UG PH S3-01
(DSC)

FOUR YEAR BACHELOR'S DEGREE COURSE

(CBCS in Accordance with NEP 2020)

SEMESTER - III

LOGIC - 2

PHILOSOPHY

Surya Kumar Bhuyan School of Social Sciences

KRISHNA KANTA HANDIQUI STATE OPEN UNIVERSITY

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COURSE INTRODUCTION

This course deals with Logic. Logic is an important branch of philosophy. The word 'logic' is derived from the Greek word 'logos'. It means thought or reason or law. Logic is defined as the science of reasoning. This definition is not accepted as the real definition of logic because reasoning implies a special kind of inferring or inference. Again, we can define logic as the study of the methods and principles used in distinguishing correct from incorrect arguments. This helps us to make a distinction between correct and incorrect argument. That is why; it is clear that logic is a tool or strategy by which we can find out the distinction between good reasoning and bad reasoning. The study of logic, therefore, enhances our reasoning power to testify the arguments, whether they are valid or invalid. It also helps us in our domestic discourse to argue systematically with others.

The first unit starts with 'Standard Form of Categorical Syllogism'. After going through this unit, you will be able to know what standard form of categorical syllogism is and what its structure. Mood and figure are two important things of the standard categorical syllogism. These two are also discussed in this unit. Apart from these two, you will also be able know to about venn-diagrametic technique.

The second unit is 'Venn-diagram'. After going through this unit, you will be able to know what the four standard forms of categorical propositions are and their class relations, distribution of terms etc. You will also be able to know, what Venn diagram is and what the use of it is. This unit is also dealt with history of Venn diagram and Aristotelian and Boolean standpoint of it.

The third unit is 'Venn-diagrametic Technique for Testing the Categorical Syllogism'. It discusses the concept of syllogism, kinds of syllogism, standard form of categorical syllogism, Venn-diagram etc.

The fourth unit, 'Rules of Categorical Syllogism'. This unit basically includes concepts like categorical syllogism, standard form of it, figure and mood, and different rules of categorical syllogism.

The fifth unit, 'Basic Categorical Fallacies' discusses what is categorical fallacy, what are different types of basic categorical fallacies.

The sixth unit starts with 'Formal Proof of Validity'. After going through this unit, you will be able to know what formal proof of validity is. You will also be able to know how the formal proof of validity of an argument can be constructed easily. The nineteen rules namely, the rules of inference and rules of replacement will also be discussed in this unit.

The seventh unit is 'Rules of Inference.' After going through this unit, you will be able to know what rules of inference are and what the nine rules of inference are. You will also be able to know how by applying these rules validity of the arguments can be proved.

The name of the eighth unit is 'Rules of Replacement'. After going through this unit you will be able to know what rules of replacement are and what the twenty rules of inference are. You will be further able to know about the difference between 'rules of inference' and 'rules of replacement.'

The ninth unit is ‘Elementary Formal Deductions by Applying These Rules’. This unit basically includes concepts like categorical syllogism, standard form of it, figure and mood and different rules of categorical syllogism. By reading this unit you will be able to know what elementary formal Deduction is and about the strategy for constructing formal deduction and certain examples of formal deduction.

The name of the tenth unit is ‘Decision Procedure’. After going through this unit you will be able to know that decision procedure is basically concerned with the concept of decidability. You will also be able to know Truth-Table method, the Method of Reductio Ad Absurdum or the Indirect Truth-Table Method, Reduction to Conjunctive Normal Form.

The eleventh unit starts with ‘Nature and Kinds of Induction’. After going through this unit, you will be able to know what is nature and characteristics of Inductive Inferences and stages of Induction. You will be further able to know the Problem of Induction, Kinds of Induction, and the Inductive Fallacies.

The twelfth unit is ‘Causality: Cause & Conditions, Necessary & Sufficient Conditions, Constant Conjunction’. After going through this unit, you will be able to know the definitions causality and difference between Cause and Condition. You will be again able to know the Necessary and Sufficient Condition and Cause and Constant Conjunction.

The name of the thirteenth unit is ‘Hypothesis: Kinds and Uses’. After going through this unit you will be able to know what is hypothesis. You will be further able to know the purpose and function of hypothesis and the characteristics of hypothesis. You will be further able to know the Steps of hypothesis and Kinds of hypothesis. Origin of hypothesis, Verification, and proof of hypothesis will be also known by you.

The fourteenth unit is ‘Analogy’. This unit basically includes concepts like what is analogy, use of it in its different contexts, especially in logic. This unit will further discuss about refutation of analogy.

The name of the fifteenth unit is ‘Problem of Justification of Induction’. After going through this unit you will be able to know what the problem of induction is. You will be further able know whether a demonstrative proof of principle of inductive reasoning be offered and whether the principle of inductive reasoning be justified by inductively or not.

While going through this course, you will come across some along-side boxes, which are put on the left side or right side of the text. These boxes will give us the meanings of some words and concepts within the text. Apart from this, there will be some broad and short questions included under Activity and Check Your Progress in every unit. Activities will increase our thinking capacity because questions put in Activity are not directly derived from the text. But answers to the short questions are put in the section Answers to Check Your Progress. Besides, there are some text-related questions which are put in Model Questions. These questions will help your attention on the probable topics for the examination so that you can prepare for the examination with confidence.

UNIT 1: STANDARD FORM OF CATEGORICAL SYLLOGISM

UNIT STRUCTURE

1.1 Learning Objectives

1.2 Introduction

1.3 Standard form of Categorical Syllogism

1.3.1 Structure of Categorical Syllogism

1.4 Mood: What it is

1.5 Figure: What it is

1.6 Venn Diagram Technique for Testing Syllogism

1.7 Let us Sum up

1.8 Further Reading

1.9 Answers to Check Your Progress

1.10 Model Questions

1.1 LEARNING OBJECTIVES

After going through unit, you will be able to:

- define standard form of categorical syllogism
- explain the structure of categorical syllogism
- discuss what is mood
- describe what is figure
- illustrate the Venn diagram technique for testing syllogism.

1.2 INTRODUCTION

This unit introduces to you standard form of categorical syllogism. It includes the definition as well as the structure of the categorical syllogism. Apart from this, you get the concepts like mood, figure of syllogism and the Venn diagram technique for testing syllogism.

A standard form of categorical syllogism is a deductive argument which contains two categorical propositions. And from two categorical propositions taken jointly conclusion is drawn. As for instance:

All Philosophers are rationalists.

All mathematicians are philosophers.

∴ All mathematicians are rationalists.

This is an example of standard form categorical syllogism.

1.3 STANDARD FORM CATEGORICAL SYLLOGISM

A syllogism is a kind of deductive argument in which a conclusion is drawn from two premises taken jointly. A categorical syllogism is a deductive argument consisting only of categorical propositions.

1.3.1 Structure of Categorical Syllogism

A categorical syllogism contains three terms, each of which occur in the two premises from which the conclusion follows. A standard form categorical syllogism consists of standard form categorical propositions. Its premises and conclusion are all standard form categorical propositions. The three terms of the standard form categorical propositions occur twice in the syllogism. The three terms are called the major term, the minor term and the middle terms respectively. The term that occurs as the predicate of the conclusion is called the major term of the conclusion is called the major term of the syllogism, and the term that occurs as the subject term of the conclusion is called the minor term of the syllogism. The term that occurs in both the premises but not in the conclusion is called the middle term. An example of the standard form categorical syllogism is given below–

All Greeks are humans.

All Athenians are Greeks.

∴ All Athenians are humans.

The term ‘humans’ is the predicate of the conclusion and thus it is the major term of the syllogism. ‘Athenians’ is the subject of the conclusion and as such it is called the minor term. The term ‘Greek’ occurs in both the premises and as such it is the middle term of the syllogism.

The premise 'All Greeks are humans' is the major premise because the major term 'human' occurs in this premise. The premise 'All Athenians are Greeks' is the minor premise since the minor term 'Athenians' occur in this premise.

In the standard form of categorical syllogism, the major premise is stated first, the minor premise the second and the conclusion last. Major premise and the minor premise are not defined in terms of their position but in terms of their possession of major term and minor term respectively.

The standard form of categorical syllogism is said to have a form. The form of standard form of categorical syllogism is determined by the form of the premises and the conclusion it contains. There are three letters that indicate the form of the syllogism. The first letter represents the term of the major premise, the second letter represents the form of the minor premise and the third letter represents the form of the conclusion. For example, the syllogism–

All Greeks are humans.

All Athenians are Greeks.

∴ All Athenians are humans.

Consists of major premise which is an A proposition, minor premise which is also an A proposition and the conclusion which is an A proposition.

1.4 MOOD: WHAT IT IS

The mood of a standard form of categorical proposition is determined by the forms of the standard form of categorical propositions it contains. Since the above syllogism contain all A propositions i.e., both the premises and the conclusion are A propositions, therefore, the mood of the syllogism is AAA.

However, the mood of a standard form of categorical syllogism does not wholly or completely determine its form. Two syllogisms may have the same mood but may differ in form. Let us take two syllogisms and see that they have the same mood but differ in form.

1) All philosophers are intellectuals.
Some scientists are intellectuals.
∴ Some scientists are philosophers.

2) All scientists are philosophers.
Some scientists are mathematicians.
∴ Some mathematicians are philosophers.

In both the syllogisms major premise are universal affirmative i.e., A propositions, minor premises are particular affirmative i.e., I propositions and the conclusions are particular affirmative propositions i.e., I propositions. Therefore, the syllogisms are of mood 'All'. But their form is different. In order to show how these two syllogisms differ in forms, we have to replace the terms of the propositions i.e., the major terms, minor terms and the middle terms by the letters P, S, and M respectively. The forms of the two syllogisms are as follows-

1) All P is M
some S is M
∴ Some S is P

2) All M is P
Some M is S
∴ Some S is P

Their difference lies in the fact that the position of the middle term is different in the two syllogisms. In the first, the middle term is the predicate in both the premises where as in the second the middle term is the subject in both the premises. Thus, it is seen that these two syllogisms inspite of having the same mood are different in form.

The mood of a syllogism can not completely define the form of a syllogism. The form of a syllogism may be completely described by stating its mood and figure.

1.5 FIGURE: WHAT IT IS

Figure indicates the position of the middle term in the syllogism. Now as we know that middle term is that term which occurs in both the premises. Depending on the position of the middle term in the premises, we have four

different figures. Those syllogisms may have. The middle term may be the subject of the major premise and predicate term of the minor premise or it may be the predicate term of both the premises or it may be the subject term of both the premises or it may be the predicate term of major premise and the subject term of minor premise. These different possible positions of the middle term constitute the first, second, third and fourth figures respectively. The four figures are schematized in the following array–

M – P	P M	M – P	P – M
S – M	S M	M – S	M – S
– P	∴ S M	∴ M – S	∴ M – S
First figure	Second figure	Third figure	Fourth figure

Thus, the mood and the figure together constitute the form of the standard form categorical syllogisms.

There are four kinds of propositions– A.E.I and O. A syllogism has got two premises. Hence we may have sixteen possible moods in each figure. Again, there are four figures so we have 16 x 4 = 64 possible moods in all the four figures. Again, each of the 64 combinations may have four forms. Thus, in this sense there are 64 x 4 = 256 moods in all the four figures. Out of these 256 moods some are valid and some are invalid.



CHECK YOUR PROGRESS

Q.1: What is standard form of categorical syllogism?

.....

.....

.....

Q.2: What are the three terms occur in 'Standard form of categorical syllogism'?

.....

Q.3: What is mood?

.....

Q.4: How many figures in syllogism?

.....

Q.5: Fill in the blanks:

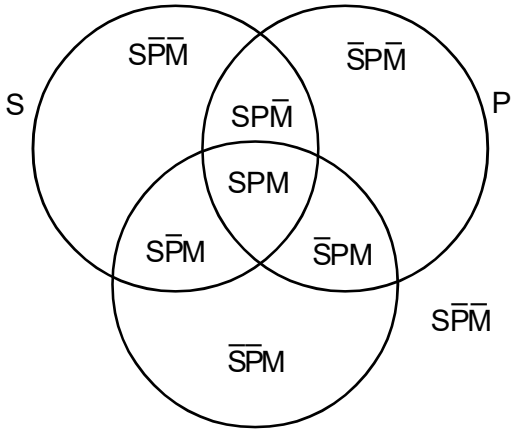
a) A categorical syllogism contain ..... terms.

b) Figure indicates the position of the ..... term.

c) ..... and ..... constitute the standard form of categorical syllogism.

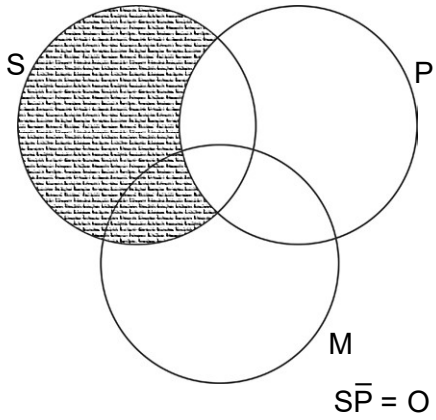
1.6 VENN DIAGRAM TECHNIQUE FOR TESTING SYLLOGISM

This section explains the Venn diagram technique to diagram the standard form of categorical propositions. In order to diagram the standard form of categorical syllogism, it is necessary to take three circles for three terms i.e., major, minor, and middle term. Three circles, abbreviated as S, P and M representing minor term, major term and middle term respectively, are drawn in such a way that they overlap. We first draw the two over lapping circles i.e., circle S and the circle P and then we draw the third circle 'M' overlapping the first two circles as shown in the diagram below. The diagram of three overlapping circle give the diagram of eight class. They are: $S\bar{P}\bar{M}$, $SP\bar{M}$, $\bar{S}P\bar{M}$, $S\bar{P}M$, SPM, $\bar{S}PM$, $\bar{S}\bar{P}M$, $\bar{S}\bar{P}\bar{M}$.

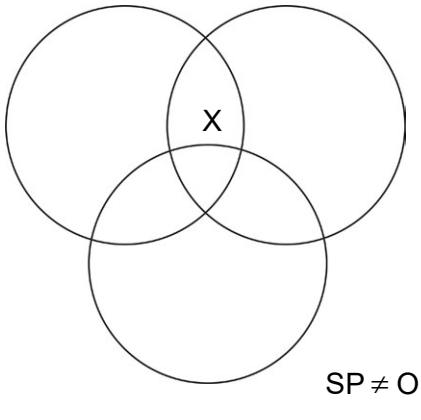


This diagram can be interpreted in terms of various classes. For example, we can take the letter 'S' for the class of all 'scientists', the letter 'P' for the class of all 'philosophers' and the letter M for the class of all mathematicians. Now the part $S\bar{P}\bar{M}$ in the diagram represents the class of all scientists who are neither philosophers nor mathematicians i.e., it is the

product of three class-scientists and non philosophers and non mathematicians. $SP\bar{M}$ is the product of the class scientists and philosophers and non mathematicians. $S\bar{P}M$ is class which is the product of scientists and mathematicians and philosophers. The part SPM represents the class of all those people who are scientists and at the same time philosophers and mathematicians. $\bar{S}PM$ is the product of all people who are both philosophers and mathematicians but not scientists. $\bar{S}P\bar{M}$ is product of the class philosophers with the complement of the class scientists and mathematicians. $\bar{S}\bar{P}\bar{M}$ is the class of all those who are non scientists, non philosophers but mathematicians. It is the product of the class mathematicians with the complement of the two classes scientists and philosophers. The part $\bar{S}\bar{P}\bar{M}$ is the product of the class non scientists, non philosophers and non mathematicians. It is the class of all those people who are neither scientists, nor philosophers nor mathematicians. By shading out or by inserting an 'X' we can diagram any standard form categorical propositions whose terms are S, P and M. For example to diagram the proposition "All S are P" ($S\bar{P} = O$) we have to shade out all of S that is not contained in P as shown in below. To diagram the proposition some S are P ($SP \neq O$) we have to put an X in the overlapping part of the circle S and P.



$S\bar{P} = O$



$SP \neq O$

The general technique of using Venn Diagrams to lest the validity of any standard form of categorical syllogisms are the following:

1) In a standard form of categorical syllogism we have 3 terms represented by S, P and M. Three circles represent three terms i.e., S, P and M. First we draw three over lapping circles and they are labeled as S, P

and M respectively. The circle S stands for minor forms, the circle P stands for minor term and the circle M stands for middle term.

2) We diagram both the premises. If one premise is universal and the other premise is particular then it is necessary to diagram the universal premise first and then to diagram the particular premise.

3) It is to be noted that we need not diagram the conclusion. Rather, we must inspect the diagram to see whether or not the diagram of the premises contains in it the diagrams of the conclusion. If the diagram of the premises contain the diagram of the conclusion then, the given syllogism is valid; if it does not, the syllogism is invalid. In other words, the syllogism is valid if and only if the two premises imply or entail the conclusion. The premises must say what is said by the conclusion. The diagram of the premises of a valid argument should be sufficient enough to include the diagram of the conclusion such that no further marking of the circles is needed.

Let us apply the Venn Diagram technique to test the validity of the following syllogism.

All mathematicians are philosophers.

All scientists are mathematicians.

$\therefore$ All scientists are philosophers.

This syllogism consists of all A propositions. It is of the form AAA. Now we symbolize the syllogism in the following way–

$M \cap \sim P = \wedge$ or $M\bar{P} = O$

$S \cap \sim M = \wedge$ or $S\bar{M} = O$

$\therefore$ $S \cap \sim P = \wedge$ or $\therefore$ $S\bar{P} = O$

Now let us apply the Venn diagram technique. We draw the following figures.

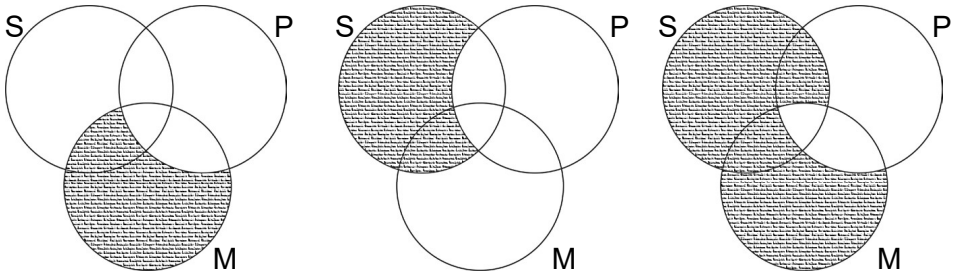


Figure I represents the diagram of the premise $M\bar{P} = O$. $M\bar{P}$ consists of two classes i.e. $M\bar{P}\bar{R}$ and $MS\bar{P}$. These two parts are shaded horizontally to indicate that $M\bar{P} = O$. Like wise figure II represents the diagram of the premise $S\bar{M} = O$. $S\bar{M}$ consists of two parts, that is, $S\bar{P}\bar{M}$ and $SP\bar{M}$. These two parts are horizontally shaded to represent the diagram of the premise $S\bar{M} = O$. The conclusion of the syllogism is $S\bar{P} = O$. The diagram of the conclusion i.e., $S\bar{P} = 0$ demands that the parts $S\bar{P}M$ and $SM\bar{P}$ must be shaded as $S\bar{P} = O$. The diagram of the two premises contain in them the diagram of the premises. Therefore, the given syllogism is valid.

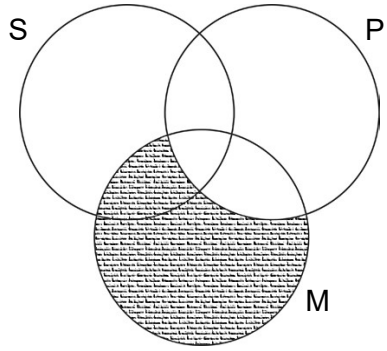
Let us lake another example of syllogistic reasoning:

$M\bar{P} = O$

$MS \neq O$

$\therefore$ $SP \neq O$

The following is the diagram of the above syllogism.



To diagram the premise $M\bar{P} = O$, the part or the area $SM\bar{P}$ and $S\bar{M}\bar{P}$ are shaded. The diagram of the premise $MS \neq O$ consists of two parts i.e., $S\bar{P}M$ and SPM. Since the part $SM\bar{P}$ is already shaded and only the part SMP is left therefore the diagram of the premise $MS \neq O$ is given by inserting an X to the overlapping pant of, S P and M and so there is an X in the region SP. Thus, the diagram of the premises diagram the conclusion or we can say that the diagram of the premises contain in them the diagram of the conclusion and hence the diagram is valid.

Let us take another example.

All scientists are mathematicians.

Some philosophers are mathematicians.

$\therefore$ Some philosophers are scientists.

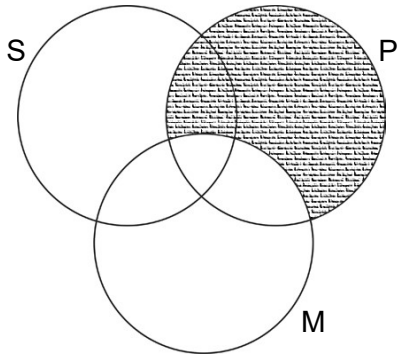
First we symbolize the syllogism in the following way by means of set notation:

$P\bar{M} = O$

$S\bar{M} \neq O$

$SP \neq O$

Now we diagram these two premises by drawing three overlapping circles in the following way as shown in the figure below –



We diagram the premise $P\bar{M} = O$ by shadding the part $SP\bar{M}$ and $S\bar{P}\bar{M}$. The premise $SM \neq O$ says that the part or the area $SM\bar{P}$ or the part SMP is non empty. Since it says that any one of the two parts is non empty therefore we insert an X in the overlapping part of the circle S and M. The conclusion $SP \neq O$ demands that the overlapping part of the circle S and P must be inserted with an X.

Since there is no X in the overlapping part of the circle S and P, therefore, it follows that the diagram of the conclusion does not follow from the diagram of the premises. As the diagram of the premises does not entail the diagram of the premises therefore the given syllogism is invalid.



CHECK YOUR PROGRESS

Q.6: What is Venn diagram technique?

.....

.....



1.7 LET US SUM UP

- A standard form of categorical syllogism consists of standard form categorical propositions.
- The form of a standard form of categorical syllogism is determined by its mood and figure.
- To test the validity of standard form of categorical syllogism three overlapping circles are drawn. We diagram both the premises but not the conclusion. The diagram of the premises must entail in it the diagram of the conclusion. An argument is valid if and only if the diagram of the conclusion follow from the diagram of the premises.



1.8 FURTHER READING

1) Bason and O'connor; *Introduction to Symbolic Logic*.
2) Copi, M. Irving; *Introduction to Logic* (Sixth edition).
3) Copi, M. Irving; *Symbolic Logic* (Fifth edition).



1.9 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q. No. 1: A standard form of categorical proposition is deductive argument which consists of two categorical propositions out of which the conclusion is drawn.

Ans. to Q. No. 2: Middle term, Minor term and Major term

Ans. to Q. No. 3: Four figures.

Ans. to Q. No. 4: Four figures

Ans. to Q. No. 5: a) Three terms, b) Middle term, c) Figure and Mood

Ans. to Q. No. 6: It is a technique of testing the validity of standard form of categorical prepositions with diagrams.



1.10 MODEL QUESTIONS

A) Very short questions:

Q.1: Define the concept of figure in syllogism.

Q.2: What is mood?

Q.3: What do you mean by categorical syllogism?

Q.4: What is venn diagram?

Q.5: What do you mean by standard form of categorical syllogism?

Q.6: What are the figures of syllogism?

Q.7: Give an example of standard form of categorical syllogism.

B) Short questions: (Answer each question in about 150 words)

Q.1: What is standard form of categorical syllogism? Briefly explain.

Q.2: Briefly explain the concept of Mood.

Q.3: What do you mean by the concept of Mood? Explain in brief.

Q.4: Writes short notes on:

a) Standard form of categorical syllogism

b) Figure

c) Venn diagram technique

Q.5: Briefly explain the structure of categorical syllogism.

C) Long questions: (Answer each question in about 300-500 words)

Q.1: What is standard form of categorical syllogism? Explain the structure of categorical syllogism.

Q.2: Explain the concept of mood in syllogism.

Q.3: What is figure? Do the concepts of mood and figure play an important role to determine the form of a syllogism? Discuss

Q.4: What is Venn diagram technique? Discuss this technique for testing syllogism? With suitable examples.

*** ***** ***

UNIT 2: VENN DIAGRAM

UNIT STRUCTURE

2.1 Learning Objectives

2.2 Introduction

2.3 Categorical Proposition and Standard Form of Categorical Proposition

2.4 Classes and Relation

2.5 The Four Standard Form of Categorical Propositions and Their Class Relations

2.6 Distribution of Terms

2.7 Venn Diagram

2.8 Use of Venn Diagram

2.9 History

2.10 Aristotelian and Boolean Standpoint

2.11 Symbolism and Diagrams for Standard Form Categorical Propositions

2.12 Let us Sum up

2.13 Further Reading

2.14 Answers to Check Your Progress

2.15 Model Questions

2.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- define a categorical proposition
- explain the four standard form of categorical propositions
- describe different types of class relation
- describe distribution of terms
- illustrate Venn diagram and its uses
- discuss the four standard form of categorical propositions by means of Venn diagram.

2.2 INTRODUCTION

The standard form of categorical propositions is the typical form of propositions recognised in Aristotelian logic. Classes and their relationships form the content of the categorical propositions. There are various ways in which classes may be related to each other. For example, one class is also a member of one class is also a member of another class then the first class is said to be included in the second class and this kind of relation is called total class inclusion relation. The standard form of categorical propositions exclusively expresses the various relations between classes.

Further, a proposition may refer to all members of a class or some of the members of a class designated by its subject and predicate term and these are the 'distribution of terms' that refers to the ways in which the terms can occur in the categorical propositions.

Venn diagram is a method of representing classes and their relationships expressed in standard form of categorical propositions. So, in order to discuss about Venn diagram, we must deal with distribution of terms, standard-form of categorical proposition, and symbolic expression of categorical propositions in set notation.

2.3 CATEGORICAL PROPOSITION AND STANDARD FORM CATEGORICAL PROPOSITION

A proposition that relates two classes is called a **Categorical proposition.** That is**,** a categorical proposition asserts something about the classes**.** Classes and their relationship form the content of the categorical propositions. The classes are denoted by the subject term and the predicate term. A categorical proposition asserts that either all or part of the class denoted by the subject term is included in or excluded from the class denoted by the predicate term. Thus, a proposition which asserts something about the relationships between the classes referred to by their subject and predicate terms is known as categorical proposition. Aristotelian logic is exclusively concerned with this kind of statements and with arguments that are formed out of these propositions. Some of the examples of categorical propositions are:

1) All ducks swim
2) A fish is not a mammal
3) A few soldiers are heroes
4) All prisoners are not violent

In other words a categorical proposition is one in which the relation between the subject and the predicate is without any condition. In all the above examples it is seen that the relation between the subject and the predicate is not subject to any condition.

A categorical proposition that expresses these relations with complete clarity is called a **standard-form of categorical proposition**. For example;

All men are animal

No man is perfect

Some students are intelligent

Some men are not wise.

Here the subject and the predicate term refer to two classes and connected by the verb 'to be'. A categorical proposition is **in standard form** if and only if it is a substitution instance of one of the following four forms:

All S are P

No S are P

Some S are P

Some S are not P.

Many categorical propositions, of course, are not in standard form because; they do not begin with the words 'all' 'no', 'or', 'some'. Categorical propositions have to be translated into standard form before it can be considered in logic.

The words 'all', 'no', and 'some' are called **quantifiers** because they specify how much of the subject class is included in or excluded from the predicate class. The letters **S** and **P** stand for the subject and the predicate terms respectively, and the words 'are' and 'are not' are called the **copula** because they link the subject and the predicate term. Some of the important points about the standard form categorical proposition are:

- The 'subject term' and the 'predicate term' do not mean the same thing in logic that 'subject' and 'predicate' mean in grammar. The

subject of the example statements includes the quantifier words such as 'all', 'some', but the subject term do not. Similarly the ***predicate*** includes the copula 'are' but the predicate term does not.

- There are exactly three forms of quantifiers (all, no, some) and two forms of copula (is/are, is not/are not). But in ordinary statements various forms of the verb 'to be' are considered as copula.
- Standard form of categorical proposition thus represent an ideal of clarity in language.



LET US KNOW

The theory of categorical propositions originated by Aristotle has constituted one of the core topics in logic for over 2,000 years. It remains important even today.

2.4 CLASSES AND RELATION

A class or category is a group of objects having some recognizable common properties or characteristics which can be predicated about all of them. Classes can also be referred to as *categories* or sets. Aristotelian logic deals with the logic of classes. Aristotle defined a class as a collection of objects or entities. Thus, a category is simply a group, a set of things. For example, class of players, class of college students etc. But it is not a random collection of members. The members of a class are supposed to have a common property that can be predicated about all of them. Terms like 'class', ' sets', 'collection', 'aggregate' etc. are used synonymously.

There are various ways in which classes may be related to each other. If every member of one class is also a member of another class then the first class is said to be included in the second class. For example, suppose we take two classes viz. 'Indians' and 'Asians'. Here, we know that all the members of the class 'Indians' are also the members of the class 'Asians'. The class 'Indians' is included in the class 'Asians. This kind of inclusion is called **total class inclusion relation** as the whole class 'Indians' is included in the class 'Asians'. If some, not all members of one class are

also members of another, then the first class may be said to be partially included in the second class. For example, *some men are poet.* If some of the members of the first class are excluded from being the members of the second class, then the first class is said to be partially excluded from the second class. For example, *some men are not poet. Of* course, there are two classes having no members in common. For example, No triangles are circles. Such kind of relation is said to be total class exclusion relation.

2.5 THE FOUR STANDARD FORM OF CATEGORICAL PROPOSITIONS AND THEIR CLASS RELATIONS

Quality and quantity are attributes of categorical proposition. According to quality, propositions are divided into two-affirmative and Negative. Again according to quantity, propositions are divided into two-Universal and Particular. Both universal and particular propositions may be affirmative and negative. Hence, combining the quality and quantity principles, we get four kinds of categorical propositions in Aristotelian logic. They are universal affirmative, Universal negative, particular affirmative and particular negative. The letters A, E, I and O are used as names for these four kinds of categorical propositions respectively.

The Universal affirmative proposition (A) asserts that the subject class is wholly contained in the predicate class. That is, every member of the subject class is also a member of the predicate class. Because it refers to 'every member', its quantity is **universal**. Since it asserts a relationship, its quality is **affirmative**. In other words, in a universal affirmative proposition predicate is affirmed of the whole subject. Suppose *S* and *P* are any two classes, its schematic form is '*All S is P'*. It may be illustrated by an actual example: 'All swans are white'. It expresses total class inclusion relation.

The universal negative proposition (E) asserts that there is no relation between the two classes. That is, no member of one class is a member of the other class. Its quality is negative and quantity is universal. In other words, in a universal negative proposition predicate is denied of the whole subject. Its schematic form is– '*No S is P'.* It may be illustrated by an actual example: 'No children are cruel'. Here, the first class is wholly excluded

from the second class. Thus, a universal negative proposition expresses the total class exclusion relation.

Particular Affirmative Proposition (I) affirms that only some, not all members of one class are also members of another class. Its quality is affirmative and quantity is particular. In other words, in a particular negative proposition predicate is affirmed of the part of the subject. For example: Some flowers are yellow. Its schematic form is 'Some S is P'. This kind of proposition expresses partial class inclusion relation.

Particular Negative Proposition (O) asserts that some members of a class are not members of another class. Its quality is particular and quantity is negative. Hence, in a particular negative proposition predicate is denied of the part of the subject. For example: 'Some flowers are not yellow'. Its schematic form is 'Some S is not P'. Here, quality is negative and quantity is particular.

Thus, the four categorical propositions, named A, E, I and O stand for universal affirmative, universal negative, particular affirmative, particular negative, respectively. They are known as standard-form categorical propositions. Standard-form categorical propositions refer to two classes.

2.6 DISTRIBUTION OF TERMS

Like quality and quantity, which are the attributes of propositions, **distribution** is an attribute of the terms of propositions. A proposition may refer to all members of a class or some of the members of a class designated by its subject and predicate term.

The term *'distribution'* refers to the ways in which the terms can occur in the categorical propositions. In other words, by the 'distribution' of a term, it is meant that *'taking it universally or referring to all or parts of it'*. A term is said to be distributed if the proposition makes an assertion about every member of the class denoted by the term; otherwise it is undistributed. That means, in a proposition a term is distributed if it refers to all the members of a class designated by the term. Any 'A' proposition of the form 'All S is P' refers to all members of the class designated by its subject term 'S' but does not refer to all the members of the class designated by its predicate

term 'P'. Hence, 'A' proposition distributes its subject term but not the predicate term.

The form of universal negative proposition is 'No S is P'. The universal negative proposition, i.e. 'E' proposition refers to all the members of the class designated by its subject term as well as by its predicate term. Thus 'E' proposition distributes both the subject and the predicate term.

The form of particular affirmative proposition is 'Some S is P'. The particular affirmative proposition i.e. 'I' proposition refers to some members of the class designated by its subject term and some members of the class designated by its predicate term. Thus 'I' proposition distributes neither the subject nor the predicate.

The form of particular negative proposition is 'Some S is not P'. The particular negative proposition i.e. 'O' proposition distributes the predicate term but not its subject term. Thus,

'A'– proposition distributes its subject term but not the predicate term.

'E'– proposition distributes both the subject and the predicate term.

'I'– proposition distributes neither the subject nor the predicate.

'O'– proposition distributes the predicate term but not the subject term.

The various relations between classes such as class inclusion or class exclusion either affirmed or denied by the four standard form categorical propositions can be represented by using the Venn diagram.



LET US KNOW

Category: A predicate or a fundamental class of things.

Class: A collection of objects or entities with a common characteristic.

Some: In logic, 'some' means at least one.



CHECK YOUR PROGRESS

Q.1: Answer briefly:

a) What is categorical proposition?

.....

.....

b) Give an example of categorical proposition?

.....

c) What do you mean by a class?

.....

.....

.....

d) What are the four different standard form categorical propositions?

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Q.2: Define:

a) Universal affirmative proposition.

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b) Particular negative proposition.

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.....

.....

Q.3: State whether the following statements are true (T) or False (F):

a) 'A' proposition distributes subject. (True/False)

b) 'E' proposition distributes predicate.(True/False)

Q.4: Fill in the blanks:

a) Standard-form categorical propositions refer to ..... classes.

b) The words 'are' and 'are not' are called the ..... because they link the subject and the predicate term.



ACTIVITY 2.1

Do you find any distinction between a categorical proposition and a standard form categorical proposition?

.....

.....

.....

2.7 VENN DIAGRAM

A Venn diagram is an illustration of the relationships between sets and among sets, indicated by the arrangement of the circles. Venn diagram uses circles to represent sets, as for example by drawing one circle within another to indicate that the first set is a subset of a second set. Venn diagram is thus a method of representing classes and their relationship expressed in propositions. Copi has defined Venn diagram as an "*Iconic representations of categorical propositions, and of arguments to display their logical forms using overlapping circles*". The Venn diagram is named after its inventor, British mathematician and logician John Venn (1834-1923).

2.8 USE OF VENN DIAGRAM

The Venn diagram is used in many fields such as in statistics, computer science, linguistics, logic, and probability. Fundamentally they are used to teach elementary set theory, as well as illustrate simple set relationships in probability, statistics, and computer science. The uses of Venn diagram in formal logic can be brought out in the following way.

- Venn diagram is used to represent the information expressed by the four kinds of categorical propositions. That is, the relationship between the classes designated by the subject and the predicate term of categorical proposition is represented by means of Venn diagram.
- The technique of Venn diagram is used to determine the validity or invalidity of syllogistic argument. Thus it enables us to distinguish valid syllogistic argument from invalid syllogistic argument.
- Venn diagrams are used to explain how the two statements relate to each other.
- The logical relations between sets and operations on sets can be represented by using Venn diagrams.

2.9 HISTORY

The history of the Venn diagram, which is otherwise known as the set diagram goes back to 1880, when British logician and philosopher John Venn introduced it in his research paper “On the Diagrammatic and Mechanical Representation of Propositions and Reasonings”, which was published in the ‘Philosophical Magazine and Journal of Science’. Venn himself did not use the term “Venn diagram” and referred to his invention as “Eulerian Circles”. The first to use the term “Venn diagram” was Clarence Irving Lewis in 1918, in his book “A Survey of Symbolic Logic”. At that time, John Venn referred to the Venn diagram as the Eulerian circle, because it resembles Euler diagrams, invented by Leonhard Euler in the 18th century. Venn diagrams differ from the use of Euler circles in a few important ways.

2.10 ARISTOTELIAN AND BOOLEAN STANDPOINT

Before going to discuss about the diagrammatic representation of the standard form categorical propositions through Venn diagram, we are to concentrate at the meaning of universal affirmative (A) and universal negative propositions (E) from **Aristotelian and Boolean standpoints.**

The Aristotelian standpoint differs from the Boolean standpoint only with regard to universal (**A** and **E**) propositions. The two standpoints are identical with regard to particular (I and O) propositions.

Aristotle held that universal propositions have **existential import**. On the other hand, the nineteenth-century English mathematician and logician, George Boole held that no universal propositions have existential import.

George Boole is one of the founders of modern symbolic logic. Boole is now chiefly remembered for Boolean Algebra, named after him. He applied algebraic techniques to traditional Aristotelian logic. Boole proposed a calculus for syllogism. It involved translating each syllogism in arithmetical notation. This idea led to invent propositional calculus and Boolean algebra. His chief works were: *The Mathematical Analysis of Logic* and *An Investigation of the Laws of Thought*. The four standard form categorical propositions have the following meaning from the Boolean standpoint.

A– All S are P = No members of S are outside P

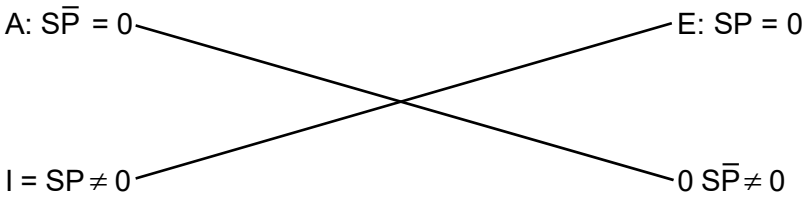
E– No S are P = No members of S are inside P

I– Some S are P = A least one S exists, and that S is a P

O– Some S are not P = At least one S exists, and that S is not a P.

John Venn has adopted this interpretation of categorical propositions, while he developed the diagrammatic technique for representing the information expressed by categorical propositions. In Venn diagram, there is an arrangement of overlapping circles in which each circle represents the class denoted by a term in a categorical proposition.

The Boolean square of opposition or the modern square of opposition: Drawing on some ideas of Boole we can build a square of opposition. In Boolean symbolic interpretation, the interrelation among the four standard form categorical propositions appears very clearly. It is obvious that A and O propositions are contradictories. Analogously, E and I propositions contradict each other. This relationship of mutually contradictory pairs of propositions is represented in a diagram called Modern Square of opposition or Boolean square of opposition. The diagram can be represented by the following figure–



***F**irst, we notice that both I and O propositions have existential import, because both assert the existence of at least one entity. On the other hand, A and E propositions both do not have existential import.

***S**econdly, if two propositions are related by the contradictory relation, they necessarily have opposite truth value. Thus, if certain **A** proposition is given as true, the corresponding **O** proposition must be false. Similarly, if certain **I** proposition is given as false, the corresponding **E** proposition must be true. But no other inferences are possible. In particular, given the truth value of an **A** or **O** proposition, nothing can be determined about the truth value of the corresponding E or I propositions. These propositions are said to have **logically undetermined truth value.** Similarly, given the truth value of an E or I proposition, nothing can be determined about the truth value of the corresponding **A** or **O** propositions. They, too, are said to have logically undetermined truth value.



LET US KNOW

Existential Import: A statement is said to have existential import if it asserts or presupposes the existence of certain kinds of objects

2.11 SYMBOLISM AND DIAGRAMS FOR STANDARD FORM OF CATEGORICAL PROPOSITIONS

The symbolisation of the standard form categorical propositions involves **Set Notation**. The symbols used for set notation are: ~, ≡ , Λ, ⋃ ⋂ The Boolean interpretation of categorical propositions depends upon the notion of an empty set. The symbols that represent **Empty Class** are **Λ (Lambda)** or **O (Zero).** For example, when we say that “the class ‘S’ is empty”, it means that “the class ‘S’ does not have any member”. We

symbolise it as **S = Λ** or **S = O.** On the other hand, to say that the class designated by 'S' is not empty is to say the class 'S' have members. We symbolise the denial by drawing line through the equality sign. For example**,** **S** ≠ **Λ** or **S** ≠ **O**. It means that the class S is not empty.

The concept of **Complementary Class** is necessary to explain in this connection. A complementary class is the collection of all things that do not belong to the original class. For example, if S designates the class of 'all students', then its complementary class would be "all things that are not students" which is to be symbolised as $\bar{S}$. It means not *S.*

If two classes, suppose S and P, have some members in common, this common membership will be called the **Product** or **Intersection** of the two classes; it is symbolised as **SP** or **S** ⋂ **P** and it is read as S intersection P**.** The symbol ⋂ is known as 'intersection'. Thus the statement that claims that the product of the two classes is empty is symbolised as **SP = Λ** or **SP = O** or **S** ⋂ **P = Λ**.

On the other hand the statement which expresses that the product of two classes is not empty is symbolised as **SP** ≠ **Λ** or **SP** ≠ **O** or **S** ⋂ ~P **= Λ**.

Symbolisation: The Standard form categorical propositions are symbolised in set notation as:

A– All S is **P** = $\mathbf{S\bar{P}}$ **= O** or $\mathbf{S\bar{P}}$ **= Λ**

The **A** proposition asserts that the part of the S circle that lies outside the P circle is empty.

E– No S is **P** = **SP = O** or **SP = Λ**

The **E** proposition asserts that the area where the two circles overlap is empty.

I– Some S is **P** = **SP** ≠ **O** or **SP** ≠ **Λ**

The **I** proposition asserts that the area where the two circles overlap is not empty.

O– Some S is not **P** = $\mathbf{S\bar{P}}$ ≠ **O** or $\mathbf{S\bar{P}}$ **= Λ**

The **O** proposition asserts that the part of the S circle that lies outside the P circle is not empty.

Venn Duagram: After discussing the symbolization procedure, we still need to learn how to diagram a standard form categorical proposition.

In Venn diagram, we represent a class by a circle labelled with the term that designates the class. Thus, the class 'S' is diagrammed as shown below:

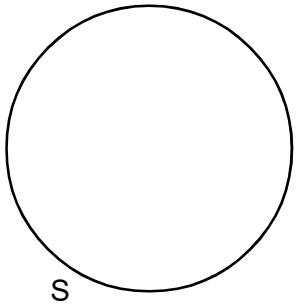
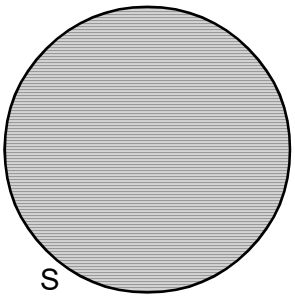


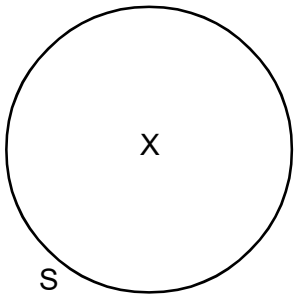
Fig. 2.1

It represents the class 'S' but says nothing about it. Hence to represent the information by the class, two kinds of marks are used: (1) shading an area and (2) Placing an X in an area. Shading an area means that the shaded area is empty and placing an X in an area means that at least one thing exists in that area. Thus, we shade the interior of the circle to indicate that the class 'S' has no members. And we place an X any where in the interior of the circle to represent that there is at least one member of the class 'S'. This is represented by the following diagram:



S = O

Fig. 2.2



S ≠ O

Fig. 2.3

Standard form of categorical propositions have exactly two terms. Therefore, to diagram the standard form categorical propositions two overlapping circles are required. In all the A, E, I and O propositions the subject and the predicate terms that represent classes are abbreviated by S and P. Usually, the left hand circle represents the subject term (S) and the right hand circle represents the predicate (P) term. Such a diagram looks like this:

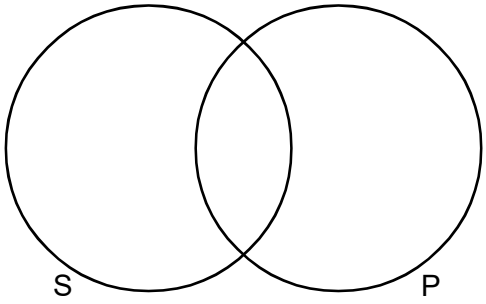


Fig. 2.4

This figure diagrams only the two classes but does not give any information about their class relationship. The above diagram of the two overlapping circles represents different types of class relationship expressed by the four kinds of categorical propositions.

Firstly, the area that designates those members of S that are *not* members of P is symbolised as $S\overline{P}$. The symbol $\overline{P}$ (P-bar) has been used to indicate the regions which are *not P*

Secondly, the area that designates those members of P that are not members of S is symbolised as $\overline{S}P$. The symbol $\overline{S}$ (S-bar) has been used to indicate the regions which are *not S.*

Thirdly the area designating members of S that are at the same time members of P is symbolised as *SP*. This part represents the intersection of the class S and P or the product of S and P i.e., all things that belong to both S and P.

Finally, the area that designates where no members of either S or P can be found is symbolised as $\overline{S}\overline{P}$. Diagrammatically, we may represent this as follows:

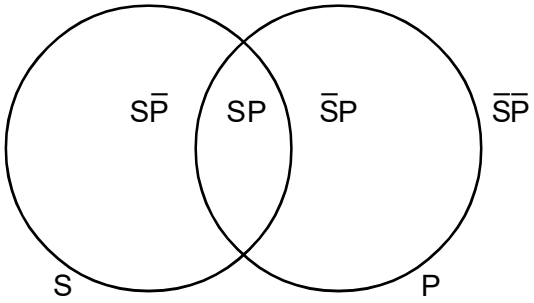


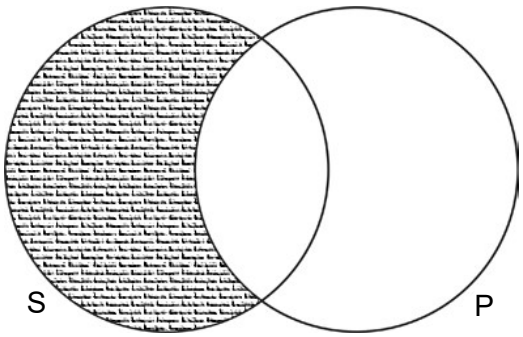
Fig. 2.5

Thus, this diagram that represents various classes is known as Venn diagram.

Universe of Discourse: The rectangle outlining the diagram represents the Universe of Discourse (U.D.) which means the context or everything that is assumed or believed for the discussion. There are many kinds of things in the world, such as people, plants, animals, physical objects etc. The collection of things that we are talking about on a given occasion constitutes the universe of discourse for that occasion.

Diagrams for Standard Form of Categorical Propositions: Now we are in position to learn how to complete the diagrams for our four standard form categorical propositions. So, let us first diagram the **Universal Affirmative (A) Proposition.**

The 'A' proposition asserts that no members of S are outside P. In other words, all members of the class S are also the members of class P. Its schematic form is– "All S is P". It is symbolised as SP = O or SP = Λ. It means the part of the circle S which is not in P is empty. This is represented by shading the part of the S circle that lies outside the P circle. The diagram for the 'A' proposition is given below:

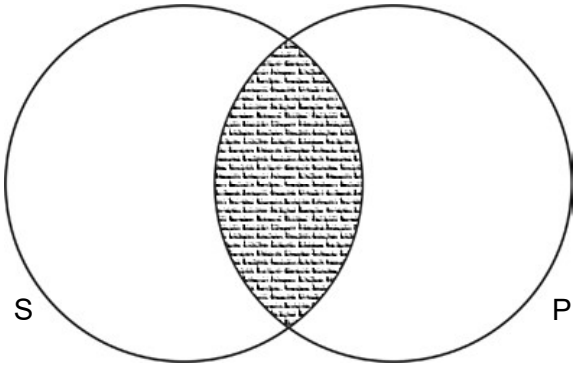


A: All S is P

$S\overline{P} = O$

Fig. 2.6

E Proposition: 'E' Proposition is the universal negative proposition. Its schematic form is– ''No S is P". It asserts that no members of S are inside P. In other words, the class S is totally excluded from the class P. It is symbolised as SP = Λ or SP = O. It means the common area of the circle S and P is empty. This is represented by shading the part of the S circle that lies inside the P circle. The diagram for the 'I' proposition is given below:

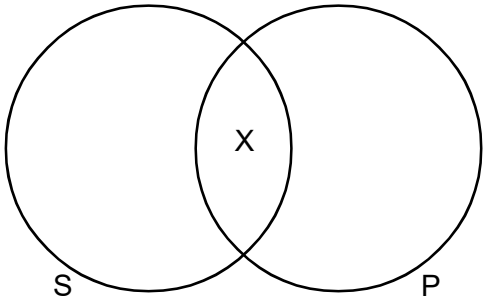


E = No S is P

SP = O

Fig. 2.7

I Proposition: 'I' proposition is the particular affirmative proposition. Its schematic form is– "Some S is P". It asserts that there exists at least one member in the class S and that is also a member of the class P. In other words, the class of S is partly included in the class P. It implies the product of the class S and P is not empty. It is symbolised as $SP \neq O$ or $SP \neq \Lambda$. This is represented by placing an X in the area where the S and P circles overlap. The diagram for the 'I' proposition is given below:

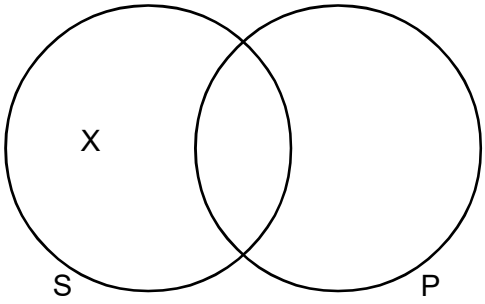


I: Some S is P

$SP \neq O$

Fig. 2.8

O Proposition: 'O' proposition is the particular negative proposition. Its schematic form is– "Some S is not P". The 'O' proposition asserts that at least one S exists, and that S is not a P. It is symbolised as $S\bar{P} \neq O$ or $S\bar{P} \neq \Lambda$. The 'O' proposition asserts that at least one S exists, and that S is not a P. This is represented by placing an X in the part of the S circle that lies outside the P circle. This X represents an existing thing that is an S but not a P. The diagram for the 'O' proposition is given below:



O: Some S is not P

Fig. 2.9



CHECK YOUR PROGRESS

Q.5: Give brief answer:

a) State the symbol which represents the empty class.

.....

b) State the symbol which represents the intersection of the two classes.

.....

c) Symbolise universal negative proposition.

.....

Q.6: What is Venn diagram?

.....

.....

Q.7: Mention two uses of Venn diagram.

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Q.8: Draw Venn diagram for universal negative proposition.

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2.12 LET US SUM UP

- Proposition is the basic unit of logical thinking. There are different ways of classifying a proposition. A proposition that relates two classes or categories is called a categorical proposition. The classes are denoted respectively by the subject term and the predicate term. For example, 'All students are honest', 'grass is green'.
- The subject and the predicate term express different types of class relations such as total class inclusion, total class exclusion, partial class inclusion and partial class exclusion relations. These relations are expressed by the four standard form categorical propositions.
- The four standard form categorical propositions are: A-Universal affirmative (All S is P), E-universal negative (No S is P), I-particular affirmative (some S is P) and O-particular negative (some S is not P). Thus, these four propositions are referred to as standard form of categorical propositions.
- The nineteenth century English logician and mathematician John Venn developed a system of diagrams to represent the categorical propositions. These diagrams have come to be known as **Venn Diagrams**.



2.13 FURTHER READING

1) Baronett, S. and Sen, M.; *Logic*.
2) Chakraborti. C.; *Logic-Informal, Symbolic & Inductive*.
3) Copi, I. M. & Cohen, C.; *Introduction to Logic*.
4) Hurley, J. P.; *Introduction to Logic*.
5) Sharma. B. and Deka J.; *A text Book of logic*.
6) (https://www.lucidchart.com/blog/2013/01/17/a-history-of-the-venn-diagram)

2.14 ANSWER TO CHECK YOUR PROGRESS

Ans. to Q. No. 1: a) A proposition that relates two classes or categories is called a Categorical proposition. For example: All ducks swim.

b) Some students are honest.

c) A class or category is a group of objects having some recognizable common properties or characteristics which can be predicated about all of them.

d) The four standard form categorical propositions are universal affirmative Universal negative, particular affirmative, particular negative. The letters A, E I and O are used as names for these four kinds of categorical propositions respectively.

Ans. to Q. No. 2: a) The Universal affirmative proposition is one which asserts that the subject class is wholly contained in the predicate class. In other words, in a universal affirmative proposition predicate is affirmed of the whole subject.

b) Particular negative proposition is one which asserts that some members of a class are not members of another class. Hence, in a particular negative proposition predicate is denied of the part of the subject.

Ans. to Q. No. 3: a) True, b) False.

Ans. to Q. No. 4: a) Two, b) quantifiers

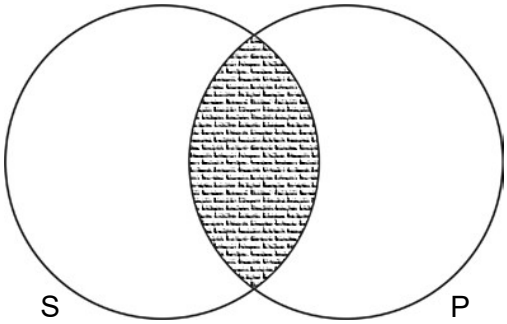
Ans. to Q. No. 5: a) Λ, b) $\cap$, c) A-All S is P = $S\bar{P}$ =O

Ans. to Q. No. 6: A Venn diagram is an illustration of the relationships between sets and among sets, indicated by the arrangement of the circles.

Ans. to Q. No. 7: i) Venn diagram is used to represent the four standard form categorical propositions.

ii) The technique of Venn diagram is used to determine the validity or invalidity of syllogistic argument.

Ans. to Q. No. 8:



E: No S is P

SP = O



2.15 MODEL QUESTIONS

A) Very short questions:

Q.1: Define categorical propositions.

Q.2: What is standard form of categorical proposition?

Q.3: Give an example of categorical proposition.

Q.4: What is venn diagram?

Q.5: What do you mean by total class inclusion relation?

B) Short questions (Answer each question in about 150 words)

Q.1: Write short notes on:

a) Class and relation

b) Uses of Venn diagram

c) Symbolisation of A, E, I and O propositions in Boolean interpretation.

Q.2: Briefly explain the concept of categorical proposition.

Q.3: What is standard form categorical proposition? Briefly explain.

Q.4: What is Boolean square of opposition? Briefly explain.

c) Long questions: (Answer each question in about 300-500 words)

Q.1: What is categorical proposition? Explain

Q.2: Explain the four standard form of categorical propositions with examples.

Q.3: What is Venn diagram? Explain how to represent the four standard form of categorical propositions by Venn diagram.

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UNIT 3: VENN DIAGRAMMATIC TECHNIQUE FOR TESTING THE CATEGORICAL SYLLOGISM

UNIT STRUCTURE

3.1 Learning objectives

3.2 Introduction

3.3 Syllogism

3.4 Kinds of Syllogism

3.5 Standard Form of Categorical Syllogism

3.6 Structure of Standard Form Categorical Syllogism

3.7 Figure of Syllogism

3.8 Moods of Syllogism

3.9 Venn Diagram Technique for Testing Syllogism

3.10 Let us Sum up

3.11 Further Reading

3.12 Answer to Check Your Progress

3.13 Model Questions

3.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- define standard form of categorical syllogism
- explain the structure of categorical syllogism
- discuss the rules of categorical syllogism
- define figure and mood of syllogism
- describe what Venn diagram is.

3.2 INTRODUCTION

This unit introduces to you syllogistic reasoning. Arguments are of two types– Deductive and Inductive. In deductive argument conclusion necessarily follows from the premises. That is, in deductive argument the premises provide conclusive support for accepting the conclusion. Aristotelian

deductive logic is primarily concerned with syllogistic reasoning. In the general sense of the term, a syllogism is a deductive argument consisting of two premises and one conclusion. This unit includes the definition as well as the structure of the categorical syllogism. A categorical syllogism is a deductive argument which contains two premises and one conclusion and conclusion necessarily follows from the two premises taken jointly. A standard form of categorical syllogism consists of three standard form categorical propositions and three terms which are arranged in a specified standard order. Apart from these, you will able to conceive the meaning of figure and mood of syllogism and Venn diagram technique for testing syllogism.

3.3 SYLLOGISM

Logic is generally described as a science of reasoning. Inferences or reasonings are of two types. They are Deductive and Inductive. In a deductive inference, the conclusion can never be more general than its premise or premises. That is, the conclusion is contained or implied in the premises. On the other hand, in an inductive inference, the conclusion is always more general than the premises. That is, the conclusion goes beyond the premises.

Deductive inference is of two types-mediate deductive inference and immediate deductive inference. **A syllogism is a form of mediate deductive inference, in which the conclusion necessarily follows from two premises, taken jointly.** From this definition we can grasp the nature and structure of syllogism.

Firstly, a syllogism is a mediate deductive inference. As a kind of inference, its conclusion expresses something other than those which are expressed in the premises. Further, as it is *deductive* in character, its conclusion can never be more general than either of its two premises, rather the conclusion contained in the premises. It is a *mediate* form of inference, the conclusion being drawn from more than one premise.

Secondly, the two premises of a syllogism are logically interconnected and thus the conclusion necessarily follows from the two premises taken jointly.

Thirdly, since the conclusion follows necessarily from the premises, so the conclusion of a syllogistic argument becomes formally true or valid.

Fourthly, a syllogism is purely formal. It is concerned with the formal truth, but not with material truth. That is, in a syllogism, we are not concerned with the question as to whether the given propositions are as a matter of fact, true or false. The only question is, granting the premises to be true, the conclusion must necessarily be true.

3.4 KINDS OF SYLLOGISM

Syllogisms have been classified into **Pure** and **Mixed**. In a pure syllogism, all the constituent propositions are of the same relation. If all of them are categorical proposition, the syllogism is known as Pure Categorical; if all the constituent propositions are hypothetical, the syllogism is Pure Hypothetical; and lastly, if all of them are disjunctive, the syllogism is Pure Disjunctive.

In a Mixed Syllogism, on the other hand, the constituent propositions are of different relations. Mixed syllogisms are of three kinds. They are: Hypothetical –Categorical, Disjunctive Categorical and Dilemma.

This unit will focus on categorical syllogism and Venn diagram technique for testing categorical syllogism.

3.5 STANDARD FORM OF CATEGORICAL SYLLOGISM

A Categorical Syllogism is one where all its premises and conclusion are categorical propositions. As such, all the constituent propositions of a categorical syllogism are categorical propositions. According to Copi & Cohen, a *categorical syllogism is a deductive argument consisting of three categorical propositions that together contain exactly three terms, each of which occurs in exactly two of the constituent propositions.*

In the preceding section we have explained about categorical propositions and the Venn diagram technique to diagram the standard form categorical propositions. This unit introduces to you the Venn diagram technique for testing categorical syllogism.

A Standard-Form of Categorical Syllogism is a categorical syllogism in which the premises and conclusion are all standard –form categorical propositions i.e. either of A, E, I, or O and are arranged in specified standard order. In other words, a standard form categorical syllogism is a form of mediate deductive inference, consisting of three standard form categorical propositions, having between them three and only three terms which are interconnected in such a way that the first two propositions jointly imply the third proposition.

For example, All men are mortal.
All kings are men.
Therefore, all kings are mortal.

3.6 STRUCTURE OF STANDARD FORM OF CATEGORICAL SYLLOGISM

A standard form of categorical syllogism consists of two premises and one conclusion. Its premises and conclusion are all standard form categorical propositions. Each proposition consists of two terms. A standard form categorical syllogism has a definite structure that can be explained in the following way.

- Firstly, it is composed of only three standard form categorical propositions.
- Secondly, it contains exactly three terms, each of which occurs twice in the constituent propositions.
- Each term is used in the same sense throughout the argument.
- The major premise is listed first, the minor premise second and the conclusion last.

Major, Minor and Middle Terms: A syllogism consists of three terms. They are: major term, minor term and middle term. The term that occurs as the predicate of the conclusion is called the **major term** of the syllogism, and the term that occurs as the subject term of the conclusion is called the **minor term** of the syllogism. The term that occurs in both the premises but not in the conclusion is called the **middle term**. The major and minor terms are called **Extremes**, to distinguish them from the middle term.

The premises of a standard-form syllogism are named after the terms that appear in them. The premise in which the major term occurs with the middle term is called the **major premise**. The premise in which the minor term occurs with the middle term is called the **minor premise**. For example, in the following syllogism–

All men are mortal.

All poets are men.

Therefore, all poets are mortal.

In this syllogism 'mortal' is the major term, being the predicate of the conclusion. The term 'poets' is minor term, because it is the subject of the conclusion; the term 'men' is the middle term, because it occurs in both the premises, but is absent from the conclusion. Further, the first premise-'All men are mortal' is the major premise, because the major term 'mortal' occurs in it. The second premises 'All poets are men' is the minor premise, because the minor term 'poets' occurs in it.

The symbol '**P'** stands for major term**, 'S'** stands for minor term and '**M'** stands for middle term.



CHECK YOUR PROGRESS

Q.1: What is standard form categorical syllogism?

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Q.2: Explain briefly the structure of categorical syllogism.

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Q.3: Define:

a) Middle term

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b) Major term

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c) Minor premise

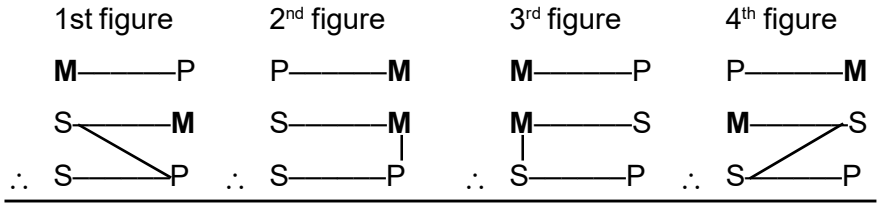
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3.7 FIGURE OF SYLLOGISM

As we know that in a syllogism the middle term occurs in both the premises. But the position of middle term is not the same in all syllogisms. Depending on the position of the middle term in both the premises we have different forms of syllogism what is known as figure of syllogism. Figure indicates the position of the middle term in the syllogism. Thus, figure is the form of a syllogism as determined by the position of the middle term in the premises. There are four possible arrangements of the middle term in the two premises and therefore, there are four figures of syllogism viz. first figure, second figure, third figure and fourth figure.

In the first figure the position of the middle term is the subject in the major premise and the predicate in the minor premise. In the second figure the position of the middle term is the predicate in both the premises. In the third figure the middle term is the subject in both the premises and in the fourth figure the middle term is the predicate in the major premise and subject in the minor premises. The four figures schematized in the following ways–



3.8 MOOD OF SYLLOGISM

The mood of a standard form categorical syllogism is determined by the quality and quantity of the constituent standard form categorical propositions. For example, since the following syllogism contains all 'A' propositions (both premises and the conclusion), therefore the mood of the syllogism is AAA.

A– All men are mortal.

A– All poets are men.

A– Therefore all poets are mortal.

Thus the mood of every syllogism is represented by three vowels stand for major premises, minor premises and the conclusion. According to the quality and quantity combined there are four kinds of propositions-A, E, I and O. A syllogism has got two premises. If we take into account the quality and quantity of the premises only, we have 16 possible moods in each figure and 64 (16 X 4)possible moods in all the figures. Again, if we take into account the quality and quantity of all the three constituent propositions, we have 64 possible moods in each figure and 256 (64 X 4) possible moods in all the figures. Out of these 256 moods some are valid and some are invalid. In Aristotelian logic 19 moods are considered to be valid. But under the Boolean interpretation, if we apply Venn diagram for testing the validity or invalidity of these possible moods we get only 15 valid standard form categorical syllogisms.

The valid moods of first figure are: **AAA (BARBARA), AII (DARII), EAE (CELARENT),** and **EIO (FERIO)**

The valid moods of the second figure are: **EAE (CESARE) AEE (CAMESSTRES), EIO (FESTINO),** and **AOO (BAROCO)**

The valid moods of the third figure are: **AAI (DARAPTI), IAI (DISAMIS), AII (DATISI), EAO (FELAPTON) OAO (BOCARDO)** and **EIO (FERISON).**

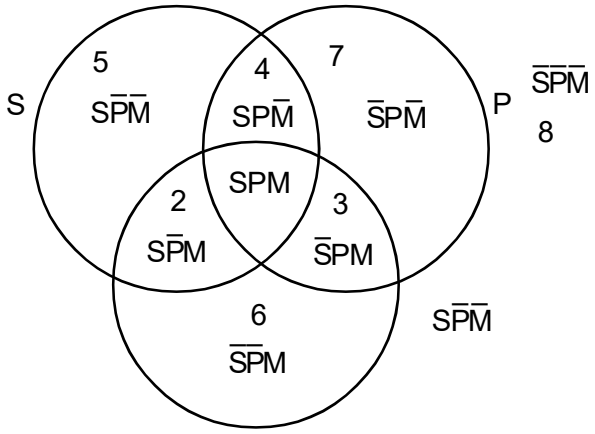
The valid moods of the fourth figure are: **AAI (BRAMANTIP), AEE (CAMENES) IAI (DIMARIS), EAO (FESAPO)** and **EIO (FRESISON).** But the four Aristotelian valid moods, namely, DARAPTI, FELAPTON, BRAMANTIP and FESAPO are invalid moods under Boolean system.

3.9 VENN DIAGRAM TECHNIQUE FOR TESTING SYLLOGISM

In the preceding chapter we have discussed how to draw Venn diagram to represent standard form categorical propositions. Now let us apply the Venn diagram technique for testing the standard form categorical syllogism. Venn diagram technique requires the following strategy.

- Since, categorical syllogisms have three terms we have to draw three overlapping circles.
- We have to label the circles by the letters S, P, and M representing the minor term, major term and middle term respectively.
- We first draw the two overlapping circles i.e. circle 'S' and circle 'P' and then we draw the third circle 'M' overlapping the first two circles as shown in the diagram below. The three overlapping circles give the diagram of eight classes or areas. The areas are– $S\bar{P}\bar{M}$, $SP\bar{M}$, $\bar{S}P\bar{M}$, $S\bar{P}M$, SPM, $\bar{S}PM$, $\bar{S}\bar{P}M$, $\bar{S}\bar{P}\bar{M}$.
- If the syllogism contains both universal and particular premises, it is necessary to draw the universal premise first irrespective of their given order.
- Finally, to decide whether the argument is valid or not, we see the conclusion whether it is covered by the premises or not.

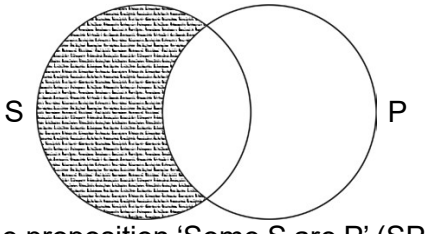
The following diagram can be interpreted in terms of various classes or areas. For example, we can take the letter 'S' for the class of all 'scientists', the letter 'P' for the class of all 'philosophers' and the letter 'M' for the class of all 'writer'. Thus the areas are:



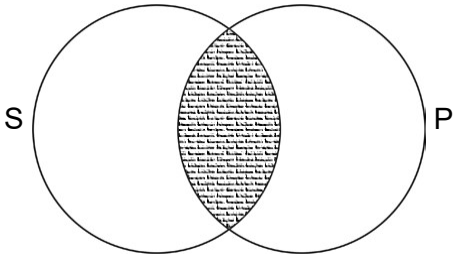
1) SPM– the class of all those people who are scientists and at the same time philosophers and writers.
2) Scientists and S$\bar{P}$M– the class which is the product of scientists and writers and the complement of philosophers.
3) $\bar{S}$PM– is the product of all people who are both philosopher and writer but not scientists.
4) SP$\bar{M}$– is the product of the class scientists and philosophers and not writers.
5) S$\bar{P}\bar{M}$ – is the class of all scientists who are neither philosophers nor writers.
6) $\bar{S}$P$\bar{M}$ – is the class of all those who are non-scientists, non-philosophers but writers.
7) $\bar{S}$P$\bar{M}$– the class of philosophers who are not scientists and not writers.
8) $\bar{S}\bar{P}\bar{M}$ – is the product of the class non-scientists, non philosophers and non writers.

By shading out or by inserting an 'X' we can diagram any standard form categorical proposition. The diagrams to represent the A, E, I and 0 propositions have already been shown in the proceeding chapter. Thus, to diagram the proposition 'All S are P' ($S\bar{P} = 0$) proposition we have to shade out all S that is not contained in P as shown below.

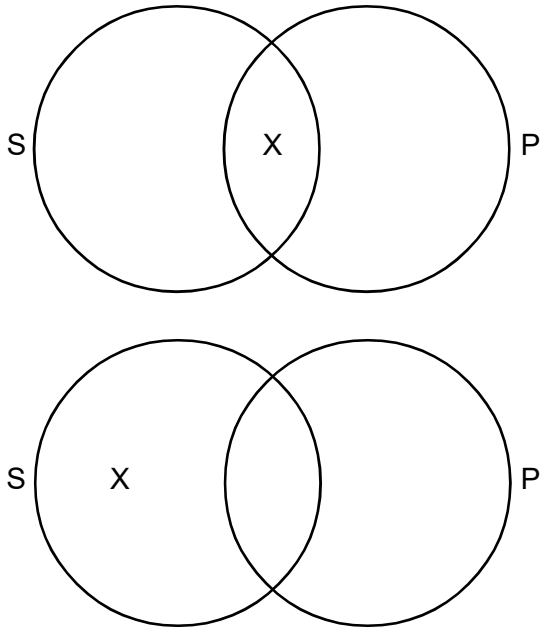
To diagram the proposition 'No S are P' (SP = O) we have to shade out the overlapping part of the circle S and P as shown below.



To diagram the proposition 'Some S are P' ($SP \neq O$) we have to put an X in the overlapping part of the circle S and P.



To diagram the proposition 'Some S are not P' (SP ≠ O) we have to put an X in the part of the circle S that is not contained in P.



Now let us apply the Venn diagram technique to test the validity of the following syllogism.

Example 1:

A– All writers are philosophers.

M P

A– All scientists are writers.

S M

Therefore all scientists are philosophers.

S P

This syllogism consists of all A propositions. We symbolise the syllogism in the following way by means of set notation or from the Boolean standpoints.

Major premise– $M \cap \sim P = \Lambda$ or $M\bar{P} = O$

Minor premise– $S \cap \sim M = \Lambda$ or $S\bar{M} = O$

Conclusion– $S \cap \sim P = \Lambda$ or $S\bar{P} = O$

Now let us apply the Venn diagram technique for testing the validity of the argument. We draw the following figures.

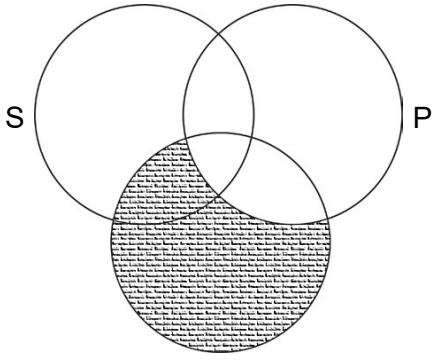


Figure I

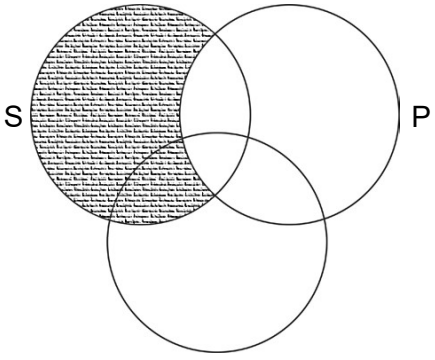


Figure II

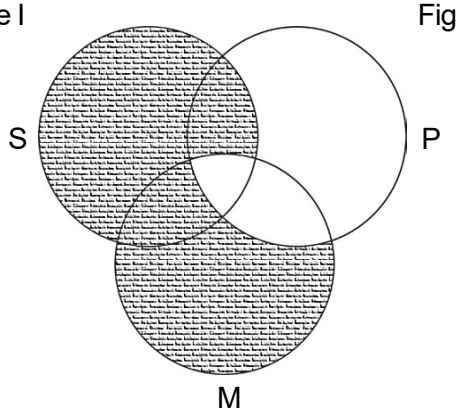


Figure III

Figure I the represents the diagram of the major premise $M\bar{P} = O$. In this figure the area of the circle M which is not contained in P is horizontally shaded out to diagram the major premise $M\bar{P} = O$. Likewise, the figure II represents the diagram of the minor premise $S\bar{M} = O$. In this figure the area of the circle S which is outside M is horizontally shaded out to diagram the minor premise $S\bar{M} = O$. Now we have to see whether the conclusion is entailed by the premises or not. We have seen that the S that is outside the P is shaded out by the premises. It means, conclusion ($S\bar{P} = O$) is entailed by the premises and hence the argument is valid.

Now let us take another example of syllogistic reasoning.

Example 2:

All men are honest

Some men are scientists

$\therefore$ Some scientists are honest.

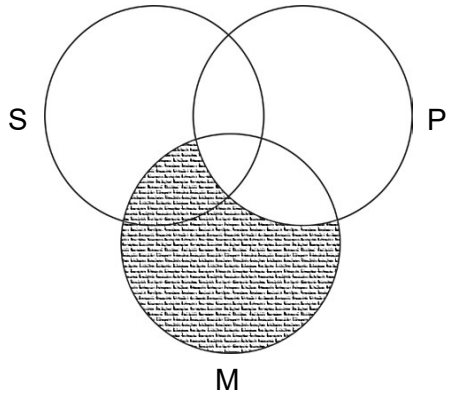
First we symbolise the argument in the following way by means of set notation.

$M\bar{P} = O$

$MS \neq O$

$\therefore$ $SP \neq O$

Now we diagram these two premises by drawing three overlapping circles in the following way as shown in the figure below–



We diagram the major premise $M\bar{P} = O$ by shading the area which are in M but not in P that is $SM\bar{P}$ and $S\bar{P}\bar{M}$. The minor premise $MS \neq O$, being an I proposition, we have to put an X in the overlapping part of the circle S and M. The conclusion $SP \neq O$ demands that the overlapping part of the circle S and P must be inserted with an X. In the above diagram we have seen that there is an X in the overlapping part i.e. in the common part of the circle S and P. That means the conclusion is entailed by the premises and therefore the given syllogism is considered to be valid.

Example 3:

All poets are creative

Some men are creative

$\therefore$ Some men are poets

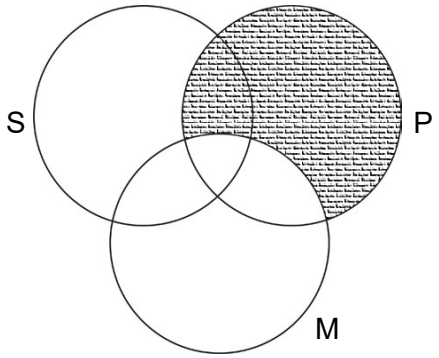
This argument has three propositions, one of which is particular proposition. It is important to note that if one of the premises is a particular proposition then we have to insert an x in the appropriate areas depending on the nature of the proposition. Now let us symbolise the propositions of the argument.

Major premise: A proposition– $P\bar{M} = 0$

Minor Premise: I proposition– $SM \neq 0$

Conclusion: I proposition– $SP \neq 0$

Let us represent the syllogism diagrammatically–



In the diagram the major premise $P\bar{M} = 0$, which is a universal proposition, is drawn first. To diagram this proposition we have to shade out the portion $SP\bar{M}$ and $\bar{S}P\bar{M}$. The minor premise is an I proposition and to put the x in the any of the region of $S\bar{P}M$ and SPM is puzzling. So x must be put somewhere in the overlapping part of the circles labelled. But the x cannot be put in both the areas because then it would go beyond what the premises assert. So we have to put the x on the line between these two areas. The conclusion $SP \neq 0$ indicates that the inserting area is not empty. But the x on the line does not show it. As such the argument is invalid.

Example 4:

No politicians are happy.

All happy people are honest

$\therefore$ Some honest people are not politician.

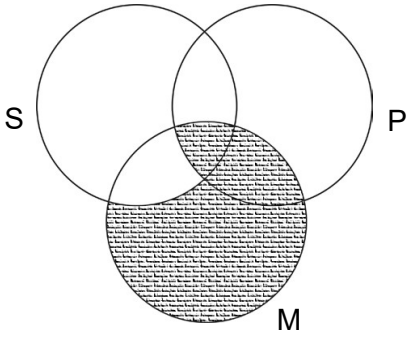
The symbolic expression of the syllogism is the following.

Major premise: E proposition– $PM = 0$

Minor premise: A proposition– $M\bar{S} = 0$

Conclusion: 0 proposition– $S\bar{P} \neq 0$

Let us represent the syllogism diagrammatically–



The above diagram clearly shows that there is no x in any of the areas in the projected conclusion. As the premises are universal propositions, so there is no necessity of inserting any x anywhere in the diagram. As such the argument is invalid and it commits the *existential fallacy*. Because, here both the premises are universal propositions but the conclusion is a particular proposition.

Example 5:

No players are philosophers.

Some scientists are not philosophers.

$\therefore$ Some scientists are not players.

Major premise: E proposition– $MP = 0$

Minor premise: 0 proposition– $SM \neq 0$

Conclusion: 0 proposition– $SP \neq 0$

Let us represent the Syllogism diagrammatically–

In the diagram we have seen that major premise $MP = 0$, being a universal proposition shaded out the two interesting areas of SPM and $\bar{S}PM$. The minor premise $\bar{S}M \neq 0$ shows that the area which is in S but not in M is not empty. So, we have to put an x in either of $S\bar{P}\bar{M}$ or $SP\bar{M}$ area. So, we have to put an x between these two areas. Thus, we put the x on the line between these two areas. But this x does not necessarily show that it is the area for the conclusion $S\bar{P} \neq 0$. As such the given argument is invalid.



CHECK YOUR PROGRESS

Q.4: What is mood?

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Q.5: What is figure?

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Q.6: What is first figure?

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Q.7: Name the valid moods of first figure.

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Q.8: Fill in the blanks:

a) DARAPTI is a valid mood of ..... figure.

b) BRAMANTIP is a valid mood of ..... figure.



3.10 LET US SUM UP

- Aristotelian deductive logic is primarily concerned with syllogistic reasoning.
- A syllogism is any deductive inference that has exactly two premises and a conclusion.
- Syllogistic reasonings are of various types. A categorical syllogism is an inference constructed entirely of categorical statements. A Standard-form of categorical syllogism is such type of categorical syllogism which is constructed entirely of standard-form of categorical propositions (either A, E, I or O) and arranged in specified standard order. It contains exactly three terms, each of which is used two times.
- The form of a standard form of categorical syllogism is determined by its mood and figure.
- To test the validity of standard form of categorical syllogism by the Venn diagram technique three overlapping circles are drawn. We diagram both the premises but not the conclusion. Then we check whether the conclusion is entailed by the premises or not. If the conclusion is entailed by the premises then the argument is considered to be valid.



3.11 FURTHER READING

1) Baronett, S. and Sen, M.; *Logic*.

2) Chakraborti. C.; *Logic-Informal, Symbolic & Inductive*.

3) Copi, I. M. & Cohen, C.; *Introduction to Logic*.

4) Hurley, J. P.; *Introduction to Logic*.

5) Sharma. B. and Deka J.; *A text Book of logic*.

3.12 ANSWER TO CHECK YOUR PROGRESS

Ans. to Q. No. 1: A Standard-form categorical syllogism is a categorical syllogism in which the premises and conclusion are all standard – form categorical propositions (either A, E, I or O) and are arranged in specified standard order.

Ans. to Q. No. 2: A standard form categorical syllogism consists of two premises and one conclusion. It consists of three terms. They are: major term, minor term and middle term. The premise in which the major term occurs with the middle term is called the major premise. The premise in which the minor term occurs with the middle term is called the minor premise.

Ans. to Q. No. 3: a) The term that occurs in both the premises but absent from the conclusion is called the middle term.

b) The term that occurs as the predicate of the conclusion is called the major term of the syllogismc.

c) The term that occurs as the subject term of the conclusion is called the minor term of the syllogism.

Ans. to Q. No. 4: Mood is a form of syllogism determined by the quality and quantity of the constituent premises of a standard form categorical syllogism

Ans. to Q. No. 5: Figure is a form of syllogism determined by the position of the middle term in the premises.

Ans. to Q. No. 6: In the first figure the position of the middle term is the subject in the major premise and the predicate in the minor premise.

Ans. to Q. No. 7: BARBARA, DARII, CELARENT, and FERIO.

Ans. to Q. No. 8: a) Third, b) Fourth



3.13 MODEL QUESTIONS

A) Very short questions:

Q.1: What is mood?

Q.2: What is figure?

Q.3: Define middle term.

Q.4: What do you mean by standard form of categorical syllogism?

B) Short questions: (Answer each question in about 150 words)

Q.1: Write short notes on:

a) Middle term

b) Venn diagram technique

Q.2: Explain briefly the concept of figure.

Q.3: What do you mean by the concept of mood in syllogism? Explain in brief.

C) Long questions: (Answer each question in about 200-300 words)

Q.1: What is standard form of categorical syllogism? Explain its nature and structure.

Q.2: What is figure? Describe the figures of a syllogism with proper symbolic form.

Q.3: What is Venn diagram technique? Discuss this technique for testing syllogism with suitable examples.

Q.4: What is 'mood'? Explain.

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UNIT 4: RULES OF CATEGORICAL SYLLOGISM

UNIT STRUCTURE

4.1 Learning Objectives

4.2 Introduction

4.3 Categorical Syllogism

- 4.3.1 Figures and Moods of Syllogism
- 4.3.2 Standard form Categorical Syllogism

4.4 Rules of Categorical Syllogism

4.5 Let us Sum up

4.6 Further Reading

4.7 Answers to Check Your Progress

4.8 Model Questions

4.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- define categorical syllogism
- explain figure and moods of syllogism
- describe standard form categorical syllogism
- discuss rules of categorical syllogism.

4.2 INTRODUCTION

This unit introduces to you the rules of categorical syllogism. The concepts discussed in this unit are: figure and moods of syllogism, standard form categorical syllogism, and rules of categorical syllogism.

4.3 WHAT IS CATEGORICAL SYLLOGISM

A Categorical syllogism is a kind of deductive argument which contains two categorical propositions as premises and one categorical proposition as conclusion. As for instance,

All humans are mortal.

All Indians are humans.

Therefore, All Indians are mortal.

This is the example of categorical syllogism. There are only three propositions and three terms in a categorical syllogism. The above example of categorical syllogism can be logically represented as:

All M is P
All S is M
Therefore, All S is P

The letters M, P, and S stand for Middle term, Major term, and Minor term respectively. The middle term is that which is present in both the premises but not in the conclusion. The predicate term ('P') of the conclusion is the major term. While, the subject term ('S') of the conclusion is known as minor term. There are two premises in a categorical syllogism, major premise and minor premise. The premise which contains the major term is known as major premise, on the other hand the premise which contains the minor term is called minor premise.

As for instance: All M is P Major Premise
All S is M Minor premise
Therefore, All S is P Conclusion

From the above example, it is clear that in a categorical syllogism, major premise comes first. And the minor premise comes after the major premise. And the last proposition, that is, conclusion comes in the end.

The main characteristic of a categorical syllogism is that all the three propositions are categorical propositions. As for instance,

All men are mortal
Aristotle is a man
Therefore, Aristotle is mortal

This categorical syllogism consists of three universal affirmative proposition that is 'A'.

4.3.1 Figures and Moods of Syllogism

By the concept of figure of a syllogism, we understand the position of a middle term in a categorical syllogism. There are four figures of a categorical syllogism on the basis of the position of middle term. The four figures can be symbolically presented as:

First figure: M P
S M
Therefore, S P

In the first figure, middle term is subject in the major premise and predicate in the minor premise.

Second figure: P M
S M
Therefore, S P

In the second figure, middle term is a predicate in both the premises.

Third figure: M P
M S
Therefore, S P

In the third figure, middle term is subject in both the premises.

Fourth figure: P M
M S
Therefore, S P

In the fourth figure, middle term is predicate in the major premise and is subject in the minor premise.

Every syllogism has a mood, so mood of a syllogism means arrangement of premises and conclusion. The mood of a syllogism is represented by the quality and quantity of the constituent propositions that is A, E, I, and O.

As for instance, AAA mood in the first figure is expressed as:

All M is P
All S is M
Therefore, All S is P

4.3.2 Standard Form of Categorical Syllogism

The validity of a syllogism can be determined only when the syllogism is in a standard form. Again, the rules of validity of a syllogism can be applied only when the syllogism is in a proper standard form. The major premise must first occur in a standard

form of categorical syllogism and the minor premise comes after the major premise. And the conclusion comes in the end. But, in some cases the conclusion may not in the right place, it may be either in the beginning or in the midst of the premises. Thus, in this situation we have to rearrange the argument into standard form of categorical syllogism.

As for instance: Socrates is mortal and all men are mortal, so, Socrates is man. This argument is to be arranged in strict syllogistic order or expressed in standard form of categorical syllogism as:

All men are mortal
Socrates is a man
Therefore, Socrates is mortal

4.4 RULES OF CATEGORICAL SYLLOGISM

In order to determine whether a syllogism is valid or invalid, Aristotle and other logicians formulated certain rules of syllogism. If any of these rules is violated, the syllogism becomes invalid. If none of the rules is broken, the syllogism is valid.

Rule 1: A valid standard form of categorical syllogism must contain exactly three terms. Each of which occurs in the same sense in the argument.

In every categorical proposition the conclusion denotes a relationship between major (predicate) and minor (subject) terms. Again, the conclusion can be justified only when the premises assert the relationship of the major and the minor term to the middle term. If the premises fail to establish the relationship among major term, minor term, and middle term consistently in a syllogism, the syllogism becomes invalid. Therefore, it is clear that each valid syllogism must contain three terms. If more than three terms are involved in a syllogism, the syllogism is invalid.

Rule 2: A valid categorical syllogism holds the view that the middle term must be distributed in at least one of the two premises.

If the middle is not distributed in any of the premises, the syllogism becomes invalid. As for instance–

All P is M
All S is M
Therefore, All S is P

This syllogism is invalid because the middle term is undistributed in both the premises. As a result we can draw the conclusion that the above example of syllogism is invalid. Therefore, in a valid syllogism the middle term must be distributed in at least one premise.

Rule 3: Any term distributed in the conclusion must be distributed in the premises.

This rule shows that if a term is distributed in the conclusion but undistributed in the premises, the syllogism is invalid. But, a valid syllogism is that in which the premises logically entail the conclusion. A term that is distributed in the conclusion but not distributed in the premises means that the conclusion has gone beyond its premises or the conclusion is not necessarily derived from the premises. Therefore, this rule clears that the term which is distributed in the conclusion must necessarily be distributed in the premises.

Rule 4: From two negative premises, we cannot draw the conclusion

Any negative proposition E and O denies class inclusion. It asserts that some or all of one class is excluded from the whole of the other class. So, if two premises are negative in a syllogism, we cannot derive a valid conclusion in the context that the two negative premises cannot make a link among the terms- major term, minor term, and middle term. Therefore, a valid conclusion cannot be derived from the two negative premises.

Rule 5: If one of the premises is negative, the conclusion must be negative.

If the conclusion is affirmative, it always asserts that the S class is contained either wholly or partially in the P class. The affirmative conclusion is followed from the premises only when if the S class is contained either wholly or partially in the M class, and the M class wholly in the P class. Therefore, an affirmative conclusion is followed only when both the premises are affirmative. On the other hand, if the S class is contained either partially or wholly in the M class, and the M class is excluded either wholly or partially

from the P class, then such a conclusion will never follow. Therefore, an affirmative conclusion cannot be drawn from negative premises.

Conversely, a negative conclusion asserts that the S class is excluded either wholly or partially from the P class. But, if both premises are affirmative, they assert class inclusion rather than class exclusion. Thus, an affirmative conclusion cannot be drawn from negative premises.

Rule 6: If both premises are universal, the conclusion cannot be particular.

This rule is not found in the traditional or Aristotle account of the categorical syllogism. The rules of categorical syllogism formulated by Aristotle do not pay attention to the existential import. Rather the concept of existential import of A, E, I and O proposition is found in the Boolean standpoint.

In the Boolean interpretation of categorical propositions, we find that universal propositions– A and E have no existential import. On the other hand, particular propositions– I and O have existential import. Therefore, it can be derived the point that if the premises of a categorical syllogism do not assert the existence of anything at all, the conclusion cannot assert the existence of something. Therefore, the conclusion is unwarranted.



CHECK YOUR PROGRESS

Q.1: What is categorical Syllogism? (Answer in about 20 words)

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Q.2: How many propositions are there in a categorical syllogism? Name them.

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Q.3: How many terms are there in a categorical syllogism?

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Q.4: Define middle term.

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Q.5: Define major term.

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Q.6: Define minor term.

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Q.7: What is figure of a syllogism?

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Q.8: How many figures are there in a categorical syllogism?

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Q.9: Name one logician, who has formulated certain rules in order to determine the validity or invalidity of a syllogism.

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Q.10: Can affirmative conclusion be drawn from two negative premises?

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Q.11: How many terms contained by a valid standard form of categorical syllogism?

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Q.12: Is it possible to draw a particular conclusion from two universal premises?

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4.5 LET US SUM UP

- A categorical syllogism is a kind of deductive argument which contains two categorical propositions as premises and one categorical proposition as conclusion.

- There are only three propositions and three terms in a categorical syllogism.
- There are three terms in a categorical syllogism, viz., major term, minor term and middle term.
- The middle term is that which is present in both the premises but not in the conclusion.
- The predicate term ('P') of the conclusion is the major term.
- The subject term ('S') of the conclusion is known as minor term.
- There are two premises in a categorical syllogism, major premise and minor premise.
- The premise which contains the major term is known as major premise; on the other hand the premise which contains the minor term is called minor premise.
- By the concept of figure of a syllogism, we understand the position of a middle term in a categorical syllogism.
- There are four figures of a categorical syllogism on the basis of the position of middle term.
- In the first figure, middle term is subject in the major premise and predicate in the minor premise.
- In the second figure, middle term is predicate in both the premises.
- In the third figure, middle term is subject in both the premises.
- In the fourth figure, middle term is predicate in the major premise and subject in the minor premise.
- The validity of a syllogism can be determined only when the syllogism is in a standard form.
- The rules of validity of a syllogism can be applied only when the syllogism is in a proper standard form.
- In order to determine the validity or invalidity of a syllogism, Aristotle and other logicians formulated certain rules of syllogism and that if any of these rules is violated, the syllogism becomes invalid and if none of the rules is broken, the syllogism is valid.
- According to the first rule, a valid standard form of categorical syllogism must contain exactly three terms, where each one occurs in the same sense in the argument.

- Rule two says that a valid categorical syllogism holds the view that the middle term must be distributed in at least one of the two premises.
- Rule three says that any term distributed in the conclusion must be distributed in the premises.
- According to the rule four, conclusion cannot be drawn from two negative premises
- Rule five says that if one of the premises is negative, the conclusion must be negative
- Rule six regards that if both the premises are universal, the conclusion cannot be particular.



4.6 FURTHER READING

1) Copi, M. Irving & Cohen, Carl & Jetli, Priyedarshi Jetli & Prabhakar, M. (2006). *Introduction to Logic*. Dorling Kindersley (India) Pvt. Ltd.
2) Hurley, J. Patrick (2007); *Introduction to Logic*; Wadsworth.
3) Hausman, A. & Kahane, H. & Tidman, P. (2007); *Logic and Philosophy*; Wadsworth.
4) Jain, Krishna (1998); *A Textbook of Logic*; D. K. Printworld (P) World.



4.7 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q. No. 1: A categorical syllogism is a kind of deductive argument which contains two categorical propositions as premises and one categorical proposition as conclusion.

Ans. to Q. No. 2: Three. Major premise, minor premise and the conclusion.

Ans. to Q. No. 3: Three. Major term, minor term and the middle term.

Ans. to Q. No. 4: The middle term is that which is present in both the premises but not in the conclusion.

Ans. to Q. No. 5: The predicate term ('P') of the conclusion is the major term.

Ans. to Q. No. 6: The subject term ('S') of the conclusion is known as minor term.

Ans. to Q. No. 7: By the concept of figure of a syllogism, we understand the position of a middle term in a categorical syllogism.

Ans. to Q. No. 8: Four

Ans. to Q. No. 9: Aristotle

Ans. to Q. No. 10: No

Ans. to Q. No. 11: Three

Ans. to Q. No. 12: No



4.8 MODEL QUESTIONS

A) Very Short Questions:

Q.1: Categorical argument is a kind of Inductive argument or deductive argument?

Q.2: What is the position of middle term in the first figure?

Q.3: What is the position of middle term in the second figure?

Q.4: What is the position of middle term in the third figure?

Q.5: What is the position of middle term in the fourth figure?

Q.6: What do you mean by mood of a syllogism?

Q.7: What are the positions of major term and middle term in the conclusion of a standard form of categorical syllogism?

Q.8: Can negative conclusion be drawn from negative premises?

Q.9: Write the first rule of a categorical syllogism?

Q.10: For a valid categorical argument, distribution of middle term at least once in a premise is compulsory or not?

B) Short Questions: (Answer each question in about 150 words)

Q.1: Discuss categorical syllogism with a suitable example.

Q.2: What are the three terms of a categorical syllogism? Explain with suitable example.

Q.3: Write down the six rules of the categorical syllogism?

Q.4: What are the three premises of the categorical syllogism? Explain with suitable examples?

Q.5: Briefly discuss about figure and moods of a categorical syllogism.

C) Short Notes on: (Answer each question in about 150 words)

Q.1: Standard form categorical syllogism.

Q.2: Four figures of a syllogism.

Q.3: Propositions of a categorical syllogism.

D) Distinguish between: (Answer each question in about 150 words)

Q.1: Premises and terms

Q.2: Figure and mood

Q.3: Second figure and fourth figure

Q.4: Boolean interpretation of categorical propositions

E) Long Questions: (Answer each question in about 200-300 words)

Q.1: Explain broadly the categorical syllogism.

Q.2: What do you mean by figure and moods of a categorical syllogism?

Q.3: Explain in details about the rules of the categorical syllogism.

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UNIT 5: BASIC CATEGORICAL FALLACIES

UNIT STRUCTURE

5.1 Learning Objectives

5.2 Introduction

5.3 Categorical Fallacies

5.4 Let us Sum Up

5.5 Further Reading

5.6 Answers to Check Your Progress

5.7 Model Questions

5.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- discuss categorical fallacies.

5.2 INTRODUCTION

This unit basically deals with the categorical fallacies. By going through this unit, you will be able to discuss fallacies involved in categorical syllogism.

5.3 CATEGORICAL FALLACIES

In unit no. 4, we have already discussed the rules of syllogism. In this unit we will discuss exclusively the fallacies of categorical syllogism.

A syllogism becomes valid when it conforms to certain rules. In contrast to this, if any of the rule is violated in syllogism, we call it a fallacy.

According to the rule of syllogism, a valid standard form of categorical syllogism must contain exactly three terms. And the three terms must be used in the same sense in the argument. This rule asserts that in every categorical syllogism, the conclusion establishes a relation between two terms, minor (subject) and major term (predicate term). Besides, the conclusion in categorical syllogism asserts the relationship of the two terms-major term and minor term to the middle term. If a syllogism does not make

a relation of the two terms (major term and minor term) to the third term (middle term), the conclusion cannot be established. And finally, the syllogism will be invalid. Therefore, it is clear that every categorical syllogism must contain exactly three terms, and if more than three terms or less than three terms are involved, the syllogism becomes invalid and commits a fallacy that is called the fallacy of four terms.

As for instance: Plato is the teacher of Aristotle.
Socrates is the teacher of Plato.
Therefore, Socrates is the teacher of Aristotle.

In this syllogism, there are four terms namely: i) Plato 2. Teacher of Aristotle 3. Socrates 4. teacher of Plato. This syllogism is not valid because it contains four terms, so it violates the rule that is - each and every syllogism must contain only three terms or not less than three terms. Therefore, this syllogism commits a fallacy that is fallacy of four terms.

The second rule of categorical syllogism asserts that the middle term must be distributed at least once in the premises. And, if the middle term is not distributed in any of the premises, the syllogism becomes invalid and commits a fallacy that is the fallacy of undistributed middle.

As for instance: All cows are four-footed animals.
All cats are four-footed animals.
Therefore, All cats are cows.

This syllogism is invalid because the middle term is not distributed in any of the premises. Therefore, this syllogism commits a fallacy that is fallacy of undistributed middle.

The third rule says that a term which is distributed in the conclusion must be distributed in the premises. A syllogism is invalid, if a term is distributed in the conclusion but undistributed in the premises. There are two types of fallacy involved in syllogism by violating this rule. They are: Fallacy of illicit minor and fallacy of illicit major.

Fallacy of Illicit Minor: If a minor term is distributed in the conclusion but not distributed in the premises, we commit a fallacy, that is, fallacy of Illicit minor.

As for instance: No philosophers are artists.
All philosophers are thinkers. (Undistributed)
Therefore, All thinkers are artists.

This syllogism is invalid because the minor term (thinkers) is distributed in the conclusion without being distributed in the premise. Therefore, a fallacy is involved in this syllogism that is fallacy of illicit minor.

Fallacy of Illicit major: If a major term is undistributed in the conclusion without being distributed in the premise, the syllogism is invalid. This invalid syllogism is called fallacy of illicit major.

As for instance: All philosophers are logicians.
Some men are not philosophers.
Therefore, Some men are not logicians.

This example shows that this syllogism is invalid because the major term (logicians) is distributed in the conclusion but undistributed in the premise. Therefore, this syllogism commits a fallacy of illicit major.

According to the fourth rule of syllogism, a valid syllogism must not have two negative premises. A syllogism is invalid, if the conclusion is derived from two negative premises. In this context a fallacy is committed in a syllogism that is Fallacy of Exclusive premises.

As for instance: No fish are mammals.
Some cats are not fish.
Therefore, Some cats are not mammals.

This syllogism is invalid because the conclusion is derived from two negative premises. The logic behind this rule is that two negative propositions have nothing in common. There is no connector which can make a relation between two premises. The function of the middle term is to connect major and minor term, but the middle term cannot function its role, if the two premises of a syllogism are negative.

The fifth rule of syllogism holds the view that if one of the premises of a syllogism is negative, the conclusion is negative. But, if we draw the affirmative conclusion by violating the rule in a syllogism, we commit a fallacy of drawing affirmative conclusion from a negative premise.

As for instance: No potters are rich man.
Some engineers are potters.
Therefore, Some engineers are rich man.

This syllogism is invalid because the S class (minor term) is contained either wholly or partially in the P class (major term). But, the affirmative conclusion can be derived only if the S class is contained either wholly or partially in the M class, and the M class wholly in the P class. So, we can derive an affirmative conclusion, if both the premises are affirmative. But, if the S class is contained either wholly or partially in the M class, and the M class is excluded (separated) either wholly or partially from the P class, an affirmative conclusion cannot be drawn from negative premises. On the other hand, a negative conclusion asserts that the S class is excluded (separated) either wholly or partially from the P class. But, if both the premises are affirmative, both the terms assert class inclusion rather than class exclusion. Thus, a negative conclusion cannot be followed from negative premises.

According to the sixth (6^{th}) rule of categorical syllogism, no particular conclusion can be drawn from two universal premises. This rule is not necessary for traditional or Aristotelian account of categorical propositions because it does not pay attention to the problem of existential import. Rather, it can be said that Boolean interpretation of categorical propositions asserts the existence of something. If the premises of an argument do not assert the existence of something at all, the conclusion will be unwarranted and commits a fallacy that is existential fallacy.

As for instance: All mammals are animals.
All lions are mammals.
Therefore, Some lions are animals.

This syllogism is invalid because it commits the existential fallacy from the Boolean standpoint. The fallacy is committed because the conclusion asserts that lions exist. Universal premises are not recognized to be possessed of existential import from the Boolean standpoint.



CHECK YOUR PROGRESS

Q.1: How does fallacy involve in a syllogism? (Answer in 20 words)

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Q.2: What are the two terms involved in a categorical syllogism?

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Q.3: If a categorical syllogism contains four terms, what type of fallacy occurs?

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Q.4: What is the second rule of categorical syllogism?

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Q.5: What is the fallacy of illicit major?

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Q.6: What is the fallacy of illicit minor?

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Q.7: What is the fallacy of Exclusive premises?

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Q.8: What is sixth rule of categorical syllogism?

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Q.9: Write the name of the logician, who has given existential fallacy.

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Q.10: What is the fallacy of drawing affirmative conclusion from a negative premise?

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5.4 LET US SUM UP

- A syllogism becomes valid when it conforms to certain rules but in contrast to this, if any of the rules is violated in syllogism, we call it a fallacy.
- According to the rule of syllogism, a valid standard form of categorical syllogism must contain exactly three terms.
- Every categorical syllogism must contain three terms, and if more than three terms are involved, the syllogism becomes invalid and commits a fallacy that is called the fallacy of four terms.
- If the middle term is not distributed in any of the premises, the syllogism becomes invalid and commits a fallacy that is the fallacy of undistributed middle.
- If a minor term is distributed in the conclusion but not distributed in the premises, we commit a fallacy, that is, fallacy of Illicit major.
- If a major term is distributed in the conclusion but not distributed in the premises, we commit a fallacy, that is, fallacy of Illicit major.
- A syllogism is invalid, if the conclusion is derived from two negative premises and in this context a fallacy is committed in a syllogism that is Fallacy of Exclusive premises.
- If the premises of an argument do not assert the existence of something at all, the conclusion will be unwarranted and commits a fallacy that is existential fallacy.

- If we draw the affirmative conclusion by violating the rule in a syllogism, i.e., if one of the premises of a syllogism is negative, the conclusion is negative and we commit a fallacy of drawing affirmative conclusion from a negative premise.
- The sixth (6^{th}) rule of categorical syllogism, i.e., no particular conclusion can be drawn from two universal premises is not necessary for traditional or Aristotelian account of categorical propositions because it does not pay attention to the problem of existential import.
- Universal premises are not recognized to be possessed of existential import from the Boolean standpoint.



5.5 FURTHER READING

1) Copi, M. Irving, &Cohen, Carl, & Jetli, Priyedarshi Jetli & Prabhakar, M. (2006); *Introduction to Logic*; Dorling Kindersley (India) Pvt. Ltd.
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4) Jain, Krishna. (1998). *A Textbook of Logic*. D. K. Printworld (P) World.



5.6 ANSWER TO CHECK YOUR PROGRESS

Ans. to Q. No. 1: A syllogism becomes valid when it conforms to certain rules. In contrast to this, if any of the rules is violated in syllogism, we call it a fallacy.

Ans. to Q. No. 2: Minor term and major term

Ans. to Q. No. 3: Fallacy of four terms.

Ans. to Q. No. 4: The second rule of categorical syllogism asserts that the middle term must be distributed at least once in the premises.

Ans. to Q. No. 5: If a major term is distributed in the conclusion but not distributed in the premises, we commit a fallacy, that is, fallacy of Illicit major.

Ans. to Q. No. 6: If a minor term is distributed in the conclusion but not distributed in the premises, we commit a fallacy, that is, fallacy of Illicit minor.

Ans. to Q. No. 7: A syllogism is invalid, if the conclusion is derived from two negative premises and in this context a fallacy is committed in a syllogism that is Fallacy of Exclusive premises.

Ans. to Q. No. 8: According to the sixth (6^{th}) rule of categorical syllogism, no particular conclusion can be drawn from two universal premises.

Ans. to Q. No. 9: Boole

Ans. to Q. No. 10: If we draw the affirmative conclusion by violating the rule, i.e., 'if one of the premises of a syllogism is negative, the conclusion is negative in a syllogism', we commit a fallacy of drawing affirmative conclusion from a negative premise.



5.7 MODEL QUESTIONS

A) Very Short Questions:

Q.1: When does a syllogism become valid?

Q.2: What are the names of subject term and predicate term in a syllogism?

Q.3: Give an example of fallacy of four terms.

Q.5: Give an example of fallacy of illicit major.

Q.6: Give an example of fallacy of illicit minor.

B) Short questions: (Answer each question in about 150 words)

Q.1: Explain briefly fallacy of four terms.

Q.2: Explain briefly second rule of the categorical syllogism.

Q.3: What is fallacy of undistributed middle? Give suitable examples.

Q.4: Explain briefly the fallacy of illicit major.

Q.5: What is the fourth rule of categorical syllogism? Explain briefly.

C) Short Notes on: (Answer each question in about 150 words)

Q.1: Fallacy of illicit minor

Q.2: Existential Fallacy

Q.3: Sixth rule of categorical syllogism

D) Distinguish between: (Answer each question in about 150 words)

Q.1: Fallacy of illicit major and fallacy of illicit major

Q.2: Third and fourth rule of categorical syllogism.

E) Long Questions: (Answer each question in about 300-500 words)

Q.1: What is the first rule of categorical syllogism? What fallacy may occur in violation of this rule? Explain with suitable example.

Q.2: Explain the fallacies of illicit major and illicit minor.

Q.3: Explain broadly what fallacies may lead in violation of fifth and sixth rules?

Q.4: What is the sixth rule of categorical syllogism? Write the Boolean interpretation in this regard.

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UNIT 6: FORMAL PROOF OF VALIDITY

UNIT STRUCTURE

6.1 Learning Objectives

6.2 Introduction

6.3 Formal Proof of Validity

6.4 Strategy for Deduction

6.5 Derivation Rules: Rules of Inference

6.6 Derivation Rules: Rules of Replacement

6.7 Differences between Replacement and Substitution

6.8 Differences between Rules of Inference and Rules of Replacement

6.9 Test of Formal Proof

6.10 General Suggestions for Formal Deduction

6.11 Let us Sum Up

6.12 Further Reading

6.13 Answers to Check Your Progress

6.14 Model Questions

6.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- define the method of deduction
- define the formal proof of validity
- explain the strategy of deduction
- discuss the rules of inference
- describe the rules of replacement
- distinguish between replacement and substitution
- explain the test of formal proof.

6.2 INTRODUCTION

This unit introduces to you the formal proof of validity. Truth tables provide an effective method for demonstrating the validity of arguments. In theory, truth tables are adequate to test the validity of any argument of general

type. But, in practice, truth tables become unwieldy and tiresome when the number of component statements increases. Thus, the truth table method is inconvenient for testing the validity of arguments containing a large number of statements. A more convenient method for establishing the validity of arguments is the Method of Deduction or what may be called the method of Derivation by Substitution. The method of deduction is applicable to those arguments which are valid. In this method we simply construct a proof of their validity in order to construct a formal proof of their validity. We have to apply certain rules to the given premises and go on deducing consequences from them unless and until we get the original conclusion. Since in this method we deduce conclusion from the original premises to get the original conclusion through the application of certain rules, thereafter, this method is called the method of deduction.

This deductive procedure is called formal; because, it relies on the valid argument forms to show as to how the conclusion can be deduced from the given set of premises. The valid argument forms are used as the logical rules to determine the consequences which can be validity inferred from the premises.

As we all know that there is a distinction between argument and argument form. If an argument form is valid then any argument of that form will always be valid. An elementary valid argument is an argument which is valid.

6.3 FORMAL PROOF OF VALIDITY

A formal proof of validity for a given argument may be defined “to be a sequence of statements, each of which is either a premiss of that argument or follows from preceding statements by an elementary valid argument and such that the last statement in the sequence is the conclusion of the argument whose validity is being proved.” (I.M. Copi.)

A substitution instance of an elementary argument forms.

$$(F \supset \sim G) \supset (\sim H \vee \sim I)$$

$$F \supset \sim G$$

$$\therefore \quad \sim H \vee \sim I$$

Is an elementary valid argument; because, it is a substitution instance of the elementary valid argument form, Modus Ponens (M.P). It results from–

$P \supset Q$

P

$\therefore q$

by substituting $(F \supset \sim G)$ for P and $(\sim H \vee \sim I)$ for q.

Therefore, it is of that form even though Modus Ponens is not the specific form of the given argument.

6.4 STRATEGY FOR DEDUCTION

The formal proof of validity of an argument can be constructed easily. The strategy for deduction will be as follows:

a) For constructing the validity of an argument formulated in ordinary language, the statements of the argument will be symbolized by using the capital letters of the alphabet, to bring out the logical form of the argument.

b) This proof of validity will be started by stating and listing the given premises of the argument in one column.

c) All the premises and the statements deduced from them must be numbered serially and must be put in on column and the conclusion must be separated from the premises and it must be written to the right of the last. Premise separated by a slanting line with the symbol '$\therefore$' which automatically marks of all the statements above it to be premises.

d) The statements that are deduced from the original premises by the application of rules must be put along with the given premises with "justifications" written beside then. The justification premise, the statement from which and the rule by which the statement in question is deduced is the second step. The statements that are deduced from the original premises by the application of rules are to be taken as premises and the deduction must continue until we get the original conclusion.

Now let us take an argument and see how the formal proof of validity of the argument can be constructed.

1) (F V G V H)
2) (G ⊃I).(H ⊃ J)
3) (I ⊃ J) ⊃ (F V H)
4) ~F / ∴H
5) (G V H) 1, 4 D.S
6) I V J 2, 5 C.D
7) F V H 3, 6 M.P
8) H 7, 4 D.S



CHECK YOUR PROGRESS

Q.1: State whether the following statements are True or False:

a) The method of deduction is applicable only to those arguments which are valid. (True/False)

b) The method of deduction is more convenient than the truth table method for establishing the validity of arguments. (Truth/False)

c) The method of deduction is also known as the method of derivation by substitution. (Truth/False)

Q.2: Fill in the blanks:

a) Rules of inference and ..... are the two sets of rules of derivation.

b) Rules of replacement consists only of .....

c) Derivation rules are also sometimes called .....

Q.3: What is the Method of Deduction?

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Q.4: Why is this derivation procedure called formal?

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Q.5: How do you define a formal proof of validity for a given argument?

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Q.6: What is an elementary valid argument?

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Q.7: What does the justification in terms of the elementary valid argument form or the logical rule for a new premise of the argument specify?

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6.5 DERIVATION RULES: RULES OF INFERENCE

Rules of Inference and Rules of Replacement are the two sets of rules for derivation. Rules of inference are nothing but some valid argument forms whose validity is established by truth tables. These rules are used in constructing formal proof of validity. The following rules are the Rules of inference.

Rules of Inference:

1) Modus Ponens (M.P.)

$p \supset q$

p

$\therefore q$

2) Modus Tollens (M.T.)

$p \supset q$

$\sim q$

$\therefore \sim p$

3) Hypothetical Syllogism (H.S.)

$p \supset q$

$q \supset r$

$\therefore\ p \supset r$

4) Disjunctive Syllogism

$p \vee q$

$\sim p$

$\therefore\ q$

5) Constructive Dilemma (C.D.)

$(p \supset q)(r \supset s)$

$p \vee r$

$\therefore\ q \vee s$

6) Destructive Dilemma

$(p \supset q)(r \supset s)$

$\sim q \vee \sim s$

$\therefore\ \sim p \vee \sim r$

7) Simplification (Simp.)

$p.q$

$\therefore\ p$

8) Conjunction (Conj.)

p

q

$\therefore\ p.q$

9) Addition (Add.)

p

$\therefore\ p \vee q$

10) Absorption (Abs.)

$p \supset q$

$p \supset (p.q)$

These rules of inference correspond to elementary argument forms, them validity of which is established easily by truth tables. These rules can be used to construct formal proofs of validity for more complicated arguments.

6.6 DERIVATION RULES

Rules of inference are not regarded as sufficient for constructing the validity of many other arguments. So additional rules are required in such cases. As it is known, here only truth functional compound statements concern us. Hence “if any part of a compound statement is replaced by any logically equivalent expression to the part replaced, the truth value of the resulting statement is the same as that of the original statement. This is sometimes called the rule of replacement and sometimes the principle of extensionality. The rule of Replacement is adopted as an additional principle of inference.

The rule of replacement consists only of logical equivalences. For example, one of rules of replacement is De Morgan’s theorem that is $\sim(p.q) \equiv \sim p \vee \sim q$. We can infer one from the other since they are logically equivalent. Therefore, $\sim(p.q)$ we can come to $\sim p \vee \sim q$

The list of replacement rules with their complete and abbreviated names is presented as follows:

Rules of Replacement:

11) De Morgan's Theorems (De.M.)
~(p.q) ≡ (~p v ~q)
~(p v q) ≡ (~p.~q)

12) Commutation (Com)
(p v q) ≡ (q v p)
(p.q) ≡ (q.p)

13) Association (Assoc.)
[p v (q v r)] ≡ [(p v q) v r]
[p.(q.r] ≡ [(p.q).r]

14) Distribution (Dist.)
p.(q v r) ≡ [(p.q) v (p.r)]
p v (q.r) ≡ [(p v q).(p.r)]

15) Double Negation (D.N.)
p ≡ ~~p

16) Transposition (Trans.)
(p ⊃ q) ≡ (~q ⊃ ~p)

17) Material Implication (Impl.)
(p ⊃ q) ≡ (~p v q)

18) Material Equivalence (Equiv.)
p ≡ q ≡ [(p ⊃ q).(q ⊃ p)]
p ≡ q ≡ [(p.q) v (~q.~p)]

19) Exportation (Exp.)
[(p.q) ⊃ r] ≡ [p ⊃ (q ⊃ r)]

20) Tautology (Taut.)
p ≡ (p v q)
p ≡ (p.q)

Each of these rules is stated as equivalences with ≡ as the main connective. This indicates that we can infer the left hand side of the ≡from the right hand side and vice versa.

6.7 DIFFERENCES BETWEEN REPLACEMENT AND SUBSTITUTION

The process of replacement is very different from that of substitution. The two important differences between replacement and substitution are the following:

1) In replacement, one statement can be replaced only by a statement logically equivalent to it where the logical equivalence of the two statements is given in rules of replacement.

   On the other hand, in substituting statements for statement variables in an argument form, any statement variable can have any statement substituted for it.

2) In substitution, every occurrence of a statement variable in an argument form must have the same statement substituted for it.

However, in replacement, one or several occurrences of a statement may be replaced by a logically equivalent statement, without other occurrences of that statement having to be replaced also.

6.8 RULES OF INFERENCE AND RULES OF REPLACEMENT

There are some major differences between rules of inference and rules of replacement.

Rules of Inference can be applied only to whole lines of proof. For example, A can be inferred from A.B. by simplification only if A.B constitutes a whole line. But, neither A nor (A $\supset$ C) follows from (A.B) $\supset$ C by simplification or by any other Rules of Inference.

On the other hand, rules of replacement can be applied either to whole lines or to parts of lines. For example, the statement A $\supset$ (B $\supset$ C) can be inferred from the whole line (A.B) $\supset$ C by Exportation. Again, the statement [A $\supset$ (B $\supset$ C)] V D can be inferred from [(A.B) $\supset$ C) V D by Exportation.

Thus, the rules of replacement can be used even where they do not constitute whole lines of a proof. But, the Rules of Inference can be used only with whole lines of a proof serving as premises.

Moreover, in contrast to the rules of inference, the rules of replacement authorize replacement of a statement by its specified logically equivalent statement. Again, the Rules of Inference work in one direction while Rules of replacement work in both directions. Replacement in both directions is indicated by the symbol '$\equiv$'.

6.9 TEST OF FORMAL PROOF

The test of whether or not a given sequence of statements is a formal proof will be discussed in this particular section. Direct observation shows whether or not, every line beyond the premises actually follows from preceding lines by one of the given rules of inference. Thus, it can be decided

mechanically in a finite number of steps whether or not a given sequence of statement constitutes a formal proof with reference to the given list of rules of inference. Thinking is not required in any case neither in thinking about what the statements mean, or in using logical intuition to check the validity of the deduction of any line. There is a finite, mechanical procedure for deciding whether or not the deduction is legitimate even where the justification of a statement is not written beside it. Each line is preceded by only a finite number of other lines, and only a finite number of Rules of Inference are adopted. Though time consuming, it can be verified by inspection whether the line in question follows from any single preceding line or from any pair of preceding lines by any one of the rules of inference listed. Again, the any legitimacy line can be decided by a finite number of observations none of which involves anything more than comparing shapes or patterns. To preserve this effectiveness, the rule is that only one Rule of Inference should be applied at a time. Strictly speaking, the explanatory notation beside each statement is not a part of the proof. However, it is helpful and should always be included. Thus, it is known that the test of whether or not a given sequence of statements is a formal proof is effective. But, constructing such a formal proof is not an effective procedure. The use of truth tables is completely mechanical. But in constructing a formal proof of validity on the basis of the rules of inference, it is necessary to think or 'figure out' where to begin and how to proceed. There is no effective or purely mechanical method of procedure. But, it is usually much easier to construct a formal proof of validity than to write out a truth table.



ACTIVITY 6.1

Do you think that Rules of Inference are sufficient for proving the validity of arguments?

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CHECK YOUR PROGRESS

Q.8: How does replacement differ from substitution?

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Q.9: What are the differences between rules of inference and rules of replacement?

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Q.10: How do you know that test of formal proof is effective?

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Q.11: How do you determine that constructing a formal proof is not an effective procedure?

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Q.12: State whether the following statements are True or False:

a) The Rules of Inference work in one direction.
(True / False)

b) The Rules of Inference are regarded as sufficient for proving the validity of arguments. (True / False)

c) The rules of replacement work in both directions.
(True / False)

d) The use of truth tables is completely mechanical.
(True / False)

6.10 GENERAL SUGGESSTIONS FOR FORMAL DEDUCTION

As we can see, there are no purely mechanical rules for constructing formal proofs. But, some general suggestions or hints should be followed while doing derivation.

The first is to begin the process by deducing conclusions from the given premises with the help of the given rules. When more and more of these sub conclusions become available as the premises for further deduction, the greater is the possibility of being able to see how to deduce the conclusion of the argument to be proved valid.

The second is to try to eliminate statements that occur in the premises, but not in the conclusion. Such elimination can be done only in accordance with the rules of inference. The rules contain many techniques for eliminating statements. Simplification, Commutation, Hypothetical Syllogism is such rules. Distribution is a useful rule for transforming a disjunction into conjunction. All these rules are helpful by various ways in eliminating statements.

The third is to introduce by means of addition a statement that occurs in the conclusion but not in the premises.

So, the most important point is to know the rules well. However, knowing the rules does not mean only memorizing the rules. Memorizing is, no doubt, necessary, but the mastery over the rules is also necessary to understand and help the process.

The fourth is to work backward from the conclusion by looking for some statement or statements from which it can be deduced and then trying to deduce those intermediate statements from the premises.

However, there is no substitute for practice as a method of acquiring facility in constructing formal Proof.



CHECK YOUR PROGRESS

Q.13: For each of the following arguments, state the Rules of Inference by which its conclusion follows from its premise or premises:

A. $(A \supset \sim B).(\sim C \supset D)$

$\therefore A \supset \sim B$

B. $K \vee (L \vee M)$

$\therefore [K \vee (L \vee M)] \vee [K \vee (L \vee M)]$

.....

Q.14: Each of the following is a formal proof of validity for the indicated argument. State the 'Justification' for each line that is not a premise

1) $(A.B) \supset [A \supset (D.E)$
2) $(A.B).C / D \vee E$
3) $A.B$
4) $A \supset (D.E)$
5) A
6) $D.E$
7) D
8) $D \vee E$

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Q.15: Construct a formal proof of Validity for each of the following arguments.

A) $A \supset B$

$C \supset D$

$(\sim B \vee D).(\sim A \vee \sim B) / \therefore \sim A \vee \sim C$

B) $A \supset B$

$C \supset D$

$A \vee C / \therefore (A.B) \vee (C.D)$

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Q.16: Construct a formal proof of validity for each of the following arguments, in each case using the suggested notation.

a) If I study, I make good grades. If I do not study, I enjoy myself. Therefore, either I make good grades or I enjoy myself. (S, G, E)

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6.11 LET US SUM UP

- The Method of deduction is a method of establishing the validity of arguments.
- The truth table method is inconvenient for testing the validity of arguments containing a large number of statements.

- So, a more convenient method for testing the validity of arguments is the Method of Deduction or what may be called the method of derivation by substitution.
- Rules of inference and Rules of replacement are the two sets of rules for derivation. By means of these rules, we can know what can be validly inferred from a certain kind of premises.
- The rules of inference work in one direction. These are: Modus Ponens, Modus tollence, Hypothetical Syllogism, Disjunctive syllogism, Constructive Dilemma, Destructive Dilemma, Simplification, Conjunction, Addition.
- Rules of inference are not sufficient for proving the validity of many other arguments. The rules of replacement are required as additional principles in such cases.
- The rules of replacement are: De Morgan's theorem, Commutation, Association, Distribution, Double Negation, Transposition, Material Implication, Material Equivalence, Exportation, and Tautology.
- Rules of replacement is indicated by the symbol '≡'.



6.12 FURTHER READING

1) Chhanda, Chakraborti (2006); *Logic: Informal, Symbolic and Inductive*; New Delhi: Prentice– Hall of India Private Limited.
2) Copi, Irving M. (1979); *Symbolic Logic* (fifth edition); Callier Macmillan International Editions.
3) Copi, Cohen (1995); *Introduction to Logic* (Ninth Edition); New Delhi: Prentice– Hall of India Private Limited.
4) Munshi, R.C. (1991); *Hand Book of Logic*; Calcutta: Baikuntha Book House.
5) Singh, S. Shyam Kishore (2000); Modern Logic (Volume one); Lamyana Press.

6.13 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q. No. 1: a) True, b) True, c) True

Ans. to Q. No. 2: a) Rule of replacement, b) Logical equivalences,
c) Rule of replacement or the principle of extentionality

Ans. to Q. No. 3: The method of deduction is a convenient method for testing the validity of arguments. This method of establishing the validity of argument is to deduce their conclusions from their premises by a sequence of shorter, more elementary arguments each of which is known to be valid.

Ans. to Q. No. 4: This deductive procedure relies on the valid argument forms to show as to how the conclusion can be deduced from the given set of premises.

Ans. to Q. No. 5: A formal proof of validity for a given argument many be defined "to be a sequence of statements, each of which is either a premiss of that argument or follows from preceding statements by an elementary valid argument, and such that the last statement in the sequence is the conclusion of the argument whose validity is being proved" (I.M. Copi)

Ans. to Q. No. 6: An elementary valid argument is any argument that is a substitution instance of an elementary valid argument form.

Ans. to Q. No. 7: The Justification in terms of the elementary valid argument form or the logical rule for a new premise specifies the preceding premises from which and the rule by which the premise in question is deduced.

Ans. to Q. No. 8: In replacement, a statement can be replaced only by a statement logically equivalent to it. In substituting statements for statements variables in an argument form, any statement variable can have any statement substituted for it. In substitution, every occurrence of a statement variable in an argument form must have the same statement substituted for it. However, in replacement one

or several occurrences of a statement may be replaced by a logically equivalent statement, without other occurrences of that statement having to be replaced also.

Ans. to Q. No. 9: Rules of Inference can be applied only to whole lines of proof whereas Rules of Replacement can be applied either to whole lines or to parts of lines. Moreover, in contrast to the Rules of Inference, the Rules of replacement authorize replacement of a statement by its specified logically equivalent statement. Again, Rules of Inference work in one direction while Rules of Replacement work in both directions.

Ans. to Q. No. 10: Because, it can be decided mechanically in a finite number of steps whether or not a given sequence of statements constitute a formal proof with reference to the Rules of Inference. Thinking is not required either in thinking about what the statements mean or in using logical intuition to check the validity of deduction. There is a finite mechanical procedure for deciding the legitimacy of deduction even where the justification of a statement is not written beside it.

Ans. to Q. No. 11: In constructing a formal proof of validity on the basis of the Rules of Inference, it is necessary to think or 'figure out' where to begin and how to proceed. There is the effective or purely mechanical method of procedure.

Ans. to Q. No. 12: a) True, b) False, c) True, d) True

Ans. to Q. No. 13: a) Simplification, b) Tautology

Ans. to Q. No. 14: A) 1) $(A. B) \supset [A \supset (D.E)]$

2) $(A.B). C / \therefore D \vee E$

3)	A.B	2. Simplification
4)	$A \supset (D.E)$	1, 3 Modus Ponens
5)	A	3 Simplification
6)	D.E	4, 5 Modus Ponens
7)	D	6 Simplification
8)	DVE	7 Addition

Ans. to Q. No. 15: A) 1) $A \supset B$

2) $C \supset D$

3) $(\sim B \vee \sim D).(\sim A \vee \sim B) / \therefore \sim A \vee \sim C$

4) $(A \supset B).(C \supset D)$ 1, 2 Conjunction

5) $\sim B \vee \sim D$ 3 Simplification

6) $\sim A \vee \sim C$ 4, 5 Destructive Dilemma

B) 1) $A \supset B$

2) $C \supset D$

3) $A \vee C \ / \therefore (A.B) \vee (C.D)$

4) $A \supset (A.B)$ 1 Absorption

5) $C \supset (C.D)$ 2 Absorption

6) $[A \supset (A.B)].[C \supset (C.D)]$ 4, 5 Conjunction

7) $(A.B) \vee (C.D)$ 6, 3 Constructive Dilemma

Ans. to Q. No. 16: 1) $S \supset G$

2) $\sim S \supset E / \therefore G \vee E$

3) $G \supset \sim S$ 1 Transportation

4) $S \supset \sim S$ 1, 3 Hypothetical Syllogism

5) $S \vee \sim S$ 4 Material Implication

6) $G \vee E$ 1, 2, 5 Constructive Dilemma



6.14 MODEL QUESTIONS

A) Very Short Questions:

Q.1: What is method of deduction?

Q.2: What is formal proof of validity?

Q.3: What are the rules of inference?

Q.4: What are the rules of replacement?

B) Short Questions: (Answer in about 150 words)

Q.1: What are the elementary valid argument forms? What is their utility in testing arguments?

Q.2: What are the basic points of difference between the rules of inference and the rules of replacement?

C) Long Questions: (Answer in about 300-500 words)

Q.1: Explain the strategy of constructing the Formal Proof of validity for arguments.

Q.2: What are the elementary valid argument forms? What is their utility in testing arguments?

Q.3: Construct formal proof of validity for the following:

i) $(A \supset B)$
$C \supset \sim B$
$\therefore A \supset \sim C$

ii) $(M \vee N) \supset (O . P)$
$\sim O$ /
$\therefore \sim M$

iii) $(M \supset N).(O \supset P)$
$\sim N \vee \sim P$
$\sim (M . O) \supset Q$
$\therefore Q$

iv) $E \supset (F \supset G)$
$\therefore F \supset (E \supset G)$

v) $E \supset (F \sim G)$
$(F \vee G) \vee \supset \sim H$
E
$\therefore H$

vi) $A \supset B$
$C \supset D$
$(\sim B \vee \sim D).(\sim A \vee \sim B)$
$\therefore \sim A \vee \sim C$

vii) $E \supset F$
$G \supset F$
$\therefore (E \vee G) \supset F$

viii) $(G \supset \sim H) \supset I$
$\sim (G.H)$
$\therefore I \vee \sim H$

ix) $A \vee (B \supset D)$
$\sim C \supset (D \supset E)$
$A \supset C$
$\sim C$
$\therefore B \supset E$

x) $E \supset (F.\sim G)$
$(F \vee G) \supset H$
E
$\therefore H$

xi) $(D . \sim E) \supset F$
$\sim (E \vee F)$
$\therefore \sim D$

Q.4: Construct formal proofs of validity for the following arguments using the suggested notation in each case:

i) Either the manager did not notice the change or else he approves of it. He noticed it alright. So, he must approve of it. (N, A.)

ii) It is not the case that she either forgot or was not able to finish. Therefore, she was able to finish (F, A.)

iii) If either George enrolls or Harry enrolls then Era does not enroll. Either Era enrolls or Harry enrolls. If either Harry enrolls or George does not enroll then Jim enrolls. George enrolls. Therefore, either Jim enrolls or Harry does not enroll. (G, H, E, J)

iv) If the police do not catch the murderer within a week, then there will be a public outcry. If there is a public outcry, then the chief of the police will resign. The chief of police will not resign. Therefore, the police will catch the murderer within a week. (C, O, R)

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UNIT 7: RULES OF INFERENCE

UNIT STRUCTURE

7.1 Learning Objectives

7.2 Introduction

7.3 Rules of Inference

7.4 Solved Examples of Rules of Inference

7.5 Let us Sum Up

7.6 Further Reading

7.7 Answers to Check Your Progress

7.8 Model Questions

7.1 LEARNING OBJECTIVES

After going through this unit, you will be to:

- discuss the rules of inference
- discuss some examples of rules of inference.

7.2 INTRODUCTION

This unit not only introduces to you the rules of inference but also the uses of the rules in case of formal proof of validity. It also provides us to go through some solved examples of the rules of inference in order to know how the rules of inference are applied to test the validity of arguments.

7.3 RULES OF INFERENCE

In the preceding chapter, you have already come to know the formal proof of validity which discusses the concepts like strategy for deduction, rules of inference, rules of replacement, differences between rules of inference and rules of replacement, test of formal proof, general suggestion for formal deduction etc. Therefore, this unit emphasizes only on the application of the rules of inference.

There are nine rules of inference. These rules are considered self-evident and are therefore valid without any proof. On the basis of these

rules we can determine or test the validity or invalidity of arguments. As we can derive a conclusion on the basis of these self-evident rules, so the method of deriving a conclusion from the rules is called deductive method.

Rules of inference are:

1) **Modus Ponens (M.P.):** $p \supset q$

p

$\therefore q$

This rule means affirmation of the antecedent and on that basis consequent is affirmed.

We can take an example of valid argument by applying this rule:

If Madhu is intelligent, he will be able to pass in the examination Madhu is intelligent.

Therefore, he will be able to pass in the examination.

This argument can be symbolised as valid argument form of the rule Modus ponens.

i) $p \supset q$

p

$\therefore q$

There are also other valid arguments forms of modus ponens. They are:

ii) $\sim p \supset \sim q$

$\sim p$

$\therefore \sim q$

2) **Modus Tollens:** $p \supset q$

$\sim q$

$\therefore \sim p$

This argument means the denial of the consequent and on that basis the antecedent is denied.

We can take an example of valid argument form by applying this rule. The argument is: If he wins in the match, he will give us a party

He will not give us a party.

$\therefore$ He does not win the match.

The symbolic form of this argument is: $p \supset q$

$\sim q$

$\therefore \sim p.$

Apart from this there are also other valid argument forms of modus ponens. They are:

i) $\sim p.\sim q$ ii) $(p.q) \supset r$

$\sim p$ $p.q$

$\therefore \sim q$ $\therefore \sim r$

This symbolic form is considered valid argument form of modus tollens. There are also other valid argument forms of modus tollens:

i) $p \supset \sim q$ ii) $p \supset (q\ r)$

q $\sim(q.r)$

$\therefore \sim p$ $\therefore \sim p$

3) Disjunctive Syllogism: $p \vee q$

$\sim p$

$\therefore q$

This argument holds that if one of the options or disjuncts is denied in the minor premise, the other option or disjunct is accepted in the conclusion. We can take real example of disjunctive syllogism.

Either He will win or defeat in the match.

He will not win in the match.

Therefore, he will defeat in the match. This real example can be symbolized in this way: $p \vee q$

$\sim p$

$\therefore q.$

Besides, there are also other valid forms of disjunctive syllogism. They are:

i) $\sim p \vee q$ ii) $\sim p \vee \sim q$ iii) $(p.q) \vee r$

p p $\sim(p.q)$

$\therefore q$ $\therefore q$ $\therefore r$

4) Hypothetical Syllogism: $p \supset q$

$q \supset r$

$\therefore p \supset r$

This rule means that both the premises are conditional statements and they have one common statement. The common statement is known as antecedent in one premise and other is consequent in another premise. We can take real example of hypothetical syllogism in order to better understand its logical form. As for instance:

If Madhu is intelligent, he will be able to pass in the examination

If he passes the examination, he will be able to get a job

Therefore, If Madhu is intelligent, he will be able to get a job

This example can be symbolized in the following way:

$p \supset q$

$q \supset r$

$\therefore\ p \supset r.$

This is a valid form of hypothetical syllogism. There are also other valid forms of hypothetical syllogism. They are respectively:

i) $q \supset r$ ii) $p \supset \sim q$ iii) $\sim p \supset \sim q$

$p \supset q$ $\sim q \supset r$ $\sim q \supset \sim r$

$\therefore\ p \supset r$ $\therefore\ p \supset r$ $\therefore\ p \supset r$

5) Conjunction: p

q

$\therefore$ p.q

This rule means that the first two premises are joined together by the word 'and'.

6) Simplification: p.q

$\therefore$ p

This rule states that if the two premises are joined by the word 'and' or by the symbol '.', the first variable or conjunct can be inferred as the conclusion. The other valid forms of simplification are:

i) $\sim p.q$ ii) $(p \vee q).r$

$\therefore\ p$ $\therefore\ p \vee q$

7) Addition: p

$\therefore\ p \vee q$

This rule means that we can add variable or variables to the premises by the symbol 'v'. The other valid forms of syllogism are:

i) p ii) p iii) p

∴ p v (q.r) ∴ p v ~q ∴ p v (q v (p.r))

8) Absorption (Abs): $p \supset q$

$\therefore p \supset (p.q)$

According to this rule, consequent carries antecedent in the conclusion.

9) Constructive Dilemma (C.D.): $(p \supset q)(r \supset s)$

$p \vee r$

$\therefore q \vee s$

This rule states that there are two conditional propositions. And the major premise possesses two different antecedents, therefore both denotes different consequent. We can take a concrete example:

If a man is honest, he must be respected; and if he is a social worker, he must get reward. A man is either an honest man or a social worker.

∴ Either he must be respected or he must get reward.

This example can be symbolized as–

$(p \supset q).(r \supset s)$

$p \vee r$

$\therefore q \vee s$

This is a valid logical form of constructive dilemma.

The other valid form of constructive dilemma is:

$(\sim p \supset q).(\sim r \supset s)$

$\sim p \vee \sim r$

$\therefore q \vee s$

10) Destructive Dilemma (D.D.): $(p \supset q).(r \supset s)$

$\sim q \vee \sim s$

$\therefore \sim p \vee \sim r$

We can take a concrete example in order to understand its logical form. If a man is honest, he must be respected; and if he is a social worker, he must get reward. A man is either not respected or does not get reward.

∴ Either he is not honest or not a social worker.

This example can be symbolized as–

$(p \supset q)(r \supset s)$

$\sim q \vee \sim s$

∴ $\sim p \vee \sim r$. This is a valid form of destructive dilemma.

7.4 SOLVED EXAMPLES OF RULES OF INFERENCE

There are some solved examples of formal proof of validity given in this section in order to understand how the rules of inference are applied to test the validity of arguments.

A) 1) $(p \supset q)$
2) $(q \supset s)$
3) $p \ / \therefore s$
4) $p \supset s$ 1st and 2nd by H.S.
5) s 4th and 3rd by M.P.

B) 1) $(p \vee q) \supset r$
2) $s \vee p$
3) $\sim s \ / \therefore r$
4) p 2st and 3nd by DS
5) $p \vee q$ 4 addition
6) r 1, 5 Modus ponens

C) 1) $(p \supset q)$
2) $(r \supset s)$
3) $p \vee r$
4) $(q \vee s) \supset t \ / \therefore t$
5) $(q \vee s)$ 1, 2 C.D.
6) t 4, 5 M.P.

D) 1) $(p \supset q)$
2) $(q \supset r)$
3) $\sim p \supset (s \vee t)$
4) $\sim r \ / \therefore s \vee t$
5) $p \supset r$ 1, 2 H.S.

6) ~p 5, 4 M.T.

7) s v t 3, 6 M.P.

E) 1) ~p ⊃ (I v E)

2) p.r

3) ~I / ∴ E

4) p 2nd sim

5) I v E 1st and 4th by D.S.

6) E 5th and 3rd by D.S.

F) 1) p ⊃ ~q

2) ~q ⊃ r

3) r ⊃ s

4) (p.s) ⊃ u / ∴ p⊃u

5) p ⊃ r 1st and 2nd by H.S.

6) p ⊃ s 5th and 3rd by H.S.

7) p ⊃ (p.s) 6th by Abs.

8) p ⊃ q 7th and 4th by H.S.

G) If rains come in time, farmers will be happy. If farmers are happy, there will be good crops. Rains come in time. Therefore, there will be good crops.

This real example can be symbolized in this way:

1) R ⊃ F

2) F ⊃ C

3) R / ∴ C

4) R ⊃ C 1, 2 H.S.

5) C 4, 3 M.P.



CHECK YOUR PROGRESS

Q.1: What is deductive method?

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Q.2: Write the symbolic form of:

a) Modus Tollens

.....

.....

b) Modus Ponens

.....

.....

c) Constructive Dilemma

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.....

d) Destructive dilemma

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.....

e) Simplication

.....

.....

f) Conjunction

.....

.....

Q.3: What do you mean by the rule of modus ponens?

.....

.....

Q.4: What do you mean by the rule of disjunctive syllogism?

.....

.....

.....

Q.5: Write a valid form of modus ponens.

.....

Q.6: Construct a formal proof of validity by applying the rules of inference.

i) $p \supset q$ ii) $q \supset r$ iii) $p \ / \therefore r$

.....

.....



7.5 LET US SUM UP

- Rules of inference are considered self-evident and are therefore valid without any proof. On the basis of these rules we can determine or test the validity or invalidity of arguments. That is why these rules are known as deductive method.
- Modus ponens means affirmation of the antecedent and on that basis consequent is affirmed.
- Modus tollens means the denial of the consequent and on that basis the antecedent is denied.
- This argument holds that if one of the options or disjuncts is denied in the minor premise, the other option or disjunct is accepted in the conclusion.
- Hypothetical syllogism means that both the premises are conditional statements and they have one common statement. The common statement is known as antecedent in one premise and other is consequent in another premise.



7.6 FURTHER READING

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6) Singh, S. Shyam Kishore (2000); Modern Logic (Volume one); Lamyana Press.

7.7 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q. No. 1: Rules of inference are considered self-evident and are therefore valid without any proof. On the basis of these rules we can determine or test the validity or invalidity of arguments. That is why these rules are known as deductive method.

Ans. to Q. No. 2:

a) Modus Ponens (M.P.)

$p \supset q$

p

$\therefore q$

b) Modus Tollens (M.T.)

$p \supset q$

$\sim q$

$\therefore \sim p$

c) Constructive Dilemma (C.D.)

$(p \supset q) (r \supset s)$

$p \vee r$

$\therefore q \vee s$

d) Destructive Dilemma

$(p \supset q) (r \supset s)$

$\sim q \vee \sim s$

$\therefore \sim p \vee \sim r$

e) Simplification (Simp.)

$p.q$

$\therefore p$

f) Conjunction (Conj.)

p

q

Ans. to Q. No. 3: Modus ponens means affirmation of the antecedent and on that basis consequent is affirmed.

Ans. to Q. No. 4: This argument holds that if one of the options or disjuncts is denied in the minor premise, the other option or disjunct is accepted in the conclusion.

Ans. to Q. No. 5: $\sim p.\sim q$

$\sim p$

$\therefore \sim q$

Ans. to Q. No. 6: i) $p \supset q$

ii) $q \supset r$

iii) $p \quad \therefore r$

iv) $p \supset r$ i) and ii) H.S.

v) r iv) and v) M.P.



7.8 MODEL QUESTIONS

A) Very short questions:

Q.1: What do you mean by method of deduction?

Q.2: State the logical form of modus ponens.

Q.3: What do you mean by the rule of disjunctive syllogism?

Q.4: State the meaning of modus tollens.

Q.5: Write the logical form of hypothetical syllogism.

Q.6: Give an concrete example of modus ponens.

B) Construct formal proof of validity for each of the following argument forms:

Q.1: p. p /∴ p

Q.2: p / pVq

Q.3: i) ~p ⊃ q
ii) ~p /∴ q

Q.4: i) p ⊃ q
ii) q ⊃ r
iii) ~ r /∴ ~ p

Q.5: i) p ⊃ (q v r)
ii) s ⊃ p
iii) s /∴ (q v r)

Q.6: i) p ⊃ q
ii) q ⊃ s
iii) p /∴ q

Q.7: i) p ⊃ q
ii) q ⊃ r
iii) r ⊃ s /∴ (p ⊃ s)

Q.8: i) s ⊃ h
ii) h ⊃ ~ l
iii) l /∴ s

Q.9: i) p ⊃ ~q
ii) ~q ⊃ r
iii) r ⊃ s
iv) (p.s) ⊃ u /∴ (p ⊃ u)

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UNIT 8: RULES OF REPLACEMENTS

UNIT STRUCTURE

8.1 Learning Objectives

8.2 Introduction

8.3 Rules of Replacement

8.4 Some Solved Examples by Rules of Replacement

8.5 Let us Sum Up

8.6 Further Reading

8.7 Answer to Check Your Progress

8.8 Model Questions

8.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- explain why rules of replacement is required
- discuss solved examples by rules of replacements
- explain how their implication is different from rules of inference.

8.2 INTRODUCTION

In the previous unit, we have discussed about the rules of inference. But there are many valid truth-functional arguments whose validity, we cannot prove by using only the nine rules of inferences. As for example, in order to construct a formal proof validity for the following problem, rules of inferences are not enough.

$$A \supset B$$
$$C \supset \sim B$$
$$\therefore\ A \supset \sim C$$

To solve this kind of problem some additional rules are required.

In any Truth-functional compound statement, if a compound statement in its being replaced by another statement having the same truth-functions, then the truth value of the compound statement will remain unchanged. But only compound statements that concern us here are truth

functional compound statements. That is why we can add some more or additional principles of rules of inference that is the rules of replacement. With the help of these rules we can replace any component of a statement by any other statement *logically* equivalent to the component replaced. For example, using the principle of Double Negation (D.N.), which asserts that 'p' is logically equivalent to '~~p', we infer from A ⊃ ~~B any of the following–

i.e., A ⊃ B, ~~A ⊃ ~~B, ~~ (A ⊃ B) or A ⊃ ~~~~B

by rules of replacement to make the ten rules definite, we list ten tautologous logically true bi-conditionals with which it can be used. These bi-conditionals provide additional rules of inference to be used in providing the validity of extended arguments.

The process of replacements is very different form substitution. We substitute statements for statements variables, whereas we replace statements with other statements with other statements. In moving from a statement form to a substitution instance of it, or from an argument form to a substitution instance of it, we can substitute any statements for any statement variable, provided that if a statement is substituted for one occurrence of a statement variable it must be substituted for every other occurrence of that statement variable. But in moving from one statement to another way of replacement, we can replace a component of the first only by a statements certified to be logically equivalent to the component by one of the logical equivalences 10 to 19, and we can replace one occurrence of that component without having to replace any other occurrence of it.

8.3 RULES OF REPLACEMENT

It is already mentioned that there are ten rules of replacement. They are stated as follows:

11) De Morgan's Theorems (De. M.)
~(p.q) ≡ (~p v ~q)
~(p v q) ≡ (~p.~q)

12) Commutation (Com.)
(p v q) ≡ (q v p)
(p.q) ≡ (q.p)

13) Association (Assoc.)
[p v (q v r)] ≡ [(p v q) v r)]
[p.(q.r)] ≡ [(p.q).r]

14) Distribution (Dist.)
[p.(q v r)] ≡ [(p.q) v (p.r)]
[p v (q.r) ≡ [(p v q).(p v r)]

15) Double Negation (D.N.)
$p \equiv \sim\sim p$

16) Transposition (Trans.)
$(p \supset q) \equiv (\sim q \supset \sim p)$

17) Material Implication (Impl.)
$(p \supset q) \equiv (\sim p \vee q)$

18) Material Equivalence (Equiv.)
$(p \equiv q) \equiv [(p \supset q).(q \supset p)]$
$(p \equiv q) \equiv [(p.q) \vee (\sim q.\sim p)]$

19) Exportation (Exp.)
$[(p.q) \supset r] \equiv [p \supset (q \supset r)]$

20) Tautology (Taut.)
$p \equiv (p \vee p)$
$p \equiv (p.p)$

8.4 SOME SOLVED EXAMPLES OF RULES OF REPLACEMENT

A) 1) $(A \supset B).(C \supset D)$ / $\therefore (A \supset B).(\sim D \supset \sim C)$
2) $(A \supset B).(\sim D \supset \sim C)$ From 1 by Trans.

B) 1) $(E \supset F).(G \supset \sim H)$ / $\therefore (E \supset F).(G \supset \sim H)$
2) $(\sim E \vee F).(G \supset \sim H)$ From 1 by Equiv.

C) 1) $(\sim M \vee \sim N) \supset (0 \supset \sim\sim P)$ / $\therefore \sim (M.N) \supset (0 \supset \sim\sim P)$
2) $\sim(M.N) \supset (0 \supset \sim\sim P)$ From 1 by De.M.

D) 1) $(X \vee Y).(\sim X \vee \sim Y)$ / $\therefore [(X \vee Y).\sim X] \vee [(X \vee Y).\sim Y]$
2) $[(X \vee Y).\sim X] \vee [(X \vee Y).\sim Y]$ From1 by Dist.



CHECK YOUR PROGRESS

Q.1: How many rules are there in the rules of replacement?

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Q.2: Write down the rule of Double Negation.

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Q.3: Give an example of Tautology.

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Q.4: Write down the rule of DeMorgan Theorem.

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Q.5: How many truth-functions are used in the rule of material implication?

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.....

Q.6: Construct a formal proof of validity by applying the rules of replacement.

$p \supset q$

$r \supset \sim q \quad / \therefore p \supset \sim r$

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.....



8.5 LET US SUM UP

- In order to construct formal proof of validity, with rules of inference we need rules of replacement too.
- There are ten rules of replacements.
- In the rules of replacement one side of the statement can be replaced with the other side of the equivalent statement.
- Rules of replacement can be applied to a part of the statement too, which is not possible in case of rules of inference.



8.6 FURTHER READING

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8.7 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q. No. 1: There are ten rules in the rules of replacement.

Ans. to Q. No. 2: The rule of Double Negation is: $p \equiv \sim\sim p$

Ans. to Q. No. 3: The example of tautology is as follows:

The rain comes tomorrow or the rain comes tomorrow

Ans. to Q. No. 4: The rule of DeMorgan's theorem is:

$\sim(p.q) \equiv (\sim p \vee \sim q)$

$\sim(p \vee q) \equiv (\sim p.\sim q)$

Ans. to Q. No. 5: Three truth-functions are used in the rule of material implication. They are: implicative function, disjunctive function and the negative function.

Ans. to Q. No. 6: 1. $p \supset q$

2. $r \supset \sim q$ $/\therefore p \supset \sim r$
3. $\sim\sim q \supset \sim r$ from 2 by Trans.
4. $q \supset \sim r$ From 3 by D.N.
5. $p \supset \sim r$



8.8 MODEL QUESTIONS

A) Very Short Questions:

Q.1: Define rules of replacement.

Q.2: How rules of replacements are different from rules of inference?

Q.3: What is the logical value of association?

Q.4: Rules of replacements are the rules of deduction—Is this statement true?

Q.5: Is substitution and replacement are same?

Q.6: Write down the logical value of Material Equivalence?

B) Construct formal proof of validity for each of the following argument forms:

Q.1: $A \supset B$

$C \supset \sim B$ / $\therefore A \supset \sim C$

Q.2: $(D.E) \supset F$

$(D \supset F) \supset G$ / $\therefore E \supset G$

Q.3: $A \supset B$
$B \supset C$
$C \supset A$
$A \supset \sim C$ / $\therefore \sim A.\sim C$

Q.4: $(H \vee I) \supset [J.(K.L)]$
I / $\therefore J.K$

Q.5: $W \supset X$
$\sim Y \supset \sim X$ / $\therefore W \supset Y$

Q.6: $(L \supset M).(N \supset M)$
$L \vee N$ / $\therefore M$

Q.7: $(D.\sim E) \supset F$
$\sim(E \vee F)$ / $\therefore \sim D$

Q.8: $Z \supset A$
$Z \vee A$ / $\therefore A$

Q.9: $(N.O) \supset P$
$\sim (P \supset Q) \supset Q$ / $\therefore N \supset Q$

Q.10: C / $\therefore D \supset C$

Q.11: $H.(I.J)$ / $\therefore J.(I.H)$

Q.12: $O \supset P$
$P \supset \sim P$ / $\therefore \sim O$

Q.13: $(M \supset N).(\sim O \vee P)$
$M \vee O$ / $\therefore N \vee P$

Q.14: $(Z \vee A) \vee B$
$\sim A$ / $\therefore Z \vee B$

Q.15: $E \vee (F.G)$ / $\therefore E \vee G$

Q.16: $T \supset U$
$\sim (U \vee T)$ / $\therefore \sim T$

Q.17: $A \supset B$
$\sim A \supset C$
$\sim(B \vee C)$ / $\therefore D$

Q.18: $B \equiv C$
$\sim D \equiv \sim C$
$D \supset (E \vee F)$
$\sim E$ / $\therefore \sim F$

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UNIT 9: ELEMENTARY FORMAL DEDUCTIONS

UNIT STRUCTURE

9.1 Learning Objectives

9.2 Introduction

9.3 Elementary Formal Deduction

9.4 Simple Introductory Deduction

9.5 The Strategy for Constructing Formal Deduction

9.6 Certain Examples of Formal deduction

9.7 Let us Sum Up

9.8 Further Reading

9.9 Answers to Check Your Progress

9.9 Model Questions

9.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- explain elementary formal deduction
- illustrate simple introductory deduction
- discuss the strategy for constructing formal deduction
- explain certain examples of formal deduction.

9.2 INTRODUCTION

The primary concern of deductive logic is validity. In deductive logic arguments are ultimately classified into two types– valid arguments and invalid arguments. An argument is valid when the conclusion of the argument is fully justified by its premises.

We have already come across certain basic rules of inference. These basic rules of inference may also be understood as elementary valid argument forms. If a particular argument is in such an elementary valid argument form then the argument is demonstrated to be valid. For example the following is the rule known as Modus Ponens (M.P.)–

$p \supset q$

p

$\therefore$ q

Let us look at the following argument–

$(A.B) \supset R$

$A.B$

$\therefore$ R

This argument conforms to the structure of modus ponens. Therefore, it is a valid argument. Longer and more complex deductive arguments may be given to us. We may discover that these arguments consist of series of steps in which every step conforms to particular rule of inference. By finding out these steps one after another and by referring to the specific rules of inference conformed by these steps we can explain that the conclusion of the argument follows from the premises of the argument. Thereby, we can establish that the argument is valid.

9.3 ELEMENTARY FORMAL DEDUCTION

An elementary formal deduction consists of a series of steps. Here, we first write down the premises. Generally the conclusion is written in the right hand side. Then premises are combined and recombined in such a way that a series of steps is constructed. In combining and recombining the premises are constantly refer to the rules of inference. We are completely guided and logically controlled by the rules of inference. Ultimately the series of steps ends in the conclusion of the argument. Every step in this step is valid. Thus this series of arguments which is constructed is a series of valid arguments. Ultimately we arrive at the conclusion. The process demonstrates that the conclusion is logically validly derivable from the premises. The entire series of continuous steps from the premises to the conclusion is known as a formal deduction. When the formal deductions are not extremely logically complicated then we call then elementary formal deductions.

Alternately the process of constructing such a formal deduction is known as constructing a derivation. When a formal deduction successfully shows that the conclusion of an argument logically follows from the premises then we have constructed a demonstration or a proof validity of the argument.

9.4 SIMPLE INTRODUCTORY DEDUCTIONS

We may begin with arguments that are simple and then we may move on to arguments of greater complexity.

Let us first take up the following arguments.

R.S /∴ R v T

This argument may be shown to be valid by using just two rules of inference. One is simplification (simp.) and the other is addition (add.)

Let us construct the formal deduction of the above argument.

1) R.S / ∴ R v T
2) R From 1 Simp.
3) R v T From 2 Add.

Again let us take the following argument–

$A \supset B$

$C \supset D$

$\sim B \vee \sim D$ / ∴ $\sim A \vee \sim C$

This argument can be shown to be valid by using just rules of inference. They are conjunction (conj) and destructive dilemma (D.D.). The complete formal deduction for this argument may be constructed as follows–

1) $A \supset B$
2) $C \supset D$
3) $\sim B \vee \sim D$ / ∴ $\sim A \vee \sim C$
4) $(A \supset B).(CD)$ From 1 and 2 by Conj.
5) $\sim A \vee \sim C$ From 3 and 5 by D.D.

There is no mechanical process to be followed while constructing formal deductions. We do not know beforehand which rule of inference is to be used first, which rule of inference is to be used next and so on. There is no set formula. Every argument will require its particular formal deduction. Every particular formal deduction will have its own specific steps. The steps

of one formal deduction of one argument will not be exactly identical with the steps of another deduction of another argument. Every argument will call for its specific deduction. The strategy of one formal deduction will not be exactly identical with the strategy found in another deduction.

9.5 THE STRATEGY FOR CONSTRUCTING FORMAL DEDUCTION

When we attempt to construct the formal deduction of an argument then first we have to carefully look at the premises of the argument and also at the conclusion. We are to think, then, from the end of the conclusion. How this conclusion can come out of the premises? This will be our guiding question.

If there are certain premises then we may as– can we combine two or more premises and thereby construct a road towards the conclusion? Again if a compound statement is there as premise then we may see whether it can be broken down into constituents by using some rules such as the rule of simplification. If there is a new proposition in the conclusion that was not present in the premises then we may be rest assured that somehow the rule of deduction has to be applied.

There are just a few thumb rules. The most important point is that if an argument is given then we have to think backwards from the conclusion.

9.6 CERTAIN EXAMPLES OF FORMAL DEDUCTION

a) 1) $\sim J \supset N$
   2) $J \supset L \; / \therefore L \vee N$
   3) $\sim L \supset \sim J$ Form 2 by Trans.
   4) $\sim L \supset N$ From 3 and 1 by H.S.
   5) $\sim\sim L \vee N$ From 4 by Impl.
   6) $L \vee N$ From 5 by D.N.

In constructing this formal deduction first we have employed the rule of transposition on the second premise. The rule of transposition is the following–

$(p \supset q) \equiv (\sim q \supset \sim p)$

Similarly, $J \supset L$ is equivalent to $\sim L \supset \sim J$. Next we have employed the rule of hypothetical syllogism on propositions 3 and 1. The rule of hypothetical syllogism is the following–

$p \supset q$

$q \supset r$

$\therefore \quad p \supset r$

By combining the lines 3 and 1 and by employing the rule of hypothetical syllogism we get the following–

$\sim L \supset \sim J$

$\sim J \supset N$

$\therefore \quad \sim L \supset N$

The conclusion is presented in the fourth line. By applying the rule of material implication upon the fourth line we get the following–

$\sim\sim L \vee N$

This is presented as the fifth line. By employing the rule of double negation on this line we may get the following line–

$L \vee N$

This is our sixth line of the formal deduction and it is also the conclusion. We have thereby demonstrated this conclusion, through a series of steps, follows from the premises. Thereby the argument is established to be a valid argument.

b) $R \supset S$

$R.T \; / \therefore S$

1) $R \supset S$
2) $R.T \; / \therefore S$
3) R From 2 by Simp.
4) S From 1 and 3 by M.P.

c) $P \vee (R.T)$

$\sim P$

$R \supset S \; / \therefore S$

1) $P \vee (R.T)$
2) $\sim P$

3) R ⊃ S / S

4) R.T From 1 and 2 by D.S.

5) R From 4 by Simp.

6) S From 3 and 5 by M.P.

d) P ⊃ R

~R.T

(~P.T) ⊃ S / ∴ S V L

1) P ⊃ R

2) ~R.T

3) (~P.T) ⊃ S / ∴ S V L

4) ~R From 2 by Simp.

5) ~P From 1 and 5 by M.T.

6) T.~R From 2 by Com.

7) T From 6 by Simp.

8) ~P.T From 5 and 7 by Conj.

9) S From 3 and 8 by M.P.

10) S V L From 9 by Add.

e) S ⊃ T

S V T / ∴ T

1) S ⊃ T

2) S V T / ∴ T

3) T V S From 2 by Com.

4) ~T ⊃ S From 3 by Impl.

5) ~T ⊃ T From 4 and 1 by H.S.

6) ~~T V T From 5 by Impl.

7) T V T From 6 by D.N.

8) T From 7 by Taut.

f) (F ⊃ R).(L ⊃ U)

(R.U) ⊃ C

~C /∴ ~F v ~L

1) (F ⊃ R).(L ⊃ U)

2) (R.U) ⊃ C

3) ~C /∴ ~F V ~L

4) ~(R.U) From 2 and 3 by M.T.

5) ~R V ~U From 4 by De.M.

6) ~F V ~L From 1 and 5 by D.D.



CHECK YOUR PROGRESS

Q.1: What is the primary concern of deductive logic?

.....

.....

Q.2: What is the other name of the basic rules of inference?

.....

.....

Q.3: What is the elementary formal deduction?

.....

.....

.....

Q.4: How can a valid argument be established?

.....

.....

.....

Q.5: Find out the validity of the following argument:

a) $D \supset A$
$A \supset (B \supset C) / \therefore (D.B) \supset C$

b) $A \vee (B.C)$
$(A \vee B) \supset D / \therefore D \vee E$

c) $C / \therefore D \supset C$

d) $R \vee S$
$\sim R.T$
$S \supset L / \therefore L \vee K$



9.7 LET US SUM UP

- The formal deduction of an argument consists of a series of steps through which it is established that the conclusion of the argument follows from the premises.

- In constructing formal deductions we take the help of the rules of inference.
- There is no fixed mechanical device by using which formal deductions are constructed. Generally we have to think backward from the conclusion and to discover the steps which will lead us from the premises to the conclusion.
- If we can show through a series of valid steps that the conclusion of an argument follows from the premises then the argument is established to be a valid argument.



9.8 FURTHER READING

1) Copi, Irving M. (2006); *Symbolic Logic* (Fifth Edition); New Delhi: Prentice Hall of India Pvt. Ltd.
2) Copi. Irving M.; Cohen, Carl; Jetli, Priyedarshi & Prabhakar, Monika (2006); *Introduction to Logic* (Twelfth Edition); Delhi: Dorling Kindersley (India) Pvt. Ltd.
3) Nolt, John; Rohatyn, Dennis Hille (2004); *Theory and Problems of Logic* (Second Edition); New Delhi: Tata McGraw-Hill.



9.9 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q. No. 1: Validity of the argument is the primary concern of deductive logic.

Ans. to Q. No. 2: The basic rules of inference are also known as elementary valid argument forms.

Ans. to Q. No. 3: When the formal deductions are not extremely logically complicated then, these rules can be regarded as elementary formal deduction.

Ans. to Q. No. 4: If it can be shown that through a series of valid steps that the conclusion of an argument follows from the premises then the argument is established to be a valid argument.

Ans. to Q. No. 5: a) 1) $D \supset A$

2) $A \supset (B \supset C)$ / $\therefore (D.B) \supset C$

3) $D \supset (B \supset C)$ From 1 and 2 by H.S.

4) $(D.B) \supset C$ From 3 by Exp.

b) 1) $A \vee (B.C)$

2) $(A \vee B) \supset D$ / $\therefore D \vee E$

3) $(A \vee B).(A \vee C)$ From 1 by Dist.

4) $(A \vee B)$ From 3 by Simp.

5) D From 2 and 4 by M.P.

6) $D \vee E$ From 5 by Add.

c) 1) C / $\therefore D \supset C$

2) $C \vee \sim D$ From 1 by Add.

3) $\sim D \vee C$ From 2 by Com.

4) $D \supset C$ From 3 by Impl.

d) 1) $R \vee S$

2) $\sim R.T$

3) $S \supset L$ / $\therefore L \vee K$

4) $\sim R$ From 2 by Simp.

5) S From 1 and 4 by D.S.

6) L From 3 and 5 by M.P.

7) $L \vee K$ From 6 by Add.



9.10 MODEL QUESTIONS

A) Construct formal deduction of the following arguments:

1) $M \supset N$

$N \supset O$

$(M \supset 0).(N \supset P)$

$(M \supset P) \supset Q$ / $\therefore Q$

2) $J \supset K$

$J \vee (K \vee \sim M)$

$\sim K$ / $\therefore \sim M.\sim K$

3) $E \supset (F . \sim G)$
   $(F \vee G) \supset H$
   $E / \therefore H$
4) $\sim(P.Q)$
   $Q \vee (R \supset S)$
   $P.\sim S / \therefore R \supset (R.S)$
5) $(G \vee X) \supset (E.K)$
   $(K \supset L)$
   $\sim L / \therefore \sim G$
6) $\sim H \vee R$
   $H / \therefore R \vee T$
7) $W \vee D$
   $W \supset F$
   $\sim F / \therefore D$
8) $(E.F) \vee (G \supset H)$
   $I \supset G$
   $\sim(E.F) / \therefore I \supset H$
9) $T \supset \sim(U \supset V) / \therefore T \supset U$
10) $(G \vee X) \supset (E.K)$
    $K \supset L$
    $\sim L / \therefore \sim G$
11) $(A \vee B) \supset (C.D)$
    $\sim C / \therefore \sim B$
12) $J \vee \sim K$
    $K \vee (L \supset J)$
    $\sim J / \therefore L \supset J$
13) $F \vee (G \vee H)$
    $(G \supset I).(H \supset J)$
    $(I \vee J).(F \vee H)$
    $\sim F / \therefore H$

B) Long Questions:

Q.1: What is formal deduction? Explain with illustrations.

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UNIT 10: DECISION PROCEDURES

UNIT STRUCTURE

10.1 Learning Objectives

10.2 Introduction

10.3 Decidability

10.4 Is Proof a Decision Procedure

10.5 Truth-Table Method

10.6 The Method of Reduction Ad. Absurdum or the Indirect Truth-Table Method

10.7 Reduction to Conjunctive Normal Form

10.8 Let us Sum Up

10.9 Further Reading

10.10 Answers to Check Your Progress

10.11 Model Questions

10.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- explain Decision Procedure as an important concept in Symbolic Logic
- analyse the concept of decidability
- analyse the different types of decision procedures
- describe Truth-Table method with example
- discuss Indirect Truth-table method with example
- explain Conjunctive Normal Formal with example.

10.2 INTRODUCTION

As the name suggests, symbolic logic is a system of logic, not very different from traditional logic, but concerned with the use of symbols. Therefore, we can say that symbolic logic has two aims– (i) to symbolise the arguments and expressions, and (ii) to evaluate them. We are concerned with the latter objective. In symbolic logic, and more precisely in the logic of

propositions, there are many methods to test the validity of arguments and argument forms and also there are ways to test or examine whether statement forms are tautologies or not. Such ways or methods that examine whether a given argument or statement forms is valid or invalid, whether it is tautologies or not, is called a decision procedure. It is such a procedure, by which we come to a "decision" i.e. we decide if the argument is valid or invalid, a tautology or not.

There are some standard decision procedures in the logic of propositions. These are–

1) Truth-Table Method
2) Shorter Truth-Table Method
3) Method of Reduction to Conjunctive Normal Form (Cnf.)

10.3 DECIDABILITY

The concept of decision procedure is basically concerned with the concept of decidability, whether a well-formed formula (wff) is valid or not, an inference is sound or not. In other words, we decide the validity or invalidity of 'wff' through a procedure. This is what we roughly mean by decidability. So, decidability requires a decision procedure. Decision procedures are effective since they are believed to be stable, reliable and finite "which can be described in advance for providing in a finite number of steps a 'yes' or 'no' answer to any class of questions". Decision procedure is regarded as mechanical. It works like a computer. If a formula, is valid applied correctly, it would at some point give a positive answer. If the formula under consideration is in fact not valid, it would give a negative answer.

10.4 IS PROOF A DECISION PROCEDURE?

Often, we think that proof and decision procedures are synonymous terms. We need to understand that with regard to 'proof', there remains nothing to be proved as false. In case of proving a formula, we know prior to it's proving that it is logically true. We need to distinguish between 'proving something' and 'testing something'. To prove something, we have no option

to make it false. But in the case of testing something, the 'given' may be either true or false and it is the task of the doer to decide effectively whether the 'given' is true or false. So, this shows that proof establishes logical truth, but cannot establish logical falsity. However, Quin is of the opinion that just because a 'proof' fails to establish logical falsity, this does not prevent us from formulating a procedure to prove validity. But he distinctly shows the difference between a proof and a decision procedure, when he says that 'a decision procedure assures an affirmative and negative answer every time, while a proof procedure assures at best an eventual affirmative answer where an affirmative answer is in order". Further, according to Quine a complete proof procedure can be distinguished from a decision procedure because a 'complete decision procedure' enables us to give both affirmative and negative answers, while a complete proof procedure give a positive or affirmative answer; it cannot give us negative answers.

Following the argument given above about a 'proof' not being a decision procedure, we can say that both Conditional Proof (CP) and Indirect Proof (IP) are not decision procedures since, the rule of CP is used is dealing with valid arguments. But, at this juncture, we face a confusing situation. Indirect Proof is equated with Reductio Ad Absurdum (RAA). But RAA is considered as a decision procedure. If this be the case, then why IP is not considered as a decision procedure? Why do we say Indirect Proof is similar to Reductio Ad Absurdum? The similarity between the two is that in either we attempt to draw contradiction of that which is "given" or we make attempts to reduce an absurdity of the "given". Their difference lies in their procedures. In Reductio Ad Absurdum, the absurdity is derived in terms of truth-values; but in case of Indirect Proof, the absurdity is derived in terms of propositions or propositional variables. In both Reductio Ad Absurdum and Indirect Proof, if the given statement is valid, then its negation is a contradiction. Therefore, the "given" has to be a tautology. But what happens when the given statement is not valid? In this case, when the given statement is not valid then its negation does not lead into a contradiction nor it is proved as a tautology. So in such cases, the given statement is either contingent or inconsistent. With regard to the truth-values, RAA helps to productively decide contingent and

inconsistent statements. In Indirect Proof, the inferential rules fail to decide if the given statement is contingent or inconsistent. Since, IP as a proof procedure, gives only a positive answer to any statement, but fails to give a negative answer. Indirect Proof (IP) cannot be regarded as a decision procedure.

10.5 TRUTH-TABLE METHOD (TT)

The Truth-Table method is an ideal example of a decision procedure. This method helps us to show that one function is equivalent to another function by analyzing the components of the truth-tables. The validity of arguments involving the truth-functional compound statements can be tested by this, truth-table method. The phrase 'truth-function is an important concept in the truth-table method. We can define truth-function as a compound expression which contains variable, where the truth-values are determined solely by the truth-values of its component variables. Through the truth-table method logicians decide whether a given compound expression is a tautology or contradictory or contingency. A detail account of these three types of logical expressions will be given in the Chapter on Truth-table. Now let us find out the validity of an argument with the help of the Truth-table method. We have already stated that the Truth-table method help us to test the validity or invalidity of arguments. Let us consider the following argument.

Premise– If the student works hard, he will be successful.

Premise– The student works hard.

Conclusion– Therefore, the student will be successful.

The logical form of this argument is: $p \supset q$

p

$\therefore q$

We can test the validity of the above argument by arranging the premises and conclusion in the following as: $[(pq).p] \supset q$

Now let us construct a truth-table for the above symbolic expression. In this expression, p, q are known as variables, and the symbols '•', '⊃' are known as constants.

1	2	3	4	5
P	q	P ⊃ q	[(p ⊃ q).P]	[(p ⊃ q).P] ⊃ q
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

The above Truth-table shows that the given symbolic expression is valid, since in the last column (5) of the truth-table, all the Truth-values are Ts (meaning true). This expression is a tautology. Similarly, an expression will be contradictory if the last column in the truth-table shows all the values as Fs (meaning false). A symbolic expression will be contingent, if the last column of the truth-table shows truth-values as having Ts and Fs.

Thus, we can say that the truth-table method is an effective mechanism, a decision procedure to decide if a given expression is valid or invalid. But this procedure may sometimes be inconvenient if we have more than three variables in a well formed formula (wff). But in spite of a symbolic expression being cumbersome, the truth-table method is regarded as an effective decision procedure.



CHECK YOUR PROGRESS

Q.1: Construct Truth-Tables for the following:

a) (p.q) ⊃ (p ∨ q)

.....

.....

.....

b) (p ⊃ q).~(p ⊃ q)

.....

.....

.....

Q.2: Fill in the blanks:

a) The Truth-table method decides whether a logical expression is a ..... and .....

10.6 THE METHOD OF REDUCTION AD ABSURDUM OR THE INDIRECT TRUTH TABLE METHOD

Another convenient method to test the validity of arguments containing large number of statements is the method of Reductio Ad Absurdum, also known as the Indirect Truth-table method. This method is shorter than the ordinary or direct Truth-table method. This method is mechanical and is used to test the validity of expression containing more number of variables and constants. In Indirect Truth-table method we count backwards. In the direct Truth-table method, we reach the main column at the end, but in the Indirect Truth table method, we start with the main column.

In this method of RAA, we begin with the hypothesis or assumption that the given argument is invalid. To do this, we place an 'F' (meaning false) below the main connective conjoining the premises and the conclusion. With the help of the principles governing the construction of the truth-tables, we assign values to the components of the argument. In the course of doing this activity of assigning values, if we find any discrepancies, then our original hypothesis about the given argument as wrong must be invalid. If such be the case, then the argument itself must be valid. But if the case be that there are no discrepancies when we assign values on the basis of our original hypothesis, then argument must be invalid. Now, let us apply the technique of RAA to test the validity or invalidity of an argument.

$(p \supset q) \supset (q \supset p)$

Step I F

$(p \supset q) \supset (q \supset p)$

Step 2 T F F

$(p \supset q) \supset (q \supset p)$

Step 3 T F T F F

$(p \supset q) \supset (q \supset p)$

Step 4 F T T F T F F

In the above example, we begin with the assumption or hypothesis that the argument is invalid. To do this, we put F under the constant $\supset$ joining $p \supset q$ and $q \supset p$ as shown in Step I. The Step 2 shows that main column can

be F when the antecedent ($p \supset q$) is true (T) and the consequent ($q \supset p$) is false (F). In Steps 3 and 4 we show that the consequent $q \supset p$ can be false (F) only if q is true (T) and p is false (F). Here we have the truth-values of p and q as false (F) and true (T) respectively. These values are assigned to p and q in the antecedent ($p \supset q$). After doing so, we find that there is no inconsistency in the expression $(p \supset q) \supset (q \supset p)$. So, our original assumption of putting F under main column is justified. But if this assumption is justified and no inconsistency arises thereby, it means that the argument is invalid.

Thus, we have seen that Reductio Ad Absurdum or the Indirect Truth-table method, when applied to argument or argument forms, is a complete decision procedure. It is a simpler method for testing arguments. Therefore, it is a convenient method to test the validity of complex logical expressions. It gives us a concrete answer to a complex logical statement, either as valid or invalid.



CHECK YOUR PROGRESS

Q.3: Prove the validity of the following arguments /expressions by applying the method of Reductio Ad Absurdum.

a) $[(p \supset q).\sim q) \supset \sim p$

.....

.....

b) $[p \supset q).\sim p) \supset \sim q$

.....

.....

c) The Direct Truth-table method/indirect direct table method is convenient to test the validity of arguments with more variables. (Tick the correct answer).

.....

d) Give the other name for Indirect Truth-Table Method.

.....

10.7 REDUCTION TO CONJUNCTIVE NORMAL FORM (CNF)

Apart from Direct and Indirect truth-table methods, we have another method to test the truth-value of complex logical expressions. This method is the Method of Reduction to Conjunctive Normal Form or what is known in brief as CNF. This method helps us to decide whether a given truth functional expression is a tautology or not. Like the other methods discussed above, this method of CNF is also a decision procedure and be conveniently applied to complex logical expressions having a large number of variables. Here, we need to introduce quite a few reference formulae for purpose of reduction to conjunctive normal form. By reference formulae, we mean equivalent logical expressions which are logically true expressions. The reference formulae are required for CNF in order to reduce a given truth functional expression to its equivalent expression. The reference formulae are:

1) **Principle of Commutation (Comm.)**
   i) $(p.q) \equiv (q.p)$
   ii) $(p \vee q) = (q \vee p)$
2) **Principle of Association (Assoc.)**
   i) $(p.(q.r)) \equiv ((p.q).r)$
   ii) $(p \vee (q \vee r) \equiv ((p \vee q) \vee r)$
3) **Principle of Distribution (Dist.)**
   i) $(p.(q \vee r) \equiv (p.q) \vee (p.r)$
   ii) $(p \vee (q.r) \equiv (p \vee q).(p \vee r)$
4) **Principle of Double Negation (D.N.)**
   $\sim\sim p \equiv p$
5) **De Morgan's Laws (De.M.)**
   i) $\sim(p.q) \equiv \sim p \vee \sim q$
   ii) $\sim(p \vee q) \equiv \sim p.\sim q$
6) **Definition of Material Implication (Impl.)**
   $p.q \equiv (\sim p \vee q)$

Let us understand CNF as a decision procedure by using the Reference Formulae to test the validity of a logical expression.

(p v q).~p) ⊃ q

~((p v q).~p) v q –by Impl.

(~(p v q) v ~~p) v q –by De.M.

((~ p.~q) v p) v q –by De.M., D.N.

((p v ~p).(p v ~q) v q –by Dist

((p v ~p) v q).(p v (q v ~q) –by Dist.

In the illustration of the given example through reduction to conjunctive Normal form, we conclude that the given truth – functional expression is a Tautology. Because, the last equivalent expression is a conjunction of two tautological disjunctions– ((p v ~p) v q) and ((p v (q v ~q). [A detailed account with illustrations will be given in the chapter on Reduction to conjunctive Normal Form]. In CNF, an argument is considered to be valid if its equivalent expression is a tautological Conjuncture Normal Form (CNF). But the argument will be invalid, if it cannot be reduced to a conjunction of tautologies.



CHECK YOUR PROGRESS

Q.4: Reduce the formulae/expressions to C.N.F.

a) ((p ⊃ q).~ q) ⊃ ~p

.....

.....

.....

b) ((p ⊃ q).(q ⊃ r)) ⊃ (p ⊃ r)

.....

.....

.....

c) What do you mean by Reference formulae?

.....

.....

d) State the method that is used to test whether a given argument is a tautology?

.....



10.8 LET US SUM UP

- Decision procedure is a purely mechanical or computing system to test the validity or invalidity of arguments. But we should remember that the Truth value assignments, is no doubt, an effective mechanism to test the validity of an argument, we can arrive at certain conclusions from certain premises. But in no way, we can ascertain how to draw the conclusions. A computer or a machine generates its actions only if its operation is conducted by certain rules. In a similar fashion, we can say that Decision procedure will be operative and adequate only if certain rules are followed. The rules alone, and not the method or process, can show how one statement is deduced from another.
- Logic is often shown to be dealing with mathematical reasoning, but this does not amount to asserting that logic exclusively relies on the basis of matrices and truth value assignments. We, no doubt, apply matrices and truth-value assignments to test Mathematical and Logical reasoning. But they can, in no way, be equivalent to mathematics and logic. We can derive a conclusion from certain premise/premises, not through assignment of truth values, but only on the basis of some rules. We can, thus, conclude that decision procedures, does not make rules redundant, but that all decision procedures operate on the basis of certain rules. The three types of Decision Procedures are convenient in allowing us to draw a conclusion, whether an argument is valid or invalid, and a tautology or not.



10.9 FURTHER READING

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10.10 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q. No. 4: I) DIRECT TRUTH-TABLE METHOD

a) $(p.q) \supset (p \vee q)$

1	2	3	4	5
P	Q	P.q	P v q	(p.q) ⊃ (p v q)
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T

The above Truth-Table proves that the given expression is valid and is a tautology. Because, the last column (5) of the table shows that the expression is true (T) under all circumstances and is therefore valid.

b) $(p.q).\sim (p \supset q)$

1	2	3	4	5
P	Q	P ⊃ q	~(p ⊃ q)	(p ⊃ q).~(~p ⊃ q)
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

From the above truth-table and as shown in the last column (5) where all are Fs, the expression is invalid and is therefore contradictory.

Ans. to Q. No. 2: Valid or invalid.

Ans. to Q. No. 3: a) Indirect Truth-table method or the Method of Reductio Ad Absurdum

$((p \supset q).\sim q) \supset \sim P$

Step 1 F

$((p \supset q).\sim q) \supset \sim P$

Step 2 T F F

$((p \supset q).\sim q) \supset \sim P$

Step 3 T T T F F

$((p \supset q).\sim q) \supset \sim P$

Step 4 T T T F F F T

$((p \supset q) . \sim q) \supset \sim P$

Step 5 T T F T T F F F T

In the Indirect truth-table constructed above we put an F under the main variable in Step 1 as required to do so in the method of Reduction Ad Absurdum. By following the logical rules we continue with the different steps. In Step 4, we find that the value of p is true and q is false. But if we assign these values to ($p \supset q$), where $p \supset q$ is already stated to be true, we are not permitted to put T under P and F under q because it violates the truth-functional use of '⊃'. Thus if we put F under the main column of the given expression, then it violates the basic rules of the truth table procedure. In other words, the original expression is valid.

b) $((p \supset q).\sim P) \supset \sim q$

Step 1 F

$((p \supset q).\sim P) \supset \sim q$

Step 2 T F F

$((p \supset q).\sim P) \supset \sim q$

Step 3 T T T F F

$((p \supset q).\sim P) \supset \sim q$

Step 4 F T T T T F F F T

In the Indirect truth table constructed for the given expression, we begin with Step 1 by putting an F under the main variable/ column. We find that in the subsequent steps, there is no violation of truth-functional procedures and the basic rules of truth-table method. Thus we can conclude that an F assumption in the main column leads to no violation of rules and therefore the given expression is invalid.

c) Indirect Truth Table Method

d) Method of Reductio Ad Absurdum.

Ans. to Q. No. 4: a) $[(p \supset q).\sim q) \sim P$

$\sim((\sim p \vee q) \vee \sim\sim q) \vee \sim P$ –by Mat Impln.

$(\sim (\sim p \vee q) \vee \sim\sim q) \vee \sim p$ –by D.N. and De.M.

$((P. \sim q) \vee q) \vee \sim P$ –by D.N. and De.M.

$((q \vee P).(q \vee \sim q)) \vee \sim P$ –by Comm. and Dist.

$((p \vee \sim P) \vee q).((\sim P \vee q) \vee \sim q$ –by Comm. and Dist.

This expression is now in Conjuncture Normal Form.

b) $((p \supset q).(q \ r)) \supset (p \supset r)$

$\sim(\sim(p \vee q).(\sim q \vee r) \vee (\sim p \vee r)$ –by Mat. Impln.

$(\sim(\sim p \vee q) \vee \sim(\sim q \vee r)) \vee (\sim p \vee r)$ –by De.M. and dropping double negation.

And a second application of the rule gives:

$((p.\sim q) \vee (q.\sim r)) \vee (\sim P \vee r)$

$((P \vee q).(p \vee \sim r).(q \vee \sim q).(\sim q \vee \sim r) \vee (\sim P \vee r)$

–by application of Distributive law to the first two members of the disjunction.

$(p \vee \sim p \vee q \vee r).(p \vee \sim p \vee r \vee \sim r).\ (\sim P \vee q \vee \sim q \vee r.(\sim P \sim q \vee r \vee \sim r)$

–by applying commutative law to the second part of the disjunction to bring it to the front of the expression and then apply Distributive Law.

The expression is shown to be in conjunctive Normal form.

c) Reference Formulae are equivalent logical expressions which are logically true expressions.

d) Reduction to Conjuncture Normal Form.



10.11 MODEL QUESTIONS

A) Very Short Questions:

Q.1: What do you mean by a Decision Procedure?

Q.2: What are the different types of Decision Procedures?

Q.3: What do you mean by Decidability?

Q.4: What are different types of statements referred to in Truth-table method?

B) Short Questions: (Answer each question within 150 words)

Q.1: What is a Proof?

Q.2: Is Proof a decision procedure?

Q.3: Explain Direct Truth-table method with the help of an example.

Q.4: Explain Indirect Truth-table method or the method of Reductio AD Absurdum.

Q.5: Construct Truth-tables, Direct and Indirect to test the validity of the given expressions.

i) $(p \supset q) \vee (\sim p \vee \sim r)$

ii) $[(p.q) \supset r] \vee (\sim p \supset (q \supset r)$

iii) $[\sim(p \supset q)] \supset (\sim P.\sim q)$

iv) $[P \supset (q \supset r)] \supset [\ (P \supset q) \supset (P \supset r)$

Q.6: Reduce the following to Conjunctive Normal Form

i) $\sim(F \supset G) \supset (E \supset \sim I)$

ii) $(p \vee q) \supset P$

iii) $(p \supset q).\sim q) \supset \sim P$

Q.7: What do you mean by Reduction to Conjunctive Normal Form? State the Reference Formula.

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