, f, 0\}$.

(iii) $A - B = \varnothing$, $B - A = \{x : x \text{ is not an odd natural number}, x \in \square\}$

$= \{\cdots, -6, -5, -3, -2, -1, 0, 2, 4, 6, 8, 10, \cdots\}$

$A \oplus B = \{x : x \text{ is not an odd natural number}, x \in \square\}$.

(iv) $A - B = \{x : x \text{ is divisible by 2 but not divisible by 3}, x \in \square\}$,

$B - A = \{x : x \text{ is divisible by 3 but not divisible by 2}, x \in \square\}$,

$A \oplus B = \{x : x \text{ is divisible by 2 or 3 but not divisible by 6}, x \in \square\}$.

(v) $A - B = \{x : x = -n \text{ for some n} \in \square\}$, $B - A = \square$,

$A \oplus B = \{\cdots, -6, -5, -4, -3, -2, -1, 1, 2, 3, 4, 5, 6, \cdots\}$.

Ans to Q No 13: We have $x \in B - A \Rightarrow x \in B$ and $x \notin A \Rightarrow x \in B$. Thus every element of $B - A$ is also an element of B. Hence $B - A \subseteq B$.

Ans to Q No 14: We have

$$\begin{aligned} & x \in (B - A) \cap (A \cap B) \\ \Rightarrow\ & x \in B - A \text{ and } x \in A \cap B \\ \Rightarrow\ & (x \in B \text{ and } x \notin A) \text{ and } (x \in A \text{ and } x \in B) \\ \Rightarrow\ & x \in B \text{ and } x \notin A \text{ and } x \in A \\ \Rightarrow\ & x \notin A \text{ and } x \in A, \text{ which is impossible.} \end{aligned}$$

Thus $x \in (B - A) \cap (A \cap B)$ is **not** possible, which gives that $(B - A) \cap (A \cap B) = \varnothing$.

Ans to Q No 15: We have

$$\begin{aligned} & x \in (A - B) \cup (B - A) \\ \Rightarrow\ & x \in A - B \text{ or } x \in B - A \\ \Rightarrow\ & (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A) \\ \Rightarrow\ & (x \in A \text{ or } x \in B) \text{ and } (x \notin A \text{ or } x \notin B) \\ \Rightarrow\ & x \in A \cup B \text{ and } x \notin A \cap B \\ \Rightarrow\ & x \in (A \cup B) - (A \cap B). \end{aligned}$$

Thus $(A-B)\cup(B-A)\subseteq(A\cup B)-(A\cap B)$. Again we have

$$
\begin{aligned}
& x\in(A\cup B)-(A\cap B)\\
\Rightarrow\ & x\in A\cup B \text{ and } x\notin A\cap B\\
\Rightarrow\ & (x\in A \text{ or } x\in B) \text{ and } (x\notin A \text{ or } x\notin B)\\
\Rightarrow\ & (x\in A \text{ and } x\notin B) \text{ or } (x\in B \text{ and } x\notin A)\\
\Rightarrow\ & x\in A-B \text{ or } x\in B-A\\
\Rightarrow\ & x\in(A-B)\cup(B-A).
\end{aligned}
$$

Thus $(A\cup B)-(A\cap B)\subseteq(A-B)\cup(B-A)$. Now

$(A-B)\cup(B-A)\subseteq(A\cup B)-(A\cap B)$ and

$(A\cup B)-(A\cap B)\subseteq(A-B)\cup(B-A)$ altogether gives

$(A\cup B)-(A\cap B)=(A-B)\cup(B-A)$.

Ans to Q No 16:

(i) A^c = the set of all odd natural numbers.

(ii) $B^c=\{1,2,3,4\}$.

(iii) $A^c-B^c=$ $\{x:x$ is an odd natural number and $x\geq 5\}$.

(iv) $\left(A\cup B\right)^c=\{1,3\}$.

(v) $A\cap B^c=\{2,4\}$.

(vi) $A^c\cap B$ = $\{x:x$ is an odd natural number and $x\geq 5\}$.

(vii) $A^c\cap B^c=\{1,3\}$.

(viii) $(A\cap B)^c$ =$\{x:x=2$ or $x=4$ or x is an odd natural number$\}$.

(ix) $A^c\cap B^c$ =$\{x:x=2$ or $x=4$ or x is an odd natural number$\}$.

(x) $B^c-A^c=\{2,4\}$

Ans to Q No 17: We have

$$
\begin{aligned}
x\in A-B &\Rightarrow x\in A \text{ and } x\notin B\\
&\Rightarrow x\notin B \text{ and } x\in A\\
&\Rightarrow x\in B^c \text{ and } x\notin A^c\\
&\Rightarrow x\in B^c-A^c.
\end{aligned}
$$

Therefore $A-B\subseteq B^c-A^c$. Again

$$
\begin{aligned}
& x \in B^c - A^c \\
\Rightarrow\ & x \in B^c \text{ and } x \notin A^c \\
\Rightarrow\ & x \notin B \text{ and } x \in A \\
\Rightarrow\ & x \in A \text{ and } x \notin B \\
\Rightarrow\ & x \in A - B.
\end{aligned}
$$

Therefore $B^c - A^c \subseteq A - B$. Now $A - B \subseteq B^c - A^c$ and $B^c - A^c \subseteq A - B$ gives $A - B = B^c - A^c$.

Ans to Q No 18: We have

$$
\begin{aligned}
x \in (A \cap B)^c \Rightarrow\ & x \notin A \cap B \\
\Rightarrow\ & x \notin A \text{ or } x \notin B \\
\Rightarrow\ & x \in A^c \text{ or } x \in B^c \\
\Rightarrow\ & x \in A^c \cup B^c.
\end{aligned}
$$

Therefore we have $(A \cap B)^c \subseteq A^c \cup B^c$. Again

$$
\begin{aligned}
x \in A^c \cup B^c \Rightarrow\ & x \in A^c \text{ or } x \in B^c \\
\Rightarrow\ & x \notin A \text{ or } x \notin B \\
\Rightarrow\ & x \notin A \cap B \\
\Rightarrow\ & x \in (A \cap B)^c.
\end{aligned}
$$

Therefore $A^c \cup B^c \subseteq (A \cap B)^c$. Now from $(A \cap B)^c \subseteq A^c \cup B^c$ and $A^c \cup B^c \subseteq (A \cap B)^c$, we get $(A \cap B)^c = A^c \cup B^c$.

Ans to Q No 19: (i) $A \supseteq A - B$ (ii) $A \cap B \subseteq A \cap (B - A)$ (iii) $A^c \supseteq B - A$ (iv) $A - B \subseteq (B - A)^c$

Ans to Q No 20: We have

$$
\begin{aligned}
(A \cup B) \cap (A \cup B^c) &= A \cup (B \cap B^c), \text{ by distributive law} \\
&= A \cup \varnothing, \text{ as } B \cap B^c = \varnothing \\
&= A.
\end{aligned}
$$

Ans to Q No 21: Given that $n(U) = 100, n(A) = 50, n(B) = 40, n(A \cap B) = 20$

We have

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 50 + 40 - 20 = 70$

(ii) $n(A^c) = n(U - A) = n(U) - n(U \cap A) = n(U) - n(A) = 100 - 50 = 50$

(iii) $n(B^c) = n(U - B) = n(U) - n(U \cap B) = n(U) - n(B) = 100 - 40 = 60$

(iv) $n(A^c \cap B^c) = n((A \cup B)^c) = n(U) - n(U \cap (A \cup B)) = n(U) - n(A \cup B) = 100 - 70 = 30$

(v) Recall that $A \oplus B = (A - B) \cup (B - A)$. Now

$n(A - B) = n(A) - n(A \cap B) = 50 - 20 = 30$ and

$n(B - A) = n(B) - n(B \cap A) = 40 - 20 = 20$

Also we know that $(A - B) \cap (B - A) = \varnothing$. Therefore we have

$$\begin{aligned} n(A \oplus B) &= n(A - B) + n(B - A) - n[(A - B) \cap (B - A)] \\ &= n(A - B) + n(B - A) - n(\varnothing) \\ &= 30 + 20 - 0 = 50. \,//\end{aligned}$$



15.10 MODEL QUESTIONS

Q 1: Let $U = \{1,2,3,4,5,6,7,8,9\}$ be a universal set and consider the sets

$A = \{1,2,3,4\}$, $C = \{4,5,6,7,9\}$, $E = \{1,2,5,9\}$,
$B = \{3,4,7,9\}$, $D = \{2,5,1,9,4\}$, $F = \{3,4,7,8\}$.

Find the following:

(i) $A \cup B$ (ii) $C \cup D$ (iii) $E \cup F$ (iv) $A \cup D$
(v) $A \cup (B \cup D)$ (vi) $U \cup F$ (vii) $C \cup F$ (viii) $D \cup F$
(ix) $C \cup E$ (x) $(A \cup B) \cup D$

Q 2: Let A, B and X be three sets such that $A \cup X = B \cup X$. Does it imply that $A = B$? Justify.

Q 3: Let $U = \{1,2,3,4,5,6,7,8,9\}$ be a universal set and consider the sets

$A = \{1,2,3,4\}$, $C = \{4,5,6,7,9\}$, $E = \{1,2,5,9\}$,
$B = \{3,4,7,9\}$, $D = \{2,5,1,9,4\}$, $F = \{3,4,7,8\}$.

Find the following:

(i) $A \cap B$ (ii) $C \cap D$ (iii) $E \cap F$ (iv) $A \cap D$
(v) $A \cup (B \cap D)$ (vi) $U \cap F$ (vii) $C \cap F$ (viii) $D \cap F$
(ix) $C \cap E$ (x) $(A \cup B) \cap (A \cup D)$

Q 4: Let A, B and X be three sets such that $A \cap X = B \cap X$. Does it imply that $A = B$? Justify.

Q 5: Prove the distributive law $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Q 6: Two sets A and B are such that their union and intersection are equal. What can you say about A and B?

Q 7: Suppose that A is a subset of B and C is a subset of D.

(i) Is it always true that $A \cup C$ is a subset of $B \cup D$?

(ii) Is it always true that $A \cap C$ is a subset of $B \cap D$?

Q 8: Prove that $\mathrm{P}(A) \cap \mathrm{P}(B) = \mathrm{P}(A \cap B)$, that is, the intersection of the power set of A and the power set of B is the power set of the intersection of A and B.

Q 9: Find three sets A, B and C such that $A \cap B, B \cap C$ and $A \cap C$ are all non-empty but $A \cap B \cap C = \varnothing$.

Q 10: Let A and B be two sets such that

$A \cap X = B \cap X = \varnothing$ and $A \cup X = B \cup X$ for some given set X. Show that $A = B$.

Q 11: Let $U = \{1,2,3,4,5,6,7,8,9\}$ be a universal set and consider the sets

$A = \{1,2,3,4\}$, $C = \{4,5,6,7,9\}$, $E = \{1,2,5,9\}$,

$B = \{3,4,7,9\}$, $D = \{2,5,1,9,4\}$, $F = \{3,4,7,8\}$.

Find the following:

(i) $A - B$ (ii) $C - D$ (iii) $E - F$ (iv) $A - D$

(v) $A - (B - D)$ (vi) $U \oplus F$ (vii) $C \oplus F$ (viii) $D \oplus F$

(ix) $C - E$ (x) $(A - B) - D$

Q 12: If $A \cap B = \varnothing$ then prove $B - A = B$.

Q 13: Prove the following: $A - B \subseteq A \cup B$ and $A^c - B^c = B - A$.

Q 14: If n = the set of real numbers, n = the set of integers, n = the set of natural numbers, then determine the sets $(n \cap n) \cup n$ and $n - (n - n)$.

Q 15: Prove the following for three arbitrary sets P, Q and R:

(i) $(P - Q) - R = P - (Q \cup R)$;

(ii) $(P - Q) - R = (P - R) - Q$;

(iii) $(P - Q) - R = (P - R) - (Q - R)$.

Q 16: Prove that $(A - B) \cap (B - A) = \varnothing$.

Q 17: Let $U = \{1,2,3,4,5,6,7,8,9\}$ be a universal set and consider the sets

$A = \{1,2,3,4,8\}, \quad B = \{4,5,7,9\}, \quad C = \{1,5,9\}.$

Find the following:

$(i)\ A^c \quad (ii)\ B^c \quad (iii)\ A - B \quad (iv)\ (A \cup B)^c \quad (v)\ A \cap B^c$

$(vi)\ A^c \cap B^c \quad (vii)\ (A \cap B)^c \quad (viii)\ A^c \cup B^c \quad (ix)\ B^c - A^c.$

Q 18: For any two sets A and B, prove that $A \cap B^c = B^c - A^c$.

Q 19: If the symmetric difference of two sets A and B is equal to the set A, what can you say about A and B?

Q 20: If A and B are two arbitrary sets, under what conditions can we conclude that the symmetric difference $(A - B) \oplus (B - A)$ is the empty set?

Q 21: What can you say about two sets A and B if $A - B$ equal to $B - A$?

Q 22: If the symmetric difference of A and B is equal to the symmetric difference of A and C, is it necessary that $B = C$?

Q 23: Let $U = \{1,2,3,4,5,6,7,8,9\}$ be a universal set and consider $A = \{1,2,3,4\}, B = \{2,4,6,8\}$ and $C = \{3,4,5,6\}$. Find the following sets:

$(i)\ A^c \quad (ii)\ B^c \quad (iii)\ (A \cup C)^c \quad (iv)\ (A \cup B)^c \quad (v)\ \left(A^c\right)^c \quad (vi)\ (B - C)^c.$

Q 24: Fill the blanks to make each of the following a true statement:

$(i)\ A \cup A^c = \ldots \quad (ii)\ A \cap A^c = \ldots \quad (iii)\ \varnothing^c \cap A = \ldots \quad (ii)\ U^c \cap A = \ldots$

Q 25: Show that if $A \subseteq B$ then $C - B \subseteq C - A$, for any set C.

Q 26: Show that for any set A and B,

$A = (A \cap B) \cup (A - B) \quad \text{and} \quad A \cup (B - A) = A \cup B$

Q 27: Write down the complements of the following sets, considering the set of natural numbers as the universal set.

(i) $\{x : x$ is an even natural number$\}$ (ii) $\{x : x$ is an odd natural number$\}$

(iii) $\{x : x$ is a positive multiple of 3$\}$ (iv) $\{x : x$ is a prime number$\}$

(v) $\{x : x$ is divisible by 3 and 5, $x \in n\}$ (vi) $\{x : x + 5 = 8,\ x \in n\}$

(vii) $\{x : 2x + 5 = 10,\ x \in n\}$ (viii) $\{x : x \geq 7,\ x \in n\}$

(ix) $\{x : 2x + 5 = 9,\ x \in n\}$ (x) $\{x : 2x + 1 > 10,\ x \in n\}$.

Q 28: Suppose that every student of your class drinks either tea or coffee. Given that the number of students who drink tea is 79, the number of

students who drink coffee is 60 and the number of students who drink both tea and coffee is 20. Find the total number of students in your class.

Q 29: Suppose that $n(U) = 70, n(A) = 30, n(B) = 45, n(A \cap B) = 10$. Find the following:

$(i)\ n(A \cup B)$ $(ii)\ n(A^c)$ $(iii)\ n(B^c)$ $(iv)\ n(A^c \cap B^c)$ $(v)\ n(A \oplus B)$

Q 30: Suppose there are 110 students in your class. Suppose 30 students can speak English, 35 students can speak Hindi and 20 students can speak both the languages. Find the following:

(i) number of students who speak either English or Hindi;

(ii) number of students who speak English but not Hindi;

(iii) number of students who speak Hindi but not English;

(iv) number of students who speak neither English nor Hindi.

*** ***** ***



REFERENCES

1) A.H. Bason & O'Connor. 1977. *Introduction To Symbolic Logic*: Oxford University Press
2) Baronett, Stan & Sen, Madhuchanda. *2009*. *Logic.* Pearson
3) Chakraborty, Chanda. 2007. *Logic Informal, Symbolic and Inductive*: Prentice-Hall India
4) Copi, I.M. 1973. *Symbolic Logic* : Macmillan Publishing Co., Inc.
5) Copi, Irving. *Symbolic Logic*: Macmillan Publishing Co., Inc. New York and Collier Macmillan Publishers London. 1979.
6) Copi, I. M. & Cohen, Carl. 1997. *Introduction to Logic*. Prentice Hall of India Pvt. Ltd, New Delhi,
7) Copi, Irving M. (2006). *Symbolic Logic* (fifth edition). New Delhi: Prentice Hall of India Private Ltd.
8) Copi, Irving M. and Cohen, Carl. 2007. *Introduction to Logic:* Prentice-Hall of India pvt.
9) Copi, M, Irving & Cohen, Carl, & Jetli, Priyedarshi, & Prabhakar, Monica. *Introduction to Logic* (Twelfth edition) Dorling Kindersley (India) Pvt. Ltd.
10) David Harel, Dexter Kozen, Jerzy Tiuryn: Dynamic Logic:Prentice - Hall India, 2007
11) Dr. Shyam kishor Sing : Modern Logic (vol. 1.) Jalukbari, Guwahati-14.
12) Hurley, Patrick J. *Introduction to Logic*. Wadsworth Cengage Learning.
13) Jain, Krishna. *A Textbook of Logic*. D.K. Printworld (P) New Delhi.
14) Jain, Krishna. 1998. *A textbook of logic* (fourth Revised Edition)
15) Korde, Ashok & Ankush Sawant: 1983. Logi.c. Himalaya Publishing House, Bombay,
16) Patrick J. Hurley. *Introduction to Logic:* Wadsworth Cengage Learning, 2007.
17) Seymour Lipschutz, 2005, *Set Theory and Related Topics*, Tata McGraw-Hill.

18) Sharma, Banajit & Deka, Dr. Jahnabi. 2014. *A Text Book of Logic*. AANK- BAAK: Guwahati-1
19) Singh, S. Shyam Kishore. *Modern Logic* (Vol.1) Lamyana Press
20) Stan Baronett and Madhucchanda Sen: Logic : Pearson 2008
21) Stebbing, L. Susan. 1969. A Modern Elementary Logic, Methuen & Co. Ltd., London,
22) V. K. Balakrishnan, 1996, *Introductory Discrete Mathematics*, Dover Publications, Inc.
23) 2006, *Mathematics, Text book for Class XI*, NCERT, New Delhi.
24) http://www.iep.utm.edu/prop-log/;InternetEncyclopaedia of Philosophy