CPADA (C1)

FOUNDATIONAL MATHEMATICS AND STATISTICS

CERTIFICATE PROGRAMME IN APPLIED DATA ANALYSIS

KRISHNA KANTA HANDIQUI STATE OPEN UNIVERSITY
Guwahati: Assam

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COURSE INTRODUCTION

This is the first course of the programme entitled “Certificate Course in Applied Data Analysis”. This course has been entitled *‘Foundational Mathematics and Statistics’.* This course has been designed in such a way that it helps you in getting the foundational concepts of quantitative techniques requried for applied data analysis. This course has been divided into two parts: Mathematics and Statistics. It is expected that you to through these basic concepts minutely. This will help you immensely in the desired progress of the programme.

This course has a total of 14 units. Of this, seven units are from Mathematics, while the rest seven units are from Statisitcs. Units 1 and 2 have been devoted on the discusison of a few preliminary concepts in Mathematics, relating to number systems, relations and functions.After going through thes units, you will be able to gain knowledge about the number system, and the various types of relations and functions. The next unit introduces you to the preliminary concepts of Sets. The empty set, finite and infinite set, equal set, subsets, power set, universal set, venn diagram and complement of a set are discussed in this unit. The fourth unit deliberates on the concept of Equations. The fifth unit of the course, entitled “Determinants” would help you basic insights of one of fundamental concepts of matrix algerbra in Mathematics. Here, the basic concept of determinants, determinants of orders 2 have been discussed. The Cramer’s Rule have also been dicussed towards the end of the Unit. Unit 6 deliberates on Matrix Algebra. In this unit, the basic concepts of matrix, viz., equality of matrices, multiplicaiton, addition, transpose have been discussed. Unit 7 discusses the concepts of limit and continuity. Thus, through this unit, you will be able to gether preliminary knowlege of limit and continuity.

On the statistics part, Units 8 and 9 introduces us to some of the basic concepts of Statistics. This unit focuses on the different methods data collection, tabulation of data and presentation of statistical information. Unit 10 basically deals with graphical representation of data. Here, the various techniques of data presentation have been discussed. Unit 11 deals with data collection. In the next unit, statistical survey as a technique of data collection has been discussed. Unit 13 and 14 deal with interview and conduct of interviews. In these two units, a few basic concept and techniques of conducting interviews have been discussed.

While going through a unit, you will notice some along-sode boxes, which have been included to help you know some of the difficult, unseen terms. Some ‘ACTIVITY’ (s) has been included to help you apply your own thoughts. Again, we have included some relevant concepts in ‘LET US KNOW’ along with the text. And, at the end of each section, you will get ‘CHECK YOUR PROGRESS’ questions. These have been designed to self-check your progress of study. It will be better if you solve the problems put in these boxes immediately after you go through the sections of the units and then match your answers with ‘ANSWERS TO CHECK YOUR PROGRESS’ given at the end of each unit.

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UNIT 1: NUMBERS

UNIT STRUCTURE

1.1 Learning Objectives
1.2 Introduction
1.3 Natural Number
1.4 Whole Number
1.5 Integers
1.6 Rational Number(or Fraction)
1.7 Irrational Numbers
1.8 Real Numbers
1.9 Imaginary Numbers
1.10 Complex Number
1.11 Prime Numbers
1.12 Concepts of Constants and its Different Types
1.13 Concept of Variables and its Different Types
1.14 Let Us Sum Up
1.15 Further Reading
1.16 Answers to Check Your Progress
1.17 Model Questions

1.1 LEARNING OBJECTIVES

After going through this unit, you will be able to-

- identify various types of numbers
- express a rational number as a terminating or non-terminating repeating decimal
- know imaginary and complex numbers
- know about constants,variables and its different types.

1.2 INTRODUCTION

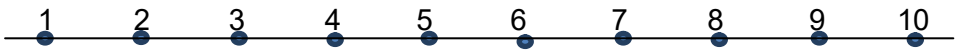
Creation of numbers is the greatest inventions in the history of

mankind.Numbers, which are a basic part of mathematics, help us to understand algebra, to measure geometric objects, and to make predictions using probability and statistics. In this unit,we will discuss diferent types numbers i.e Natural number,Whole number,Integers,Rational number,Irrational number,real number,imaginary number and Complex number.We will also discuss about the concept of constants and variables.

1.3 NATURAL NUMBER (OR COUNTING NUMBER)

Counting things is easy for us. We can count objects in large numbers, for example, the number of students in the college, and represent them through numerals. When we begin to count, we use 1, 2, 3, 4, …. These numbers come our mind naturally when we are counting. Numbers such as 1, 2, 3, 4, 5, 6, ……. .etc. which are used in day-to-day counting of things are called ***Natural Numbers*** or ***Counting Numbers***.

Gemetrical representation of Natural numbers



1.4 WHOLE NUMBER

The natural number along with zero form the collection of whole number. When 0 (zero) is included into the system of the natural number, the resultant extended system is called the system of whole numbers. Numbers such as 0, 1, 2, 3, 4, 5, 6, ……. . etc are called whole Numbers. The set of ***Whole number*** is denoted by W.

1.5 INTEGERS

There are times when we need to use numbers with a negative sign. For example, involving opposite measurements, such as, profit and loss in business, fluctuation of temperature, altitude or depth of places water level in lake or river, level of oil in tanketc. This is when we want to go below zero on the number line. These are called negative numbers.

Therefore, the negative of the natural numbers, zero and the natural

numbers together constitute the integers.

The collection of numbers …. , -4, -3, -2, -1, 0, 1, 2, 3, 4, ….. is called ***integers***. So, -1, -2, -3, -4, ……. Called negative numbers are ***negative integers*** and 1, 2, 3, 4, …. . called positive numbers are positive integers.

1.6 RATIONAL NUMBERS OR FRACTIONS

A fraction is a number representing part of a whole. The whole may be a single object or a group of objects. If x and y are two natural numbers, then $\frac{x}{y}(y \neq 0)$ is a fraction where, x and y are respectively the numerator and the denominator of the fraction.

For examples : $\frac{2}{3}, \frac{4}{9}, \frac{123}{23}$ etc.

Types of fractions :

(1) Proper fraction : If in a fraction, the numerator is always less than the denominator, then fraction is called *proper fraction*. For example : $\frac{3}{4}, \frac{1}{2}, \frac{9}{10}, \frac{5}{8}$ etc are all proper fractions.

(2) Improper fraction : The fractions, where the numerator is bigger than the denominator are called *Improper fractions*. For example : $\frac{13}{12}, \frac{12}{7}, \frac{18}{5}$ etc are all improper fractions.

(3) Mixed fraction : A *mixed fraction* has a combination of a whole and a part. We can express an improper fraction as a mixed fraction by dividing the numerator by denominator to obtain the quotient and the remainder. Thus, the mixed fraction can be written as Quotient $\frac{\text{Remainder}}{\text{Divisor}}$.

For example : $2\frac{3}{4}, 7\frac{1}{9}, 5\frac{3}{7}$ etc are all mixed fractions. These mixed fractions can be expressed as improper fractions. i.e., $2\frac{3}{4} = 2 + \frac{3}{4} = \frac{2 \times 4}{4} + \frac{3}{4} = \frac{11}{4}$.

1.7 IRRATIONAL NUMBERS

An irrational number is a number that cannot be expressed as afraction for any integers. A number which can be neither be expressed as a terminating decimal nor as a repeating decimal, is called an irrational number. Thus, nonterminating, nonrepeating decimals are irrational number.

Clearly, 10.01001000100001…… is a non-terminating ,non-repeating decimals, and therefore, it is irrational.

Irrational Numbers cannot be expressed as the quotient of two integers (ie a fraction) such that the denominator is not zero

Irrational numbers have decimal expansions that neither terminate nor repeating. All numbers that are not rational are considered irrational. An irrational number can be written as a decimal, but not as a fraction. An irrational number has endless non-repeating digits to the right of the decimal point.

One example of irrational number is the Pi (π). Pi (π) is a famous irrational number.

The value of Pi (π) is equal to 3.

14159265358979323846264338327 95 (which is a non-terminating and non-repeating decimal).

1.8 REAL NUMBERS

A number whose square is non-negative is called a real number. In fact, all rational and all irrational numbers form the set of all real numbers. Every real number is either rational or irrational.



CHECK YOUR PROGRESS

Q 1: Convert each of the following fractions to decimals

(a) 1/4(b) 2/3 (c)4/5 (d) 3/7

Q 2: Classify the following numbers as rational or irrational, terminating or recurring decimals :

(a) $\sqrt{100}$ (b) $\frac{1}{\pi}$(c) $\frac{4}{5}$ (d) 0.75 (e) $\frac{13}{99}$

Q 3: How many rational numbers and irrational numbers lie between the numbers 2 and 7?

.....

.....

1.9 IMAGINARY NUMBERS

An imaginary number is defined as any number that, when squared, results in a real number less than zero. When any real number is squared, the result is never negative, however, the square of an imaginary number is always negative. Imaginary numbers are written using the variable i.

Imaginary numbers have the form biwhere b is a non-zero real number and i is the imaginary unit, defined such that i= $\sqrt{-1}$

1.10 COMPLEX NUMBER

A number of the form x+iy where x and y are real numbers and i= $\sqrt{-1}$, is called a complex number . It is denoted by Z .

Thus, Z=x+iy is a complex number.

x is called its real part and is denoted by Re(z). Thus , Re(z) = x and y is called its imaginary part and is denoted by Im (z). Thus ,Im(z) = y.

If x=0 , then z=0+iy is a purely (or wholly) imaginary number.

If y=0 , then z=x+i.0 which is wholly is a real number.

1.11 PRIME NUMBER

The number other than 1 whose only factors are 1 and the number itself.Such number are 2, 3, 5, 7, 11 etc. These numbers are prime numbers.

The numbers other than 1 whose only factors are 1 and the number itself are called Prime Numbers.

Note : (a) 1 is not a prime number.

2 is the smallest prime number.

1.12 CONCEPT OF CONSTANTS AND ITS DIFFERENT TYPES

A symbol which represents a definite number used in any operation is called a constant. A constant retains the same value throughout a set of mathematical operation. These are generally denoted by a, b, c etc.

For example, 7, 13 , 1/12, -6 , π, e, m , c, d etc are called constants. These quantities which donot change in any mathematical operations.

Types of Constants :

Constants are of two types :

(a) Absolute constant : A constant which remains the same throughout a set of mathematical operation is known as absolute constant. The value of absolute constant remains fixed in all conditions .All numerical numbers are absolute constant. For example, 3, 6 , 7/16 , $\sqrt{3}$, π etc. are absolute constants.

(b) Arbitrary constant : A constant which remains same in a particular operation, but changes with the change of reference, is called arbitrary constant. The value of arbitrary constant remains unchanged in a particular problem.

For example, In coordinate geometry of two dimension the equation y = mx + c represents a straight line. Here But m and c are constants, but they are different for different straight lines. Therefore, m and c are arbitrary constants.

1. 13 CONCEPT OF VARIABLE AND ITS DIFFERENT TYPES

A variable is a symbol which can assume any value out of a given set values. The quantities, like height, weight, time, temperature, profit, sales etc, are examples of variables. The variables are usually denoted by x, y, z, u, v, w etc.

Types of Variable : There are two types of variables :

(a) Independent Variable : A variable which can take any arbitrary value,

is called independent variable.

(b) Dependent Variable : A variable whose value depends upon the independent variable is called dependent variable.

For example:

(i) $y = x^2$ if x=2 then y=4, so value of y depends on x. Here y is dependent and x is independent variable.

(ii) Let A be the area and r be the radius of a circle. Then $A = \pi r^2$ where A depends on r i. e area of a circle depends on radius of the circle. Here A is dependent and r is independent variable.



CHECK YOUR PROGRESS

Q 4: What is the smallest prime number ?

.....

Q 5: Find the value of i^{65}

.....

.....

Q 6: Define constants with examples.

.....

.....

.....

Q 7: Define variables with examples.

.....

.....

.....



1.14 LET US SUM UP

- The natural numbers along with zero form the collection of whole numbers.
- The positive natural number,zero and negative natural number together are called Integers.

- A rational number is a number which canbe put in the form p/q, where p and q are integers and $q \neq 0$.
- The decimal representation of a rational number is either terminating or non-terminatingrepeating.
- There exist infinitely many rational numbers between two rational numbers.
- The irrational numbers are decimal numbers that do not terminate and donot repeat. On calculators and in the solution of many problems,rational approximations are used to show values that are close to, but not equal to, irrational numbers.
- The sytem of rational numbers is extended to real numbers.
- Rationals and irrationals together constitute the system of real numbers.
- The numbers other than 1 whose only factors are 1 and the number itself are called Prime Numbers.
- A symbol which represents a definite number used in any operation is called a constant.
- A variable is a symbol which can assume any value out of a given set values.



1.15 FURTHER READING

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1.16 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: (a) 1/4=0.25 (b) 2/3=0.6666.. (c) 4/5=0.8 (d) 3/7=0.4286.

Ans to Q No 2: (a) Rational (b) Irrational (c) Irrational (d) Rational (e) Irrational.

Ans to Q No 3: Infinitely many.

Ans to Q No 4: Smallest prime number is 2.

Ans to Q No 5: $i^{65} = i^{4\times16+1} = i$

Ans to Q No 6: Try it yourself.

Ans to Q No 7: Try it yourself.



1.17 MODEL QUESTIONS

Q 1: Write down an irrational number between 6 and 7.

Q 2: Prove that the following numbers are irrational

$$\sqrt{2}, \sqrt{5}, \sqrt{7}$$

Q 3: Insert 3 rational numbers between 1/5 and 3/5.

Q 4: Write 5 rational numbers and 5 irrational numbers in decimal form.

Q 5: Prove that $\sqrt{5} - \sqrt{3}$ is not a rational number.

Q 6: Which is the greater of the numbers $\frac{7}{9}$ and $\frac{8}{11}$.

Q 7: Find a rational number between 19.9 and 20.

*** ***** ***

UNIT 2: RELATIONS AND FUNCTIONS

UNIT STRUCTURE

2.1 Learning Objectives

2.2 Introduction

2.3 Concept of Relation

- 2.3.1 Identity Relation
- 2.3.2 Inverse Relation

2.4 Concept of Function

2.5 Different Types of Function

- 2.5.1 Polynomial Function
- 2.5.2 Linear Function
- 2.5.3 Qudratic Function
- 2.5.4 Cubic Function
- 2.5.5 Power Function
- 2.5.6 Rational Function
- 2.5.7 Constant Function
- 2.5.8 Exponential Function
- 2.5.9 Logarithmic Function

2.6 Let Us Sum Up

2.7 Further Reading

2.8 Answers to Check Your Progress

2.9 Model Questions

2.1 LEARNING OBJECTIVES

After going through this unit, you will be able to-

- derive knowledge about the concept of a relation
- describe the concept of a function
- discuss the different types of function.

2.2 INTRODUCTION

In the previous unit, we discussed about a few basic concepts of

numbers, constants and variables In. this unit we will aslo discuss the concept of relations and functions. We shall define the concept of relations and functions. Apart from those, we shall also discuss the different types of functions in the general mathematial context.

2.3 CONCEPT OF RELATION

Let us consider the following sentences.

(1) 11 is greater then 10.

(2) 35 is divisible by 7.

(3) New Delhi is the capital of India.

In each of the sentences there is a relation between two 'objects'.

Now let us see what is meant by relation in set theory.

Definition Let A and B be two non-empty sets. A subset R of A x B is said to be a **relation** from A to B.

If A=B, then any subset of A x A is said to be a relation on A.

If $R \subseteq A \times B$, and $(a, b) \in R$; $a \in A$, $b \in B$, it is also written as aRb and is read as 'a is R related to b'.

Note 1. The set of the first components of the ordered pairs of R is called the domain and the set of the second components of the ordered pairs of R is called the range of R.

2. If A, B are finite sets and n(A)=x, n(B)=y; then n(AxB)=xy. So, the number of subsets of AxB is 2^{xy}.

Therefore, the number of relations from A to B is 2^{xy}.

Example 2.1: Let A ={1, 2, 3},

B={8, 9}

$\therefore$ A x B ={(1, 8), (1, 9), (2, 8), (2, 9), (3, 8), (3, 9)}

Let R = {(1, 8), (2, 9), (3, 9)}

Clearly $R \subseteq$ AxB

$\therefore$ R is a relation from A to B.

Here 1R8, 2R9, 3R9

Domain of R = {1, 2, 3}

Range of R = {8, 9}

Example 2.2: Let X be the set of odd integers.

Let R = {(x, y) : x, y $\in$ X and x+y is odd}

We know that the sum of two odd integers is an even integer.

$\therefore$ if x, y are odd, then x+y cannot be odd.

$\therefore R = \phi \subseteq X \times X$

In this case R is called a null relation on X.

Example 2.3: Let E be the set of even integers.

Let R = {(x, y) : x, y $\in$ E and x+y is even}

We know that the sum of the even integers is an even integer.

$\therefore$ if x, y are even, then x+y is always even.

$\therefore$ R = E x E $\subseteq$ E x E

In this case R is called a universal relation on E.

2.3.1 Identity Relation

Let A be a non-empty set.

$I_A = \{(a, a) : a \in A\} \subseteq A \times A$

I_A is called the identity relation on A.

Example 2.4: Let A = {1, 2, 3, 4}

Then $I_A = \{(1, 1), (2, 2), (3, 3), (4, 4)\} \subseteq A \times A$.

Clearly I_A is the identity relation on A.

2.3.2 Inverse Relation

Let A, B be two non-empty sets. Let R be a relation from A to B, i.e. $R \subseteq A \times B$. The inverse relation of R is denoted by R^{-1}, and is defined by $R^{-1} = \{(b, a) : (a, b) \in R\} \subseteq B \times A$

Clearly, domain of R^{-1} = range of R

range of R^{-1} = domain of R

Example 2.5: Let A = {1, 2, 3}

B = {5, 6}

R = {(1, 5), (2, 6), (3, 5)} $\subseteq$ A x B

R^{-1} = {(5, 1), (6, 2), (5, 3)} $\subseteq$ B x A.



CHECK YOUR PROGRESS

Q 1: Let A, B be two finite sets, and n(A)=4, n(B)=3. How many relations are there from A to B ?

.....

.....

.....

Q 2: Let A be a finite set such that n(A) =5. Write down the number of relations on A.

.....

.....

2.4 CONCEPT OF FUNCTION

A function is a technical term used to symbolise relationship between two or more variables. When two real varaibles, say, x and y are so related that that correspoding to every value of x, we get a definite value (or a set of definite values) of y, then y is said to be a function of x and symbolically we write:

$y = f(x)$.

A few examples of functions from our real life include:

- Income tax payable by a person is a function of his/her income. (t = f (Y). Here, t represents tax payable, and Y represents income.)
- Consumption is a function of disposable income. ($C = f(Y_d)$. Here C represents consumption, and Y_d represents disposable income.

Let A and B be two non-empty sets, and $f \subseteq A \times B$ such that

(i) $(x, y) \in f$, $\forall x \in A$ and any $y \in B$

(ii) $(x, y) \in f$ and $(x, y') \in f \Rightarrow y = y'$.

In this case f is said to be a function (or a mapping) from the set A to the set B. Symbolically we write it as $f : A \to B$.

Here A is called the domain and B is called the codomain of f.

Example 2.6 : Let A = {1, 2}, B = {7, 8, 9}

$f = \{(1, 8), 2, 7)\} \subseteq A \times B$.

$\therefore$ range of f = {–1, 1}

2.5 DIFFERENT TYPES OF FUNCTION

Let A, B be two non-empty sets and f : A $\rightarrow$ B be a function.

1. If there is at least one element in B which is not the image of any element in A, then f is called an "into" function.
2. If each element in B is the image of at least one element in A, then f is called an "onto" function (or a surjective function or a surjective).

Note : In case of an onto function, range of f=codomain of f.

1. If different elements in A have different images in B, then f is called a one-one function (or an injective function or an injection).
2. If two (or more) different elements in A have the same image in B, then f is called a many-one function.

2.5.1 Polynomial Function

A function of the form $f(x) = a_0 + a_1 x + a_2x^2 + ... + a_nx^n$ where n is a non-negative integer; and a_0, a_1, a_2, ... a_n are real numbers, is called a polynamial function of degree n.

For example $f(x) = x^4 – 5x^3 + 2x^2 + 6x + 7$ is a polynomial function; whereas $f(x) = 5x + 9\sqrt{x} + 7$ is not a polynamial function.

Note : The domain of a polynomial function is IR, the codomain is also IR.

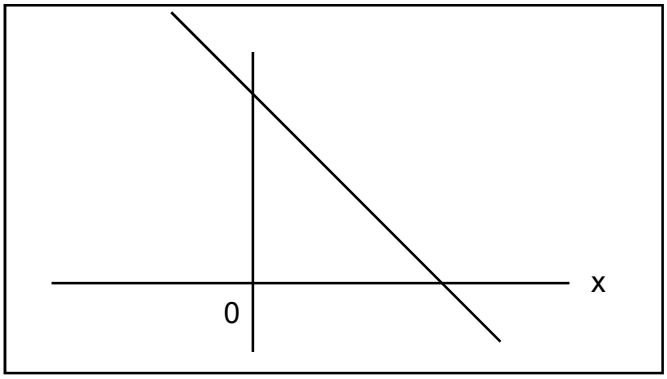
2.5.2 Linear Function

If $n = 1$,then the polynomial function is of degree 1 and is called a linear function.For $n = 1$,the function is written as $f(x) = a + bx$,where $a \& b$ are constants.

For example, f(x) = 3x + 5 is a linear function.

The graph of Linear function has been shown with the help of Figure 2.1.

Figure 2.1: Graphical Shape of a Linear Function



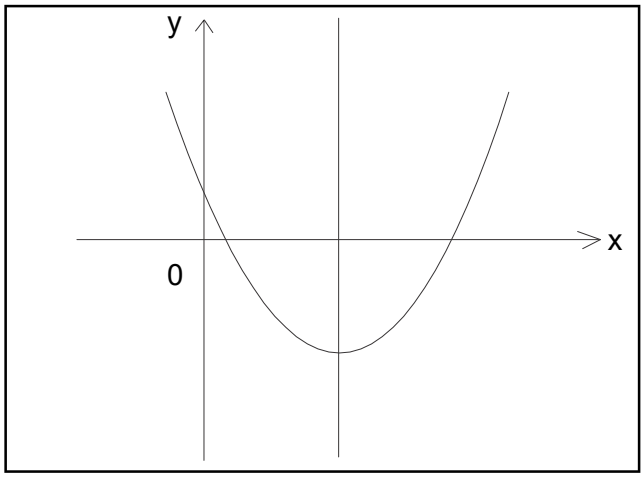
2.5.3 Qudratic Function

If $n = 2$,then the polynomial function is of degree 2 and is called a quadratic function.

A Quadratic function is of the form $f(x) = ax^2 + b x + c$,where a,b,c are constants.

The graph of quadratic function $f(x) = ax^2 + b x + c$ is parabola that opens upwards or downwards according as a>0 or a<0. It has been shown in Figure 2.2.

Figure 2.2: Graphical Shape of a Quadratic Function



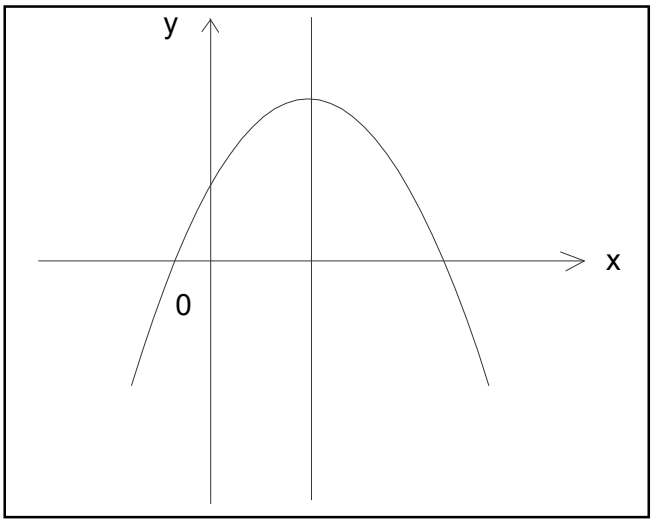
2.5.4 Cubic Function

If $n = 3$,then the polynomial function is of degree 3 and is called a cubic function.

A cubic function is of the form $f(x) = ax^3 + bx^2 + cx + d$ where

a,b,c,d are constants. The graphical shape of a cubic function has been shown in Figure 2.3.

Figure 2.3: Graphical Shape of a Cubic Function



2.5.5 Power Function

A function of the form $f(x) = ax^n$ where n is a non-negative integer; and a is constant, is called a Power function.

2.5.6 Rational Function

A function of the form $y=\frac{f(x)}{g(x)}$, where f(x) and g(x) are polynomial function and $g(x) \neq 0$, is called a rational function.

For example $\frac{x+2}{x^2-5x+6}$ a rational function. Here $x \neq 2$, $x \neq 3$, i.e. the domain of the rational function $\frac{x+2}{x^2-5x+6}$ is IR – {2, 3},

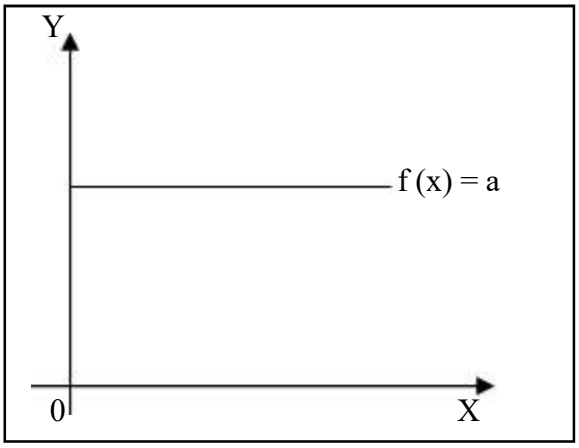
2.5.7 Constant Function

A function of the form f(x) = k, where k is a constant is called a constant function.

Note : The range of a constant function is a singleton set The graph

of a constant function is a straight line paralled to x-axis.

Figure 2.4: Graphical Shape of a Costant Function

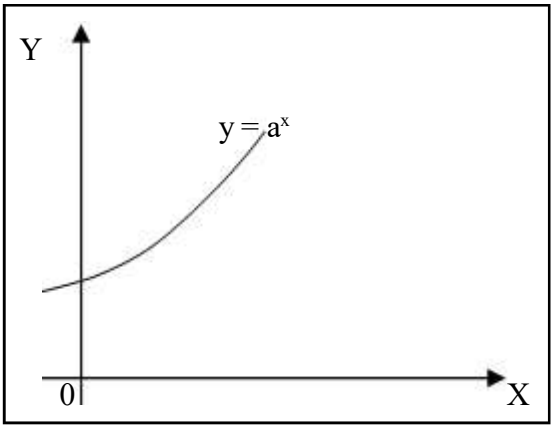


2.5.8 Exponential Function

Let a be a positive real number other than 1. then the function is of the form $y = a^x$ is called the exponential function.

Graph of an exponential function has been shown in Figure 2.5.

Figure 2.5: Graphical Shape of an Exponential Function



2.5.9 Logarithmic Function

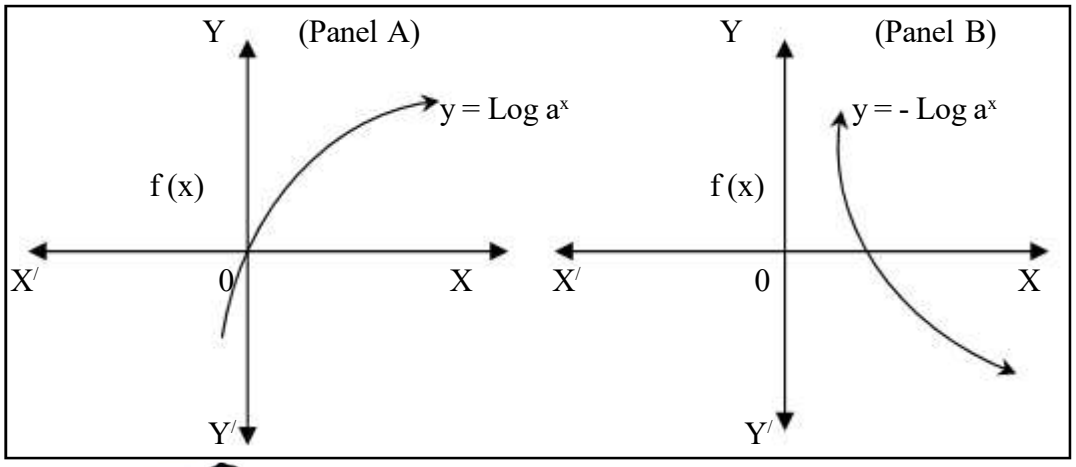
Let x be a positive real number other than 1. Then the function of the form $f(x) = \log_e x$ is called the logarithmic function.Logarithmic function is inverse to exponential function.

Remark : $\log_e x$ is also denoted by lnx.

Graph of the logarithmic functions when $y = \text{Log } a^x$ and when

$y = -\text{Log } a^x$ have been shown in Panel A and Panel B of Figure 2.6 respectively.

Figure 2.6: Graphical Representation of Logarithmic Functions





2.6 LET US SUM UP

- If A, B are two non-empty sets, a subset of A x B in said to be a relation from A to B.
- If A, B are two finite sets and n(A) = x, n(B) = y, the number of relations from A to B is 2^{xy}.
- If A is a non-empty set, $I_A = \{(a, a) : a \in A\}$ is called the identity relation on A.
- A relation R on a non-empty set A is called an equivalence relation if it is reflexive, symmetric and transitive.
- The inverse of an equivalence relation is also an equivalence relation.
- The intersection of two equivalence relations is also an equivalence relation.
- If A and B are two non-empty sets and if each element of A is associated with a unique element of B, then the rule by which this association is made, is called a function from A to B.
- Every function is a relation, but every relation is not a function.
- If different elements in domain have different images in codomain, then the function is one-one (injective).
- If each element in codomain is the image of at least one element in

domain then the function is onto (subjective).



2.7 FURTHER READING

1) Agarwal, D.K. (2012). *Business Mathematics*, New Delhi: Vrindra Publication (p) Ltd.

2) Baruah, S. (2011). *Basic Mathematics and Its Application in Economics*, New Delhi: Trinity Press Pvt. Ltd.

3) Bose, D. (2004). *Mathematical Economics;* New Delhi: Himalaya Publishing House.

4) Chiang, A.C. (2006) *Fundamental Methods of Economics Analysis*; New Delhi: MC Graw Hill Education India.

5) Kandoi, Balwant (2011). *Mathematics for Business and Economics with Application;* New Delhi: Himalaya Publishing House.



2.8 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: Try it yourself.

Ans to Q No 2: Try it yourself.



2.9 MODEL QUESTIONS

Q 1: Find a rational number between 19.9 and 20.

Q 2: Write the power function.

Q 3: Show the graphical shape of a linear function.

Q 4: Let A = {1, 2, 3}, B = {3, 4, 5}.

How many relations are there from A to B ? Write down any four relations from A to B.

*** ***** ***

UNIT 3 : THEORY OF SETS

UNIT STRUCTURE

3.1 Learning Objectives

2.2 Introduction

3.3 Sets and their Representation

3.4 The Empty Set

3.5 Finite and Infinite Sets

3.6 Equal Sets

3.7 Subsets, Super Sets, Proper Subsets

3.8 Power Set

3.9 Universal Set

3.10 Venn Diagrams

3.11 Set Operations

- 3.11.1 Union of Sets
- 3.11.2 Intersection of Sets
- 3.11.3 Difference of Sets
- 3.11.4 Complement of a Set

3.12 Let Us Sum Up

3.13 Further Reading

3.14 Answers to Check Your Progress

3.15 Model Questions

3.1 LEARNING OBJECTIVES

After going through this unit, you will be able to

- describe sets and their representations
- identify empty set, finite and infinite sets
- define subsets, super sets, power sets, universal set
- illustrate the set operations of union, intersection, difference and complement.

3.2 INTRODUCTION

Concepts of sets, though primarily discussed in the field of Mathematics also has wider use in Economics. It was developed by the German mathematician Georg Cantor (1845-1918) developed the theory of sets and subsequently many branches of modern Mathematics have been developed based on this theory. In this unit, preliminary concepts of sets, set operations and some ideas on its practical utility will be introduced. In fact, like the number systems and functions, the set theory is very fundamental in Mathematics and hence, we need to derive its basic concepts for the study of qunatitative analysis of economics as well.

3.3 SETS AND THEIR REPRESENTATION

A set is a collection of well-defined objects. By well-defined, it is meant that given a particular collection of objects as a set and a particular object, it must be possible to determine whether that particular object is a member of the set or not.

The objects forming a set may be of any sort– they may or may not have any common property. Let us consider the following collections :

i) the collection of the prime numbers less than 15 i.e., 2, 3, 5, 7, 11, 13
ii) the collection of 0, a, Sachin Tendulkar, the river Brahmaputra
iii) the collection of the beautiful cities of India
iv) the collection of great mathematicians.

Clearly the objects in the collections (i) and (ii) are well-defined. For example, 7 is a member of (i), but 20 is not a member of (i). Similarly, ‘a’ is a member of (ii), but M. S. Dhoni is not a member. So, the collections (i) and (ii) are sets. But the collections (iii) and (iv) are not sets, since the objects in these collections are not well-defined.

The objects forming a set are called elements or members of the set. Sets are usually denoted by capital letters A, B, C, ...; X, Y, Z, ..., etc., and the elements are denoted by small letters a, b, c, ...; x, y, z, ..., etc.
If ‘a’ is an element of a set A, then we write $a \in A$ which is read as ‘a belongs to the set A’ or in short, ‘a belongs to A’. If ‘a’ is not an element of A, we write

$a \notin A$ and we read as 'a does not belong to A'.

Example 3.1: For example, let A be the set of prime number less than 15.

Then $2 \in A$, $3 \in A$, $5 \in A$, $7 \in A$, $11 \in A$, $14 \in A$

$1 \notin A$, $4 \notin A$, $17 \notin A$, etc.

Representation of Sets : Sets are represented in the following two methods :

1. Roster or tabular method
2. Set-builder or Rule method

In the Roster method, the elements of a set are listed in any order, separated by commas and are enclosed within braces, For example,

A = {2, 3, 5, 7, 11, 13}

B = {0, a Sachin Tendulcar, the river Brahmaputra}

C = {1, 3, 5, 7, ...}

In the set C, the elements are all the odd natural numbers. We cannot list all the elements and hence the dots have been used showing that the list continues indefinitely.



1) It should be noted that the symbol ':' or '|' stands for the phrase 'such that'.
2) While writing a set in Roster method, only distinct elements are listed. For example, if A is the set of the letters of the word MATHEMATICS, then we write
A = {A, E, C, M, H, T, S, I}
The elements may be listed in any order.

In the Rule method, a variable x is used to represent the elements of a set, where the elements satisfy a definite property, say P(x). **Symbolically, the set is denoted by {x : P(x)} or {x | p(x)}.**

Example 3.2: A = {x : x is an odd natural number}

B = {x : $x^2 - 3x + 2 = 0$}, etc.

If we write these two sets in the Roster method, we get,

A = {1, 3, 5, ...}

B = {1, 2}

Some Standard Symbols for Sets and Numbers : The following standard symbols are used to represent different sets of numbers :

N = {1, 2, 3, 4, 5, ...}, the set of natural numbers

Z = {..., –3, –2, –1, 0, 1, 2, 3, ...}, the set of integers

Q = $\{x : x = p/q; p, q \in Z, q \neq 0\}$, the set of rational numbers

R = {x : x is a real number}, the set of real numbers

Z^+, Q^+, R^+ respectively represent the sets of positive integers, positive rational numbers and positive real numbers. Similarly Z^-, Q^-, R^- represent respectively the sets of negative integers, negative rational numbers and

negative real numbers. Z^0, Q^0, R^0 represent the sets of non-zero integers, non-zero rational numbers and non-zero real numbers.

Illustrative Examples :

3.3: Examine which of the following collections are sets and which are not :

i) the vowels of the English alphabet

ii) the divisors of 56

iii) the brilliant students degree-course of Guwahati

iv) the renowned cricketers of Assam.

Solution :

i) It is a set, V = {a, e, i, o, u}

ii) It is a set, D = {1, 2, 4, 7, 8, 14, 28, 56}

iii) not a set, elements are not well-defined.

iv) not a set, elements are not well-defined.

3.4: Write the following sets in Roster method :

i) the set of even natural numbers less than 10

ii) the set of the roots of the equation $x^2–5x+6 = 0$

iii) the set of the letters of the word EXAMINATION

Solution :

i) {2, 4, 6, 8}

ii) {2, 3}

iii) {E, X, A, M, I, N, T, O}

3.5: Write the following sets in Rule method :

i) E = {2, 4, 6, ...}

ii) A = {2, 4, 8, 16, 32}

iii) B = {1, 8, 27, 64, 125, 216}

Solution :

i) $E = \{x : x = 2n, n \in N\}$

ii) $A = \{x : x = 2^n, n \in N, n < 6\}$

iii) $B = \{x : x = n^3, n \in N, n \leq 6\}$



CHECK YOUR PROGRESS

Q 1: Express the following sets in Roster method :i) A = {x : x is a day of the week}

ii) B = {x : x is a month of the year}

iii) C = {x : x^3–1 = 0}

iv) D = {x : x is a positive divisor of 100}

v) E = {x : x is a letter of the word ALGEBRA}

Q 2: Express the following sets in Set-builder method :

i) A = {January, March, May, July, August, October, December}

ii) B = {0, 3, 8, 15, 24, ...}

iii) C = {0, ±5, ±10, ±15, ...}

iv) D = {a, b, c, ..., x, y, z}

Q 3: Write true or false :

i) 5 ∈ N ii) ½ ∈ Z iii) –1 ∈ Q

iv) $\sqrt{2} \in$ R v) $\sqrt{-1} \in$ R vi) –3 ∉ N

3.4 THE EMPTY SET

Definition : A set which does not contain any element is called an empty set or a null set or a void set. It is denoted by ϕ.

The following sets are some examples of empty sets.

i) the set {x : x^2 = 3 and x ∈ Q}

ii) the set of people in Assam who are older than 500 years

iii) the set of real roots of the equation x^2 + 4 = 0

iv) the set of Lady President of India born in Assam.

3.5 FINITE AND INFINITE SETS

Let us consider the sets

A = {1, 2, 3, 4, 5}

and B = {1, 4, 7, 10, 13, ...}

If we count the members (all distinct) of these sets, then the counting process comes to an end for the elements of set A, whereas for the elements of B, the counting process does not come to an end. In the first case we say

that A is a finite set and in the second case, B is called an infinite set. A has finite number of elements and number of elements in B are infinite.

Definition : A set containing finite number of distinct elements so that the process of counting the elements comes to an end after a definite stage is called a ***finite set***; otherwise, a set is called an ***infinite set***.

Example 3.6: State which of the following sets are finite and which are infinite.

i) the set of natural numbers N

ii) the set of male persons of Assam as on January 1, 2009.

iii) the set of prime numbers less than 20

iv) the set of concentric circles in a plane

v) the set of rivers on the earth.

Solution :

i) N = {1, 2, 3, ...} is an infinite set

ii) it is a finite set

iii) {2, 3, 5, 7, 11, 13, 17, 19} is a finite set

iv) it is an infinite set

v) it is a finite set.



A ***finite set*** can always be expressed in roster method. But an ***infinite set*** cannot be always expressed in roster method as the elements may not follow a definite pattern. For example, the set of real numbers, R cannot be expressed in roster method.

3.6 EQUAL SETS

Definition : Two sets A and B are said to be equal sets if every element of A is an element of B and every element of B is also an element of A. In otherwords, A is equal to B, denoted by A = B if A and B have exactly the same elements. If A and B are not equal, we write $A \neq B$.

Let us consider the sets

A = {1, 2}

B = {x : (x–1)(x–2) = 0}

C = {x : (x–1)(x–2)(x–3) = 0}

Clearly B = {1, 2}, C = {1, 2, 3} and hence A = B, $A \neq C$, $B \neq C$.

Example 3.7 : Find the equal and unequal sets :

i) A = {1, 4, 9}

ii) B = $\{1^2, 2^2, 3^3\}$

iii) C = {x : x is a letter of the word TEAM}

iv) D = {x : x is a letter of the word MEAT}

v) E = {1, {4}, 9}

Solution : A = B, C = D, $A \neq C$, $A \neq D$, $A \neq E$, $B \neq C$, $B \neq D$, $B \neq E$, $C \neq E$, $D \neq E$

3.7 SUBSETS, SUPERSETS, PROPER SUBSETS



According to equality of sets discussed above, the sets
A = {1, 2, 3} and
B = {1, 2, 2, 2, 3, 1, 3}
are equal, since every member of A is a member of B and also every member of B is a member of A. This is why identical elements are taken once only while writing a set in the Roster method.

Let us consider the sets A = {1, 2, 3}, B = {1, 2, 3, 4} and C = {3, 2, 1}. Clearly, every element of A is an element of B, but A is not equal to B. Again, every element of A is an element of C, and also A is equal to C. In both cases, we say that A is a subset of B and C. In particular, we say that A is a proper subset of B, but A is not a proper subset of C.

Definition : If every element of a set A is also an element of another set B, then A is called a subset of B, or A is said to be contained in B, and is denoted by $A \subseteq B$. Equivalently, we say that B contains A or B is a **superset** of A and is denoted by $B \supseteq A$. Symbolically, $A \subseteq B$ means that
for all x, if $x \in A$ then $x \in B$.

If A is a subset of B, but there exists atleast one element in B which is not in A, then A is called a proper subset of B, denoted by $A \subset B$. In otherwords, $A \subset B \Leftrightarrow (A \subseteq B$ and $A \neq B)$.

The symbol '$\Leftrightarrow$' stands for 'logically implies and is implied by' (see unit 5).

Some examples of proper subsets are as follows :

$N \subset Z$, $N \subset Q$, $N \subset R$,

$Z \subset Q$, $Z \subset R$, $Q \subset R$.

It should be noted that any set A is a subset of itself, that is, $A \subseteq A$. Also, the null set ϕ is a subset of every set, that is, $\phi \subseteq A$ for any set A. Because, if $\phi \subseteq A$, then there must exist an element $x \in \phi$ such that $x \notin A$. But $x \notin \phi$, hence we must accept that $\phi \subseteq A$.

Combining the definitions of equality of sets and that of subsets, we get $A = B \Leftrightarrow (A \subseteq B$ and $B \subseteq A)$

Illustrative Examples :

3.8: Write true or false :

i) $1 \subset \{1, 2, 3\}$

ii) $\{1, 2\} \subseteq \{1, 2, 3\}$

iii) $\phi \subseteq \{\{\phi\}\}$

iv) $\phi \subseteq \{\phi, \{1\}, \{a\}\}$

v) $\{a, \{b\}, c, d\} \subset \{a, b, \{c\}, d\}$

Solution :

i) False, since $1 \in \{1, 2, 3\}$.

ii) True, since every element of {1, 2} is an element of {1, 2, 3}.

iii) False, since ϕ is not an element of $\{\{\phi\}\}$.

iv) True, since ϕ is subset of every set.

v) False, since $\{b\} \notin \{a, b, \{c\}, d\}$ and $c \notin \{a, b, \{c\}, d\}$.

3.8 POWER SET

Let us consider a set A = {a, b}. A question automatically comes to our mind– 'What are the subsets of A?' The subsets of A are ϕ, {a}, {b} and A itself.

These subsets, taken as elements, again form a set. Such a set is called the power set of the given set A.

Definition : The set consisting of all the subsets of a given set A as its elements, is called the power set of A and is denoted by P(A) or 2^A.

Thus, $P(A)$ or $2^A = \{X : X \subseteq A\}$

Clearly,

i) $P(\phi) = \{\phi\}$

ii) if A = {1}, then $P^A = \{\phi, \{1\}\}$

iii) if A = {1, 2}, then $P^A = \{\phi, \{1\}, \{2\}, A\}$

iv) if A = {1, 2, 3}, then $P^A = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, A\}$

From these examples we can conclude that if a set A has n elements, then P(A) has 2^n elements.

3.9 UNIVERSAL SET

A set is called a Universal Set or the Universal discourse if it contains all the sets under consideration in a particular discussion. A universal set is denoted by U.

Example 2.9:

i) For the sets N={1, 2, 3}, Z={3, 7, 8} and Q = {4, 5, 6, 9}
We can take U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

ii) In connection with the sets N, Z, Q we can take U as the universal set.

iii) In connection with the population census in India, the set of all people in India is the universal set, etc.



CHECK YOUR PROGRESS

Q 4: Find the empty sets, finite and infinite sets:

i) the set of numbers divisible by zero

ii) the set of positive integers less than 15 and divisible by 17

iii) the set of planets of the solar system

iv) the set of positive integers divisible by 4

v) the set of Olympians from Assam participating in 2008, Beijing Olympic.

Q 5: Examine the equality of the following sets :

i) A = {2, 3}, B = $\{x : x^2-5x+6 = 0\}$

ii) A = {x : x is a letter of the word WOLF}
B = {x : x is a letter of the word FLOW}

iii) A = {a, b, c}, B = {a, {b, c}}

Q 6: Write true or false :

i) $\{1, 3, 5\} \subseteq \{5, 1, 3\}$ ii) $\{a\} \subset \{\{a\}, b\}$

iii) $\{x : (x-1)(x-2) = 0\} \not\subset \{x : (x^2-3x+2)(x-3) = 0\}$

Q 7: Write down the power sets of the following sets :

i) A = {1, 2, 3, 4} ii) B = {1, {2, 3}}

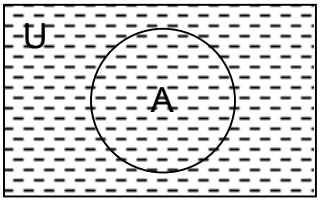
Q 8: Give examples to show that $(A \subseteq B \text{ and } B \subseteq C) \Rightarrow A \subseteq C$.

3.10 VENN DIAGRAM

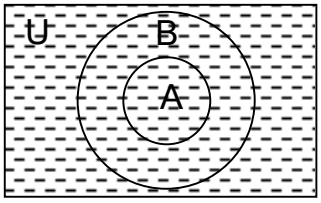
Simple plane geometrical areas are used to represent relationships between sets in meaningful and illustrative ways. These diagrams are called Venn-Euler diagrams, or simply the Venn-diagrams.

In Venn diagrams, the universal set U is generally represented by a

set of points in a rectangular area and the subsets are represented by circular regions within the rectangle, or by any closed curve within the rectangle. As an illustration Venn diagrams of $A \subset U$, $A \subset B \subset U$ are given below :



$A \subset U$



$A \subset B \subset U$

Similar Venn diagrams will be used in subsequent discussions illustrating different algebraic operations on sets.

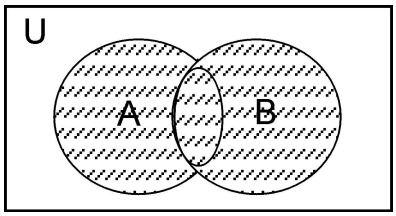
3.11 SET OPERATIONS

We know that given a pair of numbers x and y, we can get new numbers $x + y$, $x - y$, xy, x/y (with $y \neq 0$) under the operations of addition, subtraction, multiplication and division. Similarly, given the two sets A and B we can form new sets under set operations of union, intersection, difference and complements. We will now define these set operations, and the new sets thus obtained will be shown with the help of Venn diagrams.

3.11.1 Union of Sets

Definition : The union of two sets A and B is the set of all elements which are members of set A or set B or both. It is denoted by $A \cup B$, read as 'A union B' where '$\cup$' is the symbol for the operation of 'union'. Symbolically we can describe $A \cup B$ as follows :

$A \cup B = \{x : x \in A \text{ or } x \in B\}$



$A \cup B$ (Shaded)

It is obvious that $A \subseteq A \cup B$, $B \subseteq A \cup B$

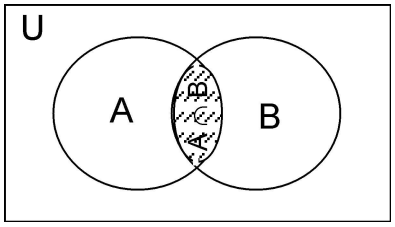
Example 3.10 : Let $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 5, 6\}$

Then $A \cup B = \{1, 2, 3, 4, 5, 6\}$

3.11.2 Intersection of Sets

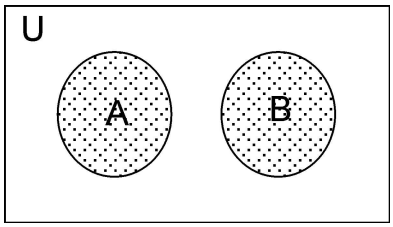
Definition : The intersection of two sets A and B is the set of all elements which are members of both A and B. It is denoted by $A \cap B$, read as 'A intersections B', where '$\cap$' is the symbol for the operation of 'intersection'. Symbolically we can describe it as follows:

$A \cap B = \{x : x \in A \text{ and } x \in B\}$



$A \cap B$ (Shaded)

From definition it is clear that if A and B have no common element, then $A \cap B = \phi$. In this case, the two sets A and B are called disjoint sets..



$A \cap B = \phi$

It is obvious that $A \cap B \subseteq A$, $A \cap B \subseteq B$.

Example 3.11 : Let A = {a, b, c, d}, B = {b, d, 4, 5}

Then $A \cap B$ = {b, d}

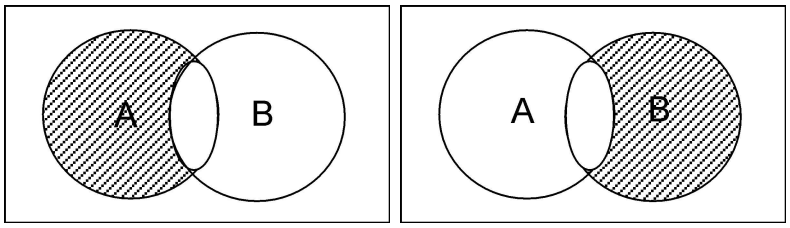
Example 3.12 : Let A = {1, 2, 3}, B = {4, 5, 6}

Then $A \cap B = \phi$.

3.11.3 Difference of Sets

Definition : The difference of two sets A and B is the set of all elements which are members of A, but not of B. It is denoted by A–B. Symbollically, $A - B = \{x : x \in A \text{ and } x \notin B\}$

Similarly, $B - A = \{x : x \in B \text{ and } x \notin A\}$



A – B (Shaded) B – A (Shaded)

Example 3.13: Let A = {1, 2, 3, 4, 5}, B = {1, 4, 5}, C = {6, 7, 8}

Then A – B = {2, 3}

A – C = A

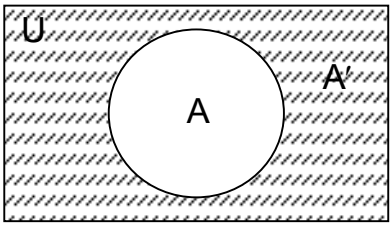
B – C = B

B – A = ϕ

3.11.4 Complement of a Set

Definition : If U be the universal set of a set A, then the set of all those elements in U which are not members of A is called the **Compliment** of A, denoted by A^C or A′.

Symbolically, A′ = {x : x $\in$ U and x $\notin$ A}.



A′ (Shaded)

Clearly, A′ = U – A.

Example 3.14: Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9} and A = {2, 4, 6, 8}

Then A′ = {1, 3, 5, 7, 9}

Illustrative Examples :

Example 3.15: If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

A = {2, 4, 6, 8, 10}

B = {3, 6, 9}

and C = {1, 2, 3, 4, 5}, then find

(i) A $\cup$ B, (ii) A $\cap$ C, (iii) B $\cap$ C, (iv) A′, (v) A $\cup$ B′, (vi) C′ $\cap$ B,

(vii) A′ $\cup$ C′, (viii) A – C, (ix) A – (B $\cup$ C)′, (x) A′ $\cap$ B′.

Solution : i) A $\cup$ B = (2, 3, 4, 6, 8, 9, 10}

ii) $A \cap C = \{2, 4\}$

iii) $B \cap C = \{3\}$

iv) $A' = \{1, 3, 5, 7, 9\}$

v) $B' = \{1, 2, 4, 5, 7, 8, 10\}$

So, $A \cup B' = \{1, 2, 4, 5, 6, 7, 8, 10\}$

vi) $C' = \{6, 7, 8, 9, 10\}$

So, $C' \cap B = \{6, 9\}$

vii) From (iv) & (vi), $A' \cup C' = \{1, 3, 5, 6, 7, 8, 9, 10\}$

viii) $A - C = \{6, 8, 10\}$

ix) $B \cup C = \{1, 2, 3, 4, 5, 6, 9\}$

$(B \cup C)' = \{7, 8, 10\}$

So, $A - (B \cup C)' = \{2, 4, 6\}$

x) From (iv) & (v), $A' \cap B' = \{1, 5, 7\}$.



CHECK YOUR PROGRESS

Q 9: If A = {a, b, c}, B = {c, d, e}, U = {a, b, c, d, e, f} then find

i) $A \cup B$ ii) $A \cap B$ iii) $A - B$ iv) $B - A$ v) A'



3.12 LET US SUM UP

- A set is a collection of well-defined and distinct objects. The objects are called members or elements of the set.
- Sets are represented by capital letters and elements by small letters. If 'a' is an element of set A, we write $a \in A$, otherwise $a \notin A$.
- Sets are represented by (i) Roster or Tabular method and (ii) Rule or Set-builder method.
- A set having no element is called empty set or null set or void set, denoted by ϕ.
- A set having a finite number of elements is called a finite set, otherwise it is called an infinite set.

- Two sets A and B are equal, i.e. A = B if and only if every element of A is an element of B and also every element of B is an element of A, otherwise $A \neq B$.
- A is a subset of B, denoted by $A \subseteq B$ if every element of A is an element of B and A is a proper subset of B if $A \subseteq B$ and $A \neq B$. In this case, we write $A \subset B$.
- A = B if and only if $A \subseteq B$ and $B \subseteq A$.
- The set of all the subsets 8 a set A is called the power set of A, denoted by P(A) or 2^A. If |A| = n, then $|P(A)| = 2^n$.
- Venn diagrams are plane geometrical diagrams used for representing relationships between sets.
- The union of two sets A and B is $A \cup B$ which consists of all elements which are either in A or B or in both. $A \cup B = \{x : x \in A \text{ or } x \in B\}$
- The intersection of two sets A and B is $A \cap B$ which consists of all the elements common to both A and B.
- For any two sets A and B, the difference set, A – B consists of all elements which are in A, but not in B. $A - B = \{x : x \in A \text{ and } x \notin B\}$
- The Universal set U is that set which contains all the sets under any particular discussion as its subsets.
- The complement of a set A, denoted by A^c or A' is that set which consists of all those elements in U which are not in A.
  $A' = \{x : x \in U \text{ and } x \notin A\} = U - A$



3.13 FURTHER READING

1) Agarwal, D.K. (2012). *Business Mathematics*, New Delhi: Vrindra Publication (p) Ltd.
2) Baruah, S. (2011). *Basic Mathematics and Its Application in Economics*, New Delhi: Trinity Press Pvt. Ltd.
3) Bose, D. (2004). *Mathematical Economics;* New Delhi: Himalaya Publishing House.

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3.14 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: i)A = {Monday, Tuesday, Wednessday, Thursday, Friday, Saturday, Sunday}

ii) B = {January, February, March, April, May, June, July, August, September, October, November, December}

iii) C = $\{1, w, w^2\}$

iv) D = {1, 2, 4, 5, 10, 20, 25, 50, 100}

v) E = {A, B, E, G, L, R}

Ans to Q No 2: i)A = {x : x is a month of the year having 31 days}

ii) B = $\{x : x = n^2 - 1, n \in N\}$

iii) C = $\{x : x = 5n, n \in Z\}$

iv) D = {x : x is a letter of the English Alphabet}

Ans to Q No 3: i) True, ii) False, iii) True, iv) True, v) False, vi) True.

Ans to Q No 4: i) ϕ, ii) ϕ, iii) finite, iv) infinite, v) ϕ.

Ans to Q No 5: i) B = {2, 3} = A

ii) A = {W, O, L, F}, B = {F, L, O, W} and so, A = B

iii) $A \neq B$; since $b \in A$ but $b \notin B$.

Ans to Q No 6: i)True

ii) False, since $\{a\} \in \{\{a\}, b\}$

iii) $\{x : (x-1)(x-2) = 0\} = \{1, 2\}$, $\{x : (x^2-3x+2)(x-3) = 0\} = \{1, 2, 3\}$ Hence $\{x : (x-1)(x-2) = 0 \subset \{x : (x^2-3x+2)(x-3) = 0\}$ and so, the given result is false.

Ans to Q No 7: i) P(A) = $\{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, A\}$

ii) P(B) = {ϕ, {1}, {{2, 3}}, B}

Ans to Q No 8: Let A = {1, 2}, B = {0, 1, 2, 3}, C = {0, 1, 2, 3, 4, 5, 7}

Ans to Q No 9: i) $A \cap B$ = {a, b, c, d, e}; ii) {C}; iii) A – B = {a, b},
iv) B – A = {d, e}; v) A' = {d, e, f}



3.15 MODEL QUESTIONS

Q 1: Give examples of i) five null sets
ii) five finite sets
iii) five infinite sets

Q 2: Write down the following sets in rule method :

i) $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$

ii) $B = \left\{\frac{1}{1.2}, \frac{1}{1.3}, \frac{1}{3.4}, \frac{1}{4.5}, \ldots\right\}$

iii) C = {2, 5, 10, 17, 26, 37, 50}

Q 3: Write down the following sets in roster method

i) A = {x : x $\in$ N, 2 < x < 10}

ii) B = {x : x $\in$ N, 4+x < 15}

iii) C = {x : x $\in$ Z, –5 $\leq$ x $\leq$ 5}

Q 4: If A = {1, 3}, B = {1, 3, 5, 9}, C = {2, 4, 6, 8} and
D = {1, 3, 5, 7, 9} then fill up the dots by the symbol $\subseteq$ or $\notin$:
i) A ... B, ii) A ... C, iii) C ... D, iv) B ... D

Q 5: Write true or false :
i) 4 $\in$ {1, 2, {3, 4}, 5}, (ii) ϕ = {ϕ}
iii) A = {2, 3} is a proper subset of B = {x : (x–1)(x–2)(x–3) = 0}
iv) A $\subseteq$ B, B $\subseteq$ C $\Rightarrow$ A $\subseteq$ C

Q 6: If U = {x : x $\in$ N}, A = {x : x $\in$ N, x is even}, B = {x : x $\in$ N, x < 10}
C = {x : x $\in$ N, x is divisible by 3}, then find
i) $A \cup B$, ii) $A \cap C$, iii) $B \cap C$, iv) A', (v) B', vi) C'.

Q 7: If $A \cup B = B$ and $B \cup C = C$, then show that $A \subseteq C$.

Q 8: If U = {–5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5},
A = {–5, –2, 1, 2, 4}

$B = \{-2, -3, 0, 2, 4, 5\}$

$C = \{1, 0, 2, 3, 4, 5\}$,

then find i) $A \cup B$, ii) $A \cap C$, iii) $A \cap (B \cup C)$, iv) $B \cap C'$,

v) $A' \cup (B \cap C')$, vi) $A - C'$, vii) $A - (B \cup C)'$, viii) $(B \cup C')$,

ix) $A' \cup C'$, x) $(C' \cup B) - A$.

Q 9: Prove the following :

i) If A, B, C are three sets such that $A \subseteq B$,

then $A \cup C \subseteq B \cup C$, $A \cap C \subseteq B \cap C$.

ii) $A \subseteq B$ if and only if $B' \subseteq A'$.

iii) $A \subseteq B$ if and only if $A \cap B = A$.

iv) If $A \cap B = \phi$, then $A \subseteq B'$.

Q 10: How many elements are there in P(A) if A has

i) 5 elements, ii) 2^n elements?

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UNIT 4: EQUATIONS

UNIT STRUCTURE

4.1 Learning Objectives

4.2 Introduction

4.3 Concept of Equation

- 4.3.1 Degree of Equation
- 4.3.2 Polynomial Equation
- 4.3.3 Simultaneous Equation

4.4 Rewriting of Equation

4.5 Solving of Equation

- 4.5.1 Solution of Polynomial Equation
- 4.5.2 Solution of Simultaneous Equation

4.6 Let Us Sum Up

4.7 Further Reading

4.8 Answers To Check Your Progress

4.9 Model Questions

4.1 LEARNING OBJECTIVES

After going through this unit, you will able to :

- discuss the concept and types of equation
- discuss how a equation is rewritten and solved.

4.2 INTRODUCTION

An equation represents a functional relationship or a theoritical relationship, which can be solved and value of the variables can be determined. It is important to note that equations can be judged or tested. Say, in Economics, hypothesis about economic behaviour is represented with the help of equations. In this unit, the concept of equation, and how it can be rewtitten and solved have been discussed in detail.

4.3.2 Polynomial Equation

The general equation of n^{th} degree in x is written as :

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_n = 0.$

It is also known as polynomial equestion of n^{th} degree, as the highest exponent of x in the equation is 'n'.

If the highest exponent of x in the equation is 1, then it is known as equation of the first degree or general linear equation and it is written as $ax + b = 0 \qquad (a \neq 0)$

For example $2x - 5 = 0$

If the highest exponent of X in the equation is 2, then it is known as equation of the second degree or general quadratic equation and it is written as,

$ax^2 + bx + c = 0 \qquad (a \neq 0)$

For example $2 + 5x + 3x^2 = 0$

It the highest exponent of X in the equation is 3, then it is known as equation of the third degree or general cubic equation and it is written as,

$ax^3 + bx^2 + cx + d = 0, \qquad (a \neq 0)$

and so on.

4.3.3 Simultaneous Equation

In a set of simultaneons equations there are atleast two or more than two equations and in each equation there are two or more variables, whose value can be determined, which simultaneously satisfy all the equations. In a set of simultaneous equations number of variables should be either equal or less than the number of equations.

(a) Two variables and two equations :

$a_{11}x_1 + a_{12}x_2 = c_1$

$a_{21}x_2 + a_{22}x_2 = c_2$

For example, $3x_1 + 5x_2 = 10$

$5x_1 + 2x_2 = 4$

(b) Three variables and three equations :

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = c_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = c_3$

For example,

$5x_1 + 2x_2 + 3x_3 = 15$

$5x_1 + 3x_2 + 6x_3 = 10$

$2x_1 + 3x_2 + 8x_3 = 12$

(c) n variables and m equation :

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = c_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = c_2$

. . . . . .

. . . . . .

$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = c_m$

4.4 REWRITING OF EQUATION

Equations are not fixed in nature. In order to make the equations simple and solvable they can be rewrite or rearranged in such a way that the equality of two sides are maintained.

For example :

(a) $x - 4 = {}^{2}/_{x}$

In order to slove or make simple the above equation is written as follows :

$x^2 - 4x = 2$

or $x^2 - 4x - 2 = 0$

Now, the equation becomes solvable.

(b) $\sqrt{x^2 + 5} = 3$

In order to solve, the above equation is written as follows :

$x^2 + 5 = 9$ (squaring both side of the equation)

$\Rightarrow x^2 + 5 = 9$

Now, the above equation becomes solvable.

4.5 SOLVING OF EQUATION

By solving of equation we simply mean to find the value of the variable of an equation that makes the equation true.

For example, the solution of the following equations are :

(a) $3x - 6 = 0$

$\Rightarrow 3x = 6$ (Rearranging the equation)

$\Rightarrow x = 6/3$

$\therefore x = 2$

(b) $x^2 - 25 = 0$

$\Rightarrow x^2 = 25$ (Rearranging the equation)

$\Rightarrow x = \sqrt{25}$ (Taking $\sqrt{}$ on both sides of the equation)

$\therefore x = \pm 5$

4.5.1 Solution of Polynomial Equation

a) Solution of 1st degree polynomial equation or general linear equation :

$ax + b = 0 \quad (a \neq 0)$

$\Rightarrow ax = -b$ (Rearranging the equation)

$\therefore x = -(b/a)$

b) Solution of 2nd degree polynomial equation or general quadratic equation :

$ax^2 + bx + c = 0, \quad (a \neq 0)$

The solution of the above equation is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

For example

$3x^2 - 4x + 1 = 0$

Here, $a = 3$, $b = -4$, $c = 1$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{16 - 4.3.1}}{2.3}$$

$$= \frac{4 \pm \sqrt{16-12}}{6}$$

$$= \frac{4 \pm \sqrt{4}}{6}$$

$$\therefore\ x = \frac{4 \pm \sqrt{4}}{6}$$

$$\Rightarrow x = \frac{4+2}{6} \qquad \text{or} \qquad \Rightarrow x = \frac{4-2}{6}$$

$$\therefore\ x = 1 \qquad\qquad \Rightarrow x = \frac{2}{6}$$

$$\therefore\ x = 1/3$$

4.5.2 Solution of Simultaneous Equation

A set of simultaneous equations can be solved by applying any of the following method :

(1) Elimination method :

In this method we eliminate any one of the variables so as to get one equation in one variable.

Let us consider one example;

2x + 3y = 13 – (I)

x + 7y = 23 – (II)

In order to solve these two simultaneous equations (I) and (II) for finding the value of x and y, we have to eliminate either x or y. Let us eliminate x and in order to eliminate x, both sides of equation (II) is multiplied with 2, and we get 2x +14 = 46 (III)

Now, solving equation (I) and (III) we get,

2x + 3y = 13

2x + 14y = 46

(–) (–) (–)

– 11y = –33

$$\Rightarrow y = \frac{33}{11}$$

$$\therefore\ y = 3$$

Now, putting the value of y in equation (1), we get,

$\Rightarrow$ 2x + 3.3 = 13

$\Rightarrow$ 2x + 9 = 13

$\Rightarrow$ 2x = 13–9

$\Rightarrow$ 2x = 4

$\therefore$ x = 2

$\therefore$ The solutions are x = 2 and y = 3

Verification : 2x + 3y = 2.2 + 3.3 = 13

or x + 7y = 2 + 7.3 = 23

(2) Substitution method :

In this method the value of one variable is found out from one equation and then substitute it in other equation so as to get one equation in one variable and we can solve the equation.

For example :

3x + 2y = 13 – (I)

9x – 3y = 21 – (II)

Rearranging the equation (I), we have,

3x = 13 – 2y

$$\therefore x = \frac{13-2y}{3} \quad -(III)$$

Now, putting the value of x from equation (III) in equation (II), we get,

$$\therefore 9\left(\frac{13-2y}{3}\right) - 3y = 21$$

$\Rightarrow$3(13–2y) –3y = 21

$\Rightarrow$39 – 6y – 3y = 21

$\Rightarrow$–9y = 21 – 39

$\Rightarrow$–9y = –18

$$\therefore y = {}^{18}\!/_{9} = 2$$

Putting the value of y in equation (I), we get,

3x + 2.2. = 13

$\Rightarrow$ 3x + 4 = 13

$\Rightarrow$ 3x = 9

$\therefore$ x = 3

$\therefore$ Solutions are x = 3 and y = 2

Varification : $3x + 2y = 3.3 + 2.2 = 13$

or $9x - 3y = 9.3 - 3.2 = 21$

(3): Cramer's method :

We will disucss this method in the next unit.

(4): Matrix method :

You will get to know this method in a higher level of study. However, the concept of matrix will be discussed in Unit 6.



CHECK YOUR PROGRESS

Q 1: What is the difference between equation and identity? (Answer in about 40 words).

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Q 2: What is a quadratic equation. (Answer in about 40 words).

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4.13 LET US SUM UP

- An equation represents a functional relationship or a theoritical relationship, which can be solved and value of the variables can be determined. It is important to note that equations can be judged or tested.
- In a set of simultaneons equations there are atleast two or more than two equations and in each equation there are two or more variables, whose value can be determined, which simultaneously satisfy all the

equations.

- In a set of simultaneous equations number of variables should be either equal or less than the number of equations.



4.14 FURTHER READING

1) Agarwal, D.K. (2012). *Business Mathematics*, New Delhi: Vrindra Publication (p) Ltd.
2) Baruah, S. (2011). *Basic Mathematics and Its Application in Economics*, New Delhi: Trinity Press Pvt. Ltd.
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5) Kandoi, Balwant (2011). *Mathematics for Business and Economics with Application;* New Delhi: Himalaya Publishing House.



4.15 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: If the two sides of an equation are true for all values of the variables involved in the equation, than the equation is called identity.

Ans to Q No 2: It the highest exponent of the indepandent variable in a polymial equation is 2, than it is known as quadratic equation and it takes the form of $ax^2 + bx + c = 0 \qquad (a \neq 0)$

Ans to Q No 3: Given, $Q_d = 40 - 5p$

(i) $Q_d = 40 - 5.2 \qquad [\because P = 2]$

$= 40 - 10$

$= 30$

$\therefore Q_d = 30$ units, when P = Rs. 2

(ii) $Q_d = 40 - 5P$

$\Rightarrow 0 = 40 - 5P \quad [\because Q_d = 0]$



4.16 MODEL QUESTIONS

Q 1: Suppose there are two consumers in a market. The demand functions are as follows :

$d_1(P) = 25 - P$ [when $P \leq 25$]

$d_1(P) = 0$ [when p>25]

and $d_2(P) = 20 - 2P$ [when $P \leq 25$]

$d_2(P) = 0$ [when p>25]

Find out the market demand function.

Q 2: Given the supply function

$Q_s = -14 + 2P$

Calculate (a) supply at price Rs. 10

(b) at what price, supply will be 0

(c) the price at which firm will be willing to supply 15 units.

Q 3: Derive the equilibrium price and equilibrium quantity of the following market model.

$Q_d = 20 - 2.5\ P$, $Q_3 = -4 + 1.5P$, and $Q_d = Q_3$

Q 4: Give the short-run average cost function

$$AC = 2q^2 - 15Q + 30 + \frac{16}{Q}$$

Find (a) TC

(b) AVC

(c) AFC

(d) MC

Q 5: Solve the following equations :

(a) $2x + 3y = 9$
$3x + 5y = 12$

(b) $5x_1 + 7x_2 = 85$
$3x_1 + 9x_2 = 75$

(c) $7x_1 - 2x_2 = 8$
$3x_1 + 6x_2 = 24$

(d) $4x - 3y = 11$
$2x + 2y = 16$

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UNIT 5: DETERMINANTS

UNIT STRUCTURE

5.1 Learning Objectives

5.2 Introduction

5.3 Determinant of order 2

5.4 Determinant of order 3

5.5 Properties of Determinants

5.6 Solution of a Set of Linear Equations by Cramer's Rule

5.7 Let Us Sum Up

5.8 Further Reading

5.9 Answers to Check Your Progress

5.10 Model Questions

5.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- define determinant
- evaluate determinants of order 2 and 3
- use the properties of determinants for evaluation of determinants
- solve determinants using Cramer's rule.

5.2 INTRODUCTION

The concept of determinants is a useful tool in solving system of linear equations in two or three variables. In this unit,we shall discusss the concept of determinants.We shall also study many properties of determinants which help in evaluation of determinants. We may use determinants to solve a system of linear equations by a method known as Cramer's rule.

5.3 DETERMINANT OF ORDER 2

Let us consider the equations $a_1x+b_1y=0$ and $a_2x+b_2y=0$.

Eliminating x and y from the equations, we have $-\frac{y}{x}=\frac{a_1}{b_1}=\frac{a_2}{b_2}$

$$\Rightarrow a_1b_2-a_2b_1=0$$

which is written in compact form as $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}=0$

In other words, we have $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}=a_1b_2-a_2b_1=0$.

The expression $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is called a determinant of the second order. It has two horizotal lines $a_1 \quad b_1$ and $a_2 \quad b_2$ and two vertical lines $\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix}$. The horizontal lines are called rows and the vertical lines are called columns.A determinant of second order has two rows and two columns.The numbers a_1, b_1, a_2, b_2 are called the elements of the determinant. $a_1b_2-a_2b_1$ is called the expansion or the value of the determinant.

Example 5.1: Evaluate (i) $\begin{vmatrix} 1 & 3 \\ -2 & -4 \end{vmatrix}$ (ii) $\begin{vmatrix} x+1 & x \\ x & x-1 \end{vmatrix}$ (iii) $\begin{vmatrix} x-1 & 1 \\ x^3 & x^2+x+1 \end{vmatrix}$

(iv) $\begin{vmatrix} a^2 & ab \\ ab & b^2 \end{vmatrix}$

Solution : (i) We have $\begin{vmatrix} 1 & 3 \\ -2 & -4 \end{vmatrix}=1.(-4)-(-2).3=-4+6=2$.

(ii) $\begin{vmatrix} x+1 & x \\ x & x-1 \end{vmatrix}=(x+1).(x-1)-x.x=x^2-1-x^2=-1$.

(iii) $\begin{vmatrix} x-1 & 1 \\ x^3 & x^2+x+1 \end{vmatrix}=(x-1)(x^2+x+1)-x^3$

$$=x^3-1-x^3$$
$$=-1.$$

(iv) $\begin{vmatrix} a^2 & ab \\ ab & b^2 \end{vmatrix}=a^2b^2-(ab)^2=0$.

Example 5.2 : Find the value of x if

(i) $\begin{vmatrix} x-3 & x \\ x+1 & x+3 \end{vmatrix} = 6$ (ii) $\begin{vmatrix} 2x-1 & 2x+1 \\ x+1 & 4x+2 \end{vmatrix} = 0$

Solution : (i) We have $\begin{vmatrix} x-3 & x \\ x+1 & x+3 \end{vmatrix} = (x-3)(x+3) - x(x+1)$

$$= (x^2 - 9) - x^2 - x$$
$$= -x - 9$$

$$\therefore -x - 9 = 6$$
$$\Rightarrow -x = 15$$
$$\Rightarrow x = -15$$

(ii) We have $\begin{vmatrix} 2x-1 & 2x+1 \\ x+1 & 4x+2 \end{vmatrix} = (2x-1)(4x+2) - (x+1)(2x+1)$

$$= 8x^2 + 4x - 4x - 2 - 2x^2 - x - 2x - 1$$
$$= 6x^2 - 3x - 3$$
$$= 3(2x^2 - x - 1)$$

$$\therefore 3(2x^2 - x - 1) = 0$$
$$\Rightarrow 2x^2 - x - 1 = 0$$
$$\Rightarrow x = \frac{1 \pm \sqrt{1+8}}{4}$$
$$= \frac{1 \pm 3}{4}$$
$$= 1, -\frac{1}{2}$$

Q 5.3. Solve for x if $\begin{vmatrix} x & 5 \\ 7 & x \end{vmatrix} + \begin{vmatrix} 1 & -2 \\ -1 & 1 \end{vmatrix} = 0$.

Solution : Given, $\begin{vmatrix} x & 5 \\ 7 & x \end{vmatrix} + \begin{vmatrix} 1 & -2 \\ -1 & 1 \end{vmatrix} = 0$

$$\Rightarrow (x^2 - 35) + (1 - 2) = 0$$
$$\Rightarrow x^2 - 35 - 1 = 0$$
$$\Rightarrow x^2 - 36 = 0$$
$$\Rightarrow x^2 = 36$$

$\Rightarrow x = \pm 6$

CHECK YOUR PROGRESS

Q 1: Evaluate :

(i) $\begin{vmatrix} 2 & -3 \\ 7 & 11 \end{vmatrix}$ (ii) $\begin{vmatrix} 2+\sqrt{3} & 3+\sqrt{11} \\ 3-\sqrt{11} & 2-\sqrt{3} \end{vmatrix}$

Q 2: Solve for x :

(i) $\begin{vmatrix} 2 & 3 \\ 1 & 4x \end{vmatrix} = \begin{vmatrix} 2x & -1 \\ 5 & x \end{vmatrix}$ (ii) $\begin{vmatrix} x & 3 \\ 4 & x \end{vmatrix} = \begin{vmatrix} 0 & -2 \\ 2x & 5 \end{vmatrix}$

Q 3: If $\begin{vmatrix} 3 & p \\ 4 & 6 \end{vmatrix} = 6$, find the value of p .

Q 4: Prove that $\begin{vmatrix} a+ib & c+id \\ c-id & a-ib \end{vmatrix} = a^2 + b^2 - c^2 - d^2$

5.4 DETERMINANT OF ORDER 3

In 5.3, We have already discussed determinant of order 2. Now, we think of a determinant which has 3 rows and 3 colums.

Let us consider the equations

$a_1x + b_1y + c_1z = 0$ ... (1)

$a_2x + b_2y + c_2z = 0$ ... (2)

and $a_3x + b_3y + c_3z = 0$ ... (3)

Soving (2) and (3), we get $\frac{x}{b_2c_3 - b_3c_2} = \frac{y}{c_2a_3 - c_3a_2} = \frac{z}{a_2b_3 - a_3b_2}$

Substituting these proportional values of x, y and z in (1), we get

$$a_1(b_2c_3 - b_3c_2) + b_1(c_2a_3 - c_3a_2) + c_1(a_2b_3 - a_3b_2) = 0$$

which can be written in compact form as

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

In other words, we have

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1(b_2c_3 - b_3c_2) + b_1(a_3c_2 - a_2c_3) + c_1(a_2b_3 - a_3b_2)$$

The expression $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is called a determinant of the third order. Since it has 3 rows and 3 columns,it is called a determinant of order 3.

$a_1(b_2c_3 - b_3c_2) + b_1(a_3c_2 - a_2c_3) + c_1(a_2b_3 - a_3b_2)$ is called the expansion or the value of the determinant.

Note : A determinant of order 3 has 9 elements.

Value of a determinant :

Determinant of order three can be determined by expressing it in terms of second order determinants.This is known as expansion of a determinant along a row (or a column).There are six ways of expanding a determinant of order 3 corresponding to each of three rows (R_1, R_2 and R_3) and three columns (C_1, C_2 and C_3) giving the same value.

Consider the determinant of order 3

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

First we expand the given determinant along first row (R_1):

Step 1 : We multiply element a_1 of R_1 by $(-1)^{1+1}$ [$(-1)^{\text{position of the element } a_1}$] and with the second order determinant obtained by deleting the elements of first row (R_1) and first column (C_1) as a_1 lies in R_1 and C_1.

$$\text{i.e., } (-1)^{1+1} a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$$

Step 2 : We multiply second element b_1 of R_1 by $(-1)^{1+2}$ [$(-1)^{\text{position of the element } b_1}$] and with the second order determinant obtained by deleting the elements of first row (R_1) and second column (C_2) as b_1 lies in R_1 and C_2.

i.e., $(-1)^{1+2} b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}$

Step 3 : We multiply third element c_1 of R_1 by $(-1)^{1+3}$ [$(-1)^{\text{position of the element } c_1}$] and with the second order determinant obtained by deleting the elements of first row(R_1) and third column (C_3) as c_1 lies in R_1 and C_3.

i.e., $(-1)^{1+3} c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$

Step 4 : The expansion of determinant is written as the sum of all the three terms obtained in step 1, 2 and 3 above and is given by

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = (-1)^{1+1} a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} + (-1)^{1+2} b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + (-1)^{1+3} c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$

$$= a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_3b_1c_2 + a_2b_3c_1 - a_3b_2c_1 .$$

Expansion along second row (R_2):

Consider the determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Expanding along R_2, we get

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = (-1)^{2+1} a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + (-1)^{2+2} b_2 \begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix} + (-1)^{2+3} c_2 \begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix}$$

$$= -a_2(b_1c_3 - b_3c_1) + b_2(a_2c_3 - a_3c_1) - c_2(a_1b_3 - a_3b_1)$$

$$= -a_2b_1c_3 + a_2b_3c_1 + a_1b_2c_3 - a_3b_2c_1 - a_1b_3c_2 + a_3b_1c_2 .$$

Expanding along C_1, we get

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = (-1)^{1+1} a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} + (-1)^{2+1} a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + (-1)^{3+1} a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

$$= a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1$$

Similarly, we can expand the determinant along R_3, C_2 and C_3.

Note : Expanding a determinant along any row or column gives same value.

Example 5.4 : Evaluate the determinant $\begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$

Solution : Expanding along R_1 we get

$$\begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix} = 1\begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} - 2\begin{vmatrix} -1 & 0 \\ 4 & 0 \end{vmatrix} + 4\begin{vmatrix} -1 & 3 \\ 4 & 1 \end{vmatrix}$$

$$= 1(0-0) - 2(0-0) + 4(-1-12)$$

$$= -52 .$$

Example 5.5 : Evaluate the determinant $\Delta = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & 4 \\ 1 & -1 & -2 \end{vmatrix}$

(i) by expanding about any row

(ii) by expanding about any column.

Solution : (i) (a) Expanding about the first row

$$\Delta = 1\begin{vmatrix} 3 & 4 \\ -1 & -2 \end{vmatrix} - (-2)\begin{vmatrix} -2 & 4 \\ 1 & -2 \end{vmatrix} + 3\begin{vmatrix} -2 & 3 \\ 1 & -1 \end{vmatrix}$$

$$= 1[3(-2) - (-1)4] + 2[(-2)(-2) - (1)(4)] + 3[(-2)(-1) - (3)(1)]$$

$$= -6 + 4 + 2(4-4) + 3(2-3)$$

$$= -2 - 3$$

$$= -5$$

(b) Expanding about the second row

$$\Delta = (-1)(-2)\begin{vmatrix} -2 & 3 \\ -1 & -2 \end{vmatrix} + 3\begin{vmatrix} 1 & 3 \\ 1 & -2 \end{vmatrix} - 4\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix}$$

$$= 2(4+3) + 3(-2-3) - 4(-1+2)$$

$$= 14 - 15 - 4$$

$$= -5$$

(c) Expanding about the third row

$$\Delta = 1\begin{vmatrix} -2 & 3 \\ 3 & 4 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 3 \\ -2 & 4 \end{vmatrix} + 2\begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix}$$

$$= -8 - 9 + 4 + 6 - 2(3 - 4)$$
$$= -17 + 10 + 2$$
$$= -5$$

(ii) (a) Expanding about first column

$$\Delta = 1\begin{vmatrix} 3 & 4 \\ -1 & -2 \end{vmatrix} + 2\begin{vmatrix} -2 & 3 \\ -1 & -2 \end{vmatrix} + 1\begin{vmatrix} -2 & 3 \\ 3 & 4 \end{vmatrix}$$
$$= -6 + 4 + 2(4 + 3) + 1(-8 - 9)$$
$$= -2 + 14 - 17$$
$$= -5$$

(b) Expanding about the second column

$$\Delta = (-1)(-2)\begin{vmatrix} -2 & 4 \\ 1 & -2 \end{vmatrix} + 3\begin{vmatrix} 1 & 3 \\ 1 & -2 \end{vmatrix} + (-1)(-1)\begin{vmatrix} 1 & 3 \\ -2 & 4 \end{vmatrix}$$
$$= 2(4 - 4) + 3(-2 - 3) + (4 + 6)$$
$$= -15 + 10$$
$$= -5$$

(c) Expanding about the third column

$$\Delta = 3\begin{vmatrix} -2 & 3 \\ 1 & -1 \end{vmatrix} - 4\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} + (-2)\begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix}$$
$$= 3(2 - 3) - 4(-1 + 2) - 2(3 - 4)$$
$$= -3 - 4 + 2$$
$$= -5$$

5.5 PROPERTIES OF DETERMINANTS

There are some properties of determinants, which are very much useful in solving problems. Here we are going to discuss the properties only for the determinant of order 3.

Property 1 : The value of determinant remains unchanged if a rows and columns are interchanged.

i.e., $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Note : If R_i = *i*th row and C_i = *i*th column, then for interchange of row and columns, we will symbolically write $C_i \leftrightarrow R_i$.



The system of writing equations within [ ] is known as Matrix. We shall discuss Matrix algebra in the next unit.

Property 2 : If any two adjacent rows (or columns) of a determinant are interchanged, then sign of determinant changes.

i.e., $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = -\begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Note : We can denote the interchange of rows by $R_i \leftrightarrow R_j$ and interchange of columns by $C_i \leftrightarrow C_j$.

Property 3 : If any two rows (or columns) of a determinant are identical (all corresponding elements are same), then value of determinant is zero.

i.e., $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

Property 4 : If each element of a row (or a column) of a determinant is multiplied by a constant *k*, then its value gets multiplied by *k*.

i.e., $\begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = k\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Note :The value of the determinant remains unchanged.by applying $R_i \leftrightarrow kR_i$ or $C_i \leftrightarrow kC_i$ to the determinant .

Property 5 : If to any row(or column) is added k times the corresponding elements of another row (or column), the value of the determinant remains unchanged.

i.e., $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 + ka_2 & b_1 + kb_2 & c_1 + kc_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Note : The value of determinant remain same if we apply the operation $Ri \leftrightarrow R_i + kR_j$ or $C_i \leftrightarrow C_i + k\,C_j$.

Property 6 : If any row (or column) is the sum of two or more elements, then the determinant can be expressed as sum of two or more determinants.

i.e., $\begin{vmatrix} a_1+d_1 & b_1+d_2 & c_1+d_3 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} d_1 & d_2 & d_3 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Illustrative Examples :

Example 5.6 : Prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

Solution : $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix}$

(Apply $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$)

$$= (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix}$$

$$= (b-a)(c-a)(c+a-b-a)$$
$$= (b-a)(c-a)(c-b)$$
$$= (a-b)(b-c)(c-a).$$

Example 5.7 : Show that

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$$

Solution : $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

Apply $C_3 \to C_3 + C_2$

$$= \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & b+c+a \\ 1 & c & c+a+b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

$= (a+b+c).0$ (C_1 and C_3 are identical)

$= 0$.

Example 5.8 : Find x if $\begin{vmatrix} 1 & x & -4 \\ 5 & 3 & 0 \\ -2 & -4 & 8 \end{vmatrix} = 0$.

Solution : Expanding by 1st row

$$\begin{vmatrix} 1 & x & -4 \\ 5 & 3 & 0 \\ -2 & -4 & 8 \end{vmatrix} = 1\begin{vmatrix} 3 & 0 \\ -4 & 8 \end{vmatrix} - x\begin{vmatrix} 5 & 0 \\ -2 & 8 \end{vmatrix} + (-4)\begin{vmatrix} 5 & 3 \\ -2 & -4 \end{vmatrix}$$

$$= 1(24) - x(40) - 4(-20+6)$$

$$= 24 - 40x + 56$$

$$= -40x + 80$$

$\Rightarrow -40x + 80 = 0$

$\Rightarrow x = 2$.

Example 5.9 : Solve for x if $\begin{vmatrix} 0 & 1 & 0 \\ x & 2 & x \\ 1 & 3 & x \end{vmatrix} = 0$.

Solution : Given , $\begin{vmatrix} 0 & 1 & 0 \\ x & 2 & x \\ 1 & 3 & x \end{vmatrix} = 0$

$$\Rightarrow 0\begin{vmatrix} 2 & x \\ 3 & x \end{vmatrix} - 1\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} + 0\begin{vmatrix} x & 2 \\ 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 0 - 1(x^2 - x) + 0 = 0$

$\Rightarrow -x^2 + x = 0$

$\Rightarrow x(1-x) = 0$

$\Rightarrow x = 0,1$.



CHECK YOUR PROGRESS

Q 5: Find the value of the following determinants:

(a) $\begin{vmatrix} 2 & 1 & -3 \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix}$ (b) $\begin{vmatrix} 4 & -1 & 5 \\ 5 & 7 & -3 \\ 3 & 4 & 1 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & -1 & 2 \\ 4 & 1 & 1 \\ 5 & -1 & 8 \end{vmatrix}$

(i) by expanding about any one row

(ii) by expanding about any one column.

Q 6: Factorise the determinants and solve the equations :

(a) $\begin{vmatrix} x & x^2 & x^2 \\ 3 & 9 & 27 \\ -1 & -2 & -3 \end{vmatrix} = 0$ (b) $\begin{vmatrix} 1 & x^3 & 2 \\ 1 & x^2 & 3 \\ 1 & x & 4 \end{vmatrix} = 0$

5.6 SOLUTION OF A SET OF LINEAR EQUATIONS BY CRAMER'S RULE

Let the system of n non-homogenous linear equations in n-unknowns linear

$a_{11} x_1 + a_{12} x_2 + \ldots\ldots\ldots + a_{1n} x_n = b_1 \rightarrow (1)$

$a_{21} x_1 + a_{22} x_2 + \ldots\ldots\ldots + a_{2n} x_n = b_2 \rightarrow (2)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$a_{n1} x_1 + a_{n2} x_2 + \ldots + a_{nn} x_n = b_n \rightarrow (n)$

The system can be written as –

$$\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots\ldots & \ldots\ldots & & \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}_{n\times n} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix}_{n\times 1} = \begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ b_n \end{bmatrix}_{n\times 1}$$

i.e. AX = B

Determinant of the co-efficient matrix A

=|A| = D (say)

Multiplying the equations (1), (2), ……., (n) respectively by the co-factors of $a_{11}, a_{21}, \cdots$ i.e. $A_{11}, A_{21}, \cdots, A_{n1}$ and adding we get

$$A_{11}(a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n) + A_{21}(a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n) + \cdots$$
$$+ A_{n1}(a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n) = b_1A_{11} + b_2A_{21} + \cdots + b_nA_{n1}$$
$$\Rightarrow (a_{11}A_1 + a_{21}A_{21} + \cdots + a_{n1}A_{n1})x_1 = b_1A_{11} + b_2A_{21} + \cdots + b_nA_{n1}$$
$$\Rightarrow Dx_1 = D_1$$

Where

$$D = \begin{vmatrix} a_{11} & a_{12} \ldots\ldots\ldots & a_{1n} \\ a_{21} & a_{22} \ldots\ldots\ldots & a_{2n} \\ \ldots\ldots\ldots\ldots & & \\ a_{n1} & a_{n2} \ldots\ldots\ldots & a_{nn} \end{vmatrix} \text{ and } D_1 = \begin{vmatrix} b_1 & a_{12} \ldots\ldots\ldots & a_{1n} \\ b_2 & a_{22} \ldots\ldots\ldots & a_{2n} \\ \ldots\ldots\ldots\ldots & & \\ b_n & a_{n2} \ldots\ldots\ldots & a_{nn} \end{vmatrix}$$

D_1 is the determinant obtained from D by replacing the elements of 1st column by corresponding b's

$$\Rightarrow \frac{x_1}{D_1} = \frac{1}{D} \text{ Provided } D \neq 0$$

Similarly multiplying the equations (1), (2), …… (n) by co-factors of the elements of 2nd column of |A| and adding, we get –

$$Dx_2 = D_2 \quad \text{where} \quad D_2 = \begin{vmatrix} a_{11} & b_1 \ldots\ldots a_{1n} \\ a_{21} & b_2 \ldots\ldots a_{2n} \\ a_{n1} & b_n \ldots\ldots a_{nn} \end{vmatrix}$$

$$\Rightarrow \frac{x_2}{D_2} = \frac{1}{D}, D \neq 0$$

As above we will get

$$\frac{x_1}{D_1} = \frac{x_2}{D_2} = \ldots\ldots\ldots = \frac{x_n}{D_n} = \frac{1}{D}, D \neq 0$$

The unique solution of the given system of equation provided $D \neq 0$ in the coefficient matrix is non-singular.

i.e. the rank of the co-efficient matrix is n = number of variables.

Note : For a system of n non-homogeneous linear equations with n-unknowns

$$\frac{x_1}{D_1} = \frac{x_2}{D_2} \ldots\ldots\ldots \frac{x_n}{D_n} = \frac{1}{D}, D \neq 0$$

Example 5.10 : Solve the equations

$3x + y + 2z = 3$

$2x - 3y - z = -3$

$x + 2y + z = 4$

Using Cramer's rule.

Solution : We have

$$D = |A| = \begin{vmatrix} 3 & 1 & -2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$= 3(-3 + 2) - 1(2+1) + 2(4+3)$
$= -3 - 3 + 14$
$= 8 \neq 0$

$$D_1 = \begin{vmatrix} 3 & 1 & 2 \\ -3 & -3 & -1 \\ 4 & 2 & 1 \end{vmatrix}$$

$= 3(-3+2) - 1(-3+4) + 2(-6+12)$
$= -3 - 1 + 12$
$= 8$

$$D_2 = \begin{vmatrix} 3 & 3 & 2 \\ 2 & -3 & -1 \\ 1 & 4 & 1 \end{vmatrix}$$

$= 3(-3+4) - 3(2+1) + 2(8+3)$
$= 3 - 9 + 22 = 16$

$$D_3 = \begin{vmatrix} 3 & 1 & 3 \\ 2 & -3 & -3 \\ 1 & 2 & 4 \end{vmatrix}$$

$= 3(-12+6) - 1(8+3) + 3(4+3)$
$= -18 - 11 + 21$
$= -8$

$$\therefore x = \frac{D_1}{D} = \frac{8}{8} = 1$$

$$y = \frac{D_2}{D} = \frac{16}{8} = 2$$

$$z = \frac{D_3}{D} = \frac{-8}{8} = -1$$

Example 5.11 : Solve the equations using Cramer's rule:

x+2y+3z = 6

3x–2y+z=2

4x+2y+z=7

Solution :

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{vmatrix} = 1(-2-2) - 2(3-4) + 3(6+8)$$
$$= -4 + 2 + 42 = 40 \neq 0$$

$$D_1 = \begin{vmatrix} 6 & 2 & 3 \\ 2 & -2 & 1 \\ 7 & 2 & 1 \end{vmatrix} = 6(-2-2) - 2(2-7) + 3(4+14)$$
$$= -24 + 10 + 54 = 40$$

$$D_2 = \begin{vmatrix} 1 & 6 & 3 \\ 3 & 2 & 1 \\ 4 & 7 & 1 \end{vmatrix}$$

= 1(2–7)–6(3–4)+3(21–8)
= –5+6+39
= 40

$$D_3 = \begin{vmatrix} 1 & 2 & 6 \\ 3 & -2 & 2 \\ 4 & 2 & 7 \end{vmatrix}$$

= 1(–14–4)–2(21–8)+6(6+8)
= –18–26+84
= 84 – 44
= 40

$$\therefore x = \frac{D_1}{D} = \frac{40}{40} = 1$$

$$y = \frac{D_2}{D} = \frac{40}{40} = 1$$

$$z = \frac{D_3}{D} = \frac{40}{40} = 1$$



CHECK YOUR PROGRESS

Solve the following system of equations by using Cramer's rule :

Q 7: $3x + 5y = 8$, $-x+2y-z = 0$, $3x-6y+4z=1$

Q 8: $x_1+x_2+x_3=7$, $x_1- x_2+ x_3 = 2$, $2x_1 - x_2+ 3x_3 = 9$



5.7 LET US SUM UP

- The value of determinant is unaltered, when its rows and columns are interchanged.
- If any two adjacent rows (columns) of a determinant are interchanged, then the value of the determinant changes only in sign.
- If the determinant has two identical rows (columns), then the value of the determinant is zero.
- If all the elements in a row or in a (column) of a determinant are multiplied by a constant k($k > 0$) then the value of the determinant is multiplied by k.
- The value of the determinant is unaltered when a constant multiple of the elements of any row (column), is added to the corresponding elements of a different row (column) in a determinant.
- If each element of a row (column) of a determinant is expressed as the sum of two or more terms, then the determinant is expressed as the sum of two or more determinants of the same order.



5.8 FURTHER READING

1) Agarwal, D.K. (2012). *Business Mathematics*, New Delhi: Vrindra Publication (p) Ltd.

2) Baruah, S. (2011). *Basic Mathematics and Its Application in Economics*, New Delhi: Trinity Press Pvt. Ltd.

3) Bose, D. (2004). *Mathematical Economics;* New Delhi: Himalaya

Publishing House.

4) Chiang, A.C. (2006) *Fundamental Methods of Economics Analysis*; New Delhi: MC Graw Hill Education India.

5) Kandoi, Balwant (2011). *Mathematics for Business and Economics with Application;* New Delhi: Himalaya Publishing House.



5.9 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: (i) $\begin{vmatrix} 2 & -3 \\ 7 & 11 \end{vmatrix} = 22 + 21$

$= 43.$

(ii) $\begin{vmatrix} 2+\sqrt{3} & 3+\sqrt{11} \\ 3-\sqrt{11} & 2-\sqrt{3} \end{vmatrix} = \left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right) - \left(3+\sqrt{11}\right)\left(3-\sqrt{11}\right)$

$= (4-3) - (9-11)$

$= 3.$

Ans to Q No 2: (i) Given, $\begin{vmatrix} 2 & 3 \\ 1 & 4x \end{vmatrix} = \begin{vmatrix} 2x & -1 \\ 5 & x \end{vmatrix}$

$\Rightarrow 8x - 3 = 2x^2 + 5$

$\Rightarrow 2x^2 - 8x + 8 = 0$

$\Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow (x-2)^2 = 0$

$\Rightarrow x = 2,2.$

(ii) Given, $\begin{vmatrix} x & 3 \\ 4 & x \end{vmatrix} = \begin{vmatrix} 0 & -2 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow x^2 - 12 = 0 + 4x$

$\Rightarrow x^2 - 4x - 12 = 0$

$\Rightarrow x^2 - 6x + 2x - 12 = 0$

$\Rightarrow x(x-6) + 2(x-6) = 0$

$\Rightarrow (x-6)(x+2) = 0$

$\Rightarrow x = -2,6.$

Ans to Q No 3: Given, $\begin{vmatrix} 3 & p \\ 4 & 6 \end{vmatrix} = 6$

$\Rightarrow 18 - 4p = 6$

$\Rightarrow -4p = -12$

$\Rightarrow p = 3$.

Ans to Q No 4: L.H.S $= \begin{vmatrix} a+ib & c+id \\ c-id & a-ib \end{vmatrix}$

$= (a+ib)(a-ib) - (c+id)(c-id)$

$= \{a^2 - (ib)^2\} - \{c^2 - (id)^2\}$

$= a^2 + b^2 - c^2 - d^2$

Ans to Q No 5: (a) (i) Expanding about the first row

$$\begin{vmatrix} 2 & 1 & -3 \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 2\begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} - 3\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}$$

$= 2(2-1) - 1(-1-2) - 3(1+4)$

$= 2 + 3 - 15$

$= -10$

(ii) Expanding about the first column

$$\begin{vmatrix} 2 & 1 & -3 \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 2\begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & -3 \\ 1 & -1 \end{vmatrix} + 2\begin{vmatrix} 1 & -3 \\ -2 & 1 \end{vmatrix}$$

$= 2(2-1) - 1(-1+3) + 2(1-6)$

$= 2 - 2 - 10$

$= -10$.

(b) Try yourself

(c) Try yourself.

Ans to Q No 6: (a) Given, $\begin{vmatrix} x & x^2 & x^2 \\ 3 & 9 & 27 \\ -1 & -2 & -3 \end{vmatrix} = 0$

$$\Rightarrow 3x\begin{vmatrix}1 & x & x^2\\ 1 & 3 & 9\\ -1 & -2 & -3\end{vmatrix}=0$$

$$\Rightarrow 3x\begin{vmatrix}3 & 9\\ -2 & -3\end{vmatrix}-3x\begin{vmatrix}x & x^2\\ -2 & -3\end{vmatrix}-3x\begin{vmatrix}x & x^2\\ 3 & 9\end{vmatrix}=0$$

$$\Rightarrow 3x(-9+18)-3x(-3x+2x^2)-3x(9x-3x^2)=0$$

$$\Rightarrow 3x(9+3x-2x^2-9x+3x^2)=0$$

$$\Rightarrow 3x(x^2-6x+9)=0$$

$$\Rightarrow 3x(x-3)^2=0$$

$$\Rightarrow x=0,3$$

(b) Try yourself.

Answer Q No 7:

3x + 5y + 0z = 8
–x + 2y – z = 0
3x – 6y + 4z = 1

$$D=\begin{vmatrix}3 & 5 & 0\\ -1 & 2 & -1\\ 3 & -6 & 4\end{vmatrix}$$

$$=3\begin{vmatrix}2 & -1\\ -6 & 4\end{vmatrix}-5\begin{vmatrix}-1 & -1\\ 3 & 4\end{vmatrix}$$

= 3 (8-6) – 5(–4+3)
= 6+5 = $11\neq 0$

$$D_1=\begin{vmatrix}8 & 5 & 0\\ 0 & 2 & -1\\ 1 & -6 & 4\end{vmatrix}=8\begin{vmatrix}2 & -1\\ -6 & 4\end{vmatrix}-5\begin{vmatrix}0 & -1\\ 1 & 4\end{vmatrix}$$

= 8(8–6) –5x1=16–5=11

$$D_2=\begin{vmatrix}3 & 8 & 0\\ -1 & 0 & -1\\ 3 & 1 & 4\end{vmatrix}$$

$$=3\begin{vmatrix}0 & -1\\ 1 & 4\end{vmatrix}-8\begin{vmatrix}-1 & -1\\ 3 & 4\end{vmatrix}$$

= 3(0+1)–8(–4+3)
= 3+8 = 11

$$D_3 = \begin{vmatrix} 3 & 5 & 8 \\ -1 & 2 & 0 \\ 3 & -6 & 1 \end{vmatrix}$$

$$= 3\begin{vmatrix} 2 & 0 \\ -6 & 1 \end{vmatrix} - 5\begin{vmatrix} -1 & 0 \\ 3 & 1 \end{vmatrix} + 8\begin{vmatrix} -1 & 2 \\ 3 & -6 \end{vmatrix}$$

$= 3(2–0) –5(–1–0) +8(6–6)$

$= 6+5 = 11$

$$\therefore x = \frac{D_1}{D} = \frac{11}{11} = 1$$

$$y = \frac{D_2}{D} = \frac{11}{11} = 1$$

$$y = \frac{D_3}{D} = \frac{11}{11} = 1$$

i.e. $x = 1$, $y = 1$, $z = 1$.

Answer Q No 8:

$x_1 + x_2 + x_3 = 6$

$x_1 – x_2 + x_3 = 2$

$2x_1 – x_2 + 3x_3 = 9$

$$\therefore D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = 1x\begin{vmatrix} -1 & 1 \\ -1 & 3 \end{vmatrix} - 1x\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + ax\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix}$$

$= (–3+1)–(3–2)+(–1+2)$

$= –2–1+1 = -2 \neq 0$

$$\therefore D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 2 & -1 & 1 \\ 9 & -1 & 3 \end{vmatrix} = 6\begin{vmatrix} -1 & 1 \\ -1 & 3 \end{vmatrix} - 1x\begin{vmatrix} 2 & 1 \\ 9 & 3 \end{vmatrix} + 1x\begin{vmatrix} 2 & -1 \\ 9 & -1 \end{vmatrix}$$

$= 6(–3+1)–(6–9)+(–2+9)$

$= –12+3+7 = – 2$

$$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 2 & 1 \\ 2 & 9 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 1 \\ 9 & 3 \end{vmatrix} - 6x\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 2 & 9 \end{vmatrix}$$

$= (6–9) –6(3–2) +(9–4)$

$= –3 –6 +5$

$= –9 + 5$

$= -4$

$$D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \\ 2 & -1 & 9 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 2 \\ -1 & 9 \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 2 & 9 \end{vmatrix} + 6 \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix}$$

$= (-9+2) - (9-4) + 6 \times (-1+2)$

$= -7-5+6$

$= -12+6$

$= -6$

$$x_1 = \frac{D_1}{D} = \frac{-2}{-2} = 1$$

$$x_2 = \frac{D_2}{D} = \frac{-4}{-2} = 2$$

$$x_3 = \frac{D_3}{D} = \frac{-6}{-2} = 3$$



5.10 MODEL QUESTIONS

Q 1: Evaluate (i) $\begin{vmatrix} 4 & 6 \\ -2 & 3 \end{vmatrix}$ (ii) $\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix}$ (iii) $\begin{vmatrix} -2 & -4 \\ -1 & -6 \end{vmatrix}$

Q 2: Evaluate (i) $\begin{vmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \\ 1 & 2 & 4 \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Q 3: Show that

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}.$$

Q 4: Show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (b-a)(c-a)(a-b)(a+b+c)$$

Q 5: Show that $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$.

Q 6: Show that $\begin{vmatrix} a & b & c \\ a-c & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$

Q 7: Prove that $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c)$

Q 8: Show that $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = xy$

Q 9: Without expanding the determinant, prove that

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

Q 10: Solve using Cramer's rule

$$2x+3y+z=9$$
$$x+2y+3z=6$$
$$3x+y+2x=8$$

*** ***** ***

UNIT 6 : MATRICES

UNIT STRUCTURE

6.1 Learning Objectives

6.2 Introduction

6.3 Concept of a Matrix

6.4 Types of Matrix

6.5 Equality of Matrices

6.6 Addition and Subtraction of Matrices

6.7 Multiplication of a Matrix by a Scalar

6.8 Multiplication of two Matrices

6.9 Transpose of a Matrix

6.10 Symmetric Matrix

6.11 Let Us Sum Up

6.12 Further Reading

6.13 Answers to Check Your Progress

6.14 Model Questions

6.1 LEARNING OBJECTIVES

After going through this unit, you will be able to :

- understand the definition of a Matrix
- describe different types of Matrix
- learn matrix operations e.g. addition, subtraction and multiplication
- learn about adjoint, inverse and rank of Matrices

6.2 INTRODUCTION

Matrix is one of the most powerful tools of modern mathematics and is also widely used in Economics. Matrix were introduced by the English Mathematician Arthur Cayley in 1858.Matrices are initially connected with linear transformation. Matrix theory now occupies important position in Physics, Economics, Statistics, Engineering etc.

6.3 CONCEPT OF A MATRIX

A set of mn numbers (real or complex) arranged in the form of a rectangular array having m rows and n columns is called an mxn matrix (to be read as 'm' by 'n' matrix).

An m x n matrix is usually written as

$$A = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} \end{bmatrix}$$

The compact representation of the above matrix is

$A = [a_{ij}]$, i = 1, 2, ... m, j = 1, 2, ... n.

or, simply, $A = [a_{ij}]_{mxn}$. Sometimes, the braket [ ] can be replaced by ( ) .

The general element of the matrix A is a_{ij} which belongs to the i-th row and j-th column. a_{ij} is sometimes denoted by (i,j)th element of the matrix. Here i, the first suffix denotes the number of row and j, the second suffix denotes the number of column in which the element a_{ij} occurs.

Note :

1. A matrix having m-rows and n-columns is called a matrix of order mn (read as 'm' by 'n') or simply m x n matrix.

2. In a matrix, the number of rows need not be equal to the number of columns.

3. A matrix is denoted by the capital letters A, B, C… etc. whereas any elements of a matrix is denoted by small letters such as

$a_{ij}, b_{ij}, c_{ij}, \cdots\cdots$ etc.

Illustrative example :

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 4 & 3 & 5 \end{bmatrix}_{2\times3} \text{ or } \begin{pmatrix} 1 & 2 & 0 \\ 4 & 3 & 5 \end{pmatrix}_{2\times3}$$

is a 2 x3 matrix i.e. it has 2 rows and 3 columns. For A,

a_{12} = (1,2)th element = 2

a_{23} = (2,3)th element = 5

a_{13} = (1,3)th element = 0

$$B = \begin{bmatrix} 1+i & 2 & \frac{1}{3} \\ 4 & 0 & 1 \\ \sqrt{3} & 2-i & \sqrt{2} \end{bmatrix}_{3\times3}$$

is a 3x3 matrix. For B,

b_{11} = 1+ i, $b_{31} = \sqrt{3}$, b_{22} =0 etc.

6.4 TYPES OF MATRIX

There are different types of matrices. We will discuss one by one.

(i) Row Matrix : Any 1x n matrix which has only one row and n column is called a row matrix.

For example,

A = [2, 5,- 3, 0, $\sqrt{3}$] is a row matrix order 1 $\times$ 5.

In general, B = $[a_{ij}]_{1\times n}$ is a row matrix of order 1x n

(ii)**Column Matrix :** Any m x1 matrix which has m rows and only one column is called a column matrix.

For example,

$$B = \begin{bmatrix} 6 \\ 0 \\ -3 \\ \sqrt{2} \end{bmatrix}$$ is a column matrix of order 4×1

In general, $B = [b_{ij}]_{m\times 1}$ is a column matrix of order m×1.

(iii) **Square Matrix :** A matrix whose numbers of rows is equal to the number of columns is called a square matrix.

For the matrix A= $[a_{ij}]_{m\times n}$, if m = n, then the matrix A is said to be square matrix of order m.

Example :

(a)

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$ is a square matrix of order 2.

(b)

$$B = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 1 \\ 5 & 0 & 1 \end{bmatrix}$$ is a square matrix of order 3

In a square matrix A = $[a_{ij}]_{m\times m}$, the elements a_{11}, a_{22}, a_{33} ..... are called the **Diagonal Elements** and the line along which they lie is called the **Principal Diagonal** of the matrix.

(iv)**Diagonal Matrix :** A square matrix A is said to be a diagonal matrix if all its non-diagonal elements be zero.

Examples:

$$A=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}, B=\begin{bmatrix}3 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{bmatrix}$$

are diagonal matrices of order 2 and 3 respectively.

(v)**Scalar Matrix :** A diagonal matrix (i.e. all non-diagonal elements being zero) where all the diagonal elements are equal is called a Scalar Matrix.

Examples :

$$A=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}, B=\begin{bmatrix}-3 & 0 & 0\\ 0 & -3 & 0\\ 0 & 0 & -3\end{bmatrix}$$

are two scalar matrices of order 2 and 3 respectively.

Thus, the square matrix $A=[a_{ij}]_{m\times m}$ is a scalar matrix if

a_{ij} = 0, when i $\neq$ j

a_{ij} = k(say) when i = j

(vi) **Unit (or Identity) Matrix :** A square matrix in which the diagonal elements are unity and non-diagonal elements are all zero is called a unit (or Identity)matrix.

Examples :

$$I_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, I_3=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Unit matrices are denoted by I.

I_2 and I_3 are Unit matrices of order 2 and 3 respectively.

Thus, the square matrix $A=[a_{ij}]_{m\times m}$ is a unit matrix if

a_{ij} = 0 when i $\neq$ j

a_{ij} = 1 when i = j

(vii) **Null Matrix or Zero Matrix :** A matrix of order m x n in which all the

elements are zero is called a null matrix (a zero matrix). It is denoted by O.

Thus,

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ are all zero matrices of order 2 x2 and 4x3

respectively.

(viii) **Upper Triangular Matrix and Lower Triangular Matrix:** Let A be a square matrix of order n.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \ldots & a_{3n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & a_{n3} & \ldots & a_{nn} \end{bmatrix}$$

The diagonal elements of A are $a_{11}, a_{12}, \ldots, a_{nn}$ i.e. a_{ij} for i= j.

All these elements of A above the principal diagonal are a_{12}

$a_{13} \ldots\ldots a_{1n}, a_{23} \ldots\ldots a_{2n} \ldots\ldots$ etc. i.e. a_{ij} for $i < j$

All these elements of A below the principal diagonal are $a_{21}, a_{31},$

$a_{32}, \ldots\ldots a_{n1}, a_{n2}, \ldots\ldots$ i.e. a_{ij} for $i > j$.

If all a_{ij} for $i > j$ be zeros in a square matrix A, then A is called **Upper Triangular matrix.**

Example :

$A = \begin{bmatrix} 3 & 1 & 2 & 6 \\ 0 & 2 & -4 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 5 \end{bmatrix}$ is Upper Triangular Matrix

If all a_{ij} for $i < j$ be zeros is a square matrix A, the A is called **Lower Triangular Matrix.**

Example :

$$A=\begin{bmatrix} 3 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 2 & -1 & 4 & 0 \\ -2 & 1 & 3 & 5 \end{bmatrix}$$ is Lower Triangular Matrix

Remarks : A diagonal matrix is both upper and lower triangular.

6.5 EQUALITY OF MATRICES

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if

(i) they are of the same size or order

(ii) each element of one matrix is equal to the corresponding element of the other matrix i.e. $a_{ij} = b_{ij}$ for all i and j.

If A and B are equal, then we write A = B.If A and B are not equal, then we write $A \neq B$.

Example :

1. If $A=\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}_{2\times2}, B=\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}_{2\times2}$ then A = B .

2. If $A=\begin{bmatrix} 1^2 & 3^2 \\ 2^2 & 4^2 \end{bmatrix}_{2\times2}, B=\begin{bmatrix} 1 & 9 \\ 4 & 16 \end{bmatrix}_{2\times2}$ then A = B.

3. If $A=\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}_{2\times2}, B=\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}_{2\times3}$ then $A \neq B$.

4. If $\begin{bmatrix} x & a \\ y & b \\ z & c \end{bmatrix}_{3\times2} = \begin{bmatrix} -3 & 0 \\ 1 & 2 \\ 4 & \sqrt{3} \end{bmatrix}_{3\times2}$

then, x = - 3, y = 1, z = 4, a = 0, b = 2, c = $\sqrt{3}$

Remarks: The inequality of two matrices arised due to the following reasons :

(i) This orders may not be equal.

(ii) Elements in the corresponding places may not be equal.



CHECK YOUR PROGRESS

Q 1: If a matrix has 10 elements what are the possible orders it can have?

Q 2: Find the values of a, b, c, d when

$$\begin{pmatrix} 2a+b & 5c-d \\ a-2b & 4c+d \end{pmatrix} = \begin{pmatrix} 4 & 11 \\ -3 & 25 \end{pmatrix}$$

Q 3: Are the following matrices equal where

$$A = \begin{pmatrix} 2 & 4 & 3 \\ 1 & 0 & 5 \\ 0 & 0 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 4 & 3 \\ 1 & 0 & 5 \\ 0 & 0 & 0 \end{pmatrix}$$

6.6 ADDITION AND SUBTRACTION OF MATRICES

Let $A = [a_{ij}]_{mxn}$ and $B = [b_{ij}]_{mxn}$ be two matrices of same order mxn. Then their sum(or difference) denoted by A+B (or A-B) is defined as another matrix $C = [c_{ij}]$ of the order m xn such that any element of C is the sum (or difference) of the corresponding element of A and B.

i.e. $c_{ij} = a_{ij} + b_{ij}$ for all i, j

Thus, $C = [c_{ij}] = [a_{ij} + b_{ij}]_{mxn}$

Example 6.1 : If $A = \begin{bmatrix} 5 & 1 \\ 3 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & -5 \\ -2 & 1 \end{bmatrix}$

Then, $C = A+B = \begin{bmatrix} 5+2 & 1-5 \\ 3+(-2) & 0+1 \end{bmatrix} = \begin{bmatrix} 7 & -4 \\ 1 & 1 \end{bmatrix}$

Similarly, $D = A - B = \begin{bmatrix} 5-2 & 1-(-5) \\ 3-(-2) & 0-1 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 5 & -1 \end{bmatrix}$

Remark : Two matrices A and B are said to be conformable for addition or subtraction if they are of the same order.

Negative of a Matrix : If A be a given matrix, then the negative of A denoted by – A is the matrix whose elements are the negative of the corresponding elements of A.

Example 6.2 :

$$\text{If } A = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ -5 & 6 \end{bmatrix}, \text{then } -A = \begin{bmatrix} -2 & 1 \\ 0 & -4 \\ 5 & -6 \end{bmatrix}$$

Example 6.3: If $A = \begin{bmatrix} 5 & 0 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$, , find A + B and A – B

Solution :

$$A + B = \begin{bmatrix} 5 & 0 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 1 \\ 9 & 3 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 5 & 0 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5-3 & 0-1 \\ 3-6 & 1-2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -3 & -1 \end{bmatrix}$$

Example 6.4: If $A = \begin{bmatrix} 1 & 0 \\ 4 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 & 3 \\ 5 & -2 \end{bmatrix}, C = \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}$

Show that [A+B] + C = A + [B+C]

Solution :

$$A+B = \begin{bmatrix} 1 & 0 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} 1+0 & 0+3 \\ 4+5 & 3-2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 9 & 1 \end{bmatrix}$$

$$[A+B]+C = \begin{bmatrix} 1 & 3 \\ 9 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 & 3+5 \\ 9+2 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 \\ 11 & 1 \end{bmatrix}$$

Again, $B+C = \begin{bmatrix} 0 & 3 \\ 5 & -2 \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ 2 & +0 \end{bmatrix}$

$$= \begin{bmatrix} 0+3 & 3+5 \\ 5+2 & -2+0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 8 \\ 7 & -2 \end{bmatrix}$$

$$\therefore A+[B+C] = \begin{bmatrix} 1 & 0 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 8 \\ 7 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 & 0+8 \\ 4+7 & 3-2 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 11 & 1 \end{bmatrix}$$

$$\therefore [A+B]+C = A+[B+C]$$



CHECK YOUR PROGRESS

Q 4: Is it possible to define A+B when–

(i) A has 3 rows, 2 columns and B has 2 rows, 2 columns.

(ii) A has 3 rows, 3 columns and B has 3 rows, 3 columns.

(iii) A and B are square matrices of same order.

6.7 MULTIPLICATION OF A MATRIX BY A SCALAR

Multiplication of a matrix by a scalar is defined as follows:

If $A = [a_{ij}]_{m\times n}$ is a matrix of order $m\times n$ and k is a scalar, then

$kA = k[a_{ij}]_{m\times n} = [ka_{ij}]_{m\times n}$

i.e., (i, j)th element of kA is k times the (i,j) th element of A .

Example 6.5:

If k = 3 and $A = \begin{bmatrix} 2 & -1 \\ -2 & 3 \\ 0 & 1 \end{bmatrix}_{2\times 3}$,then

$$3A = 3\begin{bmatrix} 2 & -1 \\ -2 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3\times 2 & 3\times(-1) \\ 3\times(-2) & 3\times 3 \\ 3\times 0 & 3\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -3 \\ -6 & 9 \\ 0 & 3 \end{bmatrix}_{3\times 2}$$

and,

$$\frac{1}{2}A = \frac{1}{2}\begin{bmatrix} 2 & -1 \\ -2 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\times 2 & \frac{1}{2}\times(-1) \\ \frac{1}{2}\times(-2) & \frac{1}{2}\times 3 \\ \frac{1}{2}\times 0 & \frac{1}{2}\times 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ -1 & \frac{3}{2} \\ 0 & \frac{1}{2} \end{bmatrix}_{3\times 2}$$

Example 6.6:

If $A = \begin{pmatrix} 3 & 9 \\ 1 & 8 \end{pmatrix}_{2\times 2}$, $B = \begin{pmatrix} 4 & 0 \\ 7 & 2 \end{pmatrix}_{2\times 2}$

Then verify that 2(A+B) = 2A + 2B

Solution :

We have

$$A + B = \begin{bmatrix} 3 & 9 \\ 1 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 7 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3+4 & 9+0 \\ 1+7 & 8+2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 9 \\ 8 & 10 \end{bmatrix}$$

$$\therefore 2\,(A+B) = 2\begin{bmatrix}7 & 9\\ 8 & 10\end{bmatrix} = \begin{bmatrix}2\times 7 & 2\times 9\\ 2\times 8 & 2\times 10\end{bmatrix}$$

$$= \begin{bmatrix}14 & 18\\ 16 & 20\end{bmatrix}$$

Again, $2A+2B = 2\begin{bmatrix}3 & 9\\ 1 & 8\end{bmatrix} + 2\begin{bmatrix}4 & 0\\ 7 & 2\end{bmatrix}$

$$= \begin{bmatrix}2\times 3 & 2\times 9\\ 2\times 1 & 2\times 8\end{bmatrix} + \begin{bmatrix}2\times 4 & 2\times 0\\ 2\times 7 & 2\times 2\end{bmatrix}$$

$$= \begin{bmatrix}6 & 18\\ 2 & 16\end{bmatrix} + \begin{bmatrix}8 & 0\\ 14 & 4\end{bmatrix}$$

$$= \begin{bmatrix}6+8 & 18+0\\ 2+14 & 16+4\end{bmatrix}$$

$$= \begin{bmatrix}14 & 18\\ 16 & 20\end{bmatrix}$$

$\therefore 2(A+B) = 2A + 2B$ verified.

Example 6.7:

If $k = 3$, $l = -2$ and $A = \begin{bmatrix}3 & 2\\ 1 & 0\\ 2 & 1\end{bmatrix}$

Then verify that $(k+l)A = kA + lA$

Solution :

$$(k+l)\,A = (3-2)\begin{bmatrix}3 & 2\\ 1 & 0\\ 2 & 1\end{bmatrix} = (1)\begin{bmatrix}3 & 2\\ 1 & 0\\ 2 & 1\end{bmatrix}$$

Again,

$$kA + lA = 3A - 2A = 3\begin{bmatrix}3 & 2\\ 1 & 0\\ 2 & 1\end{bmatrix} - 2\begin{bmatrix}3 & 2\\ 1 & 0\\ 2 & 1\end{bmatrix}$$

$$= \begin{bmatrix}9 & 6\\ 3 & 0\\ 6 & 3\end{bmatrix} - \begin{bmatrix}6 & 4\\ 2 & 0\\ 4 & 2\end{bmatrix}$$

$$= \begin{bmatrix} 9-6 & 6-4 \\ 3-2 & 0-0 \\ 6-4 & 3-2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$\therefore$(k+l) A = kA + lA. Verified

Example 6.8: If k=4, l=2 and A = $\begin{bmatrix} 3 & 2 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$, then

Verify that k(lA) = (kl)A.

Solution :

$$k(lA) = 4(2A) = 4 \begin{bmatrix} 2\times3 & 2\times2 \\ 2\times1 & 2\times0 \\ 2\times2 & 2\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 4\times2\times3 & 4\times2\times2 \\ 4\times2\times1 & 4\times2\times0 \\ 4\times2\times2 & 4\times2\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 8\times3 & 8\times2 \\ 8\times1 & 8\times0 \\ 8\times2 & 8\times1 \end{bmatrix}$$

$$= 8\begin{bmatrix} 3 & 2 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$$

= 8A

= (4×2) A

= (kl)A. verified

Example 6.9:

$$\text{If } k = 5 \text{ and } A = \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$$

Verify that (–k)A = –(kA)= k (–A)

Solution :

$$(-k)\,A = (-5)\begin{bmatrix} 3 & 2 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (-5)\times 3 & -(5\times 2) \\ (-5)\times 1) & -(5\times 0) \\ -(5\times 2) & -(5\times 1) \end{bmatrix}$$

$$= -\begin{bmatrix} (5)\times 3 & (5)\times 2 \\ (5)\times 1 & (5\times 0) \\ (5\times 2) & (5\times 1) \end{bmatrix} = -(kA)$$

Also, $k(-A) = 5\begin{bmatrix} -3 & -2 \\ -1 & 0 \\ -2 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 5\times(-3) & 5\times(-2) \\ 5\times(-1) & 5\times 0 \\ 5\times(-2) & 5\times(-1) \end{bmatrix}$$

$$= -\begin{bmatrix} 5\times 3 & 5\times 2 \\ 5\times 1 & 5\times 0 \\ 5\times 2 & 5\times 1 \end{bmatrix}$$

$$= -5\begin{bmatrix} 3 & 2 \\ 1 & 0 \\ 2 & 1 \end{bmatrix} = -(kA)$$ Verified.

Example 6.10: If $A = \begin{bmatrix} 0 & 3 & 2 \\ 1 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 & -3 \\ 0 & 1 & 3 \end{bmatrix}$, find 3A – 2B.

Solution :

$$3A - 2B = 3\begin{bmatrix} 0 & 3 & 2 \\ 1 & 4 & 2 \end{bmatrix} - 2\begin{bmatrix} 4 & 0 & -3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 9 & 6 \\ 3 & 12 & 6 \end{bmatrix} - \begin{bmatrix} 8 & 0 & -6 \\ 0 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0-8 & 9-0 & 6-(-6) \\ 3-0 & 12-2 & 6-4 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 9 & 12 \\ 3 & 10 & 2 \end{bmatrix}$$

6.8 MULTIPLICATION OF TWO MATRICES

Two matrices A and B are conformable for the product AB if and only if the number of columns of A is equal to the number of rows of B.

Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$,

Then the product A,B,C is the matrix of order m×p

i.e. $AB = C = [c_{jk}]_{m\times p}$ where

$$C_{jk} = \sum_{j=1}^{n} a_{ij}\, b_{jk} = a_{i1}\, b_{1k} + a_{i2}\, b_{2k} + \ldots\ldots\ldots\ldots + a_{in}\, b_{nk}$$

i.e. the (j, k) th element of the matrix C= AB is found by multiplying the corresponding element of the j-th row of A and the k-th column of B and then, adding the product.

The rule of multiplication is *row by column multiplication*.

Example 6.11:

$$\text{If } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3\times 2} \quad \text{and} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_{2\times 2}$$

$$\text{Then } AB = \begin{bmatrix} a_{11}\, b_{11} + a_{12}\, b_{21} & a_{11}\, b_{12} + a_{12}\, b_{22} \\ a_{21}\, b_{11} + a_{22}\, b_{21} & a_{21}\, b_{12} + a_{22}\, b_{22} \\ a_{31}\, b_{11} + a_{32}\, b_{21} & a_{31}\, b_{12} + a_{32}\, b_{22} \end{bmatrix}_{3\times 2}$$

Note : 1) If the product AB exists, then it is not necessary that the product BA will also exist.

Example 6.12: A= $[2\ \ 3]_{1\times2}$ and B = $\begin{bmatrix}4 & 0 & 5\\ 1 & 2 & 3\end{bmatrix}_{2\times3}$

Here,

$$AB = \begin{bmatrix}2 & 3\end{bmatrix}\begin{bmatrix}4 & 0 & 5\\ 1 & 2 & 3\end{bmatrix}$$

$$= \begin{bmatrix}8+3 & 0+6 & 10+9\end{bmatrix} = \begin{bmatrix}11 & 6 & 19\end{bmatrix}$$

But BA = $\begin{bmatrix}4 & 0 & 5\\ 1 & 2 & 3\end{bmatrix}\ \begin{bmatrix}2 & 3\end{bmatrix}$ which does not exist

$$\therefore AB \neq BA$$

Matrix multiplication is not commutative

2) In the case when both A and B are square matrices of the same order then also both AB and BA are defined but still $AB \neq BA$.

Example 6.13: If $A = \begin{bmatrix}2 & 1\\ 0 & 2\end{bmatrix}$ and $B = \begin{bmatrix}3 & 5\\ 4 & 1\end{bmatrix}$

Then $AB = \begin{bmatrix}2 & 1\\ 0 & 2\end{bmatrix}\begin{bmatrix}3 & 5\\ 4 & 1\end{bmatrix}$

$$= \begin{bmatrix}2x3+1x4 & 2x5+1x1\\ 0x3+2x4 & 0x5+2x1\end{bmatrix}$$

$$= \begin{bmatrix}10 & 11\\ 8 & 2\end{bmatrix}$$

$$BA = \begin{bmatrix}3 & 5\\ 4 & 1\end{bmatrix}\begin{bmatrix}2 & 1\\ 0 & 2\end{bmatrix}$$

$$= \begin{bmatrix}3x3+5x0 & 3x1+5x2\\ 4x2+1x0 & 4x1+1x2\end{bmatrix}$$

$$= \begin{bmatrix}9 & 13\\ 8 & 6\end{bmatrix}$$

It is seen that $AB \neq BA$

Example 6.14:

If $A = \begin{bmatrix}2 & 1\\ 1 & 0\end{bmatrix}$ and $B = \begin{bmatrix}2 & 3 & 0\\ -2 & 1 & 3\end{bmatrix}$

Can you find AB and BA ?

Solution :

Here, A is of order 2×2

B is of order 2×3

∴ AB will be order 2×3

Again, B is of order 2×3

A is of order 2×2

∴BA does not exist because the number of column of B is 3 and the number of rows of A is 2.

So, they are not conformable for the product BA.

Example 6.15:

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2}$, $B = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}_{2\times3}$

$C = \begin{bmatrix} 5 & 0 & 1 \\ 4 & 6 & 0 \end{bmatrix}_{2\times3}$ then show that

A(B+C) = AB + AC

Solution :

$$B+C = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 1 \\ 4 & 6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 1 & 3 \\ 4 & 8 & 1 \end{bmatrix}_{2\times3}$$

A(B+C)

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2} \begin{bmatrix} 8 & 1 & 3 \\ 4 & 8 & 1 \end{bmatrix}_{2\times3} = \begin{bmatrix} 1\times8+2\times4 & 1\times1+2\times8 & 1\times3+2\times1 \\ 3\times8+4\times4 & 3\times1+4\times8 & 3\times3+4\times1 \end{bmatrix}_{2\times3}$$

$$= \begin{bmatrix} 16 & 17 & 5 \\ 40 & 35 & 13 \end{bmatrix}$$

Again, $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1\times3+2\times0 & 1\times1+2\times2 & 1\times2+2\times1 \\ 3\times3+4\times0 & 3\times1+4\times2 & 3\times2+4\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 & 4 \\ 9 & 11 & 10 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 0 & 1 \\ 4 & 6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times5+2\times4 & 1\times0+2\times6 & 1\times1+2\times0 \\ 3\times5+4\times4 & 3\times0+4\times6 & 3\times1+4\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 12 & 1 \\ 31 & 24 & 3 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 3 & 5 & 4 \\ 9 & 11 & 10 \end{bmatrix} + \begin{bmatrix} 13 & 12 & 1 \\ 31 & 24 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 17 & 5 \\ 40 & 35 & 13 \end{bmatrix}$$

$$\therefore A(B + C) = AB + AC$$

Example 6.16:

$$\text{If} A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}_{2\times2}, B = \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}_{3\times2}$$

$$C = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}_{2\times2}$$

Find A[BC] and [AB]C and show that A[BC] $\neq$ [AB]C .

Solution :

$$BC = \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}_{3\times2} \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}_{2\times2}$$

$$= \begin{bmatrix} 1\times1+3\times2 & 1\times2+3\times0 \\ 0\times1+2\times2 & 0\times2+2\times0 \\ -1\times1+4\times2 & -1\times2+4\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 2 \\ 4 & 0 \\ 7 & 2 \end{bmatrix}$$

$$A[BC] = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} 7 & 2 \\ 4 & 0 \\ 7 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 7+1\times 4+(-1)\times 7 & 1\times 2+1\times 0+(-1)\times(-2) \\ 2\times 7+0\times 4+3\times 7 & 2\times 2+0\times 0+3\times(-2) \\ 3\times 7+(-1)\times 4+2\times 7 & 3\times 2+(-1)\times 0+2\times(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 \\ 35 & -2 \\ 31 & 2 \end{bmatrix}$$

$$\text{and } AB = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 1+1\times 0+(-1)(-1) & 1\times 3+1\times 2+(-1)4 \\ 2\times 1+0\times 0+3(-1) & 2\times 3+0\times 2+3\times 4 \\ 3\times 1+(-1)\times 0+2\times(-1) & 3\times 3+(-1)\times 2+2\times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ -1 & 18 \\ 1 & 15 \end{bmatrix}$$

$$[AB]C = \begin{bmatrix} 2 & 1 \\ -1 & 18 \\ 1 & 15 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$$

$\therefore A[BC] \neq [AB]C$

$$= \begin{bmatrix} 2\times 1+1\times 2 & 2\times 2+1\times 0 \\ (-1)\times 1+18\times 2 & (-1)\times 2+18\times 0 \\ 1\times 1+15\times 2 & 1\times 2+15\times 0 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 35 & -2 \\ 31 & 2 \end{bmatrix}$$

$\therefore A[BC] \neq [AB]C$. Proved.



CHECK YOUR PROGRESS

Q 5: Given

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 5 & 1 \\ 7 & 4 \end{bmatrix}$$

B(C+D) = BC + BD

(B+C) D = BD + CD

Q 6: For the matrices A,B,C put the numerical values of their orders, so that (A+B) C = AC + BC

Q 7: Find A and B if

$A + B = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$, $A - B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

Q 8: Given $A = \begin{bmatrix} 0 & 1 \\ 21 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, Show that

$A^2 = B^2 = I$ (unit matrix)

Q 9: If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$

6.9 TRANSPOSE OF A MATRIX

Let $A = [a_{ij}]_{m \times n}$. Then the n×m matrix obtained from A by changing the rows of A into columns and columns into rows is called the Transpose of matrix A and is denoted by A^I or A^T.

Therefore, A^I or A^T will have n-rows and m-columns.

Example :

If $A = \begin{bmatrix} 2 & 0 & 5 \\ 1 & 2 & 4 \end{bmatrix}_{2 \times 3}$ Then,

$$A^{/} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \\ 5 & 4 \end{bmatrix}_{3 \times 2}$$

6.10 SYMMETRIC MATRIX

SYMMETRIC MATRIX :

A square matrix $A=[a_{ij}]$ is said to be symmetric if for all values of i and j, i.e, $a_{ij} = a_{ji}$

i.e. $(i,j)^{th}$ element is same as the $(j,i)^{th}$ element of A

For example

$$\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

are symmetric matrix.

Theorem : The necessary and sufficient condition for a matrix A to be symmetric is that $A = A^{/}$



6.11 LET US SUM UP

- A rectangular array of mn numbers arranged in m-rows and n-columns and enclosed in square [ ] or a round bracket ( ), is called a matrix of order m×n (m by n).
- The numbers which constitute the matrix are called the elements of a matrix.
- The element which occurs in the i-th row and j-th column is called the (i,j) the element of the matrix and is denoted by a_{ij} .
- A matrix with exactly one row is called a row matrix. The order of a row matrix is of the type 1×n.
- A matrix with exactly one column is called a column matrix. The order of a column matrix is of the type m×1.
- A matrix A=[a_{ij}]$_{m\times n}$ is a square matrix if m=n.
- A square matrix a=[a_{ij}] is a scalar matrix if a_{ij} = $\begin{Bmatrix} k \text{ where } i = j \\ 0 \text{ where} i \neq j \end{Bmatrix}$
- A square matrix A = [a_{ij}] is a diagonal matrix if a_{ij} = 0 where i≠j.
- A square matrix A=[a_{ij}] is a unit matrix if a_{ij} = 1 where i = j

  = 0 where i≠j ;

  A unit matrix of order n is denoted by I or I_n.
- A matrix in which all elements are zero is called a null matrix or zero matrix. A null matrix is denoted by O.
- A matrix is said to be upper triangular if a_{ij} = 0 when i>j.
- A matrix is said to be lower triangular if a_{ij} = 0 when i < j.
- Two matrix are said to be equal if and only of they are of same order and the corresponding elements are equal.
- If A and B are two matrix of same order. Then their sum or difference denoted by A + B ,A–B or B–A can be obtained.
- For matrices A and B of the same order $(A+B)' = A' + B'$
- For matrix A and B conformable to multiplication

  $(AB)' = B'A'$
- The matrix A is called symmetric if $A'=A$



6.12 FURTHER READING

1) Agarwal, D.K. (2012). *Business Mathematics*, New Delhi: Vrindra Publication (p) Ltd.

2) Baruah, S. (2011). *Basic Mathematics and Its Application in Economics*, New Delhi: Trinity Press Pvt. Ltd.

3) Bose, D. (2004). *Mathematical Economics;* New Delhi: Himalaya Publishing House.

4) Chiang, A.C. (2006) *Fundamental Methods of Economics Analysis*; New Delhi: MC Graw Hill Education India.

5) Kandoi, Balwant (2011). *Mathematics for Business and Economics with Application;* New Delhi: Himalaya Publishing House.



6.13 ANSWERS TO CHECK YOUR PROGRESS

Answer to Q No 1: Find all possible orders of a matrix with 10 elements.

$10 = 1 \times 10$ $\quad$ $10 = 5 \times 2$

$10 = 2 \times 5$ $\quad$ $10 = 10 \times 1$

Possible orders of matrices are 1×10, 2×5, 5×2 and 10×1.

Answer to Q No 2: By equality of two matrices $\begin{pmatrix} 2a+b & 5c+d \\ a-2b & 4c+d \end{pmatrix} = \begin{pmatrix} 4 & 11 \\ 3 & 25 \end{pmatrix}$

2a+b=4 $\quad$ 5c–d=11

a–2b=–3 $\quad$ 4c+d=25

Solving these we get

a = 1, b = 2, c = 4, d = 9

Answer to Q No 3: For equality of two matrices, their orders and the corresponding elements must be same. Here, the order of the matrices are same. But (3,3)th element of $A \neq (3,3)^{th}$ element of B $\therefore A \neq B$

Answer to Q No 4:

(i) It is not possible for define A + B, since A is of order 3×2 and B is of order 2×2.

(ii) It is possible to define A+B, since A is of order 3×3 and B is of order 3×3.

(iii) It is possible to define A+B, since A and B are square matrices of the same order.

Answer to Q No 5:

$$B(C+D)=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\left(\begin{bmatrix}3 & 1\\ 4 & 2\end{bmatrix}+\begin{bmatrix}5 & 1\\ 7 & 4\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}8 & 2\\ 11 & 6\end{bmatrix}$$

$$=\begin{bmatrix}1x8+2x11 & 1x2+2x6\\ 3x8+4x11 & 3x2+4x6\end{bmatrix}$$

$$=\begin{bmatrix}30 & 14\\ 68 & 30\end{bmatrix}$$

$$BC+BD=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}3 & 1\\ 4 & 2\end{bmatrix}+\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & 1\\ 7 & 4\end{bmatrix}$$

$$=\begin{bmatrix}1x3+2x4 & 1x1+2x2\\ 3x3+4x4 & 3x1+4x2\end{bmatrix}+\begin{bmatrix}1x5+2x7 & 1x1+2x4\\ 3x5+4x7 & 3x1+4x4\end{bmatrix}$$

$$=\begin{bmatrix}11 & 5\\ 25 & 11\end{bmatrix}+\begin{bmatrix}19 & 9\\ 43 & 19\end{bmatrix}$$

$$=\begin{bmatrix}11+19 & 5+9\\ 25+43 & 11+19\end{bmatrix}=\begin{bmatrix}30 & 14\\ 68 & 30\end{bmatrix}$$

$$\therefore B(C+D)=BC+CD$$

$$\text{Again } (B+C)D=\left(\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}+\begin{bmatrix}3 & 1\\ 4 & 2\end{bmatrix}\right)\begin{bmatrix}5 & 1\\ 7 & 4\end{bmatrix}$$

$$=\begin{bmatrix}1+3 & 2+1\\ 3+4 & 4+2\end{bmatrix}\begin{bmatrix}5 & 1\\ 7 & 4\end{bmatrix}$$

$$=\begin{bmatrix}4 & 3\\ 7 & 6\end{bmatrix}\begin{bmatrix}5 & 1\\ 7 & 4\end{bmatrix}$$

$$=\begin{bmatrix}4x5+3x7 & 4x1+3x4\\ 7x5+6x7 & 7x1+6x4\end{bmatrix}$$

$$= \begin{bmatrix} 41 & 16 \\ 77 & 31 \end{bmatrix}$$

$$BD+CD= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 7 & 4 \end{bmatrix}+\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 7 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1x5+2x7 & 1x1+2x4 \\ 3x5+4x7 & 3x1+4x4 \end{bmatrix}+\begin{bmatrix} 3x5+1x7 & 3x1+1x4 \\ 4x5+2x7 & 4x1+2x4 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 9 \\ 43 & 19 \end{bmatrix}+\begin{bmatrix} 22 & 7 \\ 34 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 19+22 & 9+7 \\ 43+34 & 19+12 \end{bmatrix}=\begin{bmatrix} 41 & 16 \\ 77 & 31 \end{bmatrix}$$

$$\therefore (B+C)D = BD + CD$$

Answer to Q No 6:

If, A is of order 4×3

Then, B is of order 4×3

and C is of order 3×2

Answer to Q No 7:

$$\text{Given, } A + B = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix},\ A-B=\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\therefore A+B+A-B = 2A = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}+\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix}=\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$A = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}=\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

Again,

$$A+B-(A-B) = A+B-A+B=2B$$

$$= \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}-\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7-3 & 0-0 \\ 2-0 & 5-3 \end{bmatrix}=\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\therefore B = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}=\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

Answer to Q No 8:

$$A^2 = AA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0x0+1x1 & 0x1+1x0 \\ 1x0+0x1 & 1x1+0x0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$B^2 = BB = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0x0+(-i)i & 0x(-i)+(-i)x0 \\ ix0+0xi & ix(-i)+0x0 \end{bmatrix}$$

$$= \begin{bmatrix} -i^2 & 0 \\ 0 & -i^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Answer to Q No 9:

$A^2-5A+7I=A.A-5A+7I$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3x3+1(-1) & 3x1+1X2 \\ (-1)x3+2(-1) & (-1)x1+2x2 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$



6.14 MODEL QUESTIONS

Q 1: In a Matrix $\begin{bmatrix} 4 & -3 & 19 & \sqrt{2} \\ 11 & 24 & -7 & -5 \\ 1 & \sqrt{3} & 0 & 1 \end{bmatrix}$

Find (i) Order of the matrix

(ii) The number of elements
(iii) Write the elements a_{23}, a_{12}, a_{32}, $a_{34,}$,$a_{44.}$

Q 2: If $A = \begin{bmatrix} 4 & 5 \\ 6 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 3 & -4 \end{bmatrix}$, find $A + B$ and $A - B$

Q 3: If $A = \begin{bmatrix} 3 & 2 \\ 0 & -2 \\ 1 & 0 \end{bmatrix}$, find $3A$ and $-5A$

Q 4: Find AB and BA if $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 & 2 \\ -1 & 5 & 3 \end{bmatrix}$

Q 5: What is the additive identity of $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$?

Q 6: If $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 3 \\ -1 & 3 & 5 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 \\ 1 & 3 \\ 0 & 2 \end{bmatrix}$

$C = \begin{bmatrix} 3 & 1 & 2 \\ -2 & 2 & 0 \end{bmatrix}$, find $A(BC)$ and $(AB)C$

and show that $A[BC] = [AB]C$

Q 7: Show that $A[B+C] = AB + AC$

where $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 & 5 \\ -1 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 & -3 \\ 6 & 0 & 4 \end{bmatrix}$

Q 8: If $A = \begin{bmatrix} 4 & 3 & 7 \\ 2 & 5 & 6 \end{bmatrix}$ show that $(A')' = A$.

Q 9: If $A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $(AB)' = B'A'$

Q 10: If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then show that $A^4 = \begin{bmatrix} 1+2\times4 & -4\times4 \\ 4 & 1-2\times4 \end{bmatrix}$

Q 11: Find A^{-1} where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

Q 12: Find the inverse of the matrix $\begin{bmatrix} 4 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

Q 13: If A = $\begin{bmatrix} 2 & -4 & -2 \\ 4 & 6 & 2 \\ 0 & 10 & -4 \end{bmatrix}$, find adj A

Q 14: If A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$, find A^{-1}

*** ***** ***

UNIT 7: LIMIT AND CONTINUITY

UNIT STRUCTURE

7.1 Learning Objectives

7.2 Introduction

7.3 Limits

- 7.3.1 Finding Limits Analytically
- 7.3.2 Examples of Evaluations of Limits Using Various Rules
- 7.3.3 Limits by the Method of Substitution
- 7.3.4 A Special Limit
- 7.3.5 Some Other Special Limits
- 7.3.6 One Sided Limits

7.4 Continuity

- 7.4.1 Basic Definitions and Example
- 7.4.2 Algebra of Continuous Functions

7.5 Let Us Sum Up

7.6 Further Reading

7.7 Answer To Check Your Progress

7.8 Model Questions

7.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- define the concept of limit.
- evaluate limits by algebraic rules.
- explain the concept of continuity of a function.
- show that some well known functions are continuous.

7.2 INTRODUCTION

So far, we have discussed the Set theory and functions.

In this unit we are going to discuss about limit, continuity, differentiation and Integration, which are again very important and

fundamental in Mathematics.

The concept of limit is an abstract one. We will learn various algebraic rules to find limits at various points of well-known functions. We will also learn about left and right hand limits. We will also discuss about the continuity and differentiation of functions. Finally, we will discuss about the concept of integration. We will also discuss about two powerful methods of integration, namely, methods of substitution and the method of integration by parts.

7.3 LIMITS

We are already familiar with the concept of a real function and a real variable. The concept of limit is an abstract one. Consider the function $f(x) = \frac{x^2 - 9}{x - 3}$, which is defined at all points except $x = 3$. In the first column of following table, some values of x close to 3 are written and in the second column the corresponding values of $f(x)$ are written.

The value of the variable	The corresponding value of $f(x) = \frac{x^2 - 9}{x - 3}$
2.97	5.97
2.98	5.98
2.99	5.99
3.01	6.01
3.02	6.02
3.03	6.03

From the above table, we observe that if x is very close to 3, then $\frac{x^2 - 9}{x - 3}$ is very close to 6. It is also true that for all values of x close to 3, $\frac{x^2 - 9}{x - 3}$ is close to 6. We then say:

As x tends to 3, the function $\frac{x^2 - 9}{x - 3}$ tends to 6 or equivalently, we say: The

function $\frac{x^2-9}{x-3}$ has the limit 6 at the point 3.

In symbols, we write

$$\lim_{x\to 3}\frac{x^2-9}{x-3}=6.$$

In general, by $\lim_{x\to a} f(x)=l$ we mean that as gets close to *a* then $f(x)$ gets close to a limit of l. This can be put in a more precise way as: $|f(x)-l|$ can be made arbitrarily small, by taking $|x-a|$ to be sufficiently small.

The value of the function *f* at *a* is immaterial for finding the limit of *f* at *a*. Only the values that are close to *a* but not at *a* matter.

The following two results are simple consequences of the definition.

1. **Constant Rule:** $\lim_{x\to a} c=c$, that is, the limit of the constant function $f(x)=c$ as x tends to a is the value *c*.

2. **Identity Rule:** $\lim_{x\to a} x=a$, that is, the limit of the identity function $f(x)=x$ as x tends to *a* is the value a.

Uniqueness of Limit

If $\lim_{x\to a} f(x)$ exists, it is unique. There cannot be two distinct numbers l_1 and l_2 such that when x tends to *a* the function $f(x)$ tends to both l_1 and l_2.

7.3.1 Finding Limits Analytically

It is possible to find limits by using algebraic techniques. The limit behaves well with respect to the operations of addition, subtraction, multiplication by a constant, multiplication, division, etc. The most basic theorem governing analysis of limits is the Principal Limit Theorem.

Principal Limit Theorem:

Let *a* and *k* be real numbers, $n\geq 0$, and let f and g be functions with limits at *a* such that $\lim_{x\to a} f(x)=l$ and $\lim_{x\to a} g(x)=m$. Then

1. **Sum Rule:** $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = l + m.$

2. **Difference Rule:** $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = l - m.$

3. **Product Rule:** $\lim_{x\to a}[f(x)\bullet g(x)] = \lim_{x\to a} f(x) \bullet \lim_{x\to a} g(x) = l \bullet m.$

4. **Quotient Rule:** $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{l}{m}$ provided

$\lim_{x\to a} g(x) = m \neq 0.$

5. **Coefficient Rule:** $\lim_{x\to a} k\, f(x) = k \lim_{x\to a} f(x) = kl.$

6. **Power Rule:** $\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n = l^n.$

We can state the above rules in language. For example, Sum Rule could be expressed as **"The limit of a sum is the sum of the limits."**

7.3.2 Examples of Evaluation of Limits Using Various Rules

Example 7.1. Find $\lim_{x\to 2} x^2.$

Solution: $\lim_{x\to 2} x^2 = \left(\lim_{x\to 2} x\right)^2$ (by Power Rule)

= 2^2 (by Identity Rule)

= 4

Example 7.2. Find $\lim_{x\to a} 2x^3 + 3x^2 + 4x + 5.$

Solution:

$$\lim_{x\to a} 2x^3 + 3x^2 + 4x + 5$$

= $2\lim_{x\to a} x^3 + 3\lim_{x\to a} x^2 + 4\lim_{x\to a} x + \lim_{x\to a} 5$ (by Sum and Coefficient Rules)

= $2\left(\lim_{x\to a} x\right)^3 + 3\left(\lim_{x\to a} x\right)^2 + 4\lim_{x\to a} x + \lim_{x\to a} 5$ (by Power Rule)

= $2a^3 + 3a^2 + 4a + 5.$ (by Identity and Constant Rules)

Remark: It is clear from the above example that for all polynomial functions, the limit at a point can be obtained by substituting that

value for the variable *x*. But, for non-polynomial functions, this may not be true always. The next two examples show this fact.

Example 7.3. Find $\lim_{x\to 1}\frac{2x-1}{x^3-2}$.

Solution:

$$\lim_{x\to 1}\frac{2x-1}{x^3-2} = \frac{\lim_{x\to 1}(2x-1)}{\lim_{x\to 1}(x^3-2} \quad \text{(by Quotient Rule)}$$

$$= \frac{\lim_{x\to 1}2x-\lim_{x\to 1}1}{\lim_{x\to 1}x^3-\lim_{x\to 1}2} \quad \text{(by Difference Rule)}$$

$$= \frac{2\lim_{x\to 1}x-\lim_{x\to 1}1}{(\lim_{x\to 1}x)^3-\lim_{x\to 1}2} \quad \text{(by Coefficient and Power Rules)}$$

$$= \frac{2(1)-1}{(1)^3-2} \quad \text{(by Constant and Identity Rules)}$$

$$= -1.$$

Example 7.4. Find $\lim_{x\to 2}\frac{x^2-4}{x^2-3x+2}$.

Solution: Since the denominator function x^2-3x+2 has the limit 0 at the point 1, that is, $\lim_{x\to 2}x^2-3x+2=0,$ so Quotient Rule cannot be applied here. Therefore, we have to resort to another method. Factorizing both the numerator and the denominator, we find that

$$\frac{x^2-4}{x^2-3x+2}=\frac{(x-2)(x+2)}{(x-1)(x-2)}.$$

When $x\neq 2,$ we can cancel the common factor $x-2$.

Thus,

$$\frac{x^2-4}{x^2-3x+2}=\frac{x+2}{x-1}.$$

Therefore,

$$\lim_{x\to 2}\frac{x^2-4}{x^2-3x+2} = \lim_{x\to 2}\frac{x+2}{x-1} = \frac{\lim_{x\to 2}x+\lim_{x\to 2}2}{\lim_{x\to 2}x-\lim_{x\to 2}1} = \frac{2+2}{2-1} = 3$$



CHECK YOUR PROGRESS

Q 1: Write true (T) or false (F).

(a) $\lim_{x\to 1}(x-1)^2+5=5.$ (T/F)

(b) $\lim_{x\to 2}(x-2)(x+4)=0.$ (T/F)

(c) $\lim_{x\to 3}\dfrac{x^2-9}{x-3}=0.$ (T/F)

(d) $\lim_{x\to -1}\dfrac{x^2-1}{x+1}=2.$ (T/F)

7.3.3 Limitis by the Method of Substitution

Consider the example $\lim_{x\to 1}\dfrac{x^2-1}{x-1}$. We find that

$$\lim_{x\to 1}\frac{x^2-1}{x-1}=\lim_{x\to 1}\frac{(x-1)(x+1)}{x-1}=\lim_{x\to 1}(x+1)=2$$

The same limit can be evaluated with the help of substitution. If we substitute $y=x-1,$ i.e. $x=y+1,$ in the expression $\dfrac{x^2-1}{x-1}$, then $\dfrac{x^2-1}{x-1}=\dfrac{(y+1)^2-1}{y}=y+2.$ Also, note that when x tends 1, then y tends 0. Therefore,

$$\lim_{x\to 1}\frac{x^2-1}{x-1}=\lim_{y\to 0}(y+2)=2.$$

In general,

$$\lim_{x\to a}f(x)=\lim_{h\to 0}f(a+h).$$

Equivalently,

$$\lim_{x\to a}f(x)=\lim_{y\to a+h}f(y),\quad where\ y=x+h.$$

Example 7.5. Find $\lim_{x\to 1}\dfrac{(x-1)^4}{x^4-1}$.

Solution: We have

$$\lim_{x\to 1}\frac{(x-1)^4}{x^4-1}=\lim_{y\to 0}\frac{y^4}{(y+1)^4-1}=\lim_{y\to 0}\frac{y^4}{y^4+4y^3+6y^2+4y}.$$

$$= \lim_{y \to 0} \frac{1}{y^3 + 4y^2 + 6y + 4} = \frac{1}{4}.$$

7.3.4 A Special Limit

Let us evaluate $\lim_{x \to a} \frac{x^n - a^n}{x - a}$, where n is a positive integer.

By the method of substitution, we find that

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = \lim_{h \to 0} \frac{(a+h)^n - a^n}{a + h - a}$$

$$= \lim_{h \to 0} \frac{1}{h} \left[a^n + \binom{n}{1} a^{n-1} h + \binom{n}{2} a^{n-2} h^2 + \ldots + h^n - a^n \right]$$

[By Binomial Theorem]

$$= \lim_{h \to 0} \left[\binom{n}{1} a^{n-1} + \binom{n}{2} a^{n-2} h + \ldots + h^{n-1} \right] = \binom{n}{1} a^{n-1} = n a^{n-1}.$$

Thus,

$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$, where n is a positive integer.

The above formula remains true, when n is a rational number, provided a is positive.

Example 7.6. Find $\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9}$.

Solution:

$$\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9} = \lim_{x \to 3} \left[\frac{x^3 - 27}{x - 3} \div \frac{x^2 - 9}{x - 3} \right] = \lim_{x \to 3} \left[\frac{x^3 - 27}{x - 3} \right] \div \lim_{x \to 3} \left[\frac{x^2 - 9}{x - 3} \right].$$

$$= \lim_{x \to 3} \left[\frac{x^3 - 3^3}{x - 3} \right] \div \lim_{x \to 3} \left[\frac{x^2 - 3^2}{x - 3} \right] = 3(3)^2 \div 2(3) = 27 \div 6 = \frac{9}{2}.$$

Example 7.7. If $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 12$ and if n is a positive integer, find n.

Solution: We have $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = n 2^{n-1}$. According to the given question, $n 2^{n-1} = 12 = 3.2^2$. Therefore, $n = 3$.

7.3.5 Some Other Special Limits

The following limit rule exponential functions are very useful.

Example 7.8. Find $\lim_{x\to 0}\frac{e^{4x}-1}{x}$.

Solution: We have

$$\lim_{x\to 0}\frac{e^{4x}-1}{x}=\lim_{x\to 0}\frac{e^{4x}-1}{4x}\cdot 4=4\lim_{y\to 0}\frac{e^{y}-1}{y}=4.$$

7.3.6 One Sided Limits

In this section we will discuss one sided limits.

The notation $\lim_{x\to a+} f(x)=l$ means that when is close x to a and greater than a then $f(x)$ is close to l.. Similarly, $\lim_{x\to a-} f(x)=l$ means that when is close x to a and less than a, then $f(x)$ is close to l.

$\lim_{x\to a+} f(x)$ is called the **right limit** of f at a and $\lim_{x\to a-} f(x)$ is called the **left limit** of f at a. Both these are called **one-sided limits** of f at a

Example 7.9. Find $\lim_{x\to 0+}\frac{x}{|x|}$ and $\lim_{x\to 0-}\frac{x}{|x|}$.

Solutions: When x takes positive values, then $\frac{x}{|x|}=\frac{x}{x}=1.$

Therefore, we have $\lim_{x\to 0+}\frac{x}{|x|}=1.$

On the other hand, when takes negative values, then $\frac{x}{|x|}=\frac{x}{-x}=-1.$ So, $\lim_{x\to 0-}\frac{x}{|x|}=-1.$

If $\lim_{x\to a+} f(x)\neq\lim_{x\to a-} f(x)$ then $\lim_{x\to a} f(x)$ does not exist.

If $\lim_{x\to a+} f(x)=\lim_{x\to a-} f(x)=l$ then $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a} f(x)=l.$

Example 7.10. Decide whether $\lim_{x\to 0}\frac{x}{|x|}$ exists or not.

Solution: From the solutions of Example 7.9, we note that $\lim_{x\to 0+}\frac{x}{|x|}=1$ and $\lim_{x\to 0-}\frac{x}{|x|}=-1.$ Since the right limit and left limit are not equal, therefore $\lim_{x\to 0}\frac{x}{|x|}$ does not exist.



CHECK YOUR PROGRESS

Q 2: Write true (T) or false (F).

(a) If *f* is a real function such that *f* (*x*) = *f* (–*x*) for all *x* , then $\lim_{x\to 0+} f(x)=\lim_{x\to 0-} f(x).$ (T /F)

(b) $\lim_{x\to 1-}[x]=1.$ (T / F)

(c) $\lim_{x\to 1+}[x]=1.$ (T / F)

Q 3: Find the following limits:

(i) $\lim_{x\to 2}\frac{x-2}{x+2}$ (ii) $\lim_{x\to 0}\frac{(1+x)^6-1}{(1+x)^2-1}$ (iii) $\lim_{x\to 3}\frac{e^x-e^3}{x-3}$

7.4 CONTINUITY

In this section, we will learn about the concept of continuity of a function. We will show that a large class of well known functions is continuous.

7.4.1 Basic Definitions and Examples

Let *f* be a real function and let *a* be in the domain of *f* If the limit of *f* at *a* is same as the value of *f* at *a*, then *f* is said to be continuous at *a*.

Thus, if *f* is to be continuous at *a* the following three conditions must be satisfied.

a) *f (a)* is defined,

b) $\lim_{x\to a} f(x)$ exists,

c) $\lim_{x\to a} f(x)=f(a).$

Example 7.11. Show that every constant function is continuous at all points.

Solution: Consider a constant function $f(x) = c$ and an arbitrary point a.

Then $f(a) = c$ and $\lim_{x \to a} f(x) = \lim_{x \to a} c = c,$ and so, $\lim_{x \to a} f(x) = f(a).$

Therefore, f is continuous at a

Example 7.12. Show that the identity function is continuous at all points.

Solution: Consider the identity function $f(x) = x$ and an arbitrary point *a*.

Then, we have $f(a) = a$ and $\lim_{x \to a} f(x) = \lim_{x \to a} x = a.$

Thus, $\lim_{x \to a} f(x) = f(a),$ which shows that f is continuous at 'a'.

Example 7.13. Show that the greatest integer function [*x*] is not continuous at 0.

Solution: Let $f(x) = [x].$

Now, $\lim_{x \to 0+} f(x) = \lim_{x \to 0+} [x] = 0$ and $\lim_{x \to 0-} f(x) = \lim_{x \to 0-} [x] = -1$.

Thus, the right hand and left hand limits of f at 0 are not equal.

Which means that $\lim_{x \to 0} f(x) = \lim_{x \to 0} [x]$ does not exist. Therefore, though $f(0) = [0] = 0,$ *f* is not continuous at 0.

Some more results are listed in the following table.

(c) The function x^2 is continuous at 3.

(d) The function $\frac{x-4}{x-1}$ is continuous at 2.

(e) The function $\sin x$ is continuous at .

(f) The function $\sin x$ is continuous at 0.

(g) The function $\cos x$ is continuous at 0.

(h) The function e^x is continuous at 0.

(i) The function $\frac{x}{|x|}$ is not continuous at 0.

Definition:

A real function is said to be continuous in an open or closed interval if it is continuous at every point of the interval.

Remark: When a function f is considered on a closed interval $[a,b]$, then *f* is said to be continuous at the end point *a* if $\lim_{x\to a+} f(x) = f(a).$ Similarly, *f* is said to be continuous at the end point *b* if $\lim_{x\to b-} f(x) = f(b).$

7.4.2 Algebra of Continuous Functions

Theorem: Let *f* and *g* be two functions, continuous at *a* and let *c* be a real number. Then

a) $f + g$ is continuous at a

b) $f - g$ is continuous at a

c) *cf* is continuous at a

d) *fg* is continuous at a

e) $\frac{f}{g}$ is continuous at *a* provided $g(a) \neq 0.$

Proofs. Follow from the Principal Limit Theorem and the definition of continuity.

If f and g are continuous functions, that is, they are continuous at every point of their domains, then the above theorem can be generalized in the following way.

Theorem: If f and g are continuous functions, then

(a) $f+g$ is continuous,

(b) $f-g$ is continuous,

(c) cf is continuous at,

(d) fg is continuous at,

(e) $\frac{f}{g}$ is continuous, at those points where g does not take the value zero.

The statement (a) above means that the sum of two (or even more) continuous functions is continuous. Similarly, statement (b)

means that the difference of two continuous functions is continuous, and so on.

Example 7.14. Show that the function $2x^2 + 3x + 4$ is continuous at all points.

Solution: We have already proved that the identity function *x* is continuous at all points. Thus, x^2 is continuous at all points being the product of *x* and *x*. Again, the constant functions 2 and 4 are continuous at all points. Therefore, $2x^2$, $3x$ and 4 are continuous at all points. Finally, $2x^2 + 3x + 4$ is continuous at all points being the sum of three continuous functions.

Example 7.15. At What points is the function $\frac{x-2}{(x-3)(x-5)}$ continuous?

Solution: The given function is not defined at the points 3 and 5. The functions in the numerator, i.e., $x - 2$ and in the denominator, i.e., $(x-3)(x-5)$ are continuous at all points. Therefore, the function $\frac{x-2}{(x-3)(x-5)}$ is continuous at all those points where the denominator does not take the value zero. Thus, it is continuous at all points except at 3 and 5.



7.8 LET US SUM UP

- In this unit, we have learnt about the concept of limits and various algebraic rules to find limits at various points of well-known functions. We have also learnt about the left and right hand limits, and found that they need not be equal always.
- We have learnt about the continuity of functions. In summary, we can say that if the limit exists at a point and it is equal to the value of the function at that point then the function is continuous at that point.



7.9 FURTHER READING

1) Khanna V. K., Zameeruddin Qazi & Bhambri S.K.(1995). Business Mathematics, New Delhi, Vikas Publishing House Pvt Ltd. .

2) Hazarika P.L. Business Mathematics, New Delhi. S.Chand & Co.



7.10 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: (a) True (b) True (c) False (d) False

Ans to Q No 2: (a) True (b) False (c) True

Ans to Q No 3: (i) 0 (ii) 3, (iii) e^3.



7.11 MODEL QUESTIONS

Q 1: Evaluate the following limits:

(a) $\lim_{x\to 3}\dfrac{x^2-7x+12}{x^2-4x+3}$ (b) $\lim_{x\to 0}\dfrac{\sqrt{1+x}-1}{x}$

(c) $\lim_{x\to 0}\dfrac{\sin 10x}{\sin 11x}$ (d) $\lim_{x\to k}\dfrac{x^{10}-k^{10}}{x^5-k^5}$

Q 2: Suppose a function *f* is defined as follows.

$$f(x)=\begin{cases} x, & x\geq 0 \\ -x, & x>0. \end{cases}$$

By evaluating the left and right hand limits, show that $\lim_{x\to 0} f(x)=0.$

Q 3: If *f* is an odd function (i.e., $f(-x)=-f(x), \quad \forall\, x.$) and $\lim_{x\to 0} f(x)$ exists, then show that the value of this limit is zero.

Q 4: Decide whether the function given in Question 2 is continuous at $x=0$ or not.

UNIT 8: INTRODUCTION TO STATISTICS

UNIT STRUCURE

8.1 Learning Objectives

8.2 Introduction

8.3 Meaning and Definition of Statistics

8.4 Main Features of Statistics

8.5 Scope of Statistics

8.6 Application of Statistics in Business & Economics

8.7 Let Us Sum Up

8.8 Further Reading

8.9 Answers to Check Your Progress

1.10 Model Questions

8.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- put forward the meaning and definition of statistics
- disucss the main features of statistics
- explain the scope of statistics
- distinguish between statistical data and statistical methods
- outline the applications of statistics in business & economics
- describe the different methods of presenting statistical information
- explain how tabulation of data is done
- discuss the different parts of a table.

8.2 INTRODUCTION

This unit introduces to the field of statistics. We deal with various statistical information in our day to day life. As a branch of study, Statistics provides us the tool through which such data are collected, analyzed, summarized, and presented to facilitate the decision making process. To understand our life ans society around us, statistical information plays a pivital role. Let us step into the subject with discussion some of the basic concepts in this Unit entitled *'Introduction to Statistics'*.

8.3 MEANING AND DEFINITION OF STATISTICS

The word 'Statistics' has been derived either from Latin word Status or Italian word Statistica or German word Statistick. Each carry same meaning 'An Organized Political State'. Some people consider Statistics as Data, Facts or Measurements, simply saying numerical facts while for some Statistics is analysis of figures for prediction and for some it is presentation of facts in graphs.

In this way Statistics exists in the following three forms:

a) As Numerical Facts: In this form it includes numerical facts such as National Income, Export, Import, Production, Employment Statistics etc.

b) As a subject: It exists in Singular as well as plural form.

Plural		Singular
Numerical data relating to an aggregate of individuals	**Statistics**	The Science of collecting, analyzing & interpreting data

It is used as a subject in singular form like Economics, Physics Etc. while in plural form it is the subject matter. In other words in singular sense Statistics refers to the whole body of analytical tools that are used to collect the figures, organize and interpret them and finally to draw conclusions. In plural sense, it refers to a set of figures such as production, sale etc.

c) It may also refer to the study of, and research into the theory and principles underlying statistical methods.

According to Tate the three meanings of the word statistics are contained in the statements "You compute Statistics by Statistics from Statistics."

Definition of Statistics: According to Spiegal "Statistics is concerned with scientific method of collecting, organizing, summarizing, presenting and analyzing data as well as drawing valid conclusions and making reasonable decisions on the basis of such analysis".

Secrist highlights few major characteristics of Statistics as follows:

- Statistics are aggregates of facts. This means that a single figure is not statistics.
- Statistics are affected by a number of factors. For example, sale of product depends on number of factors such as price, demand, quality etc.
- Statistics must be reasonably accurate or in other words conclusions must be based on accurate figures.
- Statistics must be collected in a systematic manner because facts collected in haphazard manner will not be reliable and will lead to misleading conclusion.
- Finally, Statistics should be placed in relation to each other. Logical collection of data helps in avoiding confusion.

A. L. Bowley defined Statistics as: "Science of Counting, Science of averages and Science of the measurement of social organism regarded as a whole".

According to P.H. Karmel "The subject Statistics is concerned with the collection, presentation, description and analysis of data which are measurable in numerical form".

8.4 MAIN FEATURES OF STATISTICS

After going through the above definitions, you must have gathered some basic idea of statistics. Now lets discuss some of its major features:

- Statistics are aggregate of facts: A single numerical record or fact is not statistics; in fact it is the aggregate of facts. For example, salary of person for a particular month, marks of a student in Economics in some exam etc.
- Statistics are expressed numerically: Statistical facts are expressed in the form of numbers. In other words it is quantitative in nature. For example, if we say that the performance of BA students in Economics this year is Average, then it is not statistics. The qualitative characteristics like honesty, beauty, intelligence etc. expressed by giving certain scores as numerical standards can be called as

statistics.

- Statistics are affected to a marked extent by multiplicity of causes: Statistical data are influenced by a number of factors. One cause alone can not affect data to a marked extent. For example, facts related to agriculture production are affected by a number of factors like rain, quality of soil, quality of seed, methods etc.
- Statistics are collected by enumerations or estimation according to reasonable standards of accuracy: Statistical data are enumerated or estimated according to reasonable standard of accuracy. The reasonable standard of accuracy depends on the purpose which the statistics are to serve.
- Statistics are collected in a systematic manner: Statistics are collected in a systematic manner according to a well prepared plan. The purpose for which the data are to be collected is decided and well defined in advance.
- Statistics are being capable of being placed in relation to each other: Statistical data are collected mostly for the purpose of comparison. The data must be homogenous and uniform in nature. For example, if we want to compare standard of living of some class in India then it must be compared with the class of the same nature not with the other class or foreign country data.
- Statistics tests hypothesis and laws of other subjects: Nearly every law and theory, frequently used today, were based on deductive method. These laws can be tested for their validity with the help of statistics.
- Statistics estimates for the present and predicts for the future: Statistical methods are not only helpful in estimating the present but also in forecasting the future. Extrapolation and interpolation are the techniques uses for the purpose. 5 year plans and other economic plans like budget etc are the examples under this heading.

8.5 SCOPE OF STATISTICS

Statistics is a tool of all science indispensable to research and intelligent judgment and has become a recognized discipline in its own right. Moreover, the scope of applied statistics is increasing day by day. The scope of statistics stretches over all those branches of human knowledge in which a group of the significance of large numbers is looked for. The scope of statistics is wide, still it can be summarized as:

Scope of Statistics	Theoretical Statistics	Probability (It deals with the chance variation of the observation)
	Statistical Methods	Collection, classification, tabulation, presentation, analysis, Interpretation, forecasting
	Applied Statistics	Descriptive applied statistics, Scientific applied Statistics.

Theoretical Statistics: It is the basis of the science of statistics, also called as Theory of Probability which deals with the chance variation of the observations.

Statistical Methods: In the words of Johnson and Jackson "Statistical methods are the procedures used in collection, organization, summary, analysis, interpretation and presentation of data." Simply speaking they are the devices by which complex and numerical data are analyzed.

Table 8.1: Difference between Statistical Data and Statistical Methods

S.No.	Statistical Data	Statistical Methods
1	They are available in raw form called raw data.	It helps to process the raw data.
2	They are facts expressed numerically.	It is a process.
3	They are descriptive in nature.	It is a tool of analysis.
4	They are meant for processing and processed data help in decision making.	It helps in analyzing the processed data.
5	The selection of statistical methods depends on the utility of data.	It is necessary to have a knowledge of statistical methods while collecting data.
6	They are senseless without application of statistical methods.	It remains idle for want of statistical data.

8.6 APPLICATION OF STATISTICS IN BUSINESS AND ECONOMICS

In the modern era statistics serves the base for the Mathematical approach to Economics. Statistical techniques have proved immensely useful in the solution of variety of Economics problems including trend analysis, production, consumption, GDP, income etc. cost of living index, price index, wholesale price index numbers, demand curves, forecasting techniques etc. very powerful tools in statistics used in analyzing Economics facts.

According to Marshall, “Statistics are the straw out of which, I like every other economist, have to make bricks.” According to Jevons, “I know not when we shall have a perfect system of statistics but the want of it is the only insuperable obstacle in the way of making Economics an exact Science.” According to Oskar Lange, “Economics is the science which deals with the determination by statistical methods of concrete quantitative laws occurring in economic life.” According to Tippet, “It may one day happen that Economic Department, at universities, instead of being dominated by the theorists will come under the domination of the statistical laboratory just as Physics and Chemistry Departments are dominated by the experimental laboratory.”

Today, business world is changing at rapid rate. Commerce, industry and other allied fields are also expanding. Due to industrial revolution, business activities have expanded and at the same time competition in the market has also increased. Industries have crossed the local and national market to international market. Thus, in this situation statistics helps in handling the activities like sales, purchase, production, marketing, control, finance, promotion etc. For this purpose role of statistical data and statistical tools like probability, expectation, sampling procedures, significance tests, theory of estimation and techniques of projection and forecasting and many others are of paramount importance. Business forecasting techniques are of vital use for getting estimates which serves as an indicator to future economic events. The study of time series, index numbers, business cycles,

which are supposed to be Economic Barometers and are prepared by accurate statistical data, is of great importance for businessman. Boddington rightly observed that, "The successful businessman is the one whose estimates most closely approaches accuracy."

In decision making, statistical tables and charts are frequently used by managers to present numerical facts. Businessman and industrialists rely so much on statistical methods in managerial decisions.

Statistical methods also deal with researches on market and production. Market research and market surveys by sampling methods are now extremely useful for any business. Price and demand of similar products available in the present market, tastes of consumers, seasonal variations, etc, are analyzed statistically.

QC i.e. quality control is also gaining momentum in business world. Statistics has already made significant change in maintaining and improving quality through quality control techniques.

According to M. M. Blair, "If all price statistics were removed from all papers, magazines, radio and telegraphic reports for a single day, the business world would be paralysed. If all statistics now available were removed from the world for one year, utter economic chaos and ruin would result."

Thus, it can be concluded that applications of statistical methods in commerce, business and industry has become a condition of progress. Industry and business world involve risk and when it is risk, it is better to have a calculated risk.



CHECK YOUR PROGRESS

Q1: Mention any three important distinctions between statistical data and statistical methods. (Answer in about 60 words)

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8.7 LET US SUM UP

- The word 'Statistics' has been derived either from Latin word Status of Italian word Statistica or German word Statistick. Each carry same meaning 'An Organized Political State'.
- Statistics is a tool of all science indispensable to research and intelligent judgment and has become a recognized discipline in its own right.
- In the modern era statistics serves the base for the Mathematical approach to Economics.



8.8 FURTHER READING

1) Beri, G. C. (2010). *Business Statistics* (3rd Ed.). New Delhi: Tata McGraw Hill Education Private Limited.

2) Shukla, S. M. & Sahai, S. P. (2008). *Business Statistics Theory & Practice*. Agra: Sahitya Bhawan Publications.



8.9 ANSWERS TO CHECK YOUR PROGRESS

Major three distinctions between statistical data and statistical mehtods are:

(a) Statistical data are basically are available in raw form. On the other had, statistical methods help us to process such raw data.

(b) Statistical data are expressed numerically. Statistical methods are processes.

(c) Statistical data descriptive in nature. On the other hand, statistical methods are tools of analysis of data.



8.10 MODEL QUESTIONS

A) Short questions: (Answer each question in about 150 words)

Q.1: Define the term Statistics.

Q.2: Discuss the scope of Statistics.

Q.3: Differentiate between statistical data and statistical methods.

B) Long questions: (Answer each question in about 300-500 words)

Q.1: Discuss the major features of Statistics.

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UNIT 9: DATA

UNIT STRUCURE

9.1 Learning Objectives

9.2 Introduction

9.3 Meaning and Types of Data

9.4 Methods of Presenting Statistical Information

9.5 Individual, Discrete and Grouped Series

9.6 Tabulation of Data: Meaning, Importance and Objectives

9.7 Parts of Table

9.8 Let Us Sum Up

9.9 Further Reading

9.10 Answers to Check Your Progress

9.11 Model Questions

9.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- discuss the meaning, and types of data
- describe the different methods of presenting statistical information
- explain how tabulation of data is done
- discuss the different parts of a table.

9.2 INTRODUCTION

Dear learners, you must have come across various facts in your daily life involving numbers. Such as Temperature, Price of the commodity, Quantity of items, Share market etc. These facts in statistical terms are called as data. The question arises, what to do with these facts or what type of information these facts carry? The answer to the question is – it depends on person to person that what type of information they need.

Around the world, specially the business world, most of the decisions are made with the information gathered about the companies, market place, the economic and the financial environment. Such information usually comes

in the form of data or accompanied by data. This unit will provide you the foundational concepts of different methods of presentation of data which help in the decision making process.

9.3 MEANING AND TYPES OF DATA

Collection of data is the backbone of statistical investigation. Data is aggregate of facts and these facts are collected in a systematic manner for a definite and pre planned purpose.

Data can be categorized on the following bases:

1) On the basis of organization of data.
2) On the basis of sources of data.
3) On the basis of collection of data.
4) On the basis of characteristics of data.

On the basis of organization: It can be divided into two:

- **Raw data:** The data originally collected or original facts collected through questionnaire, schedules etc. are called raw data. In its original form it is also called as ungrouped data.
- **Processed data:** The raw data or ungrouped data passed through at least one statistical method, organization, classification, tabulation, computations etc. are called processed data.

On the basis of Sources: On the basis of sources it can be divided into two:

- **Internal Data:** The data collected from internal source for any specific purpose and after preparing a plan are called internal data. For example, if an economic condition of persons living in some city is to be studied, the data obtained from each person is called as internal data.
- **External data:** The data obtained from external sources are known as external data.

On the basis of Collection of data: There are two types of data based on its collection:

- **Primary data:** The data collected by the researcher itself is called as primary data. According to Horace Secrist, "By primary data are mean those data which are original, that is, those in which little or no grouping has been made, the instance being recorded or itemized as encountered. They are essentially raw materials."
- **Secondary data:** Data which have already been collected by some other investigator/person/organization for their purpose and the present investigator uses them are called secondary data. According to M. M. Blair, "Secondary data are those already in existence and which have been collected for some other purpose than answering of the question at hand." For example, if a person uses the data regarding foreign import-export published by government for studying payment-balance, then these data are secondary data.

Note: Primary data is more reliable than secondary data because it is collected by the investigator himself/herself.

Table 9.1: Comparison of Primary and Secondary data

S.No.	Basis	Primary Data	Secondary Data
1	Originality	Primary data are original i.e. collected First time by the investigator itself.	Secondary data are not original, i.e. they are already in existence and are used by investigator.
2	Organization	Primary data are like raw material.	Secondary data are in the form of finished product. They have passed through statistical methods.
3	Purpose	Primary data are according to the object of investigation and are used without correction.	Secondary data are collected for some other purpose and are corrected before use.
4	Expenditure	The collection of primary data requires large money, energy and time.	Secondary data are easily available from secondary sources. (Published and Unpublished)
5	Precautions	Precautions are not necessary in the use of primary data.	Precautions are necessary in the use of secondary data.

On the basis of characteristics: On the basis of characteristics, data are of two types:

- **Qualitative data:** The characteristics which cannot be measured but only their presence or absence in a group of individuals can be noted are called qualitative data or attribute. For example– Poverty, Unemployment, honesty etc.

  Qualitative data are of two types– Nominal data and ranked data. Categories based on gender (male, female), residential area (rural, urban) is examples of nominal data whereas the classification according to their height, short, average etc. is ranked data.
- **Quantitative data:** The characteristics which can be measured directly are called quantitative data. The measurable characteristics are also called as variable.For Height, weight, sales, production etc..

  Quantitative data or variables are again classified into two: (i) Discrete Variable & (ii) Continuous variable

  Discrete variable: A variable which takes only finite distinct values are known as discrete variable. For example– Number of employees, number of industries etc. It is expressed in whole numbers.

  Continuous variable: A variable which can theoretically assume all values within a certain interval or intervals is called continuous variable. For example– Age, Income of employees, height of persons etc. It is expressed in decimals.

9.4 METHODS OF PRESENTING STATISTICAL INFORMATION

The data collected initially from a given source, whether they are experiments, surveys, or observation, are presented with the help of the following methods:

1) Textual method: In this, the reader acquires information by reading the data simply.
2) Tabular method: Provides a more precise, systematic and orderly presentation of data in rows and columns. (We will be discussing it in the later part of the unit).
3) Semi-Tabular Method: It uses both textual and tabular methods.
4) Graphical Method: The utilization of graphs is most effective method

of visually presenting statistical results or findings. (We will be discussing it in the later part of the unit).

9.5 INDIVIDUAL, DISCRETE AND GROUPED SERIES

The process of arranging the data in a logical or systematic order is called as seriation and the data so arranged is called a statistical series. In other words when attributes or things are counted, measured or weighed and they are placed one after another in some orderly manner, they constitute a series.

According to Horace Secrist, "A series as used statistically may be defined as things or attributes of things arranged according to some logical order."

On the basis of Shape or Construction of data, statistical series are of three types:

(a) Individual, (b) Discrete & (c) Grouped

- **Individual series:** A series of individual observations where the items are listed singly is known as an individual series. For example sales of mobile of famous brand (say A) on Monday, Tuesday and Wednesday:

Day:	Monday	Tuesday	Wednesday
No. of mobiles sold:	58	62	45

An individual series arranged in an ascending or descending order of magnitude is called as array. The process of arranging the data in ascending or descending order is known as arraying of data.

- **Discrete series:** When the distinct values of a discrete variable or a continuous variable are presented in an ascending order or descending order along with their frequencies, the series is called discrete series. For example Marks obtained by 38 students of B.A. in Economics.

Marks obtained:	20	22	23	27	30	34	38	41
No. of students:	4	3	5	8	6	5	4	3

- **Grouped Series:** When the observations or measurements are arranged in ascending or descending order in groups along with

their frequencies, expressed in groups called as class intervals, is called as Grouped series. If the continuity of the group is not broken the grouped series is called as exclusive series while if the continuity is broken and the new interval starts with the next number, then it is called as inclusive series. Lets understand it with the help of examples:

Exclusive series:

Marks obtained:	0-20	20-40	40-60	60-80	80-100
No. of Students:	4	3	8	3	2

Here the continuity is maintained but when we consider a particular interval, say 20-40 then it will contain the data from 20 to less than 40 or excluding 40. Thus, it is called as exclusive series. Here the class interval is 20.

While in case of inclusive series, for example:

Marks obtained:	10-19	20-29	30-39	40-49	50-59	60-69
No. of students:	6	4	6	9	4	2

Here the both the limits of the intervals are considered, i.e. the upper as well as lower limit. Thus, it is called as **inclusive series**. While arranging them as **exclusive series** we have to subtract 0.5 from the lower limit and add 0.5 to the corresponding upper limit.

10-19 can be converted to exclusive as 9.5-19.5.

Here 0.5 is subtracted from the lower limit and 0.5 is added to upper limit to make the inclusive series an exclusive one.

Table 9.2: Comparison of Exclusive and Inclusive Series

S.No.	Exclusive Series	Inclusive Series
1	Upper limit of a class is included in the next Higher class.	Both limits are included in that class.
2	Upper limit of a class is equal to the lower limit of the next class.	Upper limit of a class is not equal to the lower limit of the next higher class.
3	This method is suitable in every situation.	This method is suitable in case of integers.
4	For further studies it is not necessary to change exclusive form to inclusive form.	Sometimes it is essential to change inclusive form to exclusive form.

9.6 TABULATION OF DATA: MEANING, IMPORTANCE AND OBJECTIVES

In broad sense, “tabulation is an orderly arrangement of data in columns and row.” In fact it is process between the collections of data on the one hand and its final analysis on the other.

According to Croxton and Cowden, “Either for one’s own use or for the use of others, the data must be presented in a suitable form.” This clearly shows the importance of systematic presentations of data.

According to Tuttle “A statistical table is the logical listing of related quantitative in vertical columns and horizontal rows of number with sufficient explanatory and qualifying words, phrases and statements in the form of title, headings and noted to make clear the full meaning of data and their origin.” Thus, a statistical table is a presentation of numbers in a logical arrangement with some brief explanation to show what they are, i.e. in rows and columns.

In a similar way, tabulation is a scientific process involving the presentation of classified data in an orderly manner so as to bring out their essential features and chief characteristics.

Objectives: Its objectives can be understand in these words, “Tabulation is one of the most important and ingenious device of presenting the data in a condensed and readily comprehensible form and attempts to furnish the maximum information contained in the data in the minimum possible space without sacrificing the quality and usefulness of data.’’

The major objectives of the tabulation are as follows:

- **Classification of the object of investigation:** Owing to presentation of data in a tabular form, the objects of investigation becomes clear and easily comprehensible. According to Bowley, “the function of tabulation in the general scheme or statistical investigation is to arrange in easily accessible form, the answer with which the investigation is concerned.”
- **Clarification of Characteristics of data:** The table, after removing the vague and irrelevant information, presents clearly the major characteristics of data.

- **Presentation of facts in minimum space:** A huge amount of data can be presented in relevant way in minimum possible space, which communicates the information in far superior way than textual material.
- **Facility of Statistical Processing:** Because of categorical classification of data presented in tabular form, statistical processing-comparability, analysis etc. becomes easier.

Importance/advantages of Tabulation: Tabulation is advantageous in many ways:

- **Simplification of facts:** The data presented in a tabular form are systematic and shows a definite pattern. This helps in understanding the trends and tendencies emerging from the information obtained under investigation in similar way. In other words, by tabulation data become simpler and easily accessible.
- **Comparability:** Tabulation makes comparison easier, and figures are presented in classified way in distinct rows and columns.
- **Classification:** It becomes easier to classify the data for the presentation in tabular form. Unless the classified data are presented in different rows and columns, the distinct characteristics of the data cannot be known.
- **Economical:** As a tabulated data requires less space and time for its presentation.
- **Referencing:** Tabulated data serves as good reference.
- **Interpretation and inference:** The data, well presented with clear caption without any ambiguity, can help in interpreting the inferring in better way.
- **Computation:** Statistical measures like mean, median, mode, dispersion etc. can easily be computed by the help of a tabled data rather than textual data. It is not possible to calculate any such statistical measures and to draw graphs in the absence of data presented in tabular form.

Disadvantages/limitations of tabulation:

- A table does not possess the description of the data presented there

in. Hence, the persons who are not accustomed of assimilating facts from table are not able to understand it easily.

- Specialized knowledge is required to understand a table.

Requisites of a good table: A good table must possess the following characteristics:

- **Appropriate size:** The table should be neither too big nor too small. It should be attractive and beautiful.
- **Meaningful information:** The table should not contain too much of information. It is always better to have several manageable size small tables instead of a big table. (Too much of details should be avoided).
- **Comparable and Accessible:** Since statistical table provides the basis for analysis and computation of various statistical parameters/ measures like mean, S.D., correlation etc., it should be free from all sorts of errors. Besides, wherever necessary, these statistical measures- averages, percentages etc. should be given in the table itself to facilitate comparison.
- **As per the objective:** The table should be according to the objective of statistical investigation.
- **Scientifically Prepared:** While preparing a table, all the rules of tabulation should be followed.
- **Clarity:** A table should be prepared that even layman can draw inferences from it.

According to Harry Jarome “A good statistical table is triumph ingenuity and technique, a masterpiece of economy of space combined with a maximum of clearly presented information.”

9.7 PARTS OF TABLE

Raw data collected are transformed into suitable table and to have proper tabulation, it is necessary to know the main parts of it. These are as follows:

- **Table number:** Every table should be numbered so that it can be located/ identified. The number is normally indicated at the left top of

the table.

- **Title:** Each table must bear a title indicating the type of data contained. The title should not be very lengthy so as to run in several lines. It should be clear and unambiguous.
- **Captions and Stubs:** A table consists of rows and columns. The headings or sub-headings given in columns are known as captions while those given in rows are stubs. It is necessary that a table should have captions and stubs to indicate what columns and rows stand for. It is also desirable to provide for an extra column and row in the table for the column and row totals.
- **Main body of the table:** As this part of the table contains data, it is the most important part of it. Its size and shape should be suitable to accommodate the data. The data are entered from the top to the bottom in columns and from left to right in rows.
- **Ruling and Spacing:** A good table should invariably have proper ruling and spacing. Vertical lines are drawn to separate columns but horizontal lines are normally not drawn except in case of totals which must be separated from the main body of horizontal lines. In addition, the horizontal lines are drawn both at the top just below the title of the table and also at the bottom. Due care must be exercised in providing proper space in accordance with the dimensions or magnitude of figures.
- **Head note:** A head note is invariably given just below the title of a table indicating the unit of measurement applicable to the data displayed. This is shown generally in bracket.
- **Footnote:** Sometimes it becomes necessary to give some explanation for the data used or to explain the meaning of the abbreviation used. This should be given in the form of a footnote at the bottom of the table just below the last horizontal line.
- **Source note:** Whenever secondary data are used, it is necessary to show the source from which such data are taken. This is also in the form of footnote. Its purpose is to facilitate the reader to refer to that source if required.

Sample of a table:

Table number _____

Title_____

Stub Heading	**Captions**		**Captions**		**Total**
	Caption	Caption	Caption	Caption	
Sub Entries		Main	Body		
Total					

Footnote:

Source:



CHECK YOUR PROGRESS

Q.1: What is meant by tabulation of data? (Answer in about 40 words)

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Q.2: Mention any two distinctions between inclusive series and exclusive series. (Answer in about 60 words).

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9.8 LET US SUM UP

- Collection of data is the backbone of statistical investigation. Data is aggregate of facts and these facts are collected in a systematic manner

for a definite and pre planned purpose.

- Qualitative data are of two types– Nominal data and ranked data.
- The process of arranging the data in a logical or systematic order is called as seriation and the data so arranged is called a statistical series.
- In broad sense, "tabulation is an orderly arrangement of data in columns and row."



9.9 FURTHER READING

1) Beri, G. C. (2010). *Business Statistics* (3rd Ed.). New Delhi: Tata McGraw Hill Education Private Limited.

2) Shukla, S. M. & Sahai, S. P. (2008). *Business Statistics Theory & Practice*. Agra: Sahitya Bhawan Publications.



9.10 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q. No. 1: Tabulation of data means an orderly arrangement of data in columns and row. In fact, it is the process between the collections of data on the one hand and its final analysis on the other.

Ans. to Q. No. 2: Two important distinctions between inclusive series and exclusive series are: (a) in inclusive series, both limits are included in that class, while in exclusive series upper limit of a class is included in the next higher class, (b) in inclusive series, the upper limit of a class is not equal to the lower limit of the next higher class, while in exclusive series the upper limit of a class is equal to the lower limit of the next class.



9.11 MODEL QUESTIONS

A) Short questions: (Answer each question in about 150 words)

Q 1: Discuss the different parts of a table.

Q 2: Discuss the concepts of individual, discrete and grouped series.

Q 3: Explain the different parts of a Table.

B) Long questions: (Answer each question in about 300-500 words)

Q 1: Discuss the process of tabulation of data.

Q 2: Discuss the different methods of presenting statistical information.

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UNIT 10: VISUAL REPRESENTATION OF DATA

UNIT STRUCTURE

10.1 Learning Objectives

10.2 Introduction

10.3 Idea of Distribution and its Need

10.4 Frequency Distribution and its Classification

10.5 Method of forming Discrete and Grouped Frequency Distribution

10.6 Graphical Representation of Data

10.7 Histogram

10.8 Frequency Polygon

10.9 Ogive or Cumutative Frequency Curve

10.10 Cartogram or Mapograph

10.11 Pictorial Representation of Data

10.12 Let Us Sum Up

10.13 Further Reading

10.14 Answer to Check Your Progress

10.15 Model Questions

10.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- discus the idea of distribution and its need
- acquire knowledge about frequency distribution and its classification
- describe the different methods of forming discrete and grouped frequency distribution
- use graphical representation of data, viz., histogram, frequency polygon, ogive and cartogram or mapograph.

10.2 INTRODUCTION

Dear learners, in this unit, we are going to discuss the distribution of data, its method and graphical representation of facts. Thus, after going through this unit, you would know the meaning of frequency distribution and

outline its need. Apart from this, the general guidelines for the use of graphs and diagrams have also been discussed in this unit. Thus, this unit will be helpful for you to be able to draw different statistical graphs like– Histogram, Frequency polygon, ogive, Cartogram and Pictograph.

10.3 IDEA OF DISTRIBUTION AND ITS NEED

A tabular presentation of the data in which the frequencies of values of a variable are given along with them is called a frequency distribution.

According to Morris Humburg, "A frequency distribution of frequency table is simply a table in which data grouped into classes and number of cases which fall in each class are recorded. The numbers in each class are referred to as frequencies."

10.4 FREQUENCY DISTRIBUTION AND ITS CLASSIFICATION

The classification according to variables can be performed in two forms:

1) Discrete Series or Discrete frequency distribution.
2) Grouped Series or grouped frequency distribution.

Frequency: The number of times each value of a variable occurs is known as its frequency.

Frequency distribution: Frequency distribution is classified (tabular form) according to the number (frequency) possessing the same values of the variable. When a frequency distribution is presented in tabular form, it is also called as frequency distribution table.

Frequency Distribution can be categorized as follows:

Univariate Frequency Distribution: A frequency distribution which shows the frequency of occurrence of different values of a single variable is called as univariate frequency distribution.

For example, marks scored by students of Class X in Economics in final exam.

Bivariate Frequency Distribution: A frequency distribution based on two variables is called as bivariate frequency distribution.

For example marks scored by students of Class X in Economic and Statistics in Final Exam.

Discrete Frequency Distribution: A frequency distribution which is formed by distinct values of a discrete variable or a continuous variable is called as discrete frequency distribution.

Grouped Frequency Distribution: A frequency distribution which is obtained by dividing the entire range of the given observations on a discrete variable or continuous variable into groups and distributing the frequencies over these groups is called a grouped frequency distribution.

Dear students for understanding lets discuss the method of forming discrete and grouped frequency distribution in the next section.

10.5 METHOD OF FORMING DISCRETE AND GROUPED FREQUENCY DISTRIBUTION

Steps for forming discrete frequency distribution

i) Make a blank table consisting of three columns with the headings– variable, tally marks and frequency.

ii) Find the smallest observation and the largest observations; write the observations in ascending order or descending order.

iii) Read off the observations one by one from the data given and for each one, record a tally mark (I) against each observation. To facilitate the counting of tally marks, arrange them in group of five like (IIII), where every fifth tally is drawn across the preceding four. This is called 'Four and crossed method'. This process is continued till all the observations are considered.

iv) In the end, count all the tally marks in a row and write their number in the frequency column.

v) Write down the total frequency in the last row of the bottom.

Example: Prepare a discrete frequency distribution from the following data:

6	12	10	8	15	20	10	16	18	15
8	10	14	12	10	19	11	10	9	7

Discrete frequency distribution:

Variable	Tally Marks	Frequency
6	I	1
7	I	1
8	II	2
9	I	1
10	IIII ⟍	5
11	I	1
12	II	2
14	I	1
15	II	2
16	I	1
18	I	1
19	I	1
20	I	1
Total	**–**	20

Steps for forming a Grouped frequency distribution: For better understanding lets understand the terminology associated with it.

Class: Various groups formed in a quantitative classification are called classes.

Class interval: The various intervals (symbols for the class) are called class-intervals (L_1–L_2).

Class frequency: The number of observations falling within a particular class or class interval is known as class frequency or frequency of the corresponding class.

Class limits: The pairs of the numbers that represent the class or class-interval are called limits. In a specified class the greatest value that a variable can take is called the upper class limit (L_2) and the least value it can take in that class is called lower class limit (L_1)

Magnitude of the class – interval or width of the class: The difference between the two limits is called as magnitude or width of the class.

Width = L_2–L_1

In case of inclusive series, the width of the class is equal to the difference between the lower class limits of the consecutive classes.

Mid-value or Mid-point of the class: The value being half way between the lower and upper class limits of a class – interval is called its mid-value or central-value or mid-point. Thus,

Mid-point = (Lower-limit + Upper-limit)/2

Now let's take one example to understand how group series is formed.

30 students appeared in an examination. The marks obtained out of 50 are as follows:

2	5	12	22	8	6	11	22	26	32	4	37	39	42	24
9	10	16	24	31	46	44	28	35	40	15	27	47	6	14

Inclusive Method			Exclusive Method		
Marks	Tally marks	No. of Students	Marks	Tally marks	No. of Students
1-10	𝍸 III	8	0-10	𝍸 II	7
11-20	𝍸	5	10-20	𝍸 I	6
21-30	𝍸 II	7	20-30	𝍸 II	7
31-40	𝍸 I	6	30-40	𝍸	5
41-50	IIII	4	40-50	𝍸	5
Total		30	Total		30



CHECK YOUR PROGRESS

Q 1: Distinguish between univariate and bivariate frequency distribution. (Answer in about 30 words)

.....

.....

.....

Q 2: What does the term class frequency mean? (Answer in about 20 words)

.....

.....

10.6 GRAPHICAL REPRESENTATION OF DATA

Tabulation is one way of presenting data. Another way of presenting it is in the form of diagrams and graphs.

A. L. Boddington observes, “The wandering of a line is more powerful in its effect on the mind than a tabulated statement; it shows what is happening and what is likely to take place, just as quickly as the eye is capable of working.”

Meaning: The word ‘Graph’ could well be associated with the word ‘graphic’, which means ‘vivid’ or ‘spraining to life’. “This is exactly the kind of function which the graphic should perform for the table from which it is drawn, because most people can grasp pictures more readily than figures. Graphic methods enables statistician to present quantitative data in simple, clear and effective manner.”

Functions of Graph: The two major functions of graph are: (a) Tools of analysis & (b) Presentation of Data

As a tool of analysis it serves as a visual guide in planning of mathematical computations and general procedure of research study. It also helps the investigators to have the knowledge of development step-by-step, which becomes a good source of using his judgment and checking the result accuracy. Freehand curves may be fitted to data in more varied and flexible form than Mathematical curves.

Through graph, it is possible to present the complex data into simple and comprehensible form. Thus making comparison of the data a much easier task, as the data can be presented side by side.

Advantages of graphical presentation of data: Lowe has expressed the advantages of graphical presentation of data in his words “When properly constructed they readily show information that might otherwise be lost amid the details or chart show more than just the value of statistics, it also shows any trend that may be present and the way in which the trends change. Although this information is inherent in a table it may be quite difficult and time consuming to determine the existence and nature of trends from tabulation of data.”

The major advantages of graphical presentation of data are as under:

- **Easy to grasp and remember:** Because of their visual impact, the data presented through graphic and diagrammatic presentation are better grasped and remembered than the tabulated data.
- **Attractive and Impressive:** Graphs are always attractive and impressive than tabular form. In graph, all the features of data becomes visible at a glance and thus making it attractive and impressive.
- **Saves time and energy:** Because of its simple nature, it saves time and energy of both the statistician as well as observer.
- **Mathematical knowledge is not required:** There is no much requirement of technical knowledge or expertise in understanding the graphs, as they are very simple and easily comprehensible.
- **It makes complex thing simpler:** Even when data show highly complex relations among variables, these devices make them much clear. They thus greatly facilitate in the interpretation and analysis of data.
- **Location of positional averages:** It is possible to determine the positional averages like median, quartiles, mode etc., with the help of special kind of graphs.
- **Study of correlation:** The movements or the tendency of points (presented as graph) can easily be taken to know the correlation or regression between the variables. In simple words one can study the correlation between two sets of data, by observing the graph.
- **Forecasting and interpretation is easy:** Interpolation, extrapolation, forecasting of the data can easily be done with the help of graphs.
- **Comparisons:** Comparisons can be made between the two or more phenomena very easily with the help of graphs.

Disadvantages and limitations of graphical presentation of data: Major disadvantages of graphical presentation are as under:

- **Accuracy is not testable:** Graphs do not reveal the accuracy of data. They only show the fluctuations of data. Hence, they cannot be used to test the accuracy of the data.

- **Incomplete information:** Graphs cannot be used to depict more than two features. It is not possible to have solutions of all the problems from the graph. Hence, they furnish incomplete and inadequate information.
- **Misleading impression:** Two different scales may show different fluctuations in data. They may often give misleading impressions.
- **Not for quotation use:** The graphs cannot be quoted in support of some statement.
- **Requires some skill:** Some special types of graphs like ratio-variation graphs etc. cannot be understood and interpreted by laymen.
- **Possibility of misuse:** A small challenge or error in measurement scale may create large difference in the shape of graph and can give misleading results.

Frequency distribution graphs in which the measurements, class-limits or mid-values are taken along X-axis and frequencies along Y-axis, are of following types:

(1) Histogram, (2) Frequency polygon, (3) Frequency curves, (4) 'Ogive' or Cummulative Frequency curves.

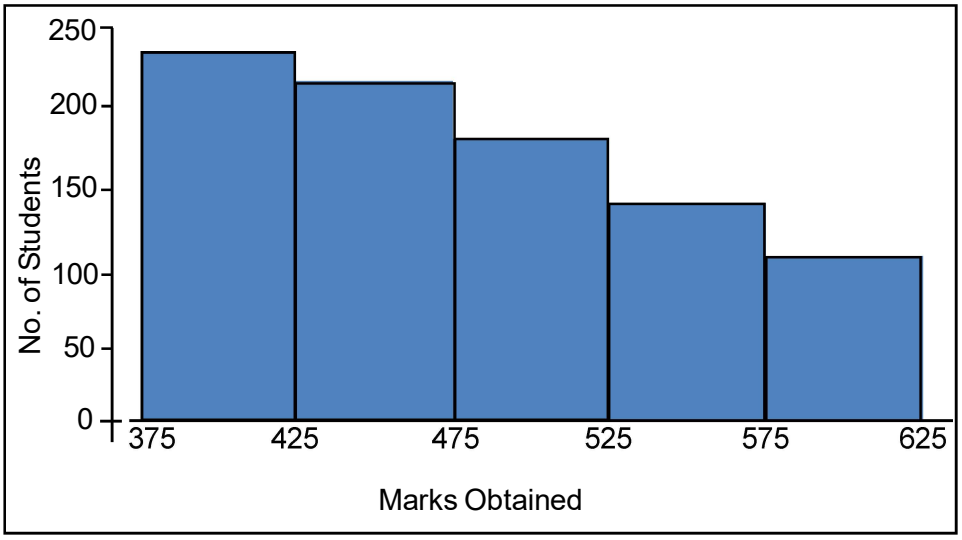
In the coming sections we will be discussing, one by one.

10.7 HISTOGRAM

Presentation of frequency distribution by a set of vertical bars-the Histograms, whose areas are proportional to the frequency represented, is the most popular and commonly used device. It consists of plotting the data as a series of rectangles. In construction of histograms, class intervals are shown on X-axis and frequencies on Y-axis. There as many rectangles as classes. The height of each rectangle represents the frequency of class. Histograms are also sometimes called as Staircase or Block Diagram.

Let's understand by taking one example:

Marks Obtained:	375-425	425-475	475-525	525-575	575-625
No. of Students:	225	180	175	125	100

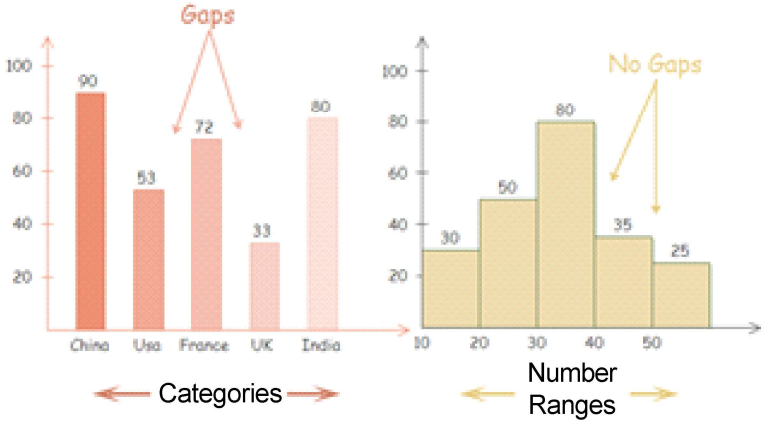


Here the marks obtained shown by Class intervals are on X-axis while the number of students falls in the corresponding groups are on Y-axis.



LET US KNOW

- The distinction between a bar chart and a histogram can be graphically represented as follows:



Bar Chart Histogram

- A histogram is a two dimensional diagram, where both the width (base) and the length (height) of the rectangle are important. On the contrary, a bar diagram is one dimensional diagram, in which only length (height) of the bar occupies significant place as the width of the bar is arbitrary.

- The bar diagrams are prepared at proper spacing between two bars, while in histograms, the bars are adjacent to each other.
- In the histogram, class frequencies are shown by the area of rectangles, whereas in bar diagrams, they are shown by the height of the bars.

10.8 FREQUENCY POLYGON

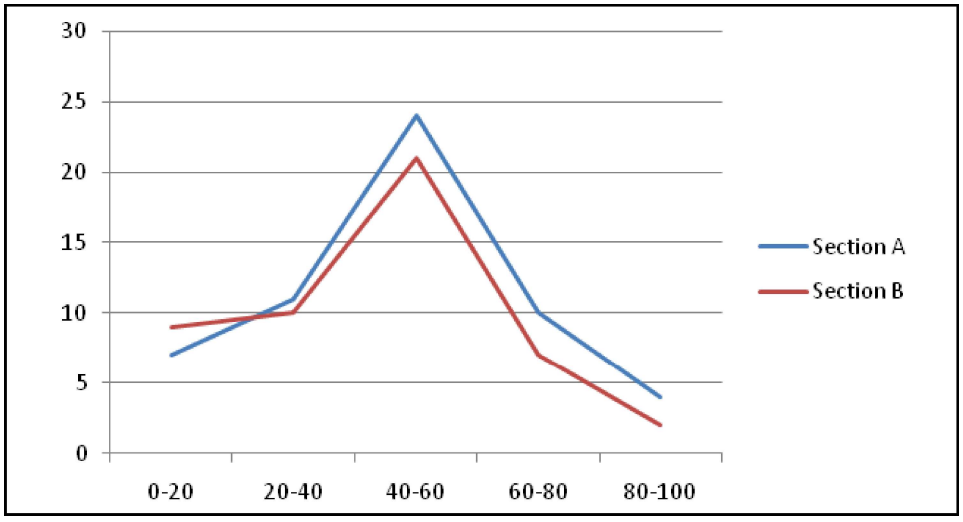
A frequency polygon is another way of graphic presentation of a frequency distribution, whether it is discrete or grouped or continuous. It can be effectively used to compare two or more frequency distribution.

A frequency polygon like any polygon consists of many angles. A histogram can be easily transformed into a frequency polygon by joining the mid-points of the rectangles by straight lines.

Even if required, one can draw straightway a frequency polygon by taking the mid-point of each class-interval and by joining the mid-points by the straight lines. Another point to be noted is that this can be done only when we have continuous series. In case of discrete distribution, this is not possible.

Let's clarify the concepts by taking one example:

Marks Obtained:	0-20	20-40	40-60	60-80	80-100
No. of Students (Section A):	7	11	24	10	4
No. of Students (Section B):	9	10	21	7	2



In the above frequency polygon the comparison can be done very easily as the two lines belonging to the two sections– Section A and Section B, are of different colours and can be easily understood.

Comparison of Frequency Polygon and Histogram:

- Frequency Polygon is a line graph where as Histogram is a collection of adjacent rectangles.
- Frequency Polygon is multi-dimensional, whereas Histogram is two-dimensional figure.
- Several frequency distributions can be plotted on the same axis as Frequency Polygon. This makes comparison- easier and meaningful. This is not possible in case of Histogram. However for the study of the relationship of the individual class frequencies to the total frequency, histogram is preferred to the frequency polygon, as the former gives a better picture.
- Frequency Polygon is a continuous curve and has all the distinct advantage of graphic representation, viz, it may be easily used to determine the scope, rate of change, estimates (interpolation and extrapolation), etc., wherever admissible. This is not possible in the case of Histogram.

10.9 OGIVE OR CUMULATIVE FREQUENCY CURVE

Till now, we have learnt about graphical representation of frequency distributions by using Histogram and Frequency Polygon. In this section, we will learn about the construction of cumulative frequency polygon or cumulative frequency curve or Ogive.

The technique of drawing the cumulative frequency polygons and cumulative frequency curves or Ogives is more or less the same. The only difference is that in case of simple frequency curves and polygons the frequencies are plotted against class marks of the class intervals where as in case of a cumulative frequency curve, the cumulative frequencies are plotted against the lower or upper limits of the class intervals, depending upon the manner in which the series has been cumulated. There are two

methods of constructing an Ogive: Less than method and More than method.

Steps of constructing less than method ogive:

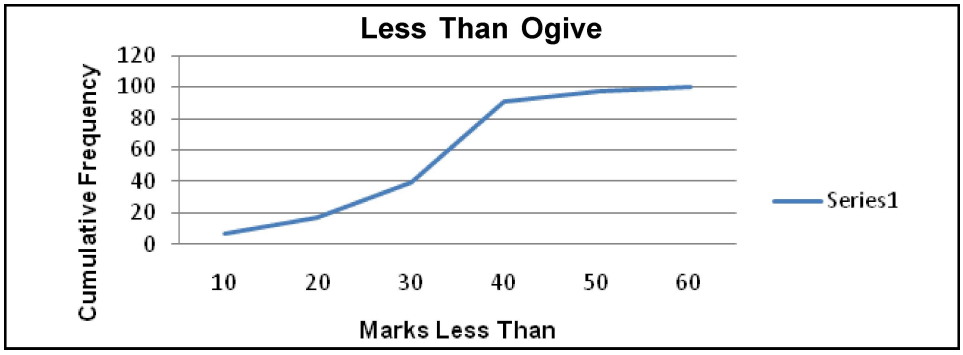
- Start with the upper limits of class intervals and add class frequencies to obtain the cumulative frequency distribution.
- Mark Upper class limits along X-axis on a suitable scale.
- Mark cumulative frequencies along Y-axis on a suitable scale.
- Plot the points (X_1, f_1), where X_1 is the upper limit of a class and f_1 is the corresponding cumulative frequency.
- Join the points obtained in Step 4 by a free hand curve to get the ogive.

Example: Draw an ogive for the following frequency distribution by less than method.

Marks:	0-10	10-20	20-30	30-40	40-50	50-60
No. of Students:	7	10	23	51	6	3

Ogive table:

Marks	No. of Students	Marks less than	Cumulative Frequency
0-10	7	10	7
10-20	10	20	17
20-30	23	30	40
30-40	51	40	91
40-50	6	50	97
50-60	3	60	100
Total	100		



Steps of constructing more than method ogive:

- Start with the lower limits of the class intervals and from the total frequency subtract the frequency of each class to obtain the

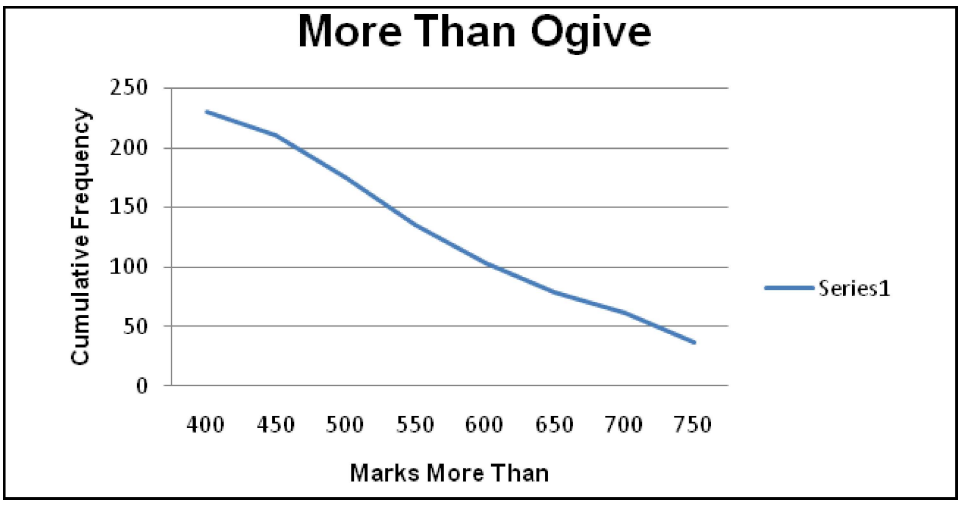
cumulative frequency distribution.

- Mark the lower class limits along X-axis on a suitable scale.
- Mark the cumulative frequencies along Y-axis on a suitable scale.
- Plot the points (X_1, f_1), where X_1 is the lower limit of a class and f_1 is the corresponding cumulative frequency.
- Join the points obtained in Step 4 by a free hand curve to get the ogive.

For example: The frequency distribution of scores obtained by 230 students of a college in an Attitude test is as follows:

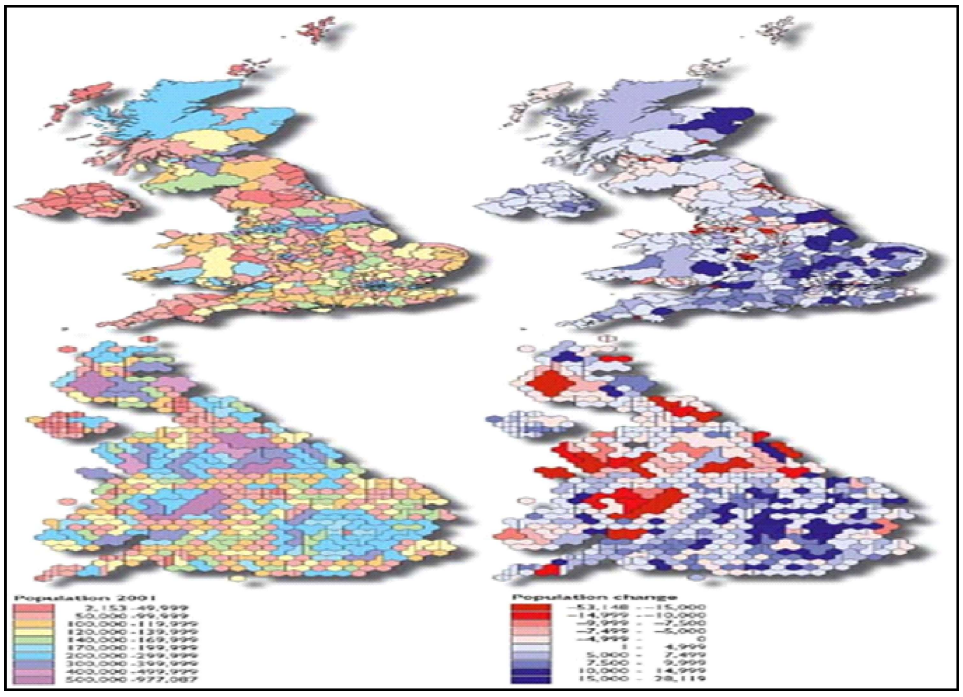
Scores:	400-450	450-500	500-550	550-600	600-650	650-700	700-750	750-800
No. of Students:	20	35	40	32	24	27	18	24

Scores	Number of Candidates	Scores More than	Cumulative Frequency
400-450	20	400	230
450-500	35	450	210
500-550	40	500	175
550-600	32	550	135
600-650	24	600	103
650-700	27	650	79
700-750	18	700	61
750-800	24	750	37
Total	230		



10.10 CARTOGRAM OR MONOGRAPH

In this technique, statistical facts are presented through maps accompanied by various types of diagrammatic presentation. They are generally used to present the facts according to geographical regions. Population and its other constituents like births, deaths, growth, density etc., production, imports, exports and several other facts are required to be compared on the regional as well as country basis. These facts can well be presented on the map with certain colours, dots, cross, points etc. Cartograms are simple and elementary form of visual presentations and easily understandable.



Description of Cartogram:

Population: The population of the UK in 2001, as counted in the 2001 Census, was the infamous One Number: 58,789,194, infamous because it was released as being the most accurate count ever made. It certainly was the most accurate, but it will still be several hundred thousand people adrift of reality in one direction or the other. The comparable figure, following counting and correcting in 1991, as 57,770,226. Thus in 10 years the population is thought to have grown by roughly one million people.The first map and cartogram pair on this page show the population of each local

authority in the UK in 2001. At most almost a million people are living in Birmingham, fewer than in 1991 as most major cities have experienced declining populations other than London . At the least, just over 2,000 people are now recorded as being resident in the Isles of Scilly.Local authorities tend to be more populous in the North as large metropolitan boroughs were created there in the 1970s. London was split in the 1960s into slightly smaller boroughs. Local authorities have their smallest populations in rural England and Northern Ireland. Outside of London it is these that are growing most in population. Authorities tend to be a little larger in population in Wales and Scotland as these were reorganised in the 1990s to be so. Within both of these countries the population is falling in some parts and rising in others. In general, however, the population of the UK is slowly moving towards the South and particular parts of the South that are now growing very quickly.

10.11 PICTORIAL PRESENTATION OF DATA

Also called as Pictogram. The technique of presenting the data through pictures was developed by Dr. Otto Neurath of Vienna. This is the reason that this method is sometimes called as Vienna Method. This is also known as Iso-type method.

According to New World Encyclopedia “A pictogram or pictograph is a symbol representing a concept, object, activity, place or event by illustration. Pictography is form of writing in which ideas are transmitted through drawing. Such pictograms are characterized by their simplified style, which omits all details that are unnecessary to the desired communication. It is the basis for some of the earliest forms of structured written languages, such as Cuneiform and, to some extent, Hieroglyphs.”

In this method the magnitude of the particular phenomenon, being studied, is drawn. The size of pictures is kept proportional to the values of the different magnitudes to be presented. Pictures are more attractive and of lasting impression on mind. Accordingly, the use of pictogram techniques is being frequently used in every field of knowledge, specially in advertising world.

Example: Information about 300 children of a school who come to school by different modes of transportation.

→1 face represents 10 children

See the picture graph or pictograph to understand the status:

Modes	School-going children
Auto-rickshaw	
Car	
Bicycle	
Bus	
On foot	

Source: http://www.math-only-math.com/examples-of-pictographs.html

Information gathered from the above table:

- Number of students going to school by different modes of transportation:

  Auto-rickshaw = 6 × 10 = 60, Car = 4 × 10 = 40,

  Bicycle = 7 × 10 = 70, Bus = 10 × 10 = 100,

  On foot = 3 × 10 = 30
- Total number = 60 + 40 + 70 + 100 + 30 = 300

The above explanation on the examples of pictographs or pictorial representation will help us to solve different types of questions on pictograph.



CHECK YOUR PROGRESS

Q 3: Why is pictorial presentation of data also called as Vienna method? (Answer in about 20 words) .....

.....

Q 4: Mention the steps involve in the construction of a less than ogive curve. (Answer in about 40 words)

.....

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10.12 LET US SUM UP

- A tabular presentation of the data in which the frequencies of values of a variable are given along with them is called a frequency distribution.
- Tabulation is one way of presenting data. Another way of presenting it is in the form of diagrams and graphs.
- Presentation of frequency distribution by a set of vertical bars-the Histograms, whose areas are proportional to the frequency represented, is the most popular and commonly used device.
- A frequency polygon is another way of graphic presentation of a frequency distribution, whether it is discrete or grouped or continuous.
- There are two methods of constructing an Ogive: Less than method and More than method.
- The technique of presenting the data through pictures was developed by Dr. Otto Neurath of Vienna.



10.13 FURTHER READING

1) Black, Ken (2007). *Business Statistics for Contemporary Decision Making*. New Delhi: Wiley India Pvt. Ltd.
2) Garrett, H. E. (2005). *Statistics in Psychology & Education*. New Delhi: Paragon International Publishers.
3) Sharma, R. D. (2009). *Mathematics for Class X*. New Delhi: Dhanpat Rai Publications.
4) Srivastava, S. C. & Srivastava, S. (2009). *Fundamentals of Statistics*. New Delhi: Anmol Publications Pvt. Ltd.



10.14 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q. No. 1: A frequency distribution which shows the frequency of occurrence of different values of a single variable is called as

univariate frequency distribution. On the other hand, a frequency distribution based on two variables is called as bivariate frequency distribution.

Ans to Q. No. 2: The number of observations falling within a particular class or class interval is known as class frequency or frequency of the corresponding class.

Ans to Q. No. 3: The technique of presenting the data through pictures was developed by Dr. Otto Neurath of Vienna. This is the reason that this method is sometimes called as Vienna Method.

Ans to Q. No. 4: The steps involved in the construction of a less than method ogive are:

1) Start with the upper limits of class intervals and add class frequencies to obtain the cumulative frequency distribution.
2) Mark Upper class limits along X-axis on a suitable scale.
3) Mark cumulative frequencies along Y-axis on a suitable scale.
4) Plot the points (X1, f1), where X1 is the upper limit of a class and f1 is the corresponding cumulative frequency.
5) Join the points obtained in Step 4 by a free hand curve to get the ogive.



10.15 MODEL QUESTIONS

Q 1: What are the advantages of Histogram?

Q 2: Draw Histogram and Frequency Polygon in the same graph, of the given data:

C.I.:	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
F:	8	5	16	14	22	14	12	9

Q 3: Write the main features of Ogive.

Q 4: What are the advantages of Cartogram?

*** ***** ***

UNIT 11: COLLECTION OF DATA

UNIT STRUCTURE

11.1 Learning Objectives

11.2 Introduction

11.3 Data Collection: Meaning

11.3.1 Data Collection: Importance

11.4 Types of Data

11.5 Sources of Secondary Data

11.6 Primary and Secondary Data: Merits and Demerits

11.7 Let Us Sum Up

11.8 Further Reading

11.9 Answer to Check Your Progress

11.10 Model Questions

11.1 LEARNING OBJECTIVES

After going through this unit, you will be able to-

- acquire knowledge about meaning and importance of data collection in research
- know about Primary and secondary data
- discuss merits and limitations of both primary and secondary data.

11.2 INTRODUCTION

In previous units, we have learnt various aspects associated with research in economics. We have learnt about review of literature, design of research and various sampling techniques used. In this unit we will learn about collection of data- an important part of research. Starting from meaning and collection of data, the unit covers about primary and secondary data along with their relative meaning.

11.3 DATA COLLECTION: MEANING

Any statistical enquiry originates from collection of related facts or information. These facts must be capable of measurement and this measurement may either be qualitative or quantitative. The facts and

figures collected for any statistical enquiry, are recorded systematically. There are three concepts that are associated with data and its collection:

- Investigator
- Respondents
- Statistical units

An **Investigator** is a person who conducts the statistical enquiry. He must be a trained person and well-acquainted with statistical analysis. He counts or measures the characteristic under study. These measurements are further analysed statistically according to the objectives of the study.

Respondents are the persons from whom the information are collected by the investigator.

The measurements or counts in any statistical investigation are done with some identifiable object or a group of objects. These are known as statistical units. These units have to be defined clearly before collection of data. (Detail in section 11.3.1).

Thus, data collection is a process. It consists not only of counting or enumerating or measurement of information but recording of the same also. It must be a systematic process. In fact, the entire structure of the statistical analysis for any enquiry is based upon systematic collection of data.

11.3.1 Data Collection: Importance

It appears that collection of data is the first step of any statistical enquiry. In fact, an efficient and well -planned statistical enquiry must ensure so many preliminaries before collection of data. Otherwise, the conclusions of the study may be misleading and biased. That is why, collection of data is so important in its operationalization and handling. Before going for collection of data, one must examine the following points:

- Objectives and scope of the enquiry.
- Statistical units to be used.
- Sources of information.

- Method of data collection.
- Degree of accuracy in the final results.
- Type of enquiry.

Let us discuss the points in some detail.

Objectives and Scope of the Enquiry: The objectives of the enquiry should be defined in clear and concrete terms. Otherwise, some irrelevant information may be collected. Similarly, some important information may be omitted.

The scope of enquiry relates to the coverage with respect to the type of information, subject matter and geographical area. It will have a great bearing upon the data to be collected and also the techniques to be used for its collection and analysis. For example, in a study of cost of living index number the area of study (a particular area or state or the country as a whole) must be specified. Moreover, the class of people (such as low -income group, middle-income group) for which they are intended must be specified clearly.

Thus, any study depends on its objectives and scope. However, the objectives and scope should be in accordance with the available resources in terms of money, manpower and time limit required for results.

Statistical Units to be Used: As mentioned in section 11.3 statistical units on which data collection is done must be clearly defined. For socio-economic surveys a unit may be an individual, a family, a block of locality etc. For these types of surveys, some arbitrary units may be used which must be defined previously. These is no ambiguity for the physical units such as metres, kilometres, kilograms quintals etc. Thus, following requisites can be summarised for a statistical units:

- **It should be un-ambiguous:** It implies that every unit of the population should be distinct and non-overlapping and the units must cover the entire population.
- **It should be specific:** In many socio-economic surveys, arbitrary units have to be taken for some characteristics/

variables. These arbitrary units varies from survey to survey, but should be specific and clear for a particular survey. For example, in an enquiry relating to wage level of industrial workers, the wages might be weekly wages or monthly wages. Similarly, workers might refer to the skilled workers or workers working on daily basis. Whatever it may be, it must be specific.

- **It should be stable:** In order to compare at different places or time, the units should be stable. For example, difference in the value of money at different times as well as difference in measurement of weight of the same item make comparisons useless.
- **It should be appropriate to the enquiry:** as discussed in point 2 that the concept and definition of arbitrary statistical units varies from study to study. However, it should be appropriate for the particular study under consideration. For example, data on whole-sale prices are taken for studying the changes in the general price level. For constructing cost of living index number, the retail prices are the appropriate unit.
- **It should be uniform:** It implies that throughout the process of investigation, unit adopted for measurement must be homogeneous. For example, we cannot compare length if we use "metre" in some occasions and "yard" in some other occasions of the same study.

11.4 TYPES OF DATA

In preceding section, we have learned about the meaning and importance of data collection. Having discussed the preliminaries we have come to the conclusion that any statistical enquiry must ensure original collection of data. That is, it should be correct and proper. Otherwise valid and useful conclusions cannot be done even after adopting sophisticated techniques of analysis and presentation of results. Data may be obtained from the following two sources:

- The investigator or the organizer may collect data first hand. This

type of data is termed as Primary Data.

- The investigator may use the data from other published sources. This type of data is termed as Secondary Data.

It is clear that primary data are originally collected by an investigator or agency for the first time for any statistical investigation and used by them in the statistical analysis. Nature of primary data depends on the objectives and scope of the enquiry. Various variables as well as statistical units for them is defined prior to the collection of data. A detailed account on collection of primary data is given in Unit 12.

On the other hand, secondary data have already been collected and processed by some agency or person, and taken over from here and used by any other agency for their statistical work. It is termed as secondary data as far as second agency is concerned. They may or may not be published. In the use of these data there exists no any problem regarding definitions of various terms and units used. It is because these type of data have already been collected under a given set of definitions of various terms and units used.

It is important to know that the distinction between primary and secondary data is a matter of degree or relativity only. The same set of data may be secondary in the hands of one and primary in the hands of others. A data set is primary to the source who collects and processes them for the first time. The same set becomes secondary for all other sources who later use it.

To clear the distinction, let us consider the data on fertility and mortality rates for India. Office of Registrar General of India, New Delhi publishes reports on various demographic rates such as birth rate and death rates etc. Similarly, United Nations Organisation (U.N.O.) publishes U.N. Statistical Abstract wherein data on various aspect are taken from individual countries. Therefore, so far as data on birth rate and death are concerned, the office of the Registrar General of India is the primary source, while U.N.O. is the secondary source.



CHECK YOUR PROGRESS

Q 1: State whether the following statements are true (T) or false (F).

(a) In collection of data, the statistical units should be stable. (T/F)

(b) Primary data is collected from a secondary source. (T/F)

11.5 SOURCES OF SECONDARY DATA

Secondary data can be obtained from the following two broad sources: (a) Published sources, and (b) Unpublished sources.

- **Published Sources:** The publications of various international organisations, government, semi-government and private organisations collects and publishes data on various aspects such as business, trade, labour, prices, consumption, production, industries, agriculture, income, currency and exchange, a number of socio-economic phenomena on a regular basis (monthly, quarterly, annually etc.). These publications are a very powerful source of secondary data. Now-a-days these are available on the official websites of the organisations. A brief summary of these sources has been outlined below.
  - **Central Government's Official Publications:** Office of the Registrar General and Census Commissioner of India ,New Delhi; Directorate-General of Commercial Intelligence and Statistics- Ministry of Commerce; Labour Bureau-Ministry of Labour; Directorate of Economics and Statistics-Ministry of Agriculture and Irrigation; The Indian Army Statistical Organisation- Ministry of Defence; National Sample Survey Organisation, Department of Statistics-Ministry of Planning; Central Statistical Organisation, Department of Statistics-Ministry of Planning-are some of the important central government organisations which publish statistical data on a number of topics of current interest-prices, wages, population, production and consumption, labour, trade, army etc. Most of

the data now-a-days are available in the official websites of the respective departments. Some of the main publications of these government agencies are:

- Monthly Abstract of Statistics: Monthly Statistics of Production of Selected Industries in India; Statistical Pocket Book, India; Annual Survey of Industries-General Review; Sample Surveys of Current Interest in India National Income Statistics-Estimates of Capital Formation in India etc. All published by the Central Statistical Organisation (C.S.O.), New Delhi.
- Census data in various census reports; Vital Statistics of India (Annual), Indian Population Bulletin (Bi-eennial)-all published by Registrar-General of India (R.G.I.)
- Various statistical reports on phenomenon relating to socio-economic and demographic conditions , prices, area and yield of different crops, as a result of the various surveys conducted in different rounds by National Sample Survey Organisation (N.S.S.O.)

In addition to the above, a number of departments both in State and Central Governments like Income Tax Department, Post and Telegraphs, Central Board of Revenues, Textlile Commissioner's Office and so on publish statistical reports on current problems and the information supplies by them.

- **Publications of Semi-Government Statistical Organisations:** Various semi-governmental organisations publish their annual/monthly reports which provide very useful information. Important among them are:
  - Annual Report of Bank, Currency and Finance; Reserve Bank of India monthly Bulletin etc.-published by Statistics and Economics Department of Reserve Bank of India
  - Publications of the Institute of Economic Growth New Delhi,
  - Publications of the Institute of Foreign Trade, New

Delhi.;etc.

In addition to these, publications of the institutions like Municipal and District Boards, Block and Panchayat Samitis on Vital Statistics, health, sanitation and other related subjects provides a fairly reliable and useful information.

- Publications of Research Institutions: Institutes like Indian Statistical Institute (I.S.I.) Calcutta and Delhi; Indian Council of Agricultural Research (ICMR), New Delhi; National Council of Educational Research and Training (N.C.E.R.T), New Delhi; The Institute of Labour Research, Mumbai etc. has been constantly publishing their findings of their research programmes in the form of monographs, research papers, journals etc. These are, of course valuable source of secondary source of data.
- Publications of Commercial and Financial Institutions: Federation of Indian Chamber of Commerce and Industries (FICCI), Institute of Chartered Accountants of India, Trade Unions, Stock Exchanges, Bank Bodies, Co-operative societies etc. are some of the important sources of secondary information on current economic, business and other phenomena.
- Reports of various Government Appointed Committees: The reports of various survey and enquiry committees like Simon-Kuznet Committee Report on National Income in India, Kothari Commission Report on Educational Reforms, Land Reforms Committee Report, Reports of NITTI Ayoug etc. are invaluable source of secondary data.
- Newspapers and Periodicals: Economic and Political Weekly, Economic Times, The Financial Express, Stateman's Year Book and the Times of India Year Book etc. are some of the important magazines, periodicals and newspapers which provide important statistical material on a number of important current socio-economic problems.
- International Publications: The publications of a number of

international agencies provide invaluable statistical information on a variety of important economic and current topics. The publications of United Nations Organisations (U.N.O.) and it's subsideries like U.N.O. Statistical Year Book, U.N. Statistical Abstracts, World Health Organisation's Report on Contagious Diseases, Reports of International Labour Organisation (I.L.O.), Reports of International Monetary Fund (I.M.F.) and so on very valued publications of secondary data.

- **Un-published sources:** Records maintained by private firms or business enterprises, various departments and offices of both central and state governments, findings of individual research scholars in universities and research institutions may remained unpublished. Private firms may not be interested in publication because of maintaining their business secrecy. Moreover, individual researchers, due to the promise of keeping confidentiality to the respondents cannot publish their findings. In spite of unpublished ness, these are sources of secondary information of great value.



CHECK YOUR PROGRESS

Q 2: What are the main concepts associated with collection of data? (Write in about 60 words)

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11. 6 PRIMARY AND SECONDARY DATA: MERITS AND DEMERITS

In previous section we have learnt in detail about primary and secondary data. We have learnt that both are totally different in collection

and follow-up treatment. Which type of data is to be used depends on the nature of study which the researcher is going to undertake. Each of these two types of data, has merit on some aspects and demerits on some other aspects. In order to differentiate both types of data as well to find out relative merits and demerits, let us have a look on the Table 11.1.

Table 11.1: Primary and Secondary Data: A comparison

Sl. No	Criteria	Primary Data	Secondary Data
1	Data Collection method	Data collected by researcher	Data collected by others
2	Originality	Original or unique information	Not original or unique information
3	Adjustment	Doesn't need adjustment, is focused	Need adjustment to suit actual aim
4	Sources	Surveys, Observations, Experiments	Internal records, Govt. published data etc
5	Methods	Observation, Experiment, Interview	Desk -research method, Searching online etc.
6	Reliability	More reliable	Less reliable
7	Time consumed	More time consuming	Less time consuming
8	Need of investigators	Needs team of trained investigators	Does not need team of investigators
9	Cost effectiveness	Costly	Economical
10	Collected when	Secondary data is inadequate	Before primary data is collected
11	Capability	More capable to solve a problem	Less capable to solve a problem
12	Suitability	More suitable to achieve objective	May or may not be suitable
13	Bias	Possibility of bias exist	Somewhat safe from bias
14	Collected by	Researcher or his agents	Persons other than who collects primary data
15	Precaution to use	Not necessary	Quite necessary

It is clear from Table 11.1 that in terms of second point, viz- originality , primary data seems better. Therefore, while taking secondary source of information it should be ascertained that the data to be taken must come from the original source (where it was collected and recorded). In terms of adjustment also (point-3) secondary data has the demerit of making adjustment to suit the objective of the survey. May be because of no need of adjustment, primary data seems more reliable (point-6) than the secondary data.

Of course points 7,8 and 9 reveal the demerits of primary data as it is more time consuming, costlier and requires more trained investigators

than the secondary data. Moreover, there may have biasness on the part of the researcher while collecting primary data(point-13). So the researcher must be very careful in the choice between two types of data.



11.7 LET US SUM UP

- Data collection is a process. It consists not only of counting or enumerating or measurement of information but recording of the same also.
- An efficient and well -planned statistical enquiry must ensure so many preliminaries before collection of data.
- The objectives of the enquiry should be defined in clear and concrete terms.
- The statistical units on which data collection is done must be clearly defined. They should also be unambiguous, specific, stable, uniform and appropriate for the enquiry.
- Two types of data are used in statistical enquiries: primary and secondary

  Secondary data can be collected from both published and unpublished sources.
- Primary data seems better in terms of adjustment ability, originality, reliability, capability and suitability. Moreover, there is no need of precaution in using these data.
- However, collection of primary data is costly in terms of time, money and trained men power needed. Also, it may suffers from personal biasness on the part of the investigator.
- Secondary data seems better in terms of less cost involved in terms of short collection period, money and trained men power required. Also, no biasness is involved here.
- However, secondary data may be less reliable, needs adjustment and can be used with great precaution.



11.8 FURTHER READING

(1) Ahuja, R. (2018). Research Methods. Jaipur: Rawat Publications.

(2) Kothari, C.R. (2011). Research Methodology, Methods and Techniques. New Age International (P) Ltd.

(3) Kumar, R. (2011). Research Methodology. New Delhi: SAGE Publications India Pvt. Ltd.



11.9 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: (a) True (b) False

Ans to Q No 2: There are three concepts that are associated with data and its collection:

a) Investigator

b) Respondents

c) Statistical units

An Investigator is a person who conducts the statistical enquiry. He must be a trained person and well-acquainted with statistical analysis. He counts or measures the characteristic under study. These measurements are further analysed statistically according to the objectives of the study.

Respondents are the persons from whom the information are collected by the investigator.

The measurements or counts in any statistical investigation are done with some identifiable object or a group of objects. These are known as statistical units.



11.10 MODEL QUESTIONS

Short Questions (Answer each question in about 40-50 words)

Q 1: What are qualities of a statistical unit that used in any statistical enquiry?

Q 2: Distinguish between Primary and Secondary data.

Q 3: Mention the main sources of secondary data.

Essay-type Questions (Answer each question in about 300-500 words)

Q 1: What is collection of data? Before collection of data, what point must be considered by the researcher?

Q 2: Discuss relative merits and demerits of primary and secondary data.

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UNIT 12: STATISTICAL SURVEY

UNIT STRUCTURE

12.1 Learning Objective

12.2 Introduction

12.3 Definition of Statistical Survey

12.4 Stages of Statistical Survey

12.5 Planning of a Statistical Survey

12.6 Execution of Statistical Survey

12.7 Basic terms used in Statistical Survey

12.8 Collection of Data

12.9 Primary Data

12.10 Secondary Data

12.11 Pilot Survey

12.12 Scruting and Editing of Secondary Data

12.13 Let Us Sum Up

12.14 Further Reading

12.15 Answer to Check Your Progress

12.16 Model Questions

12.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- put forward the meaning of Statistical Survey
- know the various stages of Statistical Survey
- know the definition of basic terms used in Statistical Survey
- state the meaning of Primary Data and Secondary data
- learn the various methods of collecting Primary data
- know about the precautions to be taken while using secondary data.

12.2 INTRODUCTION

Statistics are collected incidentally or intertionally. A considerable amount of statistics are available as a result of administrative functions of the government and are collected only incidertally, e.g. statistics of crime, accidents, taxation, imports-exports etc. These statistics are not collected primarily for research purposes, although they may be very useful for research. On the other hand, some statistics are collected for their own sake. These intertionally collected statistics are the results of statistical survey.

12.3 DEFINATION OF STATISTICAL SURVEY

Statistical Survey means some sort of investigation by an individual or agency wherein the relevant information is collected in quantitative terms rather than in words.

12.4 STAGES OF STATISTICAL SURVEY

Statistical Survey is a technical job which requires specialized knowledge and skill. The most imprtant task in any statistical survey is data collection. If the method of collecting data is not scientific, then the very purpose of the servey will not be fulfilled. A statistical survey passes through several stages. These stages can be summerised under two broad heads.

(a) Planning the survey

(b) Exceuting the survey

12.5 PLANNING OF A STATISTICAL SURVEY

Proper planning of a survey is of great importance because the quality of a survey depends mostly on planning. The matters which require careful consideration at the planning stage are :

Purpose of the survey : In any statistical survey, first essential thing is to understand the nature of the problem to be investigated and to define it in clear-cut language. This will help in identifying the relevant data.

Scope of the survey : The techniques to be adopted in collection and analysis of data depend on the scope of the survey. Scope of the survey refers to the geographical area to be coverd, the period of the study, the items to be covered, the type of information to be collected etc.

Sources of data : When the purpose and scope of a statistical survey have been stated and agreed upon, the next step is to determine the sources from which the data are to be collected. One must take decision whether the sources of data will be primary or secondary. (*A broad discussion on primary data and secondary data will be made afterwards*)

Types of survey : There are various factors that determine the type of survey to be conducted. One very important factor is object and scope and another is nature and cost factor. The cost factor determines the source of survey and its throughness.

The various type of statistical survey on which decision has to be taken are – (a) Census sample (b) Confidential or open survey (c) Direct or indirect survey (d) regular or adhoc survey (e) offical, semi-official or non-official (f) Extensive or Limited survey

Unit of data collection : To carry out a statistical survey, the statistical unit or units must be clearly defined. The unit interms of which the investigation counts or measures the variables or attributes selected for enumeration, analysis and interpretation is known as *statistical unit*. For example, in a population census, the statistical unit is a person.

To be an ideal statistical unit it must possess the following qualities:

(i) It should be clear and unambiguous

(ii) It should be specific.

(iii) It should be stable.

(iv) It should be uniform.

(v) It should be appropriate to the enquiry.

Statistical units can be broadly classified under two heads :

(a) UNITS OF COLLECTION

and

(b) UNITS OF ANALYSIS AND INTERPRETATION.

(a) **Units of Collection**: Data for enquiry are collected in terms of units of collection. The units of collection may be *simple*, *compound* or *composite*. A simple unit represents a single condition without qualifications. Examples of such units are – house, worker, hours etc. A simple unit with some qualifying words is called a compound unit. Example of such units are – skilled worker, man-hours etc.

(b) **Units of Analysis and Interpretation**: Statistical data are ultimately analysed and intimately analysed and interpreted with the units of analysis and interpretation. These units facilitate comparisions between different sets of data with respect to time and place. Genearlly the units of analysis are – rata, rations, percentages and co-efficients.

Sources of Data : For any statistical survey the sources of data may be either primary or secondary Data is termed primary when reference is to data collected for the first time by the investigator and termed secondary when the information are taken from available records on publised material or data already available. The census department, at at regular intervals, collected data on population of the country. These constitute primary data to the census department. But these data will be seconday to someone who is using these for some other investigation. The choice of the source of data depends largely on pupose and scope of the survey.

Technique of Data Collection: Mainly there are two techniques of data collection (i) census technique and (ii) sample technique. In census technique, information is obtained from each and every unit of the population which forms the subject matter of the study. Whereas, in the sample technique, information is obtained only from a representative part of the population and based on that, inference is drawn for the entire population. The census method is costlier and more time consuming. The choice of technique of data collection depends upon factors like – (a) the availability of resources (b) the time factor (c) degree of accuracy desired and (d) nature and scope of the enquiry.

2.5.8 CHOICE OF FRAME : The term frame or population frame refers to all the units in the population under study. The whole sturcture of enquiry is to a considerable extent detemined by he frame. Detailed planning of a survey can not be undertaken unless we know the nature and accuracy of the available frames. If there does not exist any frame, then the construction of a frame suitable for the purpose of the enquiry will constitute a major part of planning.

Degree of Accuracy Desired: The investigation must determine the degree of accuracy he wants to attain. In any statistical work, it is very difficult to attain 100% accuracy. The object of enquiry primarily determines the degree of accuracy. A very high degree of accuracy may mean a lot of time and cost. The necessary degree of accuracy in counting or measuring depends upon practical value of accuracy in relation to its cost.

12.6 EXECUTION OF STATISTICAL SURVEY

After planning the survey the next step is to execute the plan. The various phrases of work at execution stage are as follows :

(a) **Setting up and administrative Organisation** : An administrative organisation is needed for an investigation on which the complete control and administration of the enquiry falls. The size of the organisation depends upon the nature and scope of the enquiry.

(b) **Designing of forms** : The various forms specially the forms of questionnaires that is used in the course of enquiry should be designed with utmost care and should be drafted by skilled and experienced person.

(c) **Selection, traning and supervision of filed investigation** : The success of a survey depends upon the work of the enumerators. Therefore, these enumerators should be properly selected an throughly trained for the field work. Constant supervision is essential to achieve high quality work.

(d) **Control on the quality of field work** : Steps must be taken to ensure that the survey is under statistical control i.e. the errors if any, in the survey are due to the presence of random variation and no assignable

causes of variation are present. A system of field checks by the supervisors should also be introiduced to maintain the standard.

(e) **Follow up of non-response** : Inspite of best efforts, there may be respondents who do not supply desired information. To deal with this problem, a list of non-respondents is made and then with the help of supervisory staff efforts can be made for getting response from such non responsed members.

(f) **Processing of data** : After collection of data, by the process of classification and tabulation, the collected data are summerised an arranged in a systematic order.

(g) **Preparation of report** : After the data have been collected and analysed, the results of the survey is drafted in the form of a report, which is the final step in the execution of any survey.



CHECK YOUR PROGRESS

Q 1: Name the various types of statistical survey on which decision has to be taken.

Q 2: Fill in the blank :

(a) The unit of collection may be _____ or _____

(b) Skilled worker is an example of _____ unit.

(c) Rates, ratios and percentages are examples of units of _____

(d) Results of population data constitute _____ data for the census department.

(e) A _____ is representative part of the population.

12.7 BASIC TERMS USED IN STATISTICAL SURVEY

Associated with statistical survey there are certain terms which are frequently used :

Characteristic : A quality possessed by an individual or object is called a *characteristic*. Height, weight, temperature, nationality, honesty etc. are examples of characteristic.

Attribute : A non measurable characteristic like sex, region, honesty, nationality etc. are examples of attribute.

Variable : A measurable characteristic like height, weight, temperature, rainfall etc. are examples of variable.

Variables are of two types – (i) Discute (ii) Continious.

(i) A variable which takes specific values only (usually integral values only) between two fixed limits is called a *discrete variable*. Size of a family (i.e. no. of persons in a family), Number of rooms in a college etc. are examples of discrete variable (as the value of such variable cannot be like 4/5, 7/8 etc.)

(ii) A variable which can take any value between two fixed limits is called a continious variable. Height of students, Temperature of a particular place etc. (as the values of such variable may be like 32.8, 160.5 etc)

Population : The collection of all items related to any survey is called *population* or *universe*. For example, if our co-survey relates to the average monthly income of like workers in a factory, then the group of all the workers in that factory will constitute our population. The number of elements in a population is called its size.

Sample : A part of selected items taken from a population for a survey is called a *sample*.

Sampling : The technique of selecting sample from a population is called *sampling*.

Parameter and Statistic : Any statistical measure like mean, median, standard deviation etc relating to a population is called *parameter*. If such statistical measures are related to a sample, these are called *statistic*.

Data : Observation expressed in numerical figures are called *data.* So, variables may be termed as data.

Investigator : Investigator is a person who plans and conducts any statistical survey independent by or with the help of others.

Enumerator : *Enumerator* is a person, who actually collects the desired statistical data. Often enumerators are trained personnel hired by the investigator for collecting data.

Respondent : *Respondent* is a person who answers/responds to the set of questions included in the questionnaire/schedule.

12.8 COLLECTION OF DATA

Collection of data is the first step in any statistical survey. Utmost care must be taken while collecting data because any statistical survey are based on these data.

For any statistical survey, two types of data may be collected. These are *Primary Data* and *Secondary Data*.

12.9 PRIMARY DATA

The data which are collected for the first time from the source of origin for a specific purpose are called *Primary Data*. Some examples of Primary data are –

(i) the census statistics collected by the office of teh Registrar General and Census Commissioner, Govt of India.

(ii) Reserve Bank of India Bulletine (monthly) issued by the Reserve Bank of India, Mumbai etc.

12.10 METHODS OF COLLECTING PRIMARY DATA

The methods commonly used for collecting Primary data are –

2.9.1.1 Direct Personal Investigation

2.9.1.2 Indirect Oral Investigation

2.9.1.3 Mailed Questionnaire

2.9.1.4 Schedule sent through enumerator

2.9.1.5 By Local Agents

2.9.1.1 Direct Personal Investigation : In this method, the investigator collects the information directly from the sources concerned. For instance, if one wants to study the living conditions of the people in a village, he has to go the village, contact the people and get the required

information. Data collected by this method are original in character and very much reliable and accurate. But this method involves much cost and labour. So, this method can not be adopted when the enquiry is extensive or is to be done in large areas. This method does not give accurate result if the investigator is bias.

2.9.1.2 Indirect Oral Investigation : In this method, information is collected indirectly by interviewing persons who are supposed to have knowledge about the problem under investigation. For example, if an accident takes place in a certain area and the cause of accident is not known, then the investigator may collect information from person who might have witnessed the accident or from those living in the neighbourhood to find the possible cause of the accident. Most of the commissions of enquiry appointed by Government collect primary data by this method. This method is less expensive and time saving.

2.9.1.3 Mailed Questionnaire : In this method, a list of questions pertaining to the survey is prepared an is sent to various respondent by post. This list of questions is called a questionnaire and it provides space for answers. A mailed questionnaire should be accompanied by two things – (i) a polite covering letter – requesting the informats to fill up the questionnaire and (ii) a self addressed stamped envelope – so that the informates may post it back to the investigator.

This is the cheapest method by which data may be collected from a wide geographical area. This method can not be adopted if the respondents are illiterate. Non-response from the respondants is a major drawback of this method. It is difficult to verify the accuracy of the answers given by the respondents as there is no contact between the investigator and the respondents.

2.9.1.4 Schedule sent through enumerator : A schedule is also a list of questions pertaining to a survey. However, in this method, the enumerators contact informants personally, get replies to the questions contained in the schedule and fill them up in their own handwriting in the questionnaire form. In other words, a schedule is a list of questions where

the answers to the questions are filled by the enumerators in the presence of the respondents.

Schedule can be used when the informants are illiterate. Non-response is minimised as the enumerators ersonally go to the informants. This is the most expensive method of collecting primary data as the enumerators are paid persons. The success of this method largely depends upon the training impaited to the enumerators.

2.9.1.5 By Local Agents : Under this method, the investigator appoints local agents in different places to collect information. Generally this method is adopted by the newspaper agencies.

2.9.2 DIFFERENCE BETWEEN QUESTIONNARIRE AND SCHEDULE :

(i) A questionnaire is a list of questions which is sent to the respondents by post to be filled in by themselves	(i) A schedule is a list of questions which are carried by the enumerators personally to the respondents.
(ii) It may be used only for literate informants	(ii) It may be used for literate as well as illiterate informants.
(iii) It is the cheapest method of collecting primary data	(iii) It is the costliest method of collecting primary data.
(iv) Non-response from the respondents is a major drawback of this method.	(iv) Non response is minimised as enumerators personally go to the infomants.

2.9.3 FRAMING OF A QUESTIONNAIRE OR SCHEDULE :

Great care is to be taken in drafting a questionnaire or schedule as this is the medium through which information is collected. The success of a statistical investigation and the quality of its results depend largely upon how appropriately a questionnaire has been drafted and put to use.

The basic requirements of a good questionnaire or a schedule are as follows :

(i) The questions should be clear and brief.

(ii) The questions should be of objective type – either yes or no or multiple choice type.

(iii) The questions should be arranged in a logical manner.

(iv) The unit of measurment, if any, in which the respondent has to record the information should be clearly indicated e.g. kg, cm etc.

(v) The questionnaire should be attractive to look at.

(vi) The questions which involve numerical calculation should be avoided.

(vii) Number of quetions should not be too many.

(viii) Personal questions or sensitive questions should not be included.

(ix) Necessary instructions for filling up the questionnaire should be given

(x) Some questions which cross-check the information on vital issues should be included.

2.10 SECONDARY DATA

The data which were previoulsy collected by some individual or agency as primary data for a certain purpose and are now used by someone else for another purpose are termed as *Secondary data*.

2.10.1 SOURCE OF SECONDARY DATA : The sources of secondary data may be classified under two heads –

(a) Published sources.

(b) Unpublished sources.

(a) **Published sources** :

(I) Reports and official publication of –

(i) International bodies like WHO, ILO, World Bank, statistical office of the United Nations etc.

(ii) Central and State Governments.

(iii) Ad-hoc committees and Commissions appointed by government.

(iv) Semi govt or local bodies like Municipal Corporation.

(v) Autonomous and private institutions like the Institution of Chartered Accountants, The Institute of Foreign Trade, Institute of Economic Growth etc.

(II) Financial and Economic Jornals like Indian Economic Revdew, Bulletin of Reserve Bank of India and various National and Regional magazines and newspapers.

(b) Unpublished Sources : Records maintained by various Government and Private institutions and offices, studies conducted by research scholars and research institutions etc.

2.10.2 PRECAUTIONS IN USE OF SECONDARY DATA : Utmost care should be taken in using secondary data. The degree of reliability of the secondary data must be assessed at the source. Prof A.L. Bowley pointed out that – "*Secondary data should not be accepted at their face value*." Therefore, before using secondary data, the investigators should consider the following factors :

(a) **Suitability of the data** : Before using secondary data the investigator must ensure whether the data are suitable for the purpose of the enquiry. The suitability of the data may be judged by comparing the nature and scope of the present enquiry with that of the original enquriry. For example, if the object of the prsent enquiry is to study the cost of living of the people, and if the data provide only the wholesale peices, then such data are not suitable as for calculating C.L.I, retail price is necessary.

(b) **Adequacy of the data** : The adequacy of the secondary data may be judged by studying the geographical area covered by the original enquiry and the expected coverage of the present enquiry. For example, if we want to study the consumption pattern of the rice by the people of India, and if the available data relates only to the consumption pattern of rice by the people of Assam, then our data is not adequate. Another important factor to decide about the adequacy of the available data for the given enquiry is the time period for which the data are available.

(c) **Reliability of the data** : The reliability of the data can be judged by finding out whether the data collecting agency is free from personal bias. Moreover, it should also be verified that the data relates to a normal time period, that is free from economic boom or depression or natural calamities. If the enumeration is based on sample, it must be adequate and a representative sample. To be reliable, the data must be collected by trained and experienced investigator or enumerator.

12.11 PILOT SURVEY

It is the survey which is conducted on a small scale before conducting the main survey. Pilot survey is also known as pre-testing. Such a survey becomes useful in many ways, some of which are the following :

(i) Pilot survey becomes useful in estimating the time and cost involved in the main survey.

(ii) On the basis of pilot survey the investigator can decide whether requisite information can be collected by mailnl questionnaire or by schedule sent through enumerator.

(iii) It provides insights into the problems faced by the interviewers.

(iv) With the help of pilot survey the investigator can adopt appropriate measures to avert certain problems that might involve in the execution of main survey.

On the whole, pilot survey is extremely helpful for planning and executing a survey.

12.12 SCRUTINY AND EDITING OF SECONDARY DATA

The secondary data must be used with caution. It is usually very difficult to verify such data and to edit them to find out inconsistencies, problems errors and omission. Screetiny of the secondary data is essential because the data might be inaccurate, unsuitable or inadequate.

Data editing is defined as the process involving the review and adjustment of collected survey data. The purpose is to control the quality of

the collected data. Data editing may be performed manually, with the assistance of a computer or a combination of both.

Editing is concerned with removal of redundant data and filling of missing data.



CHECK YOUR PROGRESS

Q 3: Fill in the blanks :

(a) Data originally collected for an investigation is known as _____ data.

(b) Measurable charateristics are called _____.

(c) Population characteristics are called _____.

(d) Sample characteristics are called _____.

(e) There are _____ methods of collecting primary data.

(f) Primary data are _____ is nature.

(g) _____ survey is also known as pre-testing.



2.13 LET US SUM UP

- Statistical survey means some sort of investigation by an individual or agency where in the relevant information is collected in quantitative terms.
- Two main stages of statistical survey are – Planning the survey and executing the servey.
- There are various types of statistial servey.
- The unit in terms of which the investigation counts or measures the variable selected for enumeration, analysis and interpretation is done known as statistical unit.
- Depending on the sources, the data may be classified as primary data and secondary data.
- There are five methods of collecting Primary data.
- Precautions must be taken while using secondary data.

- Measurable characteristics are called variable and non measurable characteristics are called attribute.
- The collection of all items related to any survey is called population or universe.
- A part of the selected items taken from a population is called a sample.
- Any statistical investigation wherein data is collected from each unit of the population, is known as census survey.
- When in a statistical investigation, data is collected from a part of the population, it is known as sample survey.
- Investigator is a person who plans and conducts any statistical survey and enumerator is a person who actually collects the desired statistical data.
- The success of statistical survey depens largely upon drafting the questionnaire.
- Before using secondary data, the reliability, adequacy and accuracy of the data should be checked.
- Pilot survey is conducted on a small scale before conducting the main survey.
- The task of data editing is highly specialised and requires great care and attention. It is concerned with removal of redundant data and filling of missing data.



12.14 FURTHER READING

1) Bussiness Statistics. S.C. Gupta, Indira Gupta. Himalaya Publishing House.
2) An Introduction to commercial Mathematics and Statistics. Ajanta Mazumdar, Runjun Phookun, S. Chand and Company.
3) A text book of Bussiness Statistics : Dr Padmalochan Hazarika. S.Chand and Company.
4) Fundamentals of Statistics. S.C. Gupta. Himalya Publishing House.



12.15 ANSWER TO CHECK YOUR PROGRESS

Ans to Q No. 2: (a) Simple, compound. (b) Compound unit, (c) Analysis, (d) Primary, (e) Sample.

Ans to Q No. 3: (a) Primary, (b) Variable, (c) Parameter, (d) Statistic, (e) five, (f) orinial, (g) Pilot



12.16 MODEL QUESTIONS

Q 1: What is statistical survey? Mention the main stages of statistical survey.

Q 2: Describe the process of planning a statistical survey, with special reference to its scope and reference.

Q 3: Distinguish between (i) primary and secondary data (ii) sampling and census data.

Q 4: Define primary and secondary data. What are the precautions to be taken while using secondary data?

Q 5: Define secondary data. State their chief sources.

Q 6: Explain various methods of collecting primary data.

Q 7: Define : Primary data, sample, statistic, attribute, variable, parameter, investigator, respondent, enumerator, sample.

Q 8: What is the difference between Quetionnaire and schedule.

Q 9: Write any five essential characteristics of an ideal questionnaire.

Q 10: What is the diffrence between population and sample.

Q 11: Explain the advantages of personal invetigation as compared with other methods usually used in collecting primary data.

Q 12: Name the various types of statistical survey.

Q 13: What is meant by statistical survey. Name the various stages in such a survey.

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UNIT 13: INTERVIEW

UNIT STRUCTURE

13.1 Learning Objectives

13.2 Introduction

13.3 Meaning of Interview

13.4 Process of Interview

13.5 Types of Interview

13.6 Merits of Interview

13.7 Demerits of Interview

13.8 Let Us Sum Up

13.9 Further Reading

13.10 Answer to Check Your Progress

13.11 Model Questions

13.1 LEARNING OBJECTIVES

After going through this unit, you will be able to-

- put forward the meaning of interview
- understand the process of interview
- differentiate the different types of interviews
- discuss the merits and demerits of interview

13.2 INTRODUCTION

We have seen how the research process initiates. One of the most important aspects of any research process is data collection. There are different techniques of data collection. Out of these, interview method is a commonly used technique to collect data from a target group or sampled population. There are different types of interviews. There are certain prerequisites for the smooth conduct of the interview method. Again, it also has both advantages and disadvantages attached which we are going to discuss in detail in this unit.

13.3 MEANING OF INTERVIEW

Interview is a commonly used method of collecting information from people. In many walks of life we collect information through different forms of interaction with others. There are many definitions of interviews. According to Monette et al., 'an interview involves an interviewer reading questions to respondents and recording their answers'. According to Burns, 'an interview is a verbal interchange, often face to face, though the telephone may be used, in which an interviewer tries to elicit information, beliefs or opinions from another person'. Any person-to-person interaction, either face to face or otherwise, between two or more individuals with a specific purpose in mind is called an interview.

There are two major functions of interview technique:

- **Description:** Since interviewer enquiries about certain topics, the respondents provide certain information which helps the interviewer to attain in-depth insights into the nature of the problem he or she is concerned about.
- **Exploration:** Interviews provide insight into unexplored dimensions of the problem. Respondents get a chance to explain the problem in a better manner which is generally not possible when information is collected through questionnaires.

Some characteristics of interview are as follows-

- There is face-to-face contact, conversational exchange and verbal interaction between the interviewer and the respondent.
- The interview is not limited to two individuals only; rather it could be a group of respondents, two or more interviewers.
- The relationship between interviewer and respondents who are strangers is generally transitory.
- Questions are asked and responses are received verbally.
- The interviewer records the responses accordingly and there is flexibility in the format of interview.

13.4 PROCESS OF INTERVIEW

For smooth conduct of the interview process, the first necessary

condition is the understanding of the interviewer. The interviewer needs to be clear about the objectives and aspects of the studies to be conducted. Moreover, selecting the sample population, getting appointment from the respondents, informing the respondents about the time required for the interview, having an attitude so that the respondents feel comfortable to answer, and increasing respondents' motivation and so on are to be looked after by the interviewer. For successful conduct of the interview method, interviewers should be carefully selected, trained and briefed. They should be observant, objective, sensitive to norms and practices of the society taken for study, etc.. Occasional field checks should be made to ensure that interviewers are conducted following all the necessary requirements , not deviating from instructions given to them for performing their job efficiently. In addition, some provisions should also be made in advance so that if any selected respondent refuse to cooperate or are not available when an interviewer calls upon them, such cases are duly accommodated. In fact, interviewing is an art governed by certain scientific principles. Every effort should be made to create friendly atmosphere of trust and confidence, so that respondents may feel at ease while talking to and discussing with the interviewer. The interviewer must ask questions properly and intelligently, not only that s/he must also take into congnizance emotions of the respondents and then must record the responses accurately and completely. At the same time, the interviewer must answer legitimate question(s), if any, asked by the respondent and must clear any doubt that the respondents might have regarding purpose of the study, institutional affiliation etc. has. The interviewers approach must be friendly, courteous, conversational and unbiased. The interviewer should not show surprise or disapproval of a respondent's answer but he must keep the direction of interview in his own hand, discouraging irrelevant conversation and must make all possible effort to keep the respondent on the track.



CHECK YOUR PROGRESS

Q 1: Define interview. Answer in about 50 words.

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Q 2: Mention the characteristics of interview. Answer in about 60 words.

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13.5 TYPES OF INTERVIEW

The types of interview differ from one another in terms of structure, interviewer's role, number of respondents involved, and so on. Following are some of the important types of interview:

- **Unstructured Interviews:** In unstructured interview there are no specifications in the wording of the questions and order in which they are to be asked. The interviewer can form questions as and when required. The strength of unstructured interviews is the complete freedom they provide in terms of content and structure. We are free to order these in whatever sequence we wish. We also have complete freedom in terms of the wording we use and the way we explain questions to the respondents. We may formulate questions and raise issues on the spur of the moment, depending upon what occurs in the context of the discussion. Such interviews have no time limit for continuing the interview. Unstructured interviews are prevalent in both quantitative and qualitative research. The difference is in how information obtained through them in response to our questions is likely to be used. In quantitative research we develop response categorizations from responses which are then coded and quantified. In qualitative research the responses are used as descriptors, often in verbatim form, and can be integrated with our arguments, flow of writing and sequence of logic.

Advantages of unstructured interview: The advantages of unstructured interview include the easy flow in which the interview can be conducted in the form of natural conversations and the questions are spontaneously asked. There is greater possibility of exploring in an unrestricted manner. Knowing the interest of the respondents, the interviewer can ask questions accordingly and gain more information from that specific area.

Disadvantages of unstructured interview: There are certain disadvantages like the responses of the respondents cannot be compared with each other. With no specific sequence and format of the questions, the reliability of the data becomes doubtful. Some aspects may be left out during the interview process and at times it results in wastage of time on account of irrelevant conversations.

- **Structured interviews:** In a structured interview the researcher asks a predetermined set of questions, using the same wording and order of questions as specified in the interview schedule. An interview schedule is a written list of questions, open ended or closed, prepared for use by an interviewer in a person-to-person interaction (this may be face to face, by telephone or by other electronic media). The researcher has little freedom to make changes in the order or wordings of the questions asked.

  One of the main advantages of the structured interview is that it provides uniform information, which assures the comparability of data. Structured interviewing requires fewer interviewing skills than does unstructured interviewing. Moreover, structured interviews help to lessen the personal bias mostly seen in case of unstructured interviews.

- **Standardised and unstandardised interview:** In standardised interviews answer to each question is given in a standard form of set of responses given for this purpose. For instance, the response to be chosen by the respondents might be given as yes/no, agree/disagree, satisfied/not satisfied, illiterate/less educated/highly educated, and so on. The information attained is helpful for quantitative analysis. Whereas in unstandardised interviews there is no particular standard for the

answers, rather it is left open to the respondents. Such information is collected for qualitative research analysis.

- **Individual and group interviews:** In individual interviews the interviewer asks questions to one respondent at a time. Although it gives fair opportunity to the respondent to answer without any external influence and information on sensitive issues can be collected with the help of one to one interviews but if the number of respondents is too large, it would be a time consuming task to complete the entire process. On the other hand, in group interviews, there are more than one respondent. The group might be of two individuals like husband and wife, or a family with 10-15 members, or a group of farmers or a large group of students, and so on.
- **Personal interviews:** Personal interview method requires a person known as the interviewer asking questions generally in a face-to-face contact to the other person or persons. (At times the interviewee may also ask certain questions and the interviewer responds to these, but usually the interviewer initiates the interview and collects the information.) This sort of interview may be in the form of direct personal investigation or it may be indirect oral investigation. In the case of direct personal investigation the interviewer has to collect the information personally from the sources concerned. He has to be on the spot and has to meet people from whom data have to be collected. This method is particularly suitable for intensive investigations. But in certain cases it may not be possible or worthwhile to contact directly the persons concerned or on account of the extensive scope of enquiry, the direct personal investigation technique may not be used. In such cases an indirect oral examination can be conducted under which the interviewer has to cross-examine other persons who are supposed to have knowledge about the problem under investigation and the information, obtained is recorded. Most of the commissions and committees appointed by government to carry on investigations make use of this method.
- **Focused interview:** It is meant to focus attention on the given experience of the respondent and its effects. Under it the interviewer

has the freedom to decide the manner and sequence in which the questions would be asked and also has the freedom to explore reasons and motives. The main task of the interviewer in case of a focused interview is to confine the respondent to a discussion of issues with which he seeks conversance. Such interviews are used generally in the development of hypotheses and constitute a major type of unstructured interviews. For example, studying prisoners in jail about the restrictions on their freedom, work, recreation, interaction, food habits, etc. is an example of focused interview. One basic advantage of focused interview is that the respondents get more freedom to respond to the questions and hence there is the opportunity to gather more information.

- **Clinical interview:** Clinical interview is concerned with broad underlying feelings or motivations or with the course of individual's life experience. The method of eliciting information under it is generally left to the interviewer's discretion.
- **Non-directive interview:** Here, the interviewer's function is simply to encourage the respondent to talk about the given topic with a bare minimum of direct questioning. The interviewer often acts as a catalyst to a comprehensive expression of the respondents' feelings and beliefs and of the frame of reference within which such feelings and beliefs take on personal significance.

Advantages and Weaknesses of Personal Interview Method: Despite the variations in interview-techniques, the major advantages and weaknesses of personal interviews can be enumerated in a general way as follows:

- More information and that too in greater depth can be obtained.
- Interviewer by his own skill can overcome resistance, if any, of the respondents; the interview method can be made to yield an almost perfect sample of the general population.
- There is greater flexibility under this method as the opportunity to restructure questions is always there, especially in case of unstructured interviews.

- Observation method can as well be applied to recording verbal answers to various questions.
- Personal information can as well be obtained easily under this method.
- Samples can be controlled more effectively as there arises no difficulty of the missing returns; non-response generally remains very low.
- The interviewer can usually control which person(s) will answer the questions. This is not possible in mailed questionnaire approach. If so desired, group discussions may also be held.
- The interviewer may catch the informant off-guard and thus may secure the most spontaneous reactions than would be the case if mailed questionnaire is used.
- The language of the interview can be adopted to the ability or educational level of the person interviewed and as such misinterpretations concerning questions can be avoided.
- The interviewer can collect supplementary information about the respondent's personal characteristics and environment which is often of great value in interpreting results.

But following are some of the weaknesses of the personal interview method:

- It is a very expensive method, especially when large and widely spread geographical sample is taken.
- There remains the possibility of the bias of interviewer as well as that of the respondent; there also remains the problem of supervision and control of interviewers.
- Certain types of respondents such as important officials or executives or people in high income groups may not be easily approachable under this method and to that extent the data may prove inadequate.
- This method is relatively more-time-consuming, especially when the sample is large and recalls upon the respondents are necessary.
- The presence of the interviewer on the spot may over-stimulate the respondent, sometimes even to the extent that he may give imaginary information just to make the interview interesting.
- Under the interview method the organization required for selecting,

training and supervising the field-staff is more complex with formidable problems.

- Interviewing at times may also introduce systematic errors.
- Effective interview presupposes proper rapport with respondents that would facilitate free and frank responses. This is often a very difficult requirement.

Telephone interviews: This method of collecting information consists in contacting respondents on telephone itself. It is not a very widely used method, but plays important part in industrial surveys, particularly in developed regions. Moreover, newspapers, radios and TV personnel generally use this method to access public opinion.

Merits of Telephone Interviews: Following are the merits of such a system-

- It is more flexible in comparison to mailing method.
- It is faster than other methods i.e., a quick way of obtaining information.
- It is cheaper than personal interviewing method since not many investigators are required.
- Recall is easy; callbacks are simple and economical.
- There is a higher rate of response than what we have in mailing method; the non-response is generally very low.
- Replies can be recorded without causing embarrassment to respondents.
- Interviewer can explain requirements more easily.
- At times, access can be gained to respondents who otherwise cannot be contacted for one reason or the other.
- No field staff is required.
- Representative and wider distribution of sample is possible.

Demerits of Telephone Interviews: But this system of collecting information is not free from demerits. Following are some of the disadvantages:

- Little time is given to respondents for considered answers; interview period is not likely to exceed five minutes in most cases.
- Surveys are restricted to respondents who have telephone facilities.

- Extensive geographical coverage may get restricted by cost considerations.
- It is not suitable for intensive surveys where comprehensive answers are required to various questions.
- Possibility of the bias of the interviewer is relatively more.
- Questions have to be short and to the point; probes are difficult to handle.
- Each selected respondent in the sample may not own a telephone and thus be left out from responding.
- Respondents are often not motivated over the telephone because they can terminate the interview at his/her will.
- Since answers are to be given quickly, the respondents might not get much time to think about the answers.

Thus, we see that the interview as a method of data collection seems to be easy, yet its adequacy, reliability and validity pose important problems. Interviewers differ in skill and interest; again respondents differ in ability and motivation. Hence, for a successful interview, accessibility of information, understanding of the respondents and motivation are the prime factors.



CHECK YOUR PROGRESS

Q 3: Mention some types of interviews. Answer in about 40 words.

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Q 4: Differentiate between structured and unstructured interview. Answer in about 50 words.

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13.6 MERITS OF INTERVIEW

Interview as a method of data collection has certain specific merits as follows:

- Quick information- desired information can be obtained quickly.
- Proper interpretation- respondents interpret the questions properly.
- Flexibility- in both asking questions and responding there is flexibility in the process.
- Checking validity- validity of the information can be easily checked. It gives higher response rate and identity of the respondents is known.
- Observance - the interviewer has the opportunity to observe respondents' non-verbal behavior.
- Control- exercising control on the context of questions and answers is possible.

13.7 DEMERITS OF INTERVIEW

Following are some of the demerits or limitations of interview technique:

- Due to the fear of identity, the respondents might give improper answers or hide information and give wrong responses.
- Interviews are costly both in terms of money and time.
- Interviewer's attitude has a major role to play in the process. For instance, if he is angry or tired, he would not be able to conduct the interview properly. Again, if he is in any hurry, he would not take much time and end the process quickly. Doing so would result in inadequate information.
- It is less effective for sensitive questions.
- It offers less anonymity than other methods.
- There could be variability in responses with different interviewers, particularly when interview is unstructured.

Thus, the above are certain advantages and disadvantages involved in the technique of interview when used for data collection in a research study.



CHECK YOUR PROGRESS

Q 5: Mention some of the important merits of personal interview. Answer in about 60 words.

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Q 6: What is meant by a focused group interview? Answer in about 50 words.

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13.8 LET US SUM UP

- According to Monette et al., 'an interview involves an interviewer reading questions to respondents and recording their answers'. According to Burns, 'an interview is a verbal interchange, often face to face, though the telephone may be used, in which an interviewer tries to elicit information, beliefs or opinions from another person'.
- Any person-to-person interaction, either face to face or otherwise, between two or more individuals with a specific purpose in mind is called an interview.
- Some characteristics of interview include- There is face-to-face contact, conversational exchange and verbal interaction between the interviewer and the respondent; the interview is not limited to two individuals only; rather it could be a group of respondents, two or more interviewers. The relationship between interviewer and respondents who are strangers is generally transitory. Questions are asked and responses are received

verbally. The interviewer records the responses accordingly and there is flexibility in the format of interview.

- For successful implementation of the interview method, interviewers should be carefully selected, trained and briefed. They should be honest, sincere, hardworking, and impartial and must possess the technical competence and necessary practical experience.
- Some types of interview are- Unstructured Interviews, structured interviews, Standardised and unstandardised interview, Individual and group interviews, Personal interviews, Focused Group interview, telephonic interview, etc. All of these types have their particular advantages and disadvantages.
- Certain advantages of interview method include- quick information, proper interpretation, flexibility, checking validity, control, knowing the identity of respondents, completeness of interviews and so on.
- Possibility of hiding information or providing wrong information, costly process, less effective for sensitive questions are some of the disadvantages of interview technique.



13.9 FURTHER READING

(1) Ahuja, R. (2018). Research Methods. Jaipur: Rawat Publications.

(2) Creswell, J W and Creswell, J D. (2018). Research Design. Sage Publications.

(3) Kothari, C.R. (2011). Research Methodology, Methods and Techniques. New Age International (P) Ltd.

(4) Kumar, R. (2011). Research Methodology. New Delhi: SAGE Publications India Pvt. Ltd.



13.10 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: According to Monette et al., 'an interview involves an interviewer reading questions to respondents and recording their

answers'. According to Burns, 'an interview is a verbal interchange, often face to face, though the telephone may be used, in which an interviewer tries to elicit information, beliefs or opinions from another person'.

Ans to Q No 2: There is face-to-face contact, conversational exchange and verbal interaction between the interviewer and the respondent; The interview is not limited to two individuals only; rather it could be a group of respondents, two or more interviewers; The relationship between interviewer and respondents who are strangers is generally transitory; Questions are asked and responses are received verbally; The interviewer records the responses accordingly and there is flexibility in the format of interview.

Ans to Q No 3: Unstructured Interviews, structured interviews, Standardized and unstandardised interview, Individual and group interviews, Personal interviews, Focused Group interview, telephonic interview, etc.

Ans to Q No 4: In unstructured interview there are no specifications in the wording of the questions and order in which they are to be asked. The interviewer has complete freedom and can form questions as and when required. Whereas, in a structured interview the researcher asks a predetermined set of questions, using the same wording and order of questions as specified in the interview schedule and lacks freedom.

Ans to Q No 5: More detailed information can be obtained through personal interviews. Moreover, if the respondents have any difficulty, the interviewer can clear those instantly. In case of a face-to-face interview, the interviewer can keep motivating the respondents to answer properly.

Ans to Q No 6: Focused group interviews focus attention on the given experience of the respondent and its effects. Under it the interviewer has the freedom to decide the manner and sequence in which the questions would be asked and has also the freedom to explore reasons and motives. The main task of the interviewer in case of a focused interview is to confine the respondent to a discussion of issues with which he seeks conversance. Such interviews are used generally in the development of hypotheses and constitute a major type of unstructured interviews.



13.11 MODEL QUESTIONS

Long Questions. (Answer each question in around 300-500 words)

Q 1: Define the term 'interview'. Explain the characteristics of interview technique.

Q 2: Explain the process of interview used for data collection.

Q 3: What are the different types of interviews? Explain the types along with their advantages and disadvantages.

Q 4: What are the merits and demerits of the interview technique? Explain in details.

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UNIT 14: INTERVIEW TECHNIQUES

UNIT STRUCTURE

14.1 Learning objectives

14.2 Introduction

14.3 Questionnaire: Meaning

- 14.3.1 Preparation of a Good Questionnaire
- 14.3.2 Features of a Good Questionnaire
- 14.3.3 Types of Questionnaire
- 14.3.4 Merits of Questionnaire
- 14.3.5 Demerits of Questionnaire
- 14.3.6 Difference between Questionnaire and Schedule

14.4 Case Study

- 14.4.1 Objectives and Characteristics of Case Study Method
- 14.4.2 Forms of Case Study
- 14.4.3 Steps in Case Study
- 14.4.4 Advantages of Case Study
- 14.4.5 Disadvantages of Case Study

14.5 Focus Group- Meaning and Definition

- 14.5.1 Characteristics of Focus Group
- 14.5.2 What Focus Group can and cannot tell you?
- 14.5.3 Steps for Conducting a Focus Group Discussion
- 14.5.4 Significance of Focus Group Discussion
- 14.5.5 Advantages of Focus Group Discussion
- 14.5.6 Disadvantages of Focus Group Discussion

14.6 Let Us Sum Up

14.7 Further Reading

14.8 Answers to Check Your Progress

14.9 Model Questions

14.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- put forward the meaning of questionnaire

- acquire knowledge how to prepare a good questionnaire
- explain the different types of questionnaire
- discuss the merits and demerits of questionnaire as a technique of data collection
- explain the differences between questionnaire and schedule
- discuss case study method of conducting research
- explain how to make empirical enquiry about a contemporary phenomenon through the case study method
- discuss the advantages and disadvantages of case study method
- discuss focus group discussion as a method of conducting research
- explain the necessary steps for conducting a focus group discussion
- discuss the advantages and disadvantages of focus discussion.

14.2 INTRODUCTION

By now, we have learnt that data collection is the most vital aspect in any research and based on the nature of the study or research; there are different techniques of collecting data. Interview is one such important technique, which we have discussed, in the previous unit. From that unit we have learnt about the meaning and types of interview and the processes in which an interview is conducted. In this unit, we shall learn about questionnaire, a tool for data collection. In simple words questionnaire is a series questions printed or typed in a definite order on a form or set of forms for collecting primary data from the respondents. Here we shall discuss about how to prepare a questionnaire, what are the features of a questionnaire, the different types of questionnaire and the merit and demerits of a questionnaire.

This is the concluding unit of this course. In the previous unit, we discussed some important issues relating to construction of a questionnaire. In this Unit, we shall discuss two important techniques of conducting interviews, viz, case study and focus group discussion.

It is expected that with this Unit, you will get a proper insight into the important areas of research methodology in social sciences.

14.3 QUESTIONNAIRE: MEANING

As mentioned above, Questionnaire is a research tool used to collect primary data. According to Goode and Hatt, the word questionnaire refers to a mechanism or tool for securing answers to questions by using a form, which the respondent fills himself. Thus, anyone who has filled in a job application has had the experience of answering a questionnaire. In a research study, the researcher develops a series of questions on the basis of the objectives of his/her study. The set of questions are all logically related to the central problem of the study. The researcher then arranges the questions in a logical sequence, prints it in a form or set of forms and gives it to his/her respondents. The aim is to generate response from the respondents on the issue or the topic of the study. The respondents are expected to read and understand the questions and write down the reply in the space provided in the questionnaire itself. They are also expected to answer the questions on their own without any external influence. Thus, a questionnaire can be defined as a research instrument, which has a series of pre-set questions that are framed in a certain way to gather objective information from respondents. It is a most popular and widely used technique of data collection used in field survey. In the next section, we shall discuss how to prepare a questionnaire.

14.3.1 Preparation of a Questionnaire

Preparation of a questionnaire is a complex task. Since questionnaire has to be filled by the respondents, it is very essential to prepare the questionnaire in such a way that the respondents can easily understand what is being asked. The questions therefore need to be very precise and comprehensive. The questions must be connected to the objectives of the study and should be such that they fetch appropriate and required data for the study. Further, the questionnaire should also be very sensitive towards the respondents and should not hurt the sentiment and dignity of the respondents. Therefore preparing a questionnaire requires

cautious efforts of the researcher. The researcher must give importance on the following aspects while preparing a questionnaire.

- **General Form:** Questionnaires should have definite, concrete and predetermined questions. Each respondent should be asked the same questions and the questions should be presented with exactly the same wording and in the same order to all the respondents.
- **Question Sequence:** The researcher must be very careful about the question sequence while preparing a questionnaire. The questionnaire would be meaningful and effective only when the questions are put in a definite order or sequence. The questions must follow a specific pattern; each question following from the above. The questions must be easy; its meaning clear and there should be a flow from one question to the other so that it helps the researcher to easily understand and give his/her views/ answers. The initial or the first few questions should be the easiest to answer and should be able to arouse the interest of the respondents.
- **Question formulation and wording:** The questions put in the questionnaire should be simple and written in a lucid language. The questions should be such that they are understood easily They should not lead to any misunderstanding and miscommunication. Further, the questions should convey only one thought at a time.

14.3.2 Features of a Good Questionnaire

- Questionnaire should be precise and simple. It should comprise of short and crisp questions that are easily understandable by the respondents.
- Questions should follow a logical sequence where it starts with the easiest questions followed by the difficult ones. It should

proceed from the easy/simple to the complex questions.

- A good questionnaire should be comprehensible by the respondents. It may emphasize on the significant or important areas of the concerned research and yet the questions should be framed in a way so that the respondents do not hesitate to put forward their answers or opinions.
- The responsibility of a good questionnaire towards research is that the respondents should realize the importance of the questions asked and not brush them off as unimportant or insignificant.
- A good questionnaire should avoid questions that may be embarrassing or annoying to the respondents. Although questionnaire as a data collection tool does not leave much ground for the respondents to express themselves subjectively and also in most cases guarantee anonymity, in the ones that involve face to face interaction with the respondents, the questions should be framed with more sensitivity and care so as to not make the respondent uncomfortable. This is important for getting data, which is not coloured by uneasiness. Personal and intimate questions should be asked towards the end.
- The researcher must avoid technical or difficult terms and imprecise language, which may be incomprehensible, misunderstood or misinterpreted.
- The researcher must avoid those questions, which may hurt the sentiments and dignity of the respondents.
- The questionnaire must be prepared in a way that sufficient space is provided for the respondents to answer the open-ended questions.



CHECK YOUR PROGRESS

Q 1: Fill in the blanks.

(a) Questionnaire is a series of _____ questions to gather _____

information from respondents.

(b) A good questionnaire should be _____ by the respondents.

14.3.3 Types of Questionnaires

Two most significant dimension along which questionnaires commonly vary are: (a) degree of structure, and (b) number of response possibilities.

Degree of Structure: In light of degree of structure, questionnaires may be of three types, i.e structured, semi-structured and unstructured. A Questionnaire can be structured or semi-structured depending on the needs of the research. Structured questionnaires are also called closed questionnaires, which usually consist of pre-set options from which respondents need to choose their answer. They are those in which the questions to be asked from the respondents and the instruction to be followed by the interviewers are very precisely and concisely stated in advance.

Semi-structured or open-ended questionnaires consist of fixed questions but leaves space for the respondents to answer in their own words. Semi-structured questionnaire may result in subjective answers for some questions, and hence does not fully qualify as quantitative data.

On the other hand, unstructured questionnaires are those in which specify only the broad areas of a subject and not the form or sequence of questions. In this context, formulation of questions is left to the interviewers themselves who are free to question respondents as their own choice.

Number of response possibilities: Here, questionnaires also vary in respect of the number of response possibilities provided to the subject. If a question provides the subject with only two response possibilities it is known a dichotomous question.

14.3.4 Merits of a Questionnaire

As said above, a questionnaire is an important tool of

primary data collection. However, using questionnaire as a research tool has its own advantages and disadvantages, and hence, should only be used depending on the requirements of the research. For example, some research might demand elaborate subjective opinion from the respondents. In that case, a questionnaire would not prove to be fruitful in terms of data. Whereas, a research, which demands objective answers from respondents, would benefit a lot from a well designed, good and crisp questionnaire. Hence, as per the need of the research concerned, a questionnaire can primarily be divided into closed questionnaires and open-ended questionnaires. To elaborate further, we shall discuss a few merits and demerits of questionnaires in general that fit into both the categories.

- **Cost effective:** The best thing about a questionnaire is that it is cost effective. It is economical in terms of both time and money, to say the least, since it does not always require the researcher to interact with the respondents at a personal level. It thus saves research time, which is precious, and money as there is not a lot of travelling involved. Further, a large number of data can be collected from a large population in a very less time, making it one of the most convenient tools for conducting objective research.
- **Standardization of facts:** As a research tool, a questionnaire makes the data easy to analyze. The data collected through a questionnaire can thus be used to arrive at a generalization regarding a particular research finding. Questionnaire is the easiest way to derive patterns of social realities on the basis of information received from the data. It also helps the researcher to codify the patterns, further making it useful as quantitative data.
- **Comparable:** One important benefit of using questionnaire as a research tool is that of comparability. The data collected through multiple questionnaires on the same topic from different respondents can be compared to arrive at generalizations or

patterns of social reality. A skilful researcher may design the same set of questions in different and tactful ways for potentially diverse response from the same respondents. Though a questionnaire does not leave much room for the researcher to peak into the subjectivities of the answers provided by the respondents, the said trick proves to be useful in most cases.

- **Anonymity:** Anonymity provides most respondents with the comfort to answer certain questions that they think might be controversial or uncomfortable. A Questionnaire also ensures a kind of detachment from the answer provided, making the respondents flexible with their opinions. Without having to write one's name on it, one becomes free from the fear of unwanted associations, thus being able to contribute towards an objective data.
- **Quantifiable:** Questionnaire can help provide data which is quantifiable, that is, data which can be standardized and used for further research in similar and extended areas. A closed questionnaire is more effective in deriving quantified data.

14.3.5 Demerits of a Questionnaire

- **Lack of Personal Contact with Respondents:** One of the major demerits of questionnaire as a tool of data collection is that it does not allow the researcher to have personal interaction or contact with the respondents. This disadvantage also leads to further disadvantages throughout the process of the research, such as lack of subjective and diverse perspectives in the research, including lack of empathy, issues of validation, etc. No personal contact between the researcher and the respondents also generates a sense of detachment on the part of both the parties, resulting in a danger of the research being taken casually.
- **Poor response:** This immediately follows the above mentioned disadvantage. A Questionnaire always runs the risk

of gaining poor response among the respondents. This could be because of multiple reasons such as laziness, unwillingness, busyness and other personal reasons of the respondent, but the primary reason is generally lack of personal contact between the researcher and the respondent. This could also lead to situations like incompletely filled questionnaires or half-hearted responses.

- **Issue of validation:** Questionnaire as a research tool can face serious issues of validation. Questionnaire is always done in written format and since not all of the respondents may be well-versed in the language used, there may be misinterpretation of the questions asked. This will obviously result in poor quality data or wrong data, thus hampering the findings of the research. Further, the researcher might not be physically present while the questionnaire was being filled up (in terms of online questionnaire or postal questionnaire). Hence, there is no way that the researcher can explain a difficult question or rephrase it to make it understandable for the respondent. This also means that the researcher would generally be clueless about any manipulation of information by the respondents.
- **Misinterpretation:** In the case of questionnaire, the issue of misinterpretation can be faced by both respondents and the researcher, since the researcher is often not present while the respondent is filling the questionnaire and the respondents too are likely to not be present while the researcher is interpreting the responses. The researcher might also face the problem of legibility in responses filled in by respondents, due to bad handwriting or unclear answers (in case of open-ended questionnaires).
- As mentioned above, preparation of a questionnaire has a lot to do with the quality of the data collected. A good questionnaire is most likely to lead the researcher to a good quality data. Despite the limitations that it is susceptible to, questionnaire is

a very popular tool of collecting data in both qualitative and quantitative research methods.

14.3.6 Differences between Questionnaire and Schedule

It is a common tendency among researchers to use questionnaire and schedule interchangeably, and they do seem to be similar in certain cases. However, there are quite a few differences between the two.

- Both consist of a series of pre-set questions to be answered by the respondents, but the fundamental difference between a questionnaire and a schedule is that the questionnaire has to be filled by the respondents, while a schedule must be filled by the enumerator or the researcher herself on behalf of the respondents.
- Both are primary methods of data collection in which data is obtained by asking questions, but a questionnaire is designed to collect data from large, diverse and widely scattered groups of people whereas a schedule usually caters to a smaller population with whom the researcher interacts personally.
- A questionnaire may or may not involve personal interaction (can be sent to the respondent via mail to be filled online or by post) whereas a schedule fundamentally involves personal interaction between the researcher and the respondents.
- Both have same pre-set questions for all the respondents to ensure reliability of data. But in most cases, a questionnaire is comparatively rigid, using standardized words and phrases to get uniform results. A schedule on the other hand is more flexible, allowing the interviewer to modify the questions as per requirement of the interview scene.
- The success of a questionnaire is majorly determined by the quality of the questions asked whereas that of a schedule depends on the honesty and competence of the interviewer. However, qualities that both questionnaire and schedule must

possess as primary data collection tools are that they have to be clear and comprehensible and at the same time sensible to the respondents, and they both must be prepared responsibly so as to bring out data of rich quality.



CHECK YOUR PROGRESS

Q 2: Questionnaire is a cost effective tool for data collection. State whether true or false.

Q 3: Fill in the blanks.

(a) In a schedule, the answers of the respondents are filled up by_____.

(b) A schedule allows _____ to the interviewer in terms of reframing questions.

(c) Questionnaire and interview are both _____ tools of data collection.

14.4 CASE STUDY

In simple terms, Case study is an in depth study of a particular situation. It is a method through which a very broad area of research is narrowed down into one easily researchable topic. Case study methods attempts to find out the reasons behind the behaviour pattern of the study subjects and its relationship with the environment.

According to P.V.Young, "case study is a comprehensive study of a social unit, be it a person, a group of persons, an institution, a community or a family

Kromrey says that case study involves studying individual cases, often in their natural environment and for a long period of time.

According to Goode and Hatt, " Case study is a method of exploring and analysing the life of a social unit".

Thus, we can say that case study is a method that serves different purposes- describing, exploring and explaining a situation as well as generating a theory. It involves qualitative methods of research and presents a holistic account about a particular phenomenon.

14.4.1 Objectives and Characteristics of Case Study Method

Objectives of Case study Method

- The first and foremost objective of case study method is to analyse the behavioural patterns of a single unit and its relations with the environment.
- Another major objective of a case study is that it forms the basis for a detailed study in the future. So we can say that case study is a preliminary form of investigation.
- Case study determines and defines the research question and develops hypothesis in a quantitative study.
- Case study is used by researchers to refute generalization in research.
- One of the objectives of case study is to test whether a quantitative study can be done or not.
- Case study explores, explains and describe a particular situation.

Characteristics of Case Study Method

- In case study a small number of unit is studied.
- It studies the units in totality and not some specific aspects or variables of the units.
- Case study can be qualitative or quantitative
- In this method, one unit is considered to be one study
- Case study covers a wide cycle of time
- In this method the case is the core or centre of enquiry.

14.4.2 Forms of Case Study

For different purposes of evaluation, researchers opt for different types of case study methods. Some of those are explained below-

- **Intrinsic Case study:** The study of a case (person, group, department, community) where the case itself is the primary

interest in exploration is known as intrinsic case study. Here the researcher does not have to select one particular case. For example, evaluating the implementation of a new principle in an organization.

- **Instrumental Case study:** In this kind of case study, the researcher begins with a research question and make use of a case that simplifies or illuminates his analysis. So we can say that in Instrumental case study, the case is simply a preliminary technique that paves way for further research. For example, a researcher interested in women and food habits might set up a study with middle aged women and calcium intakes in their food. In this case, the women and the food habits are not the focus. The focus is learning the relationship between women and the food habits, and why certain women have weaker bone.
- **Pilot case study:** Sometimes a researcher conducts some condensed studies before implementing a large-scale investigation. Such studies are called pilot or exploratory case studies. The main objective of this kind of case study is to have identifying research question and selecting types of measurement.
- **Cumulative Case study:** In cumulative case study the researcher aggregate information about the same unit that has been previously collected at different times. This allows him to generalize information on the basis of past data. It also saves time, money and energy.
- **Illustrative Case study:** Illustrative case studies are descriptive in nature. In this type of case study one or two instances of the same events is used to describe an existing situation. Such as study familiarises the researcher with the research area.
- **Historical case study:** Historical case study is dependent on recordings, documents and interviews. This type of study traces the development of a person, organization or a system

over time. For example, the study of a political party from its formation till its rise to power.

- **Situational case study:** Situational case study studies a particular event or situation. All participants of the event are sought and their views are recorded. That helps the researcher understand and analyse the situation as a whole. For example, the reasons behind strike of workers in a factory. Here the management, the participant workers, the police forces who arrived at seen to control the situation are all interviewed.

14.4.3 Steps in Case Study Method

A good case study can be conducted if the following steps are followed:

Step 1: Selection of the Case - Whether or not to opt for a Case Study depends on the type of research a person is conducting. If a case study is required, then he/she has to identify the main objective of conducting the study. The selection of a particular case depends on whether the study is simply a preliminary reading for further investigation or whether the researcher is interested in solving a particular problem through the Case Study.

Step 2: Collection of Data - After selecting the case, the researcher has to make preparations for date collection through the case. Here, he/she can use different methods such as personal interviews, survey of records and interviews etc. Generally quantitative techniques of data collection are applicable in this procedure.

Step 3: Preliminary Data Analysis - The third stage in a case study is systematically analysing the preliminary data collected from the case. This will help the researcher to raise more important questions relating to the research as well as to form relevant hypothesis.

Step 4: Thorough study of the case - After identifying the important questions and constructing the hypothesis the researcher conducts a thorough study of the case. Here, he/she uses the method of prolonged observation and formal as well as informal interviews.

The researcher keeps on investigating until he/she finds satisfactory answers to the problem and can draw a clear picture of the investigation.

Step 5: Follow-Up Investigation- In order to ensure the reliability of the data so far collected, the researcher repeats the process of observation and interview and reaises questions whenever and wherever necessary. This follow -up helps him/her to arrive at reliable and valid conclusions.

Step 6: Generalization of the findings-Through the conclusion of the case study, the researcher ensures a generalization of the data collected and analysed. Through this generalization he/she can draw a clear picture of the investigation and extend this understanding to arrive at a conclusion.

14.4.4 Advantages of Case Study Method

- Case study method is an intensive investigation and exploration of a unit through which a researcher can understand an event deeply and thoroughly.
- Case studies expose the researcher to real life situations which is otherwise difficult. That's why we can say that case study simplifies complex concept.
- Through case study the continuous analysis of the life of a social unit can be conducted in order to dig-out the facts.
- It is a very important tool of hypothesis formulation in a research area.
- Case study helps the researcher to make comparison about the different types of facts collected during the study.
- Case study improves analytical thinking, communication skill, tolerance, capability and logical mindset of a researcher.
- It increases the researcher knowledge about a particular social phenomenon
- Case study method is a comprehensive method of data collection in research. It is an inclusive study that enables the

researcher to understand a broad range of topics.

- Case study method helps the researcher to develop a questionnaire and interview schedule.

14.4.5 Disadvantages of Case Study Method

- It is difficult to find appropriate case studies for all types of research.
- Case study has limited representatives because it focuses on single units as a whole. That is why generalization through case study may not always be correct.
- Due to his/her deep connection with the participants, the researcher may develop subjectivity while conducting a case study.
- Case study is not an easy and simple method of research. It is very difficult for a lay-man to conduct a case study.
- Case study is a time consuming instrument of data collection. Hence it may not be the best medium for short term research
- As it is a narrow study, discrimination and biased can occur while investigating a social unit.
- In case study method, there is a huge possibility of memory and judgement errors both on the part of the participants and the observer.
- It is very difficult to draw a definite cause/effect from case studies.



CHECK YOUR PROGRESS

Q 4: Who introduced the method of case study in research?

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Q 5: Mention any two objectives of case study

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14.5 FOCUS GROUP: MEANING AND DEFINITION

In simple terms, a focus group is a small group of six to ten people who are led through an open discussion by a skilled moderator. The main goal of the moderator is to carry forward an open and spontaneous discussion among the participants so that all the participants can express maximum number of ideas and opinions.

A focus group discussion is a good way to gather people from similar experiences and backgrounds and discuss a specific topic of interest. This kind of interviews were originally called "Focussed interviews" or "Group-depth interviews". This technique of interview was developed after World War II. Since then it has become an important method of data collection in social science.

According to Kruger, "Focus group discussion is a carefully planned discussion design to obtain perception on a defined area of interest in a permissive, non-threatening environment."

Focus group discussions are defined as semi structured group discussions, which yield qualitative data on the community level by facilitating interaction between participants.

14.5.1 Characteristics of Focus Group

- A focus group discussion usually consist of six to ten participants.
- The moderator engages the participants in small talks and gradually builds rapport amongst them.
- The moderator does not join the conversation between the participants.
- In a focus group interview, the participants should talk no more than about an hour.
- Recording the focus group discussion is a must
- In a focus group discussion small issues should be discussed and bigger issues must be avoided.
- The moderator should always maintain impartiality particularly

in facial expression and verbal comments.

- In a focus group discussion the participants are always explained the purpose and outcome of the discussion.
- Focus group discussion is a voluntary process where conversation flows among the participants.
- This type of discussion is relaxed, comfortable and often enjoyable for participants as they share their ideas and perception.

14.5.2 What Focus Groups Can and Cannot Tell You

As a method of data collection, focus group can be fun and useful but often an inexperienced moderator may be led astray by focus group discussion because opinions of the participants can often be misleading and subjective. Therefore if a researcher strictly relies on such a discussion as a research strategy, then he/she should have a clear idea about what focus groups can and can not tell him/her. These can be pointed out as follws-

What focus groups can tell you?

- Give an idea on how some people think or feel about a certain issue.
- Give an insight on why people held certain opinions
- Help improve the research planning and research design
- Gives a scope for evaluating the problem at hand
- Gives an insight for developing the research strategy

What focus groups cannot tell you?

- Valid information about individual
- A definite information about how things have changed over time with regards to a particular issue.
- Information that the researcher can applied generally to other groups of people as well
- Can not provide random samples for the study.

14.5.3 Steps for conducting a Focus Group Discussion

Generally there are four distinct step used to conduct focus group discussion. These are-

- Question designing
- Selection and preparation of participants
- Organizing the interview
- Analyzing the data

These are discussed in some detail as below:

Question designing: The first and foremost task of the researcher or moderator in a focus group discussion is to design the questions to be addressed during the discussion. Generally eight to twelve questions are prepared by the researcher. The questions should be simple, short, open-ended and to the point because the participants wont have a chance to see the questions. Generally three types of questions are used in focus group discussion- engagement questions (introduction of participants to make them comfortable with the topic of discussion), exploratory questions (in order to gather as much information as possible) and finally exit questions (to verify whether anything important was left out during the Discussion).

Selection and preparation of participants: Ideally in a focus group discussion the participants do not know each other but should have a common background and must be comfortable with one another. So we can say that in order to get maximum information out of a focus group, the researcher should select a homogenous group of participants. In this regard some important criteria's that are focussed on are gender, age, position in society, economic background etc. On the basis of these criteria's the researcher includes or excludes individual.

The selection process is arranged using a number of ways such as- i) Nomination (individuals with knowledge about the research area nominate people who they think would make good participants),ii) Random selection (participants from a large but defined group can be randomly selected until the researcher attains

the desired number of required participants), iii) All members of a group (in some cases all members of an existing group can participate in a focus group discussion, for instance the owners of an MNC), iv) Similar position (here the researcher selects people on the basis of their roll, position, job title or condition), v) Voluntary participants (in case where the selection criteria is broad)

After selecting the focus group, the researcher must contact all the participants and conform their interest and availability. On the basis of that he/she should organize the time and location of the focus group discussion and inform everyone concerned. The participants should also be informed about the duration of the interview and the issues to be discussed. Efforts should be made to make them feel easy and comfortable.

Organizing the Interview: An ideal focus group is conducted by a moderator with the help of an assistant moderator. Generally, the researcher plays the role of the moderator and carries forward the different discussion in the focus group. The assistant moderator takes notes and records the entire focus group discussion. The first thing a moderator should do in the beginning of the discussion is to take consent of all participants. He/she should create a friendly and easy environment for all the participants to open-up and put forward their views. He also addresses the purpose of the discussion, sets ground rules and tries to adequately cover all issues within the time limit of the focus group. After the participants leave, the moderator and assistant moderator checks the notes and recording device used label them along with date, time and name of the group.

Analysing the data: Analysing the data gathered is the most important and critical job for the moderator. At first all information gathered from the participants must be boiled down to select the one's useful and essential for the research. The researcher then may use different methods to analyse that information such as: labelling them under different columns, identifying categories and sub-categories of issues or summarizing different view point on the

same issue. One's these findings are systematically organized they are ready for presentation.

14.5.4 Significance of Focus Group Discussion

Focus group discussions are often used as a tool of research so that the researcher can familiarise himself or herself with the situation at hand. The most important objective of a focus group is to find solution to different problems through flows of voluntary discussion. Group attitudes and believes can also be investigated through such a discussion which is difficult to achieve through any other technique of data collection.

Generally, a researcher employs the use of focus group discussion in the beginning of studying the problem. This helps him/ her to generate ideas at the exploratory stage of the study. Moreover, focus groups are very useful when the researcher aims to investigate complex ideas and behaviours. The open-ended questioning of a focus group helps the researcher to arrive at the depth of the problem. At the same time the different opinions of the participants can help the researcher to explore differences and commonalities about the situation.

Focus group discussion can help the researcher to develop the framework for conducting further research. His understanding of quantitative data will also depend through analysis of the results of these discussions. However, the researcher must always be aware that the participants might not be entirely honest and truthful about their opinions. At the same time misinterpretation and biased findings can be very damaging for further research. Therefore, the results of focus group discussion can be most fruitful only when the researcher is experienced, intuitive and does not take the opinions of the participants for granted.

14.5.5 Advantages of Focus Group Discussion

- The researcher can take advantage of the fact that people

naturally interact and are influenced by others

- During the discussion participants often agree or disagree with others and thus create more energy and more data.
- Focus group discussion can provide more scope to understand perceptions, attitudes and experiences of the research subjects.
- Data can be provided more quickly and at lower cost than individual interviews
- The researcher can directly interact with the respondent as well as gain information from non-verbal responses during the interview
- This type of interview is very flexible in nature and can involve a wide range of individuals, settings and topics.
- The data analysis process is very easy to conduct and results can be easily understood.

14.5.6 Disadvantages of Focus Group Discussion

- Focus group discussions are more difficult to manage than one on one interviews. Therefore the interviewer must be an experienced person
- The discussion may be inhabited by unexpected conflict, power struggle and other group dynamics among the participants.
- One or two person may dominate the whole discussion while ignoring or intimidating the other participants.
- Focus group discussion require trained moderator who knows about group dynamics otherwise he/she may knowingly or unknowingly misinterpret the responses of the participants
- The participants may not always be honest or sure about their responses about a particular issue.



CHECK YOUR PROGRESS

Q 6: What is meant by focus group? Answer in about 60 words.

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Q 7: What does a focus group describe? Answer in about 50 words.

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14.6 LET US SUM UP

- A questionnaire is one which consists of a number of questions printed or typed in a definite order or on a form or set of forms. It is given to the respondents who are expected to read, understand, and write down their reply in the space provided.
- Questionnaire can primarily be divided into closed questionnaires and open ended questionnaires.
- Questionnaire can be structured or semi-structured, depending on the needs of the research.
- Structured or closed questionnaires usually consist of pre-set options from which respondents need to choose their answer.
- Semi-structured or open-ended questionnaires consist of fixed questions but leave space for the respondents to answer in their own words.

- A good questionnaire should be comprehensible by the respondents.
- A good questionnaire should consist of logical questions looking for logical answers.
- A good questionnaire should avoid questions that may be embarrassing or annoying for the respondents.
- A good questionnaire should be designed in such a way that it draws the interest of the respondents. Failure to do so might lead to failure of the questionnaire to collect adequate data.
- Case study method of research is an intensive study of a phenomenon.
- In this method the case is the core or centre of enquiry.
- For different purposes of evaluation, researchers opt for different types of case study methods such as Intrinsic Case study, Instrumental Case study, Pilot case study, Cumulative Case study, Illustrative Case study, Historical case study, Situational case study etc.
- Case studies help the researcher to connect with his/her research problem. These are in-depth analysis that makes the researcher simplify and understand the background of a particular case.
- Case study method allows the exploration, simplification and understanding of complex issues.
- Focus group discussion is a form of interview that occurs in a group setting and can reveal detailed information and deep insight about a particular situation.
- A focus group discussion is a good way to gather people from similar experiences and backgrounds and discuss a specific topic of interest Generally, there are four step used for conducting focus group discussion. These are- Question designing, Selection and preparation of participants, Organising the Interview, Analysing the data.
- Focus group discussions are often used as a tool of research so that the researcher can familiarise himself or herself with the situation at hand

- Focus group discussion can help the researcher to develop the framework for conducting further research.



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14.8 ANSWERS TO CHCK YOUR PROGRESS

Ans to Q No 1: (a) Pre-set (b) Comprehensible

Ans to Q No 2: True

Ans to Q No 3: (a) Interviewer (b) Flexibility (c) Primary

Ans to Q No 4: An English philosopher Frederic Leplay introduced the case study method.

Ans to Q No 5: Two objectives of case study are mentioned as below-

(1) The first and foremost objective of case study method is to analyse the behavioural patterns of a single unit and its relations with the environment.

(2) Another major objective of a case study is that it forms the basis for a detailed study in the future. So we can say that case study is a preliminary form of investigation.

Ans to Q No 6: A focus group is a small group of six to ten people who are led through an open discussion by a skilled moderator.

Ans to Q No 7: Focus group discussion can describes following:

- Give an idea on how some people think or feel about a certain issue.
- Give an insight on why people held certain opinions
- Help improve the research planning and research design
- Gives a scope for evaluating the problem at hand
- Gives an insight for developing the research strategy.



14.9 MODEL QUESTIONS

Short Questions (Answer each question in around 150 words).

Q 1: Distinguish between questionnaire and schedule

Q 2: Briefly explain the different types of questionnaire.

Q 3: State the meaning of Pilot Case study.

Q 4: Why is case study an important technique of collecting data for research?

Q 5: Explain the concept of focus group discussion.

Q 6: Explain the process of designing question for focus group discussion with example.

Q 7: What are the essential qualities for a good moderator?

Long Questions. (Answer each question in around 300-500 words)

Q 1: Discuss how to prepare a good questionnaire.

Q 2: Discuss the merits and demerits of questionnaire.

Q 3: Explain the features of a good questionnaire.

Q 4: Discuss the various steps of case studies which are important in research.

Q 5: Explain the Significance of case study in research.

Q 6: Discuss the advantage and disadvantages of case study.

Q 7: Briefly discuss the step for conducting focus group discussion.



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