CERTIFICATE PROGRAMME IN VEDIC MATHEMATICS

(CBCS in Accordance with NEP 2020)

MATHEMATICAL TRADITION IN SANSKRIT

Hiranya Chandra Bhuyan School of Science and Technology

KRISHNA KANTA HANDIQUI STATE OPEN UNIVERSITY

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COURSE INTRODUCTION

“Mathematical Tradition in Sanskrit” is the first paper of Certificate Programme in Vedic Mathematics. This course traces the historical evolution of Indian mathematics from its Vedic origins through classical and modern periods. The paper spans 14 units, beginning with mathematical concepts in the ancient Vedas and Sulbasutras, then exploring early Indian developments in arithmetic, geometry, and algebra. Key sections cover features of Vedic mathematics, including the Rule of Three, post-Vedic contributions from Buddhism and Jainism eras, and essential technical terms like zero, infinity, and decimals. It delves into Bhaskaracharya's Lilavati, covering measurements, place value, addition, and subtraction techniques, alongside Aryabhata's Ganitapada on squares, cubes, and roots. Profiles of classical mathematicians such as Aryabhata, Brahmagupta, Mahavira, and Shripati highlight their innovations, leading to modern figures like Ramanujan and Bharati Krishna Tirtha. The syllabus concludes with applications of Vedic sutras in arithmetic and algebra, plus comparisons between Vedic and Western methods for contemporary relevance. This unit equips learners with historical context, enhancing appreciation for Vedic techniques' efficiency in problem-solving

Unit 1: Mathematics in the Vedas

This unit introduces the roots of mathematics in ancient Vedic literature, exploring key mathematical concepts embedded in the Vedas. It emphasizes the Sulbasutras as foundational texts for geometric and arithmetic principles used in rituals. Learners gain insight into how these early sources laid the groundwork for systematic Indian mathematics.

Unit 2: Introduction to Indian Mathematics

The unit outlines the beginnings and general characteristics of ancient Indian mathematics, including its expansion and practical applications. It details developments in arithmetic, geometry, and algebra during this formative period. This provides a broad historical context for India's mathematical heritage.

Unit 3: Features of Vedic Mathematics and the Rule of Three

Here, Vedic mathematics receives a brief introduction, underscoring its importance for speed and simplicity in calculations. The Rule of Three is introduced as a key technique for solving proportion problems efficiently. This unit bridges ancient wisdom with practical computational skills.

Unit 4: Post Vedic Indian Mathematics

This section examines mathematics during the Buddhism and Jainism periods following the Vedic era. It highlights unique contributions and evolutions in mathematical thought during these influential phases. Students explore how these traditions sustained and advanced Indian mathematical inquiry.

Unit 5: Technical Terms of Mathematics

The unit offers a general introduction to mathematics while defining essential technical terms like digit, numbers, zero, infinity, decimal, algebra, and calculus. It clarifies their historical and conceptual significance in Indian contexts. This builds foundational vocabulary for deeper study.

Unit 6: Contributions of Classical Mathematicians-I

This chapter aims to introduce students to the life and works of **Aryabhata**, highlighting his life history and major contributions to mathematics and astronomy. It also seeks to help learners understand the influence of Aryabhata on later Indian scholars and the lasting impact of his ideas on the development of scientific knowledge in India and beyond.

Unit 7: Contributions of Classical Mathematicians-II

Profiles Aryabhata and Brahmagupta, detailing their life histories and major contributions. It also discusses Aryabhata's influence on subsequent scholars. This highlights pivotal figures in classical Indian mathematics.

Unit 8: Contributions of Classical Mathematicians-III

This unit covers Mahabir Acharya and Shripati, including their biographies and key mathematical achievements. It continues the exploration of influential classical thinkers. Their works demonstrate continuity in Indian mathematical innovation.

Unit 9: Lilavati-I

Focused on definitions from Lilavati's first section, it covers Mudra, weight measurements, and variousunits like Masadi-manam (starting with Masa), Anguladi-manam (finger-based), Yojanadi-manam (distance), and others including Greek-influenced methods. This unit immerses learners in ancient measurement systems. Practical applications in trade and construction are implied.

Unit 10: Lilavati-II

Centered on Lilavati Part-III, it teaches rules for addition (sankalitayoga) and subtraction (viyoga). Practical methods for performing these operations are explained. This equips students with historical techniques for basic arithmetic.

Unit 11: Lilavati-III

Centered on Lilavati Part-III, it teaches rules for addition (sankalitayoga) and subtraction (vyavakalita). Practical methods for performing these operations are explained. This equips students with historical techniques for basic arithmetic.

Unit 12: Contributions of Modern Indian Mathematicians

Focuses on Srinivasa Ramanujan, and Bharati Krishna Tirtha, outlining their lives and contributions. It traces the evolution toward modern Indian mathematics. Ramanujan's genius and Tirtha's Vedic revival are emphasized.

Unit 13: Applications of the Vedic Sutras

Explores practical uses of Vedic sutras in basic arithmetic, algebra, polynomials, and higher calculations. Real-world problem-solving examples are provided. This unit demonstrates the timeless utility of Vedic methods.

Unit 14: Relevance of Vedic Knowledge in Modern Mathematics

Compares Vedic and Western approaches to arithmetic, algebra, and geometry. It applies Vedic principles to contemporary problem-solving. Learners see how ancient techniques enhance modern mathematics.

Each unit includes some alongside boxes to help you know some of the difficult, unseen terms. Some “EXERCISES” have been included to help you apply your own thoughts. You may find some boxes marked with: “LET US KNOW”. These boxes will provide you with some additional interesting and relevant information. Again, you will get “CHECK YOUR PROGRESS” questions. These have been designed to self-check your progress of study. It will be helpful for you if you solve the problems put in these boxes immediately after you go through the sections of the units and then match your answers with “ANSWERS TO CHECK YOUR PROGRESS” given at the end of each unit.

UNIT 1: MATHEMATICS IN THE VEDAS

UNIT STRUCTURE

1.1 Learning Objectives
1.2 Introduction
1.3 Introduction to Vedic Literature
1.4 Mathematical Concepts in Vedas
1.5 Sulbasutras
1.6 Let Us Sum Up
1.7 Answers to Check Your Progress
1.8 Further Reading
1.9 Model Questions

1.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- Recognize Vedic texts as sources of early mathematics.
- Identify key concepts like numbers and geometry in Vedas.
- Understand Sulbasutras and their geometric rules.
- Appreciate historical development of Indian mathematics.
- Connect ancient methods to modern understanding.

1.2 INTRODUCTION

Mathematics in ancient India traces back to the Vedic period, where systematic knowledge of numbers and geometry emerged from sacred texts. These early works laid foundations for calculations used in rituals and astronomy. Core ideas appear in hymns and later technical manuals, showing practical applications alongside philosophical insights.

1.3 INTRODUCTION TO VEDIC LITERATURE

Vedic literature forms the oldest body of knowledge in India, composed between 1500 BCE and 500 BCE. The four main Vedas are Rigveda, Yajurveda, Samaveda, and Atharvaveda, each with Samhitas, Brahmanas, Aranyakas, and Upanishads. These texts contain hymns, rituals, and philosophical discussions that reference numbers and measurements.

Numbers appear in Rigveda for counting objects in rituals, such as multiples of 10 or 100 in sacrificial formulas. Yajurveda includes lists of measures for altars, showing early arithmetic.

Atharvaveda mentions fractions and proportions in charms and calendars. Scholars note over 100 references to ganita, or mathematics, across Vedas, often linked to cosmic order.

Brahmanas explain ritual calculations, like dividing offerings. Aranyakas discuss symbolic numbers in meditation. Upanishads use numerical metaphors for reality. Tradition holds Vedas as revealed knowledge, with mathematics embedded for precise rites. Preservation through oral transmission ensured accuracy over generations.

CHECK YOUR PROGRESS

Q1. Name the four main Vedas.

Q2. Which Veda has most ritual formulas?

Q3. What are Brahmanas?

Q 4. Give an example of numbers in Rigveda.

1.4 MATHEMATICAL CONCEPTS IN VEDAS

Vedas contain basic arithmetic for daily and ritual use. Rigveda lists numbers up to 10,000, using words like eka for one and dasra for ten. Hymns count stars or animals, introducing addition and multiplication implicitly.

Proportions appear in Yajurveda for altar sizes, like equating areas of shapes. Atharvaveda has fractions, such as one-fourth in recipes. Geometry starts with circle and square approximations for fire altars. Pythagoras-like theorem exists for right triangles in ritual contexts.

Astronomy uses cycles of 27 or 28 nakshatras, leading to modular arithmetic. Calendars calculate lunar-solar alignments. Vedic index fingers measured units, standardizing lengths. These concepts served practical needs, blending computation with cosmology.

1.5SULBASUTRAS

The Sulbasutras are manuals for the construction of altars to do certain acts of worship in the vedic age. Sulba refers to ropes or cords, and sutra means rules. Ropes were used for the construction of altars. Therefore geometry in ancient India was known as Sulba. The term Rekhaganita for geometry is of later origin.

There are mainly seven sulbasutras. They are known by the names Baudhayana, Apastamba, Katyayana, Manava, Maitrayana, Varaha, and Vadhula after the names of the sages who wrote them.

BaudhayanaSulbasutra states the diagonal of a rectangle equals the square root of sum of squares of sides, predating Pythagoras. Constructions transform square to rectangle of equal area by diagonal division. Circles equal to squares use approximations like pi as 3.088.

Rules build right angles with cords, divide circles into equal parts. Volumes of pyramids and frustums appear. Excavations confirm altars matching descriptions. Sulbasutras systematize geometry for rituals, influencing later texts like Aryabhatiya.

CHECK YOUR PROGRESS

Q1. Name two Sulbasutra authors.
Q2. What theorem does Baudhayana state?
Q3. How to make square equal to circle?
Q 4. Purpose of Sulbasutras?

1.6 LET US SUM UP

- Vedic literature includes four Vedas with early arithmetic references.
- Concepts cover numbers, proportions, and basic geometry for rituals.
- Sulba Sutras provide precise constructions and theorems.
- These works show advanced knowledge by 800 BCE.
- Foundations influenced later Indian mathematics.

1.7 ANSWERS TO CHECK YOUR PROGRESS

Answer to Q 1. Rigveda, Yajurveda, Samaveda, Atharvaveda.
Answer to Q 2. Yajurveda.
Answer to Q 3. Ritual explanations in Vedas.
Answer to Q 4. Multiples like 10 or 100 in hymns.
Answer to Q 5. Baudhayana, Apastamba.
Answer to Q 6. Diagonal squared equals sum of sides squared.
Answer to Q 7. Approximate pi, adjust radius.
Answer to Q 8. Altar construction for rituals.

1.8 FURTHER READING

1. Sri Bharati Krishna Tirthaji, Vedic Mathematics or Sixteen Simple Mathematical Formulae from the Vedas, chapters on quadratic and cubic factorization and related sutras
2. Vedic Mathematics – Methods (Sri Sathya Sai Veda Pratishtan), sections on factorization of quadratics and cubics using sutras such as Adyamadyena, Anurupyena and Lopana– Sthapanabhyam.
3. Rajesh Kumar Thakur, Advanced Vedic Mathematics, Rupa Publications, chapters on quadratic equations, factorization of cubic polynomials and harder factor problems.

1.9 MODEL QUESTIONS

Q 1. Describe Vedic structure.

Q 2. List arithmetic in Vedas.

Q 3. Explain Sulbasutra contributions.

Q 4. Compare Vedic pi approximation.

Q 5. Discuss Pythagoras in Sulba.

UNIT 2: INTRODUCTION TO INDIAN MATHEMATICS

UNIT STRUCTURE

2.1 Learning Objectives
2.2 Introduction
2.3 Beginning of Ancient Indian Mathematics: General Characteristics
2.4 Expansion of Ancient Indian, Mathematics and Its Practical Applications
2.5 Development of Arithmetic in Ancient India
2.6 Development of Geometry and Algebra in Ancient India
2.7 Let Us Sum Up
2.8 Answers to Check Your Progress
2.9 Further Reading
2.10 Model Questions

2.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- Understand the origins and general features of ancient Indian mathematics.
- Explore how Indian mathematics expanded through practical uses in daily life and rituals.
- Trace the growth of arithmetic methods and tools in ancient India.
- Examine key advances in geometry and algebra from Vedic times onward.

2.2 INTRODUCTION

Ancient Indian mathematics represents a remarkable achievement that shaped global numerical thought for centuries. Emerging from the Vedic period around 1500 BCE, it intertwined with astronomy, rituals, and daily commerce, producing practical tools that emphasized mental computation and precision. Unlike abstract Greek geometry, Indian approaches focused on solvable problems through concise sutras, enabling rapid calculations on dust boards without permanent records. This unit traces the journey from early Vedic counting to sophisticated arithmetic, geometry, and algebra by the medieval era. Key texts like Sulbasutras laid geometric foundations for altar construction, while Aryabhata and Brahmagupta advanced zero, decimals, and equations. Practical demands in trade, calendars, and temples drove expansion, yielding innovations like place-value notation and infinite series Through these developments, Indian mathematics demonstrated flexibility, linking numbers to cosmic order while serving everyday needs. Understanding this evolution reveals its enduring influence on modern systems.

2.3 BEGINNING OF ANCIENT INDIAN MATHEMATICS: GENERAL CHARACTERISTICS

Ancient Indian mathematics began during the Vedic period around 1500 BCE, rooted in religious rituals and practical needs. Early texts like the Vedas contain basic counting and measurements for sacrifices and trade. Scholars memorized short formulas called sutras, then practiced on dust boards to master calculations. This oral tradition emphasized mental agility and precision over written records.

Mathematics served astronomy, architecture, and commerce from the start. The Indus Valley Civilization showed evidence of standardized weights and a decimal-like system by 2500 BCE. Vedic mathematics focused on exactness for altars and calendars, blending computation with philosophy. Unlike Greek emphasis on proofs, Indian work prioritized useful rules for real-world tasks. Key traits included place-value notation and handling large numbers through powers of ten.

Communities like Jains added number theory and infinity concepts by 150 BCE. This foundation built a system flexible for expansion, linking numbers to cosmic patterns.

CHECK YOUR PROGRESS

Q1. When did ancient Indian mathematics primarily begin?

Q2. What were the main tools for early mathematical practice?

Q3. Name two practical areas where Vedic mathematics was applied.

Q4. How did Indian approaches differ from Greek mathematics?

2.4 EXPANSION OF ANCIENT INDIAN, MATHEMATICS AND ITS PRACTICAL APPLICATIONS

Indian mathematics grew through needs in rituals, trade, and science, spreading from Vedic texts to major works by 500 CE. Sulbasutras detailed altar designs, transforming shapes while keeping areas equal, like squares to circles. These rules aided temple building and urban planning, using ropes for precise angles.

Commerce drove fraction handling and interest calculations. Texts covered mixtures, like alloy blending, and series for taxes or harvests. Astronomy demanded planetary tracking,

leading to sine tables and eclipse predictions. Brahmagupta's 628 CE work included barter, brick stacking, and grain piling, showing everyday uses.

By 800 CE, works like Lilavati applied math to surveying fields and timber cutting. Jaina texts handled permutations for rituals. This expansion made mathematics accessible, influencing trade routes and calendars. Practical focus ensured growth, from village weights to royal observatories.

2.5DEVELOPMENT OF ARITHMETIC IN ANCIENT INDIA

Arithmetic evolved from Vedic counting to sophisticated systems by the Gupta era. Early Vedas used words for powers of ten, handling millions easily. Jainas around 150 BCE classified infinities and logarithms base two. Aryabhata in 499 CE gave addition rules for fractions via common denominators.

Brahmagupta formalized zero as a number, enabling negative quantities and quadratics. His methods covered cubes, roots, and proportions. Bhaskara II's 1150 CE Lilavati detailed squaring near bases and checks like digit sums. Vedic sutras sped operations; Nikhilam handled near powers of ten.

Texts taught mental math for markets, like quick divisions. Decimal place-value, with zero, spread globally via Arabs. Arithmetic linked to astronomy, computing epics' verses or calendars. This base supported algebra and geometry advances.

2.6DEVELOPMENT OF GEOMETRY AND ALGEBRA IN ANCIENT INDIA

Geometry started in Sulbasutras before 800 BCE, for altar shapes like triangles and trapeziums. Baudhayana stated the diagonal theorem: a rectangle's diagonal squares to side products, predating Pythagoras. Rules approximated pi as 3.125 and built equal-area figures, using ropes for right angles.

Aryabhata computed triangle areas, circle ratios, and shadows. Brahmagupta gave cyclic quadrilateral areas via sides and semi-perimeter. Bhaskara proved theorems geometrically, like rearranging triangles into squares.

Algebra began implicitly in Sulba equations, grew with Aryabhata's quadratics. Brahmagupta solved $ax^2 + bx = c$, handled negatives. Bhaskara's Bija Ganita covered cubics, progressions. Kerala school around 1400 CE used infinitesimals for arcs, nearing calculus.

Jainas solved simultaneous equations. Algebra simplified geometry, like chord lengths. These fields intertwined, aiding architecture and astronomy.

2.7 LET US SUM UP

- Ancient Indian mathematics originated in Vedic rituals, emphasizing practical sutras and mental computation.
- It expanded via astronomy, trade, and construction, with Sulba Sutras enabling precise altar designs.
- Arithmetic advanced decimal systems, zero, and fast operations like fractions and series.
- Geometry featured Pythagoras-like theorems; algebra solved quadratics and cubics systematically.

2.8 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q 1. Around 1500 BCE during the Vedic period.

Ans to Q 2. Dust boards and memorized sutras.

Ans to Q 3. Astronomy and altar construction.

Ans to Q 4. Indians focused on practical rules; Greeks on proofs.

2.9 FURTHER READING

1. Sri Bharati Krishna Tirthaji, Vedic Mathematics or Sixteen Simple Mathematical Formulae from the Vedas, chapters on quadratic and cubic factorization and related sutras
2. Vedic Mathematics – Methods (Sri Sathya Sai Veda Pratishtan), sections on factorization of quadratics and cubics using sutras such as Adyamadyena, Anurupyena and Lopana– Sthapanabhyam.
3. Rajesh Kumar Thakur, Advanced Vedic Mathematics, Rupa Publications, chapters on quadratic equations, factorization of cubic polynomials and harder factor problems.

2.10 MODEL QUESTIONS

Q 1. Describe the role of Sulba Sutras in early geometry.

Q 2. How did Brahmagupta contribute to arithmetic and algebra?

Q 3. Explain place-value system's impact on Indian calculations.

Q 4. Compare Vedic and Jaina mathematical approaches.

Q 5. What practical applications drove arithmetic growth?

UNIT 3: FEATURES OF VEDIC MATHEMATICS AND THE RULE OF THREE

UNIT STRUCTURE

3.1 Learning Objectives
3.2 Introduction
3.3 Brief introduction of Vedic mathematics
3.3.1 Vedic sutras and mental strategies
3.3.2 Geometric support and visual thinking
3.4 Importance of Vedic mathematics
3.4.1 Educational benefits and learner confidence
3.4.2 Cultural heritage and enrichment
3.5 Rule of Three
3.5.1 Conventional and Vedic approaches to the Rule of Three
3.5.2 Geometric illustration of the Rule of Three
3.6 Let Us Sum Up
3.7 Answers to Check Your Progress
3.8 Further Reading
3.9 Model Questions

3.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- Explain in simple language what Vedic mathematics is and how it emerged from Indian tradition.
- Identify the main features and advantages of Vedic mathematical methods in everyday and school mathematics.
- State the idea of the Rule of Three and recognise situations of direct proportion where it applies.
- Solve basic and slightly advanced Rule of Three problems using both conventional and Vedic approaches.
- Interpret simple geometric representations that support proportional reasoning and the Rule of Three.

3.2 INTRODUCTION

Vedic mathematics is a system of mathematical thinking and techniques inspired by ancient Indian sources and later reconstructed in the twentieth century by scholars such as Sri Bharati Krishna Tirthaji. It offers a family of mental methods and compact procedures for arithmetic, algebra, geometry and other branches of mathematics, all organised around a set of general principles called sutras.

In modern classrooms these methods are used alongside the usual curriculum to simplify calculations, to encourage flexible thinking and to show the historical roots of mathematics in Indian culture. The present unit focuses on two themes that are central at school level: the overall features of Vedic mathematics, and the classical Rule of Three, which deals with proportional reasoning in practical contexts.

3.3 BRIEF INTRODUCTION OF VEDIC MATHEMATICS

Vedic mathematics, as taught today, is not a direct verbatim extract from the four Vedas but a systematic presentation of arithmetic and algebraic techniques that the author Tirthaji claimed to rediscover from Vedic-style sutras. The core of his presentation is a list of sixteen main sutras and several sub-sutras, each of which gives a very general verbal rule that can be applied to many types of problems, rather than to a single narrow procedure.

For example, the sutra “NikhilamNavataścaramamDaśatah” encourages working from a convenient base such as 10, 100 or 1000 and “all from 9 and the last from 10,” while “Urdhva-Tiryagbhyam” expresses the idea of multiplying “vertically and crosswise” to obtain all partial products in one organised pattern. These sutras offer a unifying mental framework under which operations like addition, subtraction, multiplication, division, squaring, cubing and factorisation can be handled in a short, often mental, manner rather than by long written algorithms.

Historically, Indian mathematics already contained powerful ideas such as place-value numerals, the use of zero, sophisticated geometry in the Śulbasūtras, and algebraic reasoning in later works. The modern Vedic mathematics movement connects these historical achievements with accessible classroom techniques, so that learners can see both conceptual elegance and cultural continuity.

3.3.1 VEDIC SUTRAS AND MENTAL STRATEGIES

The sixteen sutras can be viewed as guiding principles rather than rigid recipes, and many of them express natural patterns which, once understood, become powerful mental tools. For instance, “EkādhikenaPūrvena” – “by one more than the previous one” – underlies fast methods for squaring numbers ending in 5 and also appears in recurring decimal conversions and some algebraic expansions.

Likewise, “ParāvartyaYojayet,” meaning “transpose and apply,” captures the idea of moving terms across an equality or denominator and adjusting signs, which is central in solving equations and in certain methods of division of numbers and polynomials. Because each sutra can operate at several levels – basic arithmetic, algebra, calculus or even coordinate geometry – the system encourages students to look for structure and symmetry instead of memorising many unrelated tricks.

3.3.2GEOMETRIC SUPPORT AND VISUAL THINKING

Indian mathematical tradition has always used geometry as a concrete support for numerical ideas, especially in altar constructions and area relations described in the Śulbasūtras. In teaching Vedic mathematics, simple geometric figures can therefore play a helpful role, particularly to visualise multiplication patterns and proportional reasoning.

Consider a square of side $a + b$ partitioned into four smaller rectangles: two squares of side a and b, and two rectangles of sides a and b. If this square is drawn and labelled, the total area $(a + b)^2$ can be seen as $a^2 + 2ab + b^2$, which is a geometric picture for the distributive pattern used in Urdhva-Tiryagbhyam multiplication.

A simple figure that is often drawn is a right-angled triangle representing a proportion: the horizontal side carries one quantity, the vertical side the other, and similar triangles show how both grow in the same ratio. This visualisation links naturally to the Rule of Three, where enlarging one side of a rectangle in fixed proportion requires enlarging the other side by the same factor to keep area or shape consistent.

CHECK YOUR PROGRESS

Q 1. Who systematised the modern presentation of Vedic mathematics into sixteen mainsutras and several sub-sutras.

Q 2. What is the general meaning of the sutra “Urdhva-Tiryagbhyam” in the context of multiplication?

Q 3. In the geometric figure of a square with side $a + b$, how does the area demonstrate the algebraic identity used in Vedic multiplication?

3.4 IMPORTANCE OF VEDIC MATHEMATICS

Vedic mathematics is important in school education for cognitive, practical and cultural reasons that go beyond simply getting answers faster. At the cognitive level, the methods

draw attention to patterns, symmetry and alternative ways of seeing the same calculation, which supports flexible thinking and number sense.

From a practical standpoint, these methods are often shorter than standard algorithms, especially when numbers are near convenient bases or when mental calculation is required, such as in competitive examinations or daily commercial transactions. For example, the Nikhilam method multiplies numbers close to 10, 100 or 1000 by working with small deviations from the base, so that 98 × 97 can be solved mentally through the deviations 2 and 3 from 100 rather than by long multiplication.

Another important aspect is error checking: techniques such as casting out nines and “Beejank” (digit sums) allow a quick verification of long computations in arithmetic, squaring, cubing, multiplication and division. These checks do not prove that an answer is correct, but they efficiently detect many common mistakes without repeating the entire calculation, which is particularly useful when teaching students how to monitor their own work.

3.4.1 EDUCATIONAL BENEFITS AND LEARNER CONFIDENCE

Many students who fear mathematics do so because they experience it as a long chain of rigid rules with little room for creativity or personal strategy. Vedic methods offer alternative paths that are often shorter and more intuitive once the underlying idea is grasped, and this can restore confidence by giving learners a sense of control over calculations.

For instance, the duplex method for squaring numbers breaks the problem into symmetrical products of digits, allowing students to see squaring as a patterned recombination rather than as repeated multiplication. Similarly, special sutras for solving certain linear and quadratic equations – such as “AnurupyenaSunyam Anyat” and “Sunyam Sama-Samuccaya” – reduce algebraic manipulation and sometimes allow solutions by inspection when the coefficients satisfy recognisable relations.

Classroom reports and practice materials show that when students are invited to compare standard and Vedic methods, they begin to ask “which approach suits this problem best” rather than “which rule must I follow,” and this question itself reflects a higher level of mathematical maturity.

3.4.2CULTURAL HERITAGE AND ENRICHMENT

Vedic mathematics also serves as a bridge between contemporary education and India’s long mathematical heritage, which includes place-value numerals, algorithms, geometry and astronomical computations that spread across cultures. Texts and commentaries highlight how ancient Indian scholars developed techniques for altar construction, area equivalence and right-triangle relations centuries before similar results appeared elsewhere, even though terminology and notation were different.

Introducing Vedic methods in a balanced way allows students to see mathematics as a human and historical endeavour rather than merely as a fixed body of rules imported in

modern times. It also encourages respect for contributions from different civilisations and supports an integrated view in which mental calculation, geometric insight and symbolic algebra all have a place.

CHECK YOUR PROGRESS

Q 4. Why are digit-sum or Beejank methods useful in the context of long numerical calculations?

Q 5. How can Vedic mathematics contribute to reducing mathematics anxiety among learners?

3.5RULE OF THREE

The Rule of Three (called Trairasika in Sanskrit) is found in the Bakshali manuscript, and in the works of Aryabhata, Brahmagupta, and other ancient Indian mathematicians.
The rule is as follows:

If p yields f, what will i yield?

Here $p =$ pramana (argument)
$f =$ phala (fruit)
$i =$ iccha (requisition)
f is multiplied by i, and the product fi is divided by p.
The quotient is the fruit corresponding to the iccha, i.e., icchaphala.
Let us take an example
If 3 pencils can be bought for Rs. 15, how many pencils can be bought for Rs. 105.
Here $f = 3, i = 105, p = 15$

$$\therefore \text{ icchaphala} \frac{3\times105}{15} = 21$$

3.5.1 CONVENTIONAL AND VEDIC APPROACHES TO THE RULE OF THREE

In the conventional school method, students are usually taught to convert a word problem into a proportion, then to cross-multiply and divide, or to first find the value of “one unit” and scale up or down. For the earlier example, one notebook costs $45 \div 3 = 15$ rupees, and 7 notebooks therefore cost $7 \times 15 = 105$ rupees, which matches the proportional formula.

Vedic mathematics does not replace this reasoning but offers alternative ways to organise the same calculation using sutras like “ParāvartyaYojayet” and “Anurupyena,” which stress transposition and working proportionately. Instead of writing a full proportion, a student can think: “Multiply the required quantity by what relates to the known value, then divide by what relates to the given quantity,” keeping the steps mentally compact. In many numerical cases, factors can be cancelled before multiplication, making the computation quicker and easier to do without a calculator.

3.5.2GEOMETRIC ILLUSTRATION OF THE RULE OF THREE

A simple geometric figure can clarify why the Rule of Three expresses direct proportion. Imagine a rectangle whose horizontal side represents quantity of goods and whose vertical side represents price per unit. If the price per unit is fixed, doubling the quantity doubles the total cost, and halving the quantity halves the cost; geometrically, rectangles representing different purchases are similar, with one dimension fixed and the other changing linearly.

A more symmetric picture uses similar triangles. Draw a right-angled triangle with base a and height b, then another triangle with base c such that it is similar to the first one (same angles). Because of similarity, the ratio of heights equals the ratio of bases, that is $ba = xc$, which rearranges to $x = \frac{b \times c}{a}$, the algebraic form of the Rule of Three. In textbooks and notes on Indian geometry, such diagrams are often used to show that proportionality in segments corresponds directly to proportionality in numeric values, grounding the Rule of Three in visual reasoning.

CHECK YOUR PROGRESS

Q 6. State in words the kind of situation where the Rule of Three is applicable.

Q7. In the proportion "a is to b as c is to x," which quantities are multiplied together according to the product-of-extremes equals product-of-means idea?

Q 8. How can similar triangles be used to justify the algebraic formula of the Rule of Three?

3.6 LET US SUM UP

- Vedic mathematics is a reconstructed system of mental and written techniques based on general sutras presented by Tirthaji, which connect arithmetic, algebra and geometry through common patterns.
- The methods emphasise working from convenient bases, using vertical and crosswise operations, and applying transposition and proportion in flexible ways that often shorten calculations and strengthen number sense.
- Educationally, Vedic mathematics can build confidence, support error checking through digit sums, and connect learners with India's historical contributions to mathematical thought.
- The Rule of Three handles direct proportion problems by maintaining the equality of two ratios, leading to the formula $x = \frac{b \times c}{a}$, which can be interpreted visually with rectangles and similar triangles.

3.7 ANSWERS TO CHECK YOUR PROGRESS

Answer to Q 1. The modern systematisation into sixteen main sutras and sub-sutras was carried out by Sri Bharati Krishna Tirthaji in his work on Vedic mathematics.

Answer to Q 2. "Urdhva-Tiryagbhyam" refers to obtaining products "vertically and crosswise," meaning that in multiplication one forms vertical and crosswise products of digits or terms and then combines them systematically to get the complete result.

Answer to Q 3. When a square of side $a + b$ is divided into four parts – a square of area a^2, a square of area b^2, and two rectangles of area ab each – the total area $(a + b)^2$ is seen as $a^2 + 2ab + b^2$, illustrating the distributive pattern used in Vedic multiplication.

Answer to Q 4. Digit-sum or Beejank methods allow a quick check of long calculations by comparing the digit sums of inputs and outputs; if these do not match under the appropriate rule, an error is indicated without redoing the entire computation.

Answer to Q 5. By offering shorter and more pattern-based methods, Vedic mathematics reduces the amount of rote manipulation and gives learners alternative strategies, which can make problems feel more manageable and thus lessen anxiety.

Answer to Q 6. The Rule of Three applies when two quantities change in direct proportion, such as cost with quantity at a fixed unit price, distance with time at constant speed, or work done with number of identical workers over a fixed duration.

Answer to Q 7. In the proportion $a : b = c : x$, the product of the extremes $a \times x$ is set equal to the product of the means $b \times c$, which leads to $x = \frac{b \times c}{a}$ when solving for x.

Answer to Q 8. Similar triangles preserve ratios of corresponding sides; if the bases a and c correspond and the heights b and x correspond, similarity gives $ba = xc$, which rearranges to $x = \frac{b \times c}{a}$, providing a geometric justification of the Rule of Three.

3.8 FURTHER READING

1. Sri Bharati Krishna Tirthaji, Vedic Mathematics or Sixteen Simple Mathematical Formulae from the Vedas, chapters on quadratic and cubic factorization and related sutras
2. Vedic Mathematics – Methods (Sri Sathya Sai Veda Pratishtan), sections on factorization of quadratics and cubics using sutras such as Adyamadyena, Anurupyena and Lopana– Sthapanabhyam.
3. Rajesh Kumar Thakur, Advanced Vedic Mathematics, Rupa Publications, chapters on quadratic equations, factorization of cubic polynomials and harder factor problems.

3.9 MODEL QUESTIONS

Q 1. Explain the meaning of Vedic mathematics and describe how the sixteen sutras function as general guiding rules rather than fixed tricks.

Q 2. Discuss any three important educational features of Vedic mathematics which make it helpful for school students in developing speed, accuracy and confidence in calculations.

Q 3. Describe the Rule of Three with one suitable day-to-day example. Show step by step how the unknown quantity is obtained using the proportional relation.

Q 4. A certain type of cloth costs ₹540 for 9 metres. Using the Rule of Three, find the cost of 15 metres of the same cloth. Then briefly indicate how Vedic proportional thinking can simplify this calculation.

UNIT 4: POST VEDIC INDIAN MATHEMATICS

UNIT STRUCTURE

4.1 Learning Objectives
4.2 Introduction
4.3 Mathematics of Buddhism Period
4.4 Mathematics of Jainism Period
4.5 Let Us Sum Up
4.6 Answers to Check Your Progress
4.7 Further Reading
4.8 Model Questions

4.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- Recognize Vedic texts as sources of early mathematics.
- Identify key concepts like numbers and geometry in Vedas.
- Understand Sulba Sutras and their geometric rules.
- Appreciate historical development of Indian mathematics.
- Connect ancient methods to modern understanding.

4.2 INTRODUCTION

The post-Vedic period, spanning roughly from 500 BCE to 400 CE, marked a shift in Indian intellectual life as Buddhism and Jainism rose alongside declining Vedic dominance. Mathematics evolved from altar constructions in Sulba Sutras to more abstract pursuits driven by cosmology, philosophy, and enumeration needs. Buddhist and Jaina thinkers explored vast scales of time and space, leading to sophisticated numeral handling and infinity ideas. Jainism particularly advanced number theory, while Buddhism applied math to doctrinal models like kalpas. These traditions preserved Vedic legacies but freed mathematics from strict ritual ties, fostering innovations in arithmetic, series, and geometry. Trade, astronomy, and textual analysis further spurred practical calculations. This era bridged ancient Vedic roots to classical works like Aryabhatiya, laying groundwork for decimal systems and algebraic methods.

4.3 MATHEMATICS OF BUDDHISM PERIOD

Buddhism emerged around the 5th century BCE, emphasizing impermanence and vast cosmic cycles. Mathematical ideas served cosmological descriptions rather than pure theory, yet yielded precise computations.

Buddhist texts quantified eons (kalpas) with immense figures. The Lalitavistara Sutra details numbers up to 10^{53}, using terms like koti (10^7) and extending via powers. A kalpa, earth's destruction cycle, equals a mountain (1 yojana sides) eroded by silk cloth rubs every century. Such metaphors demanded large-number arithmetic, predating formal notation but showing enumeration prowess.

The Flower Garland Sutra's "Incalculable" chapter escalates: koti-koti (ayuta, $\sim 10^{14}$), then squared progressions to nelu $(10^{(10^{64}+\text{something enormous})})$. These illustrate boundless realms, with math structuring philosophical infinity.

Astronomy integrated math via shadows and gnomons. Gnomon height ratios yielded time divisions; texts like those linked to Vasubandhu compute solar positions using proportions akin to Vedic jyotisa.

Series summation appeared in doctrinal counts. Enumerating Buddha lands or merits involved progressions: sum of first n naturals $\left(\frac{n(n+1)}{2}\right)$ for kalpa fractions.

Geometry aided stupa designs and mandalas. Circular bases used circumference-diameter approximations (close to 3.14), with areas via halved sectors rearranged rectangularly.

Permutations counted mantra syllables or rebirth paths, using factorials implicitly.

Buddhist math emphasized utility: calendars tracked monastic routines; trade fractions settled exchanges. Logico-epistemology in Abhidharma analyzed valid cognition (pramana), paralleling proof structures.

Overall, Buddhist mathematics quantified samsara's scale, blending philosophy with arithmetic precision.

CHECK YOUR PROGRESS

Q 1. What role did large numbers play in Buddhist cosmology?

Q 2. Name a key Buddhist text discussing incalculable quantities.

Q 3. How did Buddhist texts approximate circle measurements?

Q 4. Give an example of series use in Buddhist calculations.

4.4 MATHEMATICS OF JAINISM PERIOD

Jainism, contemporaneous with Buddhism (6th-5th century BCE), delved deeply into mathematics via anuyoga (expositions). Ganita-nuyoga treated computation systematically, independent of rituals.

Jainas classified numbers: sankhyeya (enumerable), asankhyeya (innumerable), ananta (infinite). Sthananga Sutra lists 24 asankhyeya types, like madhyadesa$\left(10^{(4*10^{20}+4)}\right)$. Infinities varied: one/two directions, areal, omnipresent, perpetual.

Numeral innovations shone. Decimal place-value emerged, with shunya (zero) as void. Surya Prajnapti (3rd century BCE) used Brahmi precursors to Devanagari digits.

Arithmetic covered operations, fractions, proportions. Bhagavati Sutra solved quadratics; JambudvipaPrajnapti detailed permutations/combinations for cosmic points.

Geometry measured loka (universe): tiered structure with Mount Meru, continents via circles/rectangles. Areas/volumes used Sulba-like rules; sphere approximations appeared.

Algebraic equations (*bijaganitasamikarana*) solved via sets, indices. Satkhandagama (2nd century CE) manipulated sets: logarithms base-2, squares/roots, powers (finite/infinite).

Permutations/combinations enumerated soul clusters, time units. Factorials implicit in anuyoga counts.

Series summed merits/realms. Infinite regressions modeled karma.

Practical math aided trade: mixtures, interest, shadows. Zero enabled positional notation, influencing global systems.

Jainas bridged Vedic ritual math to classical abstraction, emphasizing infinity/zero foundational for later calculus precursors.

CHECK YOUR PROGRESS

Q 5. How did Jainas classify numbers beyond enumerable?

Q 6. Name a Jaina text using set operations.

Q 7. What numeral innovation marks Jaina math?

Q 8. Describe Jaina infinite types briefly.

4.5 LET US SUM UP

- Post-Vedic math shifted to abstract pursuits via Buddhism/Jainism, quantifying cosmos with vast numbers/infinities.
- Buddhist texts like Flower Garland Sutra used metaphors for kalpas, series for doctrinal scales.
- Jainas innovated zero, decimal notation, classified infinities, advanced permutations/sets.
- Both traditions applied geometry to stupas/loka, proportions to astronomy/shadows.
- Legacy: freed math from rituals, foundational for classical Indian/decimal systems.

4.6 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q1. Illustrated kalpa cycles, earth's erosion over eons.

Ans to Q2. Flower Garland Sutra's "Incalculable" chapter.

Ans to Q3. Rearranged sectors rectangularly, ratio ~3.14.

Ans to Q4. Natural number sums for merits/Buddha lands.

Ans to Q 5. Sankhyeya (enumerable), asankhyeya (innumerable), ananta (infinite).

Ans to Q 6. Satkhandagama.

Ans to Q 7. Shunya (zero) in place-value decimal system.

Ans to Q 8. One/two directions, areal, omnipresent, perpetual.

4.7 FURTHER READING

1. Sri Bharati Krishna Tirthaji, Vedic Mathematics or Sixteen Simple Mathematical Formulae from the Vedas, chapters on quadratic and cubic factorization and related sutras

2. Vedic Mathematics – Methods (Sri Sathya Sai Veda Pratishtan), sections on factorization of quadratics and cubics using sutras such as Adyamadyena, Anurupyena and Lopana– Sthapanabhyam.
3. Rajesh Kumar Thakur, Advanced Vedic Mathematics, Rupa Publications, chapters on quadratic equations, factorization of cubic polynomials and harder factor problems.
4. Sen, S.N., and A.K. Bag. The Sulbasutras. Indian National Science Academy, 1983.
5. Datta, Bibhutibhusan, and Avadesh Narayan Singh. History of Hindu Mathematics: A Source Book. Asia Publishing House, 1962.

4.8 MODEL QUESTIONS

Q1. Compare number classifications in Jaina texts with Buddhist cosmic scales.

Q2. Describe zero's emergence in Jaina mathematics and its implications.

Q3. Explain infinity types in Jainism with examples from texts.

Q4. How did post-Vedic traditions advance beyond Vedic ritual geometry?

Q5. Discuss permutations/combinations in Jaina anuyoga applications.

UNIT 5: TECHNICAL TERMS OF MATHEMATICS

UNIT STRUCTURE

5.1 Learning Objectives
5.2 Introduction
5.3 Digits and Numbers
5.4 Zero
5.5 Infinity
5.6 Decimal
5.7 Algebra
5.8 Calculus
5.9 Let Us Sum Up
5.10 Answers to Check Your Progress
5.11 Further Reading
5.12 Model Questions

5.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- Understand the basic structure and purpose of mathematics as a field of study.
- Define and explain key terms such as digit, number, zero, infinity, decimal, algebra, and calculus.
- Recognize the historical development of these terms, especially their origins in ancient Indian mathematics.
- Apply these terms in simple examples and appreciate their role in everyday calculations and advanced problem-solving.
- Connect these concepts to Vedic mathematical principles for faster computation and deeper insight.

5.2 INTRODUCTION

Mathematics serves as a universal language that helps people describe patterns, solve problems, and understand the world. From counting objects in daily life to exploring complex theories about the universe, it provides tools for reasoning and discovery. Ancient civilizations contributed key ideas that form the basis of modern mathematics. In India, texts like the Vedas and works by scholars such as Aryabhata and Brahmagupta introduced concepts that spread worldwide. These include the decimal system and the idea of zero, which made calculations simpler and more precise.

This unit examines essential technical terms: digit, numbers, zero, infinity, decimal, algebra, and calculus. Each term builds on the previous ones, creating a foundation for arithmetic and higher studies. Vedic mathematics, derived from ancient Indian sutras, offers efficient ways to use these terms in computations. For instance, sutras like "NikhilamNavatascaramamDasatah" help with operations near powers of ten, linking directly to digits and decimals. Understanding these terms reveals how simple building blocks lead to powerful applications in science, engineering, and finance.

5.3 DIGITS AND NUMBERS

A digit is the smallest unit in a number system, representing values from 0 to 9 in the decimal system. Digits combine to form numbers, which quantify amounts or positions. For example, the number 247 uses digits 2, 4, and 7 to represent two hundreds, four tens, and seven units.

In ancient India, the place-value system allowed digits to change value based on position. This innovation, evident in texts like the Bakhshali manuscript from around the 3rd to 7th century CE, made large numbers manageable. Without place value, writing 123 would require separate symbols for 100, 20, and 3. Vedic methods exploit this by treating numbers near bases like 10 or 100 for quick calculations.

Numbers come in types: natural numbers $(1, 2, 3\ldots)$, whole numbers $(0, 1, 2\ldots)$, integers $(\ldots, -2, -1, 0, 1, 2\ldots)$, rational (fractions like $\frac{1}{2}$), and irrational (like $\sqrt{2}$). Vedic sutras simplify operations on these. For instance, to multiply 98 × 97, note both are near 100. Deviations are -2 and -3. Cross-subtract: 98 - 3 = 95 or 97 - 2 = 95 (left part). Multiply deviations: (-2) × (-3) = 6 (right part). Result: 9506.

$$98 \quad -2$$

$$97 \quad -3$$

$$- - - - - -$$

$$95 \mid 06$$

This uses "Nikhilam" sutra, showing digits' power in positional systems. Numbers enable counting and measurement, from market prices to astronomical distances.

CHECK YOUR PROGRESS

Q1. What is a digit, and how does the place-value system work with an example?
Q2. List three types of numbers and give one example of each.
Q3. Explain how Vedic methods use digits near base 100 for multiplication.
Q4. Why is the Indian place-value system important in modern mathematics?

5.4 ZERO

Zero represents nothingness or absence of quantity, serving as a placeholder and number. In equations, it acts as identity: any number times zero is zero. Ancient Indians first treated

zero as a number around 500 CE. Aryabhata used it positionally, and Brahmagupta's 628 CE text Brahmasphutasiddhanta defined rules: $1 + 0 = 1$, $0 - 0 = 0$.

Zero's dot symbol evolved into the modern '0'. The Bakhshali manuscript shows early use. Vedic mathematics calls zero "shunya." Sutra "Ekadhikena Purvena" uses zero in recurring decimals, like 1/19 ≈ 0.052631578947368421 repeating.

Zero enables the decimal system. Without it, 101 means one hundred one, not one zero one. In algebra, zero solves $x + 5 = 5$as $x = 0$. Division by zero is undefined, a key concept.

Imagine a scale:

```
100   10    1    .   1/10   1/100
|-------|-------|-------|-------|-------|
1     0     1           0     5     → 101.05
```

Zero's placeholder role is clear. Indian zero reached Arabs by 773 CE, then Europe, revolutionizing math.

5.5 INFINITY

Infinity (∞) denotes boundlessness, larger than any number. In limits, as x approaches infinity, $\frac{1}{x}$ approaches zero. Ancient Indians grasped infinity philosophically in Vedas, linking to cosmic cycles. Aryabhata used infinite series for π approximations.
In Vedic terms, infinity appears in recurring decimals, endless without repeating fully. Calculus uses infinity for continuous change. Example: sum $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = 1$ (geometric series to infinity).
Infinity challenges intuition: $\infty + 1 = \infty$, ∞/∞ indeterminate. Indian texts like Sulba Sutras approximated π infinitely, using polygons.

Graphically, infinity extends lines endlessly:

$$y = \frac{1}{x}$$

```
^
|  .
|  .
|  .
---+--.---------> x
∞
```

As $x \to \infty$, $y \to 0$. Infinity enables advanced math like sets (Hilbert's hotel: infinite rooms, all full, yet accommodate more).

CHECK YOUR PROGRESS

Q5. Define zero's dual role and name an Indian scholar who defined its rules.

Q6. How does zero function in the decimal system with an example?

Q7. Explain infinity in limits with a simple series.

Q 8. Link infinity to Vedic recurring decimals.

5.6 DECIMAL

Decimals express fractions positionally after a point, like 3.14 for π approximation. India's decimal system, from 6th century BCE bamboo slips, uses base 10. Pingala and others refined it.

Brahmagupta formalized decimals. Vedic "Ekadhikena Purvena" converts fractions to decimals efficiently. Example: $\frac{1}{19}$ uses multiplier 2 (one more than previous digit 1), yielding 0.052631578947368421 repeating.

Decimals aid precision: $0.1 = \frac{1}{10}$. Operations follow integer rules, shifting point.

$$\begin{array}{r} 2.5 \\ \times\ 1.2 \\ \hline 5.0 \end{array} \quad (2 \times 2 = 4, carry\ 1;\ 2 \times 1 = 2 + carry = 3?$$

$$Wait, proper: 2.5 \times 1.2 = (25 \times 12)/10 = 300/10 = 3.00$$

Indian decimals spread via Arabs. Modern use: measurements, money.

5.7 ALGEBRA

Algebra studies symbols for quantities, solving equations like $2x + 3 = 7\ (x = 2)$. From Arabic "al-jabr," roots in India. Aryabhata solved quadratics; Brahmagupta, cubics.

Vedic sutras factor: "ParavartyaYojayet" for division. Example: factor $x^2 - 5x + 6 = (x - 2)(x - 3)$.

$$x^2 - 5x + 6$$

$$Divide by\ (x - 2): synthetic\ division$$

$$\begin{array}{r|rrr} 2 & 1 & -5 & 6 \\ & & 2 & -6 \\ \hline & 1 & -3 & 0 \end{array}$$

$$Quotient: x - 3$$

5.8 CALCULUS

Calculus studies change: derivatives (rates), integrals (accumulation). Newton/Leibniz formalized, but India foreshadowed. Bhaskara II (12th CE) used infinitesimals for tangents.

Vedic "Calana-Kalanabhyam" implies sequential motion. Derivative: $\frac{dy}{dx}$ for slope. Example: $y = x^2$, $\frac{dy}{dx} = 2x$.

Integral: area under curve. $\int x\,dx = \left(\frac{1}{2}\right)x^2 + C$.

Indians approximated π via infinite series: Aryabhata's 3.1416.

5.9 LET US SUM UP

- Mathematics uses digits (0-9) to form numbers via place-value, invented in India.
- Zero, from shunya, is placeholder/number; infinity handles endless processes.
- Decimals enable fractions; algebra solves equations; calculus studies change.
- Vedic sutras simplify using these terms for fast computation.
- Indian origins underpin modern math foundations.

5.10 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q1: Digit is 0-9 unit. Place-value: $247 = 2 \times 100 + 4 \times 10 + 7 \times 1$

Ans to Q2: Natural $(1,2,\ldots)$, whole $(0,1,\ldots)$, integers $(\ldots,-1,0,1,\ldots)$.

Ans to Q3: Near 100, deviations multiply for right; cross-subtract for left.

Ans to Q4: Enables large numbers, positional notation.

Ans to Q 5: Placeholder (101) and number ($x \times 0 = 0$). Brahmagupta.

Ans to Q 6: Separates integer/fraction: $101.05 = 101 + \frac{5}{100}$.

Ans to Q 7: $\frac{1}{2} + \frac{1}{4} + \ldots \to 1$ as $n \to \infty$.

Ans to Q 8: Endless decimals like $\frac{1}{7} = 0.142857...$

5.11 FURTHER READING

1. Sri Bharati Krishna Tirthaji, Vedic Mathematics or Sixteen Simple Mathematical Formulae from the Vedas, chapters on quadratic and cubic factorization and related sutras
2. Vedic Mathematics – Methods (Sri Sathya Sai Veda Pratishtan), sections on factorization of quadratics and cubics using sutras such as Adyamadyena, Anurupyena and Lopana– Sthapanabhyam.
3. Rajesh Kumar Thakur, Advanced Vedic Mathematics, Rupa Publications, chapters on quadratic equations, factorization of cubic polynomials and harder factor problems.
4. Sen, S.N., and A.K. Bag. The Sulbasutras. Indian National Science Academy, 1983.

5. Datta, Bibhutibhusan, and Avadesh Narayan Singh. History of Hindu Mathematics: A Source Book. Asia Publishing House, 1962.

5.12 MODEL QUESTIONS

Q1. Define digit and number. Explain place-value with a Vedic multiplication example near base 10.

Q2. Describe zero's invention in India. How does it enable decimals? Give rules from Brahmagupta.

Q3. What is infinity? Show its role in series summing to finite via Vedic recurring decimals.

Q4. Explain decimals' Indian origin. Convert $\frac{1}{19}$ using Ekadhikena Purvena.

Q5. Differentiate algebra and calculus. Factor $x^2 - 5x + 6$ Vedic-style; find $\frac{dy}{dx}$ for $y = x^2$.

UNIT 6: CONTRIBUTIONS OF CLASSICAL MATHEMATICIANS-I

UNIT STRUCTURE

6.1 Learning Objectives
6.2 Introduction
6.3 Aryabhata: Life History
6.4 Major Contributions
6.5 Influence of Aryabhata on Others
6.6 Let Us Sum Up
6.7 Answers to Check Your Progress
6.8 Further Reading
6.9 Model Questions

6.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- describe the life history of Aryabhata and place his work in the historical context of ancient India;
- explain Aryabhata's major contributions to mathematics, including concepts related to numeration, arithmetic, and trigonometry;
- discuss his important contributions to astronomy, such as his views on planetary motion, eclipses, and time measurement;
- analyse the influence of Aryabhata on later Indian scholars and the development of mathematical and astronomical traditions in India;
- explain how Aryabhata's ideas were transmitted beyond India and contributed to the growth of scientific knowledge in other civilizations;
- appreciate the enduring legacy of Aryabhata and his lasting impact on the history of mathematics and astronomy.

6.2 INTRODUCTION

Aryabhata was one of the greatest figures in the history of Indian mathematics and astronomy. He lived during the late 5th and early 6th centuries, a time when scientific thinking was still developing in many parts of the world. Through careful observation and logical reasoning, Aryabhata laid strong foundations for mathematical and astronomical studies, making him a pioneer of classical Indian science.

Aryabhata made important contributions to both mathematics and astronomy. He developed ideas related to number systems, place value, algebra, trigonometry, and methods for solving equations. He also gave clear explanations of the movement of planets, eclipses, and the rotation of the Earth. His work influenced many Indian scholars who continued to develop these ideas. Later, his knowledge spread beyond India to the Islamic world and Europe through translations and scholarly exchanges. Even today, Aryabhata's legacy continues to inspire scientists and highlights India's valuable contribution to the growth of science.

6.3 ARYABHATA: LIFE HISTORY

Aryabhata was "certainly the first of the greatest mathematicians and astronomers responsible for the renaissance of mathematics and science in ancient India."

Aryabhata was born in 476 CE, perhaps he completed his education at the school of Kusumapura near modern Patna. Aryabhata's magnum opus is Aryabhaṭiya. It was written in 499 CE when he was only 23 years old. The Aryabhaṭiya is very concise book. The book contains an introduction of few verses known as the Dashagitika, followed by a section on mathematics of 33 verses, known as the Ganitapada, then a section of 25 verses on the reckoning of time and planetary models known as the Kalakriya. The final section is of 50 verses on Sphere and eclipses known as Gola. Thus, there are altogether four sections of 118 verses.

The mathematical part of the Aryabhaṭiya includes arithmetic, algebra, plane trigonometry and spherical trigonometry.

Very little is known about Aryabhata's personal life, teachers, or institutional affiliations. However, references by later commentators such as Bhaskara I strongly suggest that Aryabhata was a leading scholar of a well-established astronomical school. His reputation during his lifetime and afterward indicates that he was widely respected as an authority in both mathematics and astronomy.

6.4 MAJOR CONTRIBUTIONS

Aryabhata's contributions span pure mathematics, applied computation, and theoretical astronomy, marking a decisive shift toward scientific rationalism in Indian thought.

6.4.1 Contributions to Mathematics

Aryabhata made foundational advances in arithmetic, algebra, and trigonometry:

- **Place Value System and Zero:** While not explicitly defining zero as a numeral, Aryabhata's work clearly employs the decimal place-value system, which later became the backbone of global arithmetic. Moreover, Aryabhata introduced alphabetic system of numeration.
- **Algorithms for Arithmetic Operations:** He provided systematic methods for extracting square roots and cube roots, as well as procedures for solving linear indeterminate equations.
- **Approximation of π(pi):** Aryabhata gave a remarkably accurate approximation of π as

$$\pi \approx \frac{62832}{20000} = 3.1416$$

and explicitly stated that this value is an approximation, demonstrating early awareness of irrational numbers.

- **Trigonometry:** He introduced the concept of ardha-jyā (half-chord), which corresponds to the modern sine function, and constructed sine tables with high numerical accuracy.The vertical line from the arc to the diameter represents

$$ardha - jyā = Rsin\theta$$

Where, R is the radius of the circle and θ is the angle at the center.

6.4.2 Contributions to Astronomy

Aryabhata's astronomical insights were revolutionary:

- **Earth's Rotation:** He asserted that the apparent daily motion of the heavens is due to the rotation of the Earth on its axis, a concept far ahead of its time.
- **Scientific Explanation of Eclipses:** Rejecting mythological interpretations, Aryabhata explained solar and lunar eclipses as shadow phenomena involving the Earth and the Moon.
- **Planetary Motion:** He developed computational models for planetary positions using epicyclic theories and introduced precise time calculations based on large cosmic cycles (yugas).
- **Sidereal Measurements:** Aryabhata calculated the length of the solar year and planetary periods with impressive precision for the era.

These contributions firmly establish Aryabhata as one of the earliest proponents of mathematical astronomy based on observation and computation.

6.5 INFLUENCE OF ARYABHATA ON OTHERS

Aryabhata's influence on subsequent scholars was profound and long-lasting, shaping Indian and global scientific traditions.

6.5.1 Influence on Indian Scholars

- Bhāskara I (7th century AD) wrote an authoritative commentary on the Āryabhaṭīya, clarifying and expanding Aryabhata's mathematical and astronomical ideas.
- Brahmagupta engaged critically with Aryabhata's theories, particularly on planetary motion and Earth's rotation, thereby advancing astronomical debate.
- Later mathematicians such as Lalla, Mahāvīra, and Bhāskara II built upon Aryabhata's trigonometry, algebraic techniques, and computational methods.

Aryabhata thus laid the foundations for the classical Indian school of astronomy (Siddhānta tradition).

6.5.2 Transmission Beyond India

Through translations into Arabic during the Abbasid period, Aryabhata's ideas entered the Islamic scientific world. His works influenced astronomers such as Al-Khwārizmī, particularly in numerical methods and astronomical tables. These ideas eventually reached medieval Europe, contributing indirectly to the Renaissance revival of mathematical astronomy.

6.5.3 Enduring Legacy

Aryabhata is remembered as:

- A pioneer of scientific reasoning over mythological explanation,
- A central figure in the development of trigonometry and numerical computation,
- A catalyst for cross-cultural transmission of mathematical knowledge.

His work represents one of the earliest systematic attempts to understand the universe through mathematics, observation, and rational analysis, securing him a permanent place in the global history of science

CHECK YOUR PROGRESS

1. State the year of birth of Aryabhata and name his principal work.

2. What is meant by ardha-jyā in Aryabhata's trigonometry?

3. Describe Aryabhata's method of finding the cube root of a number.

4. Discuss in detail the mathematical contributions of Aryabhata, with special reference to trigonometry and numerical algorithms.

6.6 LET US SUM UP

- **Aryabhata** was a renowned ancient Indian mathematician and astronomer whose work laid the foundation for scientific mathematics and astronomy in India.
- Major contributions to **mathematics** include the place value system, accurate calculation of π, algebraic methods, and the introduction of trigonometric concepts such as sine.
- Important achievements in **astronomy** include the explanation of the Earth's rotation, causes of day and night, and scientific explanations of solar and lunar eclipses.
- Aryabhata's ideas strongly **influenced Indian scholars** such as Bhaskara I and Brahmagupta, who expanded and refined these concepts.
- The transmission of Aryabhata's work **beyond India** through translations ensured an **enduring legacy** in global scientific and mathematical thought.

6.7 ANSWERS TO CHECK YOUR PROGRESS

Ans no 1: Aryabhata was born in 476 CE. His principal work is the Āryabhaṭīya, composed in 499 CE.

Ans no 2: Ardha-jyā means "half-chord" and corresponds to the sine function in modern trigonometry.

Ans no 3: Aryabhata's cube root method involves dividing the number into groups of three digits from the right, finding the largest cube less than or equal to the leftmost group, subtracting it, and then successively determining the remaining digits using a divisor based on three times the square of the root obtained so far. The method relies on place value and the algebraic expansion of $(a + b)^3$.

Ans no 4: Try Yourself

6.8 FURTHER READING

[1] 2. State Council of Educational Research and Training, Fundamentals and Applications of Vedic Mathematics, New Delhi.
[2] Kenneth R. Williams, Discover Vedic Mathematics, Vedic Mathematics Research Group.
[3] VedicMaths.Org, “Practise and Free Books on Vedic Mathematics.”
[4] Moonpreneur, “Vedic Math Sutra 1: Ekadhikena Purvena.”

6.9 MODEL QUESTIONS

Q1. Explain Aryabhata’s scientific explanation of solar and lunar eclipses.

Q2. Discuss the significance of Aryabhata’s introduction of sine tables.

Q3. Write Aryabhata’s approximation for the value of π.

Q4. Aryabhata composed the Āryabhaṭīya in which year?

Q5. Discuss the contributions of Aryabhata to astronomy. How were his ideas different from earlier beliefs?

Q6. Explain the influence of Aryabhata on Indian scholars. Name some scholars and describe how they benefited from his work.

Q7. Write a short note on the life history of Aryabhata.

Q8. Mention any three contributions of Aryabhata to mathematics.

UNIT 7: CONTRIBUTIONS OF CLASSICAL MATHEMATICIANS-II

UNIT STRUCTURE

7.1 Learning Objectives
7.2 Introduction
7.3Aryabhata: Life history
7.4Major Contributions
7.5Influence of Aryabhata on Others
7.6 Brahmagupta: Life history
7.7 Major Contributions
7.8 Let Us Sum Up
7.9 Further Reading
7.10 Answer to Check Your Progress
7.11 Model Questions

7.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- Learn the life history and scholarly environment of Aryabhata and Brahmagupta.
- Understand the major mathematical contributions of Aryabhata, particularly in arithmetic, trigonometry, and numerical algorithms.
- Understand Aryabhata's scientific approach to astronomy, including his explanation of eclipses and Earth's rotation.
- Learn the life and works of Brahmagupta in the context of the development of algebra.
- Understand Brahmagupta's contributions to quadratic equations, indeterminate equations, and geometry.

7.2 INTRODUCTION

Indian mathematics reached a remarkable level of development during the classical period, particularly between the Gupta and post-Gupta eras, when scholars combined numerical skill with theoretical insight to explain natural phenomena. This unit focuses on two of the

greatest mathematicians–astronomers of this period, Aryabhata and Brahmagupta, whose works laid the foundations of arithmetic, algebra, trigonometry, and mathematical astronomy. Aryabhata introduced systematic numerical algorithms, trigonometric concepts, and scientific explanations of celestial events, while Brahmagupta provided rigorous algebraic formulations, especially concerning zero, negative numbers, and quadratic equations. Together, their contributions represent a decisive shift from empirical computation to structured mathematical theory and played a crucial role in shaping not only Indian mathematics but also the development of mathematical thought in the Islamic world and later in Europe.

7.6 BRAHMAGUPTA: LIFE HISTORY

Brahmagupta was one of the most distinguished mathematicians–astronomers of ancient India, belonging to the post-Aryabhata period. He was born in 598 CE in Bhillamāla (modern Bhinmal, Rajasthan), which was an important center of learning and astronomical studies during the early medieval period. Brahmagupta lived and worked during a time when Indian mathematics and astronomy had already attained considerable sophistication, yet still allowed room for deep theoretical refinement.

He served as the head of the astronomical observatory at Ujjain, a city renowned as the foremost scientific center of India at that time. Ujjain attracted leading scholars, and Brahmagupta's position there reflects his exceptional reputation and authority.

Brahmagupta composed his most celebrated work, the Brāhmasphuṭasiddhānta, in 628 CE, when he was thirty years old. Later, he wrote another important treatise, the Khaṇḍakhādyaka, which served as a practical handbook for astronomical calculations. His writings reveal a scholar deeply rooted in tradition yet unafraid to critique earlier authorities, including Aryabhata, when he found conceptual or computational inconsistencies.

Brahmagupta passed away around 668 CE, leaving behind a legacy that profoundly shaped both Indian and Islamic mathematical traditions.

7.7 MAJOR CONTRIBUTIONS

Brahmagupta's contributions are of foundational importance, particularly in algebra, number theory, and astronomy. His work marks a decisive transition from computational techniques to explicit mathematical theory.

7.7.1 Contributions to Algebra and Arithmetic

Brahmagupta was the first mathematician in history to give a systematic treatment of zero and negative numbers.

Rules for Zero:

He defined operations involving zero, stating that:

- A number minus itself is zero.
- A number multiplied by zero is zero.
- A number divided by zero is undefined (though he attempted a rule, reflecting early conceptual struggle).

Negative Numbers:

Brahmagupta interpreted positive numbers as fortunes and negative numbers as debts, and clearly formulated rules such as:

- Debt minus zero is debt
- Debt multiplied by debt is fortune

This metaphorical but rigorous approach established the arithmetic of signed numbers.

7.7.2 Quadratic Equations and Algebraic Methods

Brahmagupta gave general solutions for quadratic equations, including cases with negative coefficients. His work shows a clear understanding of algebraic abstraction beyond specific numerical examples.

He also studied indeterminate equations, laying early groundwork for what later evolved into advanced number theory.Brahmagupta made notable contributions to the theory of indeterminate equations, especially equations of the form:$Nx^2 + 1 = y^2$

His work laid the foundation for what later became known as the Chakravāla method, perfected by Bhāskara II. This represents one of the earliest sophisticated approaches to number theory anywhere in the world.

7.7.3 Geometry: Brahmagupta's Formula

One of Brahmagupta's most celebrated results is the formula for the area of a cyclic quadrilateral:

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where a, b, c, d are the sides of the quadrilateral and s is the semi-perimeter.

This formula generalizes Heron's formula for triangles and remains a cornerstone of classical geometry. It illustrates Brahmagupta's ability to extend known results into broader geometric frameworks.

7.7.4 Astronomical Contributions

- In astronomy, Brahmagupta made significant refinements to earlier siddhāntic models:
- He improved methods for calculating planetary positions, conjunctions, and eclipses.
- He provided more accurate computational procedures for lunar and solar eclipses, building on but also correcting Aryabhata's methods.
- While he rejected Aryabhata's assertion of Earth's rotation, his critiques stimulated further astronomical debate and development.
- His Khaṇḍakhādyaka became especially popular for practical astronomical computation, influencing calendars and time-reckoning systems.

7.7.5 Influence Beyond India

Brahmagupta's works were translated into Arabic during the Abbasid period. Islamic scholars, including Al-Khwārizmī, drew extensively on his algebraic and numerical methods. Through this transmission, Brahmagupta's ideas entered medieval Europe, contributing indirectly to the development of Renaissance mathematics.

Brahmagupta stands as a pivotal figure in the history of mathematics for transforming arithmetic and algebra into rule-based, abstract disciplines. His systematic treatment of zero, negative numbers, and algebraic equations represents a decisive advance beyond earlier computational traditions. In astronomy, his refinements and critiques ensured the continued evolution of Indian siddhāntic science.

Together, Aryabhata and Brahmagupta form the twin pillars of classical Indian mathematics, with Brahmagupta providing the theoretical rigor that enabled the subject to mature into a fully articulated mathematical science.

7.8VARAHAMIHIRA

Varahamihira was a versatile man. According to Samarendranath Sen, Varahamihira may be called the Pliny of India. Varahamihira was a native of Avanti. His father's name was Adityadasa
who was also an astronomer. Varahamihira was a pupil of his father. Varahamihira was a mathematician and astronomer of repute of the sixth century CE. Varahamihira was one of the
greatest scholars of ancient India, renowned for his outstanding contributions to astronomy, mathematics, and astrology. He lived during the 6th century CE (approximately 505–587 CE) and was a prominent member of the scholarly circle known as the Navaratnas (Nine Gems) in
the court of the great ruler Yashodharman of Malwa (though some traditions associate him with
Vikramaditya)
Varahamihira authored several important Sanskrit texts, among which the most notable are:

- Brihat Samhita – An encyclopedic treatise covering astronomy, astrology, architecture,
- meteorology, omens, planetary movements, rainfall prediction, and various scientific
- subjects.
- Pancha-Siddhantika – A compilation and comparative study of five earlier astronomical
- systems, preserving valuable knowledge of ancient Indian astronomy.
- Brihat Jataka – A foundational text on predictive astrology.
- Laghu Jataka – A shorter treatise on astrology.

Varahamihira's works influenced Indian astronomy and astrology for many centuries. His encyclopedic approach in Brihat Samhita shows remarkable scientific curiosity and interdisciplinary knowledge. He is remembered as one of the pillars of classical Indian science.

CHECK YOUR PROGRESS

1.Where was Brahmagupta born, and which work established his mathematicalreputation?

2.How did Brahmagupta interpret positive and negative numbers?

3.Explain Brahmagupta's rules for arithmetic with negative numbers using the concept of debts and fortunes.

4.Examine Brahmagupta's role in the development of algebra, emphasizing his treatment of zero and negative numbers.

7.6 LET US SUM UP

A summary of the lessons we learnt from this unit is provided below.

- Aryabhata and Brahmagupta were two eminent mathematician–astronomers who significantly contributed to the growth of classical Indian mathematics.
- Aryabhata lived during the Gupta period and authored the Āryabhaṭīya, which laid the foundations of arithmetic, trigonometry, and mathematical astronomy.
- Aryabhata introduced the decimal place-value system in practice and developed systematic algorithms for square and cube roots.
- Aryabhata gave an accurate approximation of the value of π and introduced the concept of ardha-jyā, equivalent to the modern sine function.
- Aryabhata provided scientific explanations of solar and lunar eclipses and proposed that the apparent motion of the heavens is due to the rotation of the Earth.
- Brahmagupta belonged to the post-Aryabhata period and composed the Brāhmasphuṭasiddhānta, a landmark text in the history of algebra.
- Brahmaguptawas the first mathematician to formulate clear rules for arithmetic involving zero and negative numbers using the concepts of debts and fortunes.
- Brahmagupta made significant contributions to algebra, including the general solution of quadratic equations and early work on indeterminate equations.
- Brahmagupta established the famous formula for the area of a cyclic quadrilateral, which remains an important result in geometry.
- The works of both Aryabhata and Brahmagupta influenced later Indian scholars and were transmitted to the Islamic world, thereby contributing to the global development of mathematics.

7.7 ANSWERS TO CHECK YOUR PROGRESS

Ans no 1: Brahmagupta was born in Bhillamāla (modern Bhinmal, Rajasthan). His reputation was established by the Brāhmasphuṭasiddhānta (628 CE).

Ans no 2: Brahmagupta interpreted positive numbers as fortunes (dhana) and negative numbers as debts (ṛṇa).

Ans no 3: Brahmagupta described negative numbers as debts and positive numbers as fortunes. He stated rules such as: debt plus debt is debt, fortune minus debt is fortune, and debt multiplied by debt is fortune. These rules match modern laws of signed numbers.

Ans no 4: Try Yourself

7.8 FURTHER READING

1. The History of Ancient Indian Mathematics, C.N. Srinivasiengar, The World Press, Calcutta, 1967.
2. The History of Hindu Mathematics, Asia Publishers House, Bombay, 1962

7.9 MODEL QUESTIONS

Q1. Who first gave systematic rules for arithmetic with zero?
Q2. Name the formula associated with Brahmagupta in geometry.
Q3. Where was Brahmagupta born, and which work established his mathematical reputation?
Q4. How did Brahmagupta interpret positive and negative numbers?
Q5. State any two rules given by Brahmagupta for operations involving zero.
Q6. Explain Brahmagupta's rules for arithmetic with negative numbers using the concept of debts and fortunes.
Q7. Describe Brahmagupta's formula for the area of a cyclic quadrilateral and explain its importance.
Q8. Compare the mathematical approaches of Aryabhata and Brahmagupta in terms of computation and theory.
Q9. Compare and contrast the astronomical views of Aryabhata and Brahmagupta.

Q10. Analyze the influence of Aryabhata and Brahmagupta on later Indian and world mathematics.

UNIT 8: CONTRIBUTIONS OF CLASSICAL MATHEMATICIANS-III

UNIT STRUCTURE

8.1 Learning Objectives
8.2 Introduction
8.3 Mahabir Acharya: Life History
8.4 Mahabir Acharya: Major Contributions
8.5 Shripati: Life History
8.6 Shripati: Major Contributions
8.7 Shridharacharya: Life history
8.8 Shridharacharya: Major Contributions
8.9Let Us Sum Up
8.10 Answers to Check Your Progress
8.11 Further Reading
8.12 Model Questions

8.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- understand the historical background of important classical Indian mathematicians
- describe the life and intellectual environment of Mahabir Acharya and Shripati
- explain the major mathematical contributions of Mahabir Acharya and Shripati with suitable illustrations
- recognize the role of mathematics in medieval India
- appreciate the continuity of algebraic and arithmetic traditions in India

8.2 INTRODUCTION

The development of mathematics in India grew through centuries as scholars studied earlier works, refined ideas, and applied them to practical problems related to trade, astronomy, land measurement, and daily computation. The classical period saw scholars like Mahabir Acharya (9th century) and Shripati (11th century) who strengthened arithmetic and algebra in systematic forms. Mahabir Acharya, a Jain scholar under Rashtrakuta patronage, created the first independent mathematical text, Ganita Sara Sangraha (850 CE). Shripati blended mathematics with astronomy in works like Siddhantasekhara and Ganita Kaumudi (or

Ganitatilaka). Their clear methods, practical focus, and respect for predecessors like Aryabhata and Brahmagupta reflect Indian mathematics' maturity.

8.3 MAHABIR ACHARYA: LIFE HISTORY

Mahabir Acharya (c. 800-870 CE), also known as Mahavira, lived in southern India, likely near Mysore or Gulbarga in present-day Karnataka, during the Rashtrakuta dynasty under King Amoghavarsha I. A Jain monk and teacher, he emphasized logic and precision, traveling to teach and honoring earlier scholars like Aryabhata, Bhaskara I, and Brahmagupta. His main work, Ganita Sara Sangraha (Compendium on the Essence of Mathematics, 850 CE), spans nine chapters in Sanskrit verses, treating mathematics as a distinct subject separate from astronomy. This text organized knowledge for students, with clear explanations and practical examples, influencing southern Indian traditions and later scholars like Narayana.

8.4 MAHABIR ACHARYA: MAJOR CONTRIBUTIONS

Mahabir systematized arithmetic, algebra, geometry, and mensuration in Ganita Sara Sangraha, using place-value with zero.

Arithmetic and Fractions:

He detailed operations like addition, subtraction, multiplication, division, powers, roots, and fractions, breaking any fraction into unit fractions (e.g., $\frac{4}{7} = \frac{1}{4} + \frac{1}{28}$). He recognized division by zero as undefined, advancing zero's properties.

Algebra:

Mahabir solved linear and quadratic equations, admitting two positive roots, and stated square roots of negatives do not exist. He handled indeterminate equations, like merchant purse problems ($x + y + z = p$, etc., solved as $x = 2k$, $y = k$, $z = 3k$,$p = 4k$), and gave combinations formulas.

Geometry and Mensuration:

He defined shapes like equilateral/isosceles triangles, rhombus, circle, semicircle; approximated ellipses, conch figures; calculated areas/volumes of spheres, ditches, triangles, circles. Practical word problems covered trade, mixtures, interest

CHECK YOUR PROGRESS

Q1. Where was Mahabir likely born, and under which dynasty?

Q2. Name his major text and year.

Q3. What did he contribute to fractions/algebra?

Q4. Example of his geometric terms?

8.5 SHRIPATI: LIFE HISTORY

Shripati (c. 1019-1066 CE) lived in western/southern India (Rohinikhanda, near Maharashtra/Karnataka or Ranastambhapura), son of Nagadeva, following Lalla's school. A Brahmin scholar, he wrote in Sanskrit and early Marathi, producing works from 1039-1056 CE amid trade/temple learning centers. Key texts: Dhikotidakarana (eclipses), Dhruvamanasa (planets), Siddhantasekhara (19 chapters, astronomy), Ganita Kaumudi/Tilaka (arithmetic, based on Sridhara). He refined predecessors for calendars, predictions, balancing theory/practice

8.6 SHRIPATI: MAJOR CONTRIBUTIONS

Shripati advanced arithmetic, algebra, geometry for astronomy.

Arithmetic and Algebra:

Ganita Kaumudi/Tilaka covers operations, squares/roots/cubes (positive/negative), fractions, proportions, trade rules. Siddhantasekhara Ch.13 (arithmetic/mensuration/shadows), Ch.14 (algebra: quadratics, identity$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$), Ch.15 (spheres). Solved indeterminate equations like Brahmagupta.

Geometry, Mensuration, Astronomy:

Areas/perimeters of squares, rectangles, triangles, circles; volumes of solids. Refined trig tables for planetary motion, eclipses, transits, moon inequalities; practical for time, fields

8.7SHRIDHARACHARYA: LIFE HISTORY

Shridharacharya was a distinguished Indian mathematician who lived during the early medieval period, generally placed around the 8th or 9th century CE. Very little is known about his personal life, birthplace, or family background, as was common with many scholars of ancient and medieval India. However, his scholarly works firmly establish him as one of the important figures in the history of Indian mathematics.

Shridharacharya is best known for his celebrated mathematical treatise *Pāṭīgaṇita*, which deals primarily with arithmetic. This work was written in Sanskrit and presented

mathematical rules in verse form, making it easier for students to memorize and apply them. *Pāṭīgaṇita* systematically explains fundamental operations such as addition, subtraction, multiplication, division, fractions, zero, square roots, cube roots, and problems related to practical calculations. The text shows his concern for clarity, accuracy, and applicability of mathematics in daily life.

In addition to *Pāṭīgaṇita*, Shridharacharya is also associated with another work titled *Triśatikā*, which further elaborates algebraic concepts. He made notable contributions to algebra by providing explicit methods for solving quadratic equations. The well-known quadratic formula is often referred to as “Shridhara’s formula” in Indian mathematical tradition, highlighting his influence in this area.

Shridharacharya’s works indicate that Indian mathematics during his time had reached a high level of sophistication. His emphasis on clear rules, systematic presentation, and practical problem-solving made his texts widely used for teaching mathematics for several centuries. Through his contributions, Shridharacharya played a significant role in strengthening the foundations of arithmetic and algebra in India and helped transmit mathematical knowledge to later generations.

8.8 SHRIDHARACHARYA: MAJOR CONTRIBUTIONS

Shridharacharya made profound and enduring contributions to Indian mathematics, particularly in the areas of arithmetic and algebra, at a time when mathematical knowledge was closely connected with practical life. His most important work, *Pāṭīgaṇita*, is a comprehensive treatise that systematically explains a wide range of mathematical operations. It covers fundamental topics such as addition, subtraction, multiplication, division, fractions, zero, and the extraction of square and cube roots, along with numerous illustrative problems. The clarity and orderly presentation of these topics show his deep understanding of mathematical principles and his concern for effective teaching.

A major highlight of Shridharacharya’s contribution is his work in algebra. He provided clear and precise rules for solving quadratic equations, a method that later came to be known as “Shridhara’s formula.” This contribution demonstrates the advanced level of algebraic thinking present in India during his time. In addition to *Pāṭīgaṇita*, his work *Triśatikā* further elaborated algebraic ideas and problem-solving techniques, strengthening the theoretical foundation of mathematics.

Shridharacharya also played a significant role in shaping mathematical education through his use of Sanskrit verses to present rules and methods. This verse-based approach made mathematical concepts easier to memorize and transmit orally, ensuring their preservation over generations. His emphasis on practical applications—such as calculations related to trade, measurement, and daily life—made mathematics accessible and relevant to society. Through his systematic methods, practical orientation, and lasting influence on later scholars, Shridharacharya occupies an important place in the history of Indian mathematics.

CHECK YOUR PROGRESS

Q5. Name Shripati's main works/topics.

Q6. Siddhantasekhara math chapters?

Q7. An algebraic identity?

Q8. Contribution to indeterminate equations?

Q9. Who was Sridharacharya and during which period did he live?

Q10. State the main mathematical work of Sridharacharya.

Q11. What is Sridharacharya’s contribution to quadratic equations?

8.9 LET US SUM UP

- Mahabir Acharya (c.800-870 CE) authored Ganita Sara Sangraha (850 CE), systematizing arithmetic (fractions/zero), algebra (quadratics/negatives), geometry (shapes/volumes).
- Shripati (1019-1066 CE) wrote Siddhantasekhara/Ganita Tilaka, refining arithmetic/algebra for astronomy (eclipses/spheres).
- Both used clear rules, practical problems, place-value; influenced medieval India.
- Their works separated pure math, preserved traditions.

8.10 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q1: Possibly Mysore/Gulbarga, Karnataka; Rashtrakuta under Amoghavarsha I.

Ans to Q2: Ganita Sara Sangraha, 850 CE.

Ans to Q3: Unit fractions; quadratic solutions, no negative square roots.

Ans to Q4: Equilateral/isosceles triangles, rhombus, circle, semicircle.

Ans to Q5: Dhikotidakarana (eclipses), Dhruvamanasa (planets), Siddhantasekhara (astronomy), Ganitatilaka (arithmetic).

Ans to Q6: Ch.13 arithmetic, 14 algebra, 15 spheres.

Ans to Q7: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Ans to Q8: Rules like Brahmagupta for first-degree.

Ans to Q9: Sridharacharya was an ancient Indian mathematician who lived around the 8th–9th century CE. He is best known for his work on algebra, particularly quadratic equations.

Ans to Q10: Sridharacharya wrote the famous book “Pātīganita”, which deals with arithmetic operations, algebraic methods, and solutions of equations.

Ans to Q11: Sridharacharya gave a clear method for solving quadratic equations, which is similar to the modern quadratic formula used today.

8.11 FURTHER READING

1. State Council of Educational Research and Training, Fundamentals and Applications of Vedic Mathematics, New Delhi.
2. Kenneth R. Williams, Discover Vedic Mathematics, Vedic Mathematics Research Group.
3. VedicMaths.Org, “Practise and Free Books on Vedic Mathematics.”
4. Moonpreneur, “Vedic Math Sutra 1: Ekadhikena Purvena.”

8.12 MODEL QUESTIONS

Q1. Discuss Mahabir Acharya's life and Ganita Sara Sangraha's structure/contributions.
Q2. Explain Shripati's algebra/astronomy links with examples.
Q3. Compare their fraction/algebra methods.
Q4. Describe geometric advances by both.
Q5. Examine practical problems' role in their works.
Q6. Sridharacharya is best known for his contribution to:

a) Trigonometry

b) Geometry

c) Algebra and quadratic equations

d) Statistics

UNIT 9: LILAVATI-I

UNIT STRUCTURE

9.1 Learning Objectives
9.2 Introduction
9.3 Mudra (मुद्रा) -Symbol or Numerical Mark
9.4 Weight Measurement (Bharamana)
9.5 Masadi - manam (Weight System Beginning with Masa)
9.6 Anguladi - manam (Measurement Based on Angula/ Finger)
9.7 Yojanadi - manam (Distance Measurement System)
9.8 Ghana – hastadi - manam (Volumetric Measurement Based on Cubic Hasta)
9.9 Droṇadi - manam (Capacity/Volume Measurement System)
9.10 Yavana – pracarita - manam (Measurements Influenced by Greeks/Foreigners)
9.11 Let Us Sum Up
9.12 Answers to Check Your Progress
9.13 Further Reading
9.14 Model Questions

9.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- define **Mudra** and explain traditional concepts of **weight measurement**.
- describe **Masādi-mānam** and **Aṅgulādi-mānam** used for weight and length measurements.
- explain **Yojanādi-mānam** for distance measurement.
- describe **Ghanahastādi-mānam** and **Droṇādi-mānam** related to volume and capacity.
- understand **Yavana-pracārita-mānam** and the influence of foreign measurement systems.

9.2 INTRODUCTION

Ancient Indian mathematics, as preserved in texts such as *Lilavati* of Bhaskaracarya (12th century CE), offers a remarkable framework of measurement systems used in commerce, architecture, astronomy, and daily life. These include well-defined units for **weight, length, distance, and volume**, along with systems influenced by contact with foreign cultures such as the **Yavanas (Greeks)**.

The measurement systems described in *Lilavati* exhibit a sophisticated, hierarchical, and practical approach to quantification in ancient India. They demonstrate the intellectual

maturity of Indian mathematicians and their deep engagement with geometry, commerce, astronomy, and daily life. Understanding these systems provides not only mathematical insight but also cultural and historical appreciation of India's scientific heritage.

This chapter explains key definitions and measurement systems mentioned in the *Definition Section* of *Lilavati Part–I*, namely: **Mudra, weight measurements, Masadi - manam, Aṅguladi - manam, Yojanadi - manam, Ghana – hastadi - manam, Dronadi - manam**, and **Yavana – pracarita - manam**.

9.3 MUDRA (मुद्रा) -SYMBOL OR NUMERICAL MARK

In traditional Indian mathematical literature, mudra refers to the symbolic representation of numbers, units, or operations. Mudra functions in two principal ways:

1. **Symbolic notation**:

   Used to denote numbers, weights, or units within verses.

2. **Indicative symbol/seal**:

   A sign or token used for authentication and validation of measurements in trade.

The concept ensures clarity when expressing mathematical problems in verse form.

9.4 WEIGHT MEASUREMENT (BHARAMANA)

Indian systems of weight evolved through centuries, rooted in trade, Ayurvedic medicine, and metallurgy. The units are hierarchical and connected through fixed ratios.

Basic units included **ratti, masa, karṣa, pala, prastha, tula**, etc. These units were applied in:

- Commerce
- Medicine
- Goldsmithing
- Taxation systems

The logic of progression from small to large units forms the basis for the next system, i.e., Masadi-manam.

9.5 MASADI - MANAM (WEIGHT SYSTEM BEGINNING WITH MASA)

Masa is a fundamental weight unit from which higher measures derive.

Approximate traditional hierarchy:

- 1 Masa = 8 Ratti
- 12 Masa = 1 Karṣa (Tola in some regions)
- 16 Karṣa = 1 Pala
- 100 Pala = 1 Tula (varied by region and era)

This system was particularly significant in Ayurvedic dosage calculations and economic transactions involving metals and spices.

9.6 ANGULADI - MANAM (MEASUREMENT BASED ON ANGULA/ FINGER)

Angula means “finger-breadth” and forms the foundation of traditional Indian linear measurement.

Common sequence:

Units	**Description**
1 Angula	Finger - breadth (Width of a finger)
12 Angula = 1 Vitasti	span
24 Angula = 1 Hasta	cubit
4 Hasta = 1 Dhanus	Bow - length

Applications:

- Architecture (vastu sastra)
- Geometry in mathematics
- Construction of altars, temples, and instruments
- Textile measurements

This reflects the anthropometric nature of ancient measurement - relating units to the human body.

9.7 YOJANADI – MANAM (DISTANCE MEASUREMENT SYSTEM)

For long distances, especially in astronomy and geography, larger units were required.

Traditional distances:

- 1 Krosa / Gavyuti ≈ 3 km
- 4 Krosa = 1 Yojana (≈ 12–13 km)

These units appear frequently in:

- Epics and Puraṇas
- Astronomical treatises (*Surya Siddhanta*)
- Travel records and land measurement texts

The system enabled accurate mapping and astronomical distance calculations long before modern metric systems.

9.8 GHANA – HASTADI – MANAM (VOLUMETRIC MEASUREMENT BASED ON CUBIC HASTA)

These units measure solid volume.

- A Ghana-hasta is the volume of a cube whose side = 1 *hasta* (cubit).

Applications:

- Estimation of soil, brick, and stone volumes
- Construction (tanks, wells, pillars)
- Geometry problems involving cubes and rectangular solids

This system integrates geometric understanding with practical needs in architecture.

9.9 DROṆADI - MANAM (CAPACITY/VOLUME MEASUREMENT SYSTEM)

Used primarily for grains, ghee, milk, oil, and other bulk commodities.

Typical hierarchy:

- 1 $Kudava$

- $4\ Kudava\ =\ 1\ Prastha$
- $4\ Prastha\ =\ 1\ Aḍhaka$
- $4\ Aḍhaka\ =\ 1\ Droṇa$

This system was crucial for:

- Agriculture
- Storage and taxation
- Commerce and trade regulations

Its use persisted in India for centuries until replaced by modern metric units.

9.10YAVANA – PRACARITA - MANAM (MEASUREMENTS INFLUENCED BY GREEKS/FOREIGNERS)

“Yavana” referred to Greeks, Romans, and later western foreigners.

Greek influence arrived through:

- Astronomy (e.g., trigonometric tables, chord functions)
- Calendrical systems
- Certain units of length and weight

In the context of *Lilavati*, this term acknowledges measurement practices introduced or modified due to intercultural exchanges with the Greek world, especially during the Indo-Greek and later periods.

CHECK YOUR PROGRESS

Q1) Explain the hierarchy of Masadi - manam with its relations.

Q2) Describe the Anguladi - manam system. What does it reveal about ancient Indian metrology?

Q3) Explain the Droṇādi-mānam system and its importance in traditional Indian society.

Q4) Compare Yojanadi - manam and Anguladi - manam in terms of purpose and scale.

Q5) Discuss the role of Yavana – pracarita - manam and the influence of Greek science on Indian measurements.

Q6) How do the measurement systems described in Lilavati show the scientific development of ancient India?

Q7) Why do you think most ancient measurement systems were based on the human body?

9.11 LET US SUM UP

- Ancient Indian measurement systems included standard units for **weight, length, distance, and capacity**, ensuring precision in trade and daily life.
- **Mudra, Masādi-manam, and Angulādi-manam** formed the basis for traditional measurements of weight and length.
- **Yojanādi-manam** was used for measuring long distances, while **Ghanahastādi-manam and Dronādi-manam** measured volume and capacity.
- **Yavana-pracārita-manam** shows the influence of foreign, especially Greek, systems on Indian measurement practices.

9.12 ANSWERS TO CHECK YOUR PROGRESS

Answer to Q1: The system begins with Masa:

$$1\ Masa\ =\ 8\ Ratti$$

$$12\ Masa\ =\ 1\ Karṣa\ /\ Tola$$

$$16\ Karṣa\ =\ 1\ Pala$$

$$100\ Pala\ =\ 1\ Tula$$

It was used mainly in medicine, trade, and metal measurement.

Answer to Q2: The system uses Angula as the base unit (finger breadth).

$$12\ Angula\ =\ 1\ Vitasti$$

$$24\ Angula\ =\ 1\ Hasta$$

$$4\ Hasta\ =\ 1\ Dhanus$$

It shows that ancient Indian measurement was anthropometric, using human body parts for accuracy and practical convenience.

Answer to Q3: A capacity-measure system for grains and liquids.

$$1\ Prastha\ =\ 4\ Kudava$$

$$1\ Aḍhaka\ =\ 4\ Prastha$$

$$1\ Droṇa\ =\ 4\ Aḍhaka$$

It was vital for agriculture, trade, taxation, and storage of food items.

Answer to Q4:

Anguladi - manam	**Yojanadi - manam**
1. Based on the Aṅgula (finger). 2. Used for small to medium measurements (architecture, geometry). 3. It is anthropometric. 4. Deals with centimeters/meters	1. Based on the Krośa and Yojana. 2. Used for long distances (geography, astronomy, travel). 3. It is geographic/astronomical. 4. Deals with kilometers.

Answer to Q5: "Yavana" refers mainly to Greeks who interacted with India during Indo-Greek and later periods. Greek influence contributed:

i. New astronomical tables and methods
ii. Refinement of trigonometric and calendrical calculations
iii. Adjustments in some measurement practices

It shows India's scientific openness and the exchange of knowledge across cultures.

Answer to Q6: They reveal:

i. Precision in weight, distance, volume, and linear measures
ii. Use of geometric understanding (Ghana - hasta)
iii. Integration of practical needs (trade, architecture, agriculture)
iv. Influence from foreign scientific traditions (Yavana)
v. Sophisticated mathematical structuring of units
vi. Collectively, the systems demonstrate advanced scientific maturity.

Answer to Q7: Because body-based measures:

i. Are universally available
ii. Require no instruments
iii. Provide consistent practical reference

iv. Are convenient for everyday construction and trade
v. Such systems reflect practicality in societies without standardized tools.

9.13 FURTHER READING

1. State Council of Educational Research and Training, Fundamentals and Applications of Vedic Mathematics, New Delhi.
2. Kenneth R. Williams, Discover Vedic Mathematics, Vedic Mathematics Research Group.
3. VedicMaths.Org, “Practise and Free Books on Vedic Mathematics.”
4. Moonpreneur, “Vedic Math Sutra 1: Ekadhikena Purvena.”

9.14 MODEL QUESTIONS

Q1. What is the meaning of the term “Mudra” in mathematical texts?

Q2. How many Rattis make one Masa?

Q3. What is the basic body-measure used in Anguladi - manam?

Q4. How many Angulas are there in one Hasta?

Q5. How many Krosas make one Yojana?

Q6. What does Ghana - hasta measure?

Q7. Name the largest unit in Droṇadi - manam.

Q8. Who are referred to as “Yavanas” in ancient Indian texts?

Q9. Write any two uses of the Masadi - manam system.

Q10. Define Vitasti and state its relation with Angula.

Q11. Mention any two applications of Anguladi - manam.

Q12. What were Ghana - hasta units used for in practical life?

Q13. Write the sequence of Droṇadi - manam units.

Q14. Write a detailed note on Dronadi-manam. Explain its units, method of measurement, and its use in measuring capacity and volume, especially in agriculture and storage.

Q15. What is Yavana-pracarita-manam? Explain how foreign or Greek systems of measurement influenced Indian measurement practices and discuss their significance.

Q16. Compare Anguladi-manam and Yojanadi-manam. Highlight the differences in their basis, units, and applications.

UNIT 10: LILAVATI–II

UNIT STRUCTURE

10.1 Learning Objectives
10.2 Introduction
10.3 Mangalacharan (Invocation)
10.4 Digits and their significance
10.5 Places of numbers
10.6 Decimal place-value system
10.7 Arrangement of numbers
10.8 Dissemination of the Place Value and Number Table System Abroad
10.9 Word Numerals
10.10 Educational and Historical Significance
10.11 Let Us Sum Up
10.12 Answers to Check Your Progress
10.13 Further Reading
10.14 Model Questions

10.1 LEARNING OBJECTIVES

After studying this unit, learners will be able to:

- Understand the concept of place value and its role in number representation.
- Explain the significance of digits and zero in the decimal number system.
- Arrange numbers systematically and interpret their magnitudes.
- Appreciate the contribution of *Lilavati* to the development of arithmetic.
- Describe the global dissemination of the Indian place value system.

10.2 INTRODUCTION

Bhaskaracarya was the leading mathematician of the 12th century CE. He was born in the year 1114 CE and his monumental work Siddhantasiromani was written in 1150 CE. Bhaskara has said, “In the year 1036 of the Saka kings I was born, and at the age of 36 I have written the Siddhantasiromani.” His father’s name was Mahesvara, and he was born at a city called Bijjalabida. Bhaskara’s another work Karanakutuhala was written in 1183 CE.

The Siddhantasiromani is divided into 4 parts, viz. Lilavati, Vijaganita, Goladhyaya and Grahaganita.

The Lilavati is mainly a work on arithmetic. It also contains many useful propositions of geometry. In addition to fundamental operation, he dealt with Rule of Three, Five, Seven,

Nine, and Eleven; Permutation and Combination, Treatment of zero. Lilavati was translated into Persian by Fyzi in 1587 CE by the direction of the Emperor Akbar.
Lilavati was immensely popular. Many commentaries were written on Lilavati. It was also taught in the Tols of Assam.
This unit, Lilavati Part–II, focuses on the foundational ideas of place value, digits, and the systematic arrangement of numbers. These concepts form the backbone of arithmetic and played a decisive role in the development of the decimal system. In addition, this chapter discusses the historical dissemination of the Indian place value and numeral system beyond India, highlighting its global significance.
The treatment in this chapter follows an academic yet narrative approach, blending textual explanation with examples, Sanskrit verses, and exercises for deeper understanding.

10.3 MANGALACHARAN (INVOCATION)

Indian scholarly tradition prescribes the beginning of any serious intellectual work with a Mangalacharan, or auspicious invocation. Bhāskara II adheres to this convention in Lilavati by offering salutations to the divine and to the source of knowledge.
Illustrative Sanskrit Verse (Invocation Style):

"नत्वाशिवंज्ञानगुरुंचभक्त्या
प्रवक्ष्येलीलावतींसुपाठ्याम्।"

Explanation:
The author bows with devotion to Lord Śiva and to the teacher, regarded as the embodiment of knowledge, before commencing the exposition of Lilavati. This reflects humility, reverence, and the belief that learning flourishes through discipline and grace.
The Mangalacharan serves multiple purposes:

- It sanctifies the act of learning
- It prepares the mind of the learner
- It emphasizes the ethical responsibility of knowledge transmission

10.4 DIGITS AND THEIR SIGNIFICANCE

In Vedic Mathematics, digits are the basic parts of numbers, and each one has its own positional meaning that makes it easy to do quick mental math by recognizing patterns. The relevance of Vedic sutras like Ekadhikena Purvena comes from how they use the qualities of digits to make multiplication, squaring, and checking easier without long steps. This method turns numbers from simple symbols into strong tools that may help you solve every day and more difficult problems quickly.

The Indian numeral system employs ten digits:

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9$$

10.4.1 ROLE OF ZERO (ŚŪNYA)

In Vedic Mathematics, Śūnya (zero) signifies the profound notion of void or nothingness, emerging from ancient Indian philosophy as a crucial placeholder that facilitates the

decimal place-value system and endless numerical expansions. This new idea from Indian sages changed arithmetic by making it possible to do things like add opposites that cancel out to zero, which is both mathematically useful and spiritually infinite.

One of the most profound contributions of Indian mathematics is the concept of zero (śūnya). In Lilavati and related works, zero is treated both as:

A placeholder, and

A number capable of participating in operations

Illustrative Sanskrit Verse:

"स्थानंविनानकिमपिअंकानां
शून्येनसहार्थवत्ताभवति।"

Explanation:

Without position, digits have no value; with zero and proper placement, they acquire meaning. This verse-like formulation captures the philosophical and mathematical importance of śūnya.

The introduction of zero changes the magnitude significantly, demonstrating its indispensable role in place value notation.

10.5 PLACES OF NUMBERS

Places of numbers is a basic idea in Vedic Mathematics that focuses on understanding how the position of a digit within a number (ones, tens, hundreds, and so on) determines its value in the decimal place-value system. This concept helps learners see numbers not as isolated digits but as structured combinations of place values, which supports faster mental calculations and clearer manipulation of large numbers. By becoming comfortable with place value, students can apply Vedic methods more effectively for operations like addition, subtraction, and multiplication.

10.5.1 Concept of Place Value

The place value system is a method in which the value of a digit depends upon its position within a number. Lilavati explains this idea with remarkable clarity, emphasizing that a digit acquires magnitude only when placed in an appropriate position.

In the decimal (base–10) system, each successive place to the left represents a value ten times greater than the previous one:

Place	Value
Units	10^0
Tens	10^1
Hundreds	10^2
Thousands	10^3
Ten-thousands	10^4

Example:

Consider the number 5,736.

- 6 is in the units place → value = 6
- 3 is in the tens place → value = 30
- 7 is in the hundreds place → value = 700
- 5 is in the thousands place → value = 5000

Thus,

$$5{,}736 = 5000 + 700 + 30 + 6$$

Bhāskara stresses that without positional distinction, numerals would be ineffective for expressing large quantities.

10.6 THE DECIMAL PLACE-VALUE SYSTEM

The decimal place-value system is an epoch-making achievement of the ancient Indian mathematicians. In this system, there are only ten symbols, viz. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These are called ankas (digits) with the application of the principle of place-value, we can write any number with the help of the above ten digits. This system is commonly used throughout the world. The system originated in India and then travelled to other parts of the world.

In this system, a digit has two values, viz. face value and place value. Let us consider the number 47529. Here face value of 4 is 4, and the place value is 40000; the face value of

7 is 7, and the place value is 7000; the face value of 5 is 5, and the place value is 500; the face value of 2 is 2, and the place-value is 20; the face value of 9 is 9, and the place-value is also 9.

Thus

$$47529 = 40000 + 7000 + 500 + 20 + 9$$

10.7 ARRANGEMENT OF NUMBERS

Vedic Mathematics encompasses techniques for arranging numbers that facilitate mental calculations and enable quick problem-solving. This traditional Indian mathematical approach emphasizes the identification of patterns, allowing for rapid computation.

10.7.1 ASCENDING AND DESCENDING ORDER

Lilavati discusses the logical ordering of numbers, which is essential for comparison and computation.

- Ascending order: 125, 248, 376, 490
- Descending order: 490, 376, 248, 125

10.7.2 FORMATION OF LARGE NUMBERS

By combining digits systematically according to place value, extremely large numbers can be constructed using only ten symbols. This economy of symbols is a hallmark of the Indian system.

Example:

Form the greatest and smallest numbers using the digits 3, 5, 7, 9 (without repetition):

- Greatest: 9,753
- Smallest: 3,579

10.8 DISSEMINATION OF THE PLACE VALUE AND NUMBER TABLE SYSTEM ABROAD

The place-value and number table system spread to other countries from ancient Vedic India, where positional notation with zero changed how people did math around the 5th to 9th centuries CE. Al-Khwarizmi brought this new idea to the Arab world through trade and

intellectuals, and Fibonacci brought it to Europe later, laying the groundwork for current global mathematics.

Spread to the Arab World:

From the 8th century CE onward, Indian mathematical texts influenced scholars in the Islamic world. Indian numerals and the decimal system were studied, translated, and taught by Arab mathematicians.

Al-Khwarizmi, in particular, acknowledged Indian methods of calculation, which later formed the basis of algebra and arithmetic in the Islamic world.

Transmission to Europe:

Through Arabic–Latin translations, the Indian place value system reached Europe around the 12th century CE. The so-called Hindu–Arabic numerals gradually replaced Roman numerals, enabling more efficient computation and accelerating scientific progress.

10.9 WORD NUMERALS

A system of writing numbers by means of words was developed in India in the “early centuries of the

Christian era.” In this system the numerals are expressed “by means of things, being or concepts”. For

example zero is expressed with the help of the words sunya, kha, gagana, abhra, etc. The selection of

the proper words begins with the figure in the units place and the numerals proceed to the left. For

example the number 1036 may be expressed by ‘rasa-guna-purna-mahi’, rasa=6, guna=3, purna=0,

mahi=1.

Some commonly used words are given below

Zero(0): sunya, kha, gagana, abhra, purna, nabha, ambara etc.

One (1): adi, sasi, candra, dhara, mahi, prithivi etc.

Two (2): yama, lochana, netra, nayana, paksha, kara, karna etc.

Three (3): rama, guna, agni, vahni etc.

Four (4): veda, sruti, varna, yuga, krta etc.

Five (5): vana, sara, pandava, visaya, indriya, bhava etc.

Six (6): rasa, anga, raga, tarka etc.

Seven (7): naga, parvata, muni, vara, svara, asva, turaga etc.

Eight (8): vasu, gaja, dvipa, sarpa etc.

Nine (9): anka, nanda, graham etc.

Ten (10): dis, dik, asha, avatara etc.

Eleven (11): rudra, isvara, isa etc.

Twelve (12): ravi, surya etc.

10.10 EDUCATIONAL AND HISTORICAL SIGNIFICANCE

It is very important to the history and teaching of Vedic mathematics because it made arithmetic, algebra, and geometry easier to learn for hundreds of years. The fact that it has been used as a textbook for so long and has useful techniques shows how rich ancient Indian mathematics is, as it links Vedic sutras with classical calculations.

The concepts explained in Lilavati:

- Simplified arithmetic learning
- Encouraged logical reasoning
- Influenced global mathematical development

The place value system remains fundamental to modern mathematics, computing, finance, and science.

CHECK YOUR PROGRESS

Q1) Write the expanded form of 8,204.

Q2) How many hundreds are there in 4,768?

10.11 LET US SUM UP

- *Lilavati* presents the principles of place value and digits in a clear, systematic, and pedagogical manner.
- The decimal place value system assigns value to digits based on their position, making large-number computation possible.
- The concept of zero (śūnya) as both a placeholder and a number is a landmark contribution of Indian mathematics.
- Arrangement and formation of numbers form the basis of comparison and arithmetic operations.
- The Indian place value system spread to the Arab world and Europe, profoundly influencing global mathematical development.

10.12 ANSWERS TO CHECK YOUR PROGRESS

Answer to Q1:8,204 = 8 × 1000 + 2 × 100 + 0 × 10 + 4 × 1

Answer to Q2: There are 47 hundreds in 4,768.

10.13 FURTHER READING

1. Bhāskara II, Lilavati, translated by H. T. Colebrooke.
2. Datta, B. B. & Singh, A. N., History of Hindu Mathematics, Vol. I.
3. Boyer, Carl B., A History of Mathematics.
4. Joseph, G. G., The Crest of the Peacock: Non-European Roots of Mathematics.
5. Plofker, Kim, Mathematics in India.

10.14 MODEL QUESTIONS

Q1. Write the place value of each digit in 6,509.

Q2. Express 9,432 in expanded form.

Q3. Arrange the following numbers in ascending order: 3,205; 3,025; 3,520; 3,052.

Q4. Form the smallest and greatest numbers using the digits 1, 4, 6, 8 without repetition.

Q5. Explain the importance of zero in your own words.

Q6. Explain the place value system as described in Lilavati.

Q7. Discuss the historical spread of the Indian numeral system to the rest of the world.

Q8. Write a short note on the educational style of Bhāskara II.

Q9. What is meant by place value?

Q10. Who was the author of Lilavati?

Q11. What is the significance of zero?

Q12. What is a Mangalacharan?

Q13. Why is the Indian numeral system considered efficient?

Q14. Describe the decimal place value system with suitable examples.

Q15. Explain the role of zero as both a placeholder and a number.

Q16. Discuss how Lilavati contributed to the global development of mathematics.

Q17. Explain the importance of number arrangement in arithmetic.

UNIT 11: LILAVATI–III

UNIT STRUCTURE

11.1 Learning Objectives
11.2 Introduction
11.3 Addition (Sankalita/ Yoga)
11.4 Subtraction (Viyoga)
11.5 Importance of addition and subtraction
11.6 Let Us Sum Up
11.7 Answers to Check Your Progress
11.8 Further Reading
11.9 Model Questions

11.1 LEARNING OBJECTIVES

After studying this unit, learners will be able to:

- Understand the classical Indian approach to fundamental arithmetic operations.
- Explain the rules and procedures of addition (Sankalita/Yoga) and subtraction (Viyoga) as presented in Lilavati.
- Apply traditional methods of calculation to numerical problems.
- Interpret simple Sanskrit verses related to arithmetic operations.
- Develop accuracy and logical thinking in performing basic computations.

11.2 INTRODUCTION

The celebrated mathematical treatise Lilavati, composed by Bhaskaracarya (Bhaskara II) in the 12th century CE, occupies a distinguished place in the history of Indian mathematics. Written in elegant Sanskrit verses, the text presents arithmetic operations through poetic expression, making mathematical learning both engaging and intellectually stimulating.

In this unit, we focus on the method of calculation, particularly the rules of addition and subtraction, which form the foundation of all higher mathematical operations. These operations are referred to in Sanskrit as Sankalita (Yoga) for addition and Viyoga for subtraction. Bhaskaracarya not only explains the procedures but also emphasizes clarity, order, and verification—principles still central to modern arithmetic.

11.3 ADDITION(SANKALITA/YOGA)

11.3.1Concept of Addition

Addition is the process of combining two or more numbers to obtain their total or sum. In Lilavati, addition is termed Sankalita or Yoga, meaning “bringing together” or “union.”

11.3.2Role of Addition

The general rule followed in traditional Indian arithmetic is:

- Numbers are arranged according to place value.
- Addition begins from the rightmost digit (units place).
- Carry-over is applied wherever the sum exceeds the base (ten).

11.3.3Sanskrit Verse of Addition

“स्थानेस्थानेसमानीय

योगंकुर्याद्बुधोजनः।“

It means a wise person should place numbers correctly according to their positions and then perform addition place by place.

Illustrative Example

Add: 478 and 596

$$\begin{array}{r} 478 \\ +\ 596 \\ \hline 1074 \end{array}$$

Steps:

- Units: 8 + 6 = 14 → write 4, carry 1
- Tens: 7 + 9 + 1 = 17 → write 7, carry 1
- Hundreds: 4 + 5 + 1 = 10

Thus, the sum is 1074.

11.4 SUBTRACTION (VIYOGA)

11.4.1Concept of Subtraction

Subtraction, known as **Viyoga**, is the process of finding the difference between two numbers. It involves taking away a smaller number from a larger one.

11.4.2Rule of Subtraction

- Numbers must be placed according to place value.
- Subtraction proceeds from the **unit's place**.
- If the upper digit is smaller, borrowing is done from the next higher place.

11.4.2Sanskrit Verse on Subtraction

"अल्पेयदिमहत्नस्यात्

ऋणंकृत्वाविशोधयेत्।"

Explanation:

If the upper number is smaller, one should borrow and then perform the subtraction correctly.

Illustrative Example:

Subtract: 752 – 489

$$\begin{array}{r} 752 \\ -489 \\ \hline 263 \end{array}$$

Steps:

- Units: Borrow → 12 – 9 = 3
- Tens: Borrow → 14 – 8 = 6
- Hundreds: 6 – 4 = 2

Thus, the difference is **263**.

11.5 IMPORTANCE OF ADDITION AND SUBTRACTION

Addition and subtraction form the basis of all arithmetic operations, including multiplication, division, and algebraic reasoning. In Lilavati, these operations are not merely mechanical procedures but are connected to daily life problems involving trade, measurement, and distribution.

Bhaskaracarya's systematic exposition highlights the importance of:

- Correct placement of digits
- Mental alertness
- Verification of result

CHECK YOUR PROGRESS

Q1) Why does Bhaskaracarya emphasize place value while explaining addition and subtraction in Lilavati?

Q2) How do the methods of addition and subtraction in Lilavati compare with modern arithmetic techniques?

Q3) What pedagogical value does Sanskrit verses have in explaining mathematical operations?

11.6LET US SUM UP

- *Lilavati* presents arithmetic operations in a systematic and poetic form.
- Addition (Saṅkalita/Yoga) is the process of combining numbers based on place value.
- Subtraction (Viyoga) involves finding differences using borrowing when required.
- Sanskrit verses aid in memorization and conceptual clarity.
- These methods laid the foundation for advanced mathematical calculations in Indian mathematics.

11.7 ANSWERS TO CHECK YOUR PROGRESS

Answer to Q1: Bhaskaracrya emphasizes place value because correct alignment of digits ensures accuracy in calculation. Without proper placement, the fundamental operations of addition and subtraction become erroneous. The place value system reflects the decimal structure of numbers and allows systematic carrying and borrowing, which are essential for reliable computation.

Answer to Q2: The methods described in Lilavati closely resemble modern arithmetic techniques. Both systems rely on place value, right-to-left computation, and the use of carrying and borrowing. The main difference lies in presentation: Lilavati conveys rules through Sanskrit verses and verbal reasoning, whereas modern mathematics uses symbolic notation and algorithms.

Answer to Q3: Sanskrit verses serve as effective pedagogical tools by aiding memorization and conceptual clarity. The rhythmic and concise nature of the verses allows learners to recall rules easily. Moreover, the poetic form makes the learning process engaging and reflects the traditional Indian approach of integrating mathematics with literary expression.

11.8 FURTHER READING

1. Bhaskaracarya, *Lilavati*, Translated by H. T. Colebrooke.
2. Datta, B. & Singh, A. N., *History of Hindu Mathematics*, Vol. I.
3. Sharma, K. S., *Indian Mathematics: Concepts and Methods*.
4. Plofker, K., *Mathematics in India*, Princeton University Press.

11.9 MODEL QUESTIONS

Q1. What is meant by Saṅkalita in Lilavati?

Q2. Define Viyoga with an example.

Q3. Why is place value important in addition and subtraction?

Q4. Add:

(a) $684 + 739$

(b) $1256 + 3489$

Q5. Subtract:

(a) $905 - 478$

(b) $3000 - 1648$

Q6. Explain the method of addition as described in Lilavati.

Q7. Discuss the role of borrowing in subtraction with suitable examples.

Q8. Write and explain one Sanskrit verse related to arithmetic operations.

UNIT 12: CONTRIBUTIONS OF MODERN INDIAN MATHEMATICIANS

UNIT STRUCTURE

12.1 Learning Objectives
12.2 Introduction
12.3 Srinivasa Ramanujan
12.3.1 Life history
12.3.2 Major Contributions
12.4 Bharati Krishna Tirtha
12.4.1 Life history
12.4.2 Major Contributions
12.5 Check your progress
12.6 Let Us Sum Up
12.7 Answers to Check Your Progress
12.8 Further Reading
12.9 Model Questions

12.1 LEARNING OBJECTIVES

The course will enable the readers to

- Understand the development and growth of modern Indian mathematics and its historical background.
- Describe the life and mathematical contributions of important mathematicians such as Shridharacharya, Srinivasa Ramanujan, and Bharati Krishna Tirtha.
- Appreciate the significance of their work in the development of mathematical thought in India.
- Analyse the role of Indian mathematicians in shaping mathematical ideas during the modern period.

12.2 INTRODUCTION

Mathematics in India has a long and rich tradition, extending from ancient times to the modern period. The modern era of Indian mathematics reflects a transition from classical methods to new ideas influenced by both indigenous scholarship and global developments. This unit focuses on the evolution of modern Indian mathematics through the lives and

contributions of eminent mathematicians such as Shridharacharya, Srinivasa Ramanujan, and Bharati Krishna Tirtha. Their work not only enriched mathematical knowledge but also played a significant role in preserving and advancing India's mathematical heritage.

12.3 SRINIVASA RAMANUJAN

12.3.1 LIFE HISTORY

Srinivasa Ramanujan was one of the most extraordinary mathematicians in the history of India and the world. He was born on 22 December 1887 at Erode in Tamil Nadu, and was brought up in the town of Kumbakonam. From a very young age, Ramanujan showed exceptional ability in mathematics. While he struggled with most other school subjects, numbers and mathematical patterns fascinated him deeply. By his teenage years, he had mastered advanced mathematical concepts on his own, using a few books, especially G. S. Carr's *Synopsis of Elementary Results in Pure and Applied Mathematics*, which greatly influenced his thinking.

Ramanujan faced many hardships in his early life. Due to his intense focus on mathematics and neglect of other subjects, he was unable to complete his formal college education and lived in extreme poverty for several years. Despite these difficulties, he continued his mathematical research independently, filling notebooks with original formulas, theorems, and results. His work was largely self-developed, without access to the contemporary mathematical methods used in the West.

In 1913, Ramanujan wrote a historic letter to the British mathematician G. H. Hardy at Cambridge University, enclosing some of his original mathematical results. Hardy immediately recognized Ramanujan's exceptional genius and arranged for him to come to England. Ramanujan arrived at Cambridge in 1914, where he worked closely with Hardy and other mathematicians. During this period, he made ground breaking contributions to number theory, partitions, infinite series, continued fractions, and modular forms. His work earned him international recognition, and in 1918 he became a Fellow of the Royal Society and a Fellow of Trinity College, Cambridge—rare honors for an Indian mathematician at that time.

However, Ramanujan's health suffered greatly during his stay in England due to the cold climate, unfamiliar food, and continuous illness. After World War I, his condition worsened, and he returned to India in 1919. Despite his declining health, he continued to work on

mathematics until the end of his life. Srinivasa Ramanujan passed away on 26 April 1920 at the young age of 32.

Although his life was short, Ramanujan's contributions to mathematics were profound and far-reaching. His ideas continue to influence modern mathematical research, and his life stands as a powerful example of genius, perseverance, and dedication to knowledge.

12.3.2 MAJOR CONTRIBUTIONS

Srinivasa Ramanujan (1887–1920) was a legendary Indian mathematician whose work profoundly influenced modern mathematics, despite his lack of formal training and his very short life. His contributions are remarkable for their depth, originality, and intuitive nature. Many of his results were far ahead of their time and continue to inspire research today.

1. Number Theory

Ramanujan made outstanding contributions to number theory, especially in the study of integers and their properties. He worked extensively on prime numbers, divisor functions, and highly composite numbers. The *Ramanujan tau function*, introduced in his study of modular forms, later became a central object in number theory and played a role in major developments such as the proof of the Weil conjectures.

2. Theory of Partitions

One of Ramanujan's most celebrated achievements is in the theory of partitions of numbers, which deals with the number of ways a positive integer can be expressed as a sum of positive integers. In collaboration with G. H. Hardy, he developed the famous *Hardy–Ramanujan asymptotic formula*, which gives an approximate value for the partition function for very large numbers. This work laid the foundation for later exact formulas developed by mathematicians such as Rademacher.

3. Infinite Series and Approximations of π

Ramanujan discovered several extraordinary infinite series that converge very rapidly, especially for calculating the value of π. These series are so efficient that they are still used

in modern high-precision computations. His formulas demonstrated his deep insight into analysis and special functions.

4. Modular Forms and q-Series

Ramanujan made significant contributions to the theory of modular forms and q-series, areas that are now central to modern mathematics. His identities involving q-series revealed unexpected relationships between different branches of mathematics and later found applications in physics, especially in string theory.

5. Continued Fractions

He introduced many new continued fractions and studied their properties in depth. Some of these continued fractions are unique and had not been previously discovered, showing his exceptional originality and creativity.

6. Mock Theta Functions

In the final year of his life, Ramanujan introduced *mock theta functions*, a mysterious class of functions that puzzled mathematicians for decades. Much later, these functions were understood more fully and were found to be connected to harmonic Maass forms. Today, they have applications in number theory and theoretical physics.

7. Notebooks and Legacy

Ramanujan recorded thousands of results in his notebooks, often without proofs. Many of these results were verified and proved by later mathematicians, revealing their correctness and depth. His work, much of it produced in isolation, continues to influence research across mathematics.

12.4 BHARATI KRISHNA TIRTHA

12.4.1 LIFE HISTORY

Bharati Krishna Tirtha (1884-1960), born as Venkataraman Shastri on 14 March 1884 in Tinnevelly (now Tirunelveli), Tamil Nadu, was one of the most remarkable spiritual scholars of modern India. From an early age, he displayed extraordinary intellectual ability and a deep inclination toward learning. His father was a well-educated man, and the household atmosphere strongly encouraged academic excellence.

As a student, Venkataraman Shastri excelled in a wide range of subjects, including Sanskrit, mathematics, history, philosophy, logic, astronomy, and the natural sciences. He pursued higher education at Madras University, where he earned top ranks in several disciplines. His brilliance was widely recognized, and he was known for his prodigious memory and analytical skills.

Despite his academic success, he felt a strong spiritual calling. He came under the influence of Swami Satchidananda ShivabhinavaNrisimha Bharati, the Shankaracharya of Sringeri Sharada Peetham, who became his spiritual guru. Under his guidance, Venkataraman Shastri renounced worldly life and entered the monastic order, receiving the name Swami Bharati Krishna Tirtha.

One of the most significant phases of his life was his intensive research into ancient Indian scriptures, particularly the Vedas. Through years of deep study, meditation, and contemplation, he claimed to have rediscovered sixteen mathematical sutras and thirteen sub-sutras embedded in the Vedic texts. These sutras formed the foundation of what later became known as Vedic Mathematics, a system aimed at simplifying and accelerating mathematical calculations.

In 1925, Bharati Krishna Tirtha was appointed Shankaracharya of the Govardhana Matha in Puri, one of the four cardinal monastic centers established by Adi Shankaracharya. As Shankaracharya, he undertook extensive tours across India, delivering lectures on spirituality, philosophy, Sanskrit literature, and mathematics. He also traveled abroad, spreading Indian knowledge systems and engaging with scholars in Europe and the United States.

Throughout his life, Bharati Krishna Tirtha worked tirelessly to demonstrate that ancient Indian wisdom was both profound and practical. He emphasized that Vedic knowledge was not merely religious but scientific and universal. His teaching style was clear, logical, and inspiring, making complex ideas accessible to students and scholars alike.

Although he wrote numerous manuscripts, many were lost over time. His most influential work, “Vedic Mathematics”, was compiled from his notes and published posthumously in 1965, five years after his death. The book gained international attention and sparked renewed interest in Indian mathematical traditions, especially in schools and academic circles.

Bharati Krishna Tirtha passed away on 2 February 1960. His legacy endures through the continued study and application of Vedic Mathematics and his contributions to Indian philosophy and education. He is remembered as a saint-scholar who successfully bridged ancient wisdom and modern intellectual life, leaving an enduring imprint on both spiritual and academic traditions.

12.4.2 MAJOR CONTRIBUTIONS

Bharati Krishna Tirtha made wide-ranging and influential contributions to mathematics, education, Indian philosophy, and the revival of ancient knowledge systems. His work helped reconnect modern society with India’s intellectual heritage while offering practical tools for learning and problem-solving.

1. **Development and Systematization of Vedic Mathematics**

His most important contribution was the revival and organization of Vedic Mathematics. Through years of research in Sanskrit texts and deep contemplation, he presented mathematics as a system based on 16 Vedic sutras and 13 sub-sutras. These aphorisms provide short, intuitive, and flexible methods for solving mathematical problems in arithmetic, algebra, geometry, trigonometry, and calculus.

Vedic Mathematics emphasizes mental calculation, reducing dependence on lengthy procedures. This approach not only speeds up computation but also improves conceptual understanding and creativity in learners. Today, these methods are widely used in schools, competitive examinations, and mental-math programs worldwide.

2. **Revival of India’s Ancient Intellectual Heritage**

Bharati Krishna Tirtha played a key role in restoring pride in ancient Indian knowledge systems at a time when Western education dominated intellectual life. He argued that the Vedas contained scientific and mathematical insights, demonstrating their relevance to modern disciplines. His work encouraged scholars to re-examine traditional texts with a rational and analytical mindset.

3. Contribution to Educational Philosophy

He strongly believed that education should be holistic, integrating intellectual training with moral and spiritual development. He advocated a teaching approach that developed clarity of thought, logical reasoning, memory, and intuition. His educational philosophy influenced teachers and institutions seeking alternatives to purely mechanical learning methods.

4. Spiritual and Philosophical Leadership

As the Shankaracharya of Govardhana Matha, Puri, Bharati Krishna Tirtha made significant contributions to Advaita Vedanta. He delivered scholarly discourses on the Upanishads, Bhagavad Gita, and Brahma Sutras, strengthening the tradition of non-dual philosophy. His spiritual leadership helped bridge classical Vedantic thought with modern intellectual inquiry.

5. Global Promotion of Indian Thought

He traveled extensively across India, Europe, and the United States, delivering lectures on Indian philosophy, culture, and mathematics. These international engagements helped introduce Indian intellectual traditions to global academic audiences and fostered cross-cultural dialogue.

6. Literary and Scholarly Contributions

Although many of his manuscripts were lost, his seminal book “Vedic Mathematics”, published posthumously in 1965, remains his most influential written work. The book has inspired extensive research, debate, and pedagogical innovation in mathematics education.

Bharati Krishna Tirtha’s contributions lie in his ability to unite spirituality with science, tradition with modernity, and intuition with logic. He is remembered as a visionary scholar who transformed ancient wisdom into practical knowledge for contemporary society, leaving a lasting impact on education and intellectual thought.

CHECK YOUR PROGRESS

(1) Who was Srinivasa Ramanujan and why is he famous?
(2) Mention any two areas of mathematics to which Ramanujan contributed.
(3) What was the significance of Ramanujan’s collaboration with G. H. Hardy?
(4) What honour did Srinivasa Ramanujan receive from the Royal Society, and why was it significant?

(5) Who was Bharati Krishna Tirtha?

(6) What is Bharati Krishna Tirtha best known for in mathematics?

(7) Name the book written by Bharati Krishna Tirtha and its importance.

12.5 LET US SUM UP

- Sridharacharya was an ancient Indian mathematician (8th–9th century CE) known for his book *Pātīganita*, which deals with arithmetic and algebra.
- He made an important contribution by giving a systematic method for solving quadratic equations, which is still used in modern mathematics.
- Srinivasa Ramanujan (1887–1920) was a self-taught mathematical genius who made remarkable contributions to number theory, infinite series, and continued fractions.
- Ramanujan's work gained international recognition through his collaboration with G. H. Hardy, and he became a Fellow of the Royal Society in 1918.
- Bharati Krishna Tirtha (1884–1960) revived and popularized Vedic Mathematics, presenting simple calculation techniques through 16 sutras in his book *Vedic Mathematics*.

12.6 ANSWERS TO CHECK YOUR PROGRESS

Answer to Q1: Srinivasa Ramanujan was a brilliant Indian mathematician born in **1887**. He is famous for his extraordinary contributions to number theory, infinite series, and mathematical analysis**.**

Answer to Q2: Ramanujan made significant contributions to number theory and **infinite series**. He also worked on continued fractions and partitions of numbers.

Answer to Q3: Ramanujan's collaboration with G. H. Hardy at Cambridge helped bring his work to global recognition and led to several important mathematical discoveries.

Answer to Q4: Srinivasa Ramanujan was elected as a Fellow of the Royal Society in 1918 for his remarkable contributions to mathematics. This honour was significant because it recognized his genius at an international level and made him one of the **few Indians** to receive this distinction at that time.

Answer to Q5: Bharati Krishna Tirtha was an Indian mathematician and spiritual leader, born in 1884**,** and later became the Shankaracharya of Puri**.**

Answer to Q6: He is best known for reviving and popularizing Vedic Mathematics, a system of fast and simple mathematical techniques.

Answer to Q7: His book **"**Vedic Mathematics**"** explains 16 sutras that simplify calculations in arithmetic, algebra, and geometry, making mathematics quicker and easier.

12.7 FURTHER READING

1. Kanigel, R. (1991). *The man who knew infinity: A life of the genius Ramanujan*. New York, NY: Scribner.
2. Plofker, K. (2009). *Mathematics in India: 500 BCE–1800 CE*. Princeton, NJ: Princeton University Press.
3. Tirtha, B. K. (1965). *Vedic mathematics: Sixteen simple mathematical formulae from the Vedas*. Delhi, India: Motilal Banarsidass.
4. Tekriwal, G. (2021). *The great Indian mathematicians: 15 pioneers who put Indian mathematics on the world map*. Gurugram, India: Penguin Random House India.
5. Prince, J. (2013). *Srinivasa Ramanujan: The great mathematician*. New Delhi, India: Prabhat Prakashan.
6. Carr, G. S. (1886). *A synopsis of pure mathematics*. London, England: Deighton Bell.

12.8 MODEL QUESTIONS

Q1. Srinivasa Ramanujan was elected as a Fellow of which institution?

a) Indian Academy of Sciences

b) University of Cambridge

c) Royal Society, London

d) Indian Mathematical Society

Q2. Bharati Krishna Tirtha is associated with:

a) Aryabhatiya

b) Sulba Sutras

c) Vedic Mathematics

d) Lilavati

Q3. Name the book written by Sridharacharya and mention one topic discussed in it.

Q4. Mention any two areas of mathematics in which Srinivasa Ramanujan made significant contributions.

Q5. What is meant by Vedic Mathematics according to Bharati Krishna Tirtha?

Q6. Describe the life and major mathematical contributions of Srinivasa Ramanujan.

Q7. Explain the contribution of Sridharacharya to algebra with reference to quadratic equations.

Q8. Write a brief note on Bharati Krishna Tirtha and the importance of his book *Vedic Mathematics*.

Q9. Compare the contributions of Srinivasa Ramanujan and Bharati Krishna Tirtha to Indian mathematics.

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UNIT 13: APPLICATIONS OF THE VEDIC SUTRAS

UNIT STRUCTURE

13.1 Learning Objectives

13.2 Introduction
13.3 Applications in Basic Arithmetic
13.4 Applications in Polynomial and Higher Calculations
13.5 Check Your Progress
13.6 Let Us Sum Up
13.7 Answers to Check Your Progress
13.8 Further Reading
13.9 Model Questions

13.1 LEARNING OBJECTIVES

The course will enable the readers to

- understand the practical significance and scope of Vedic Sutras in modern mathematics.
- apply Vedic Sutras for simplifying and solving problems in basic arithmetic efficiently.
- use Vedic methods for solving algebraic expressions and equations accurately.
- perform polynomial and higher-order calculations using appropriate Vedic techniques.
- develop speed, accuracy, and logical thinking in mathematical problem-solving through Vedic mathematics.

13.2 INTRODUCTION

Vedic Mathematics is an ancient system of computation based on sixteen Sutras that offer simple, logical, and efficient methods for solving mathematical problems. This unit focuses on the actual applications of these Vedic Sutras in various areas of mathematics such as basic arithmetic, algebra, and polynomial calculations. By exploring practical examples, learners will understand how complex calculations can be performed quickly and accurately using Vedic techniques. The chapter aims to enhance problem-solving skills, improve

computational speed, and demonstrate the relevance of Vedic Mathematics in modern mathematical learning.

13.3 APPLICATIONS IN BASIC ARITHMETIC

Vedic Mathematics provides powerful and efficient techniques for performing basic arithmetic operations such as addition, subtraction, multiplication, and division. These methods are based on simple mental processes derived from Vedic Sutras, which reduce lengthy calculations and improve speed and accuracy. In basic arithmetic, Vedic techniques help simplify calculations involving large numbers, repetitive operations, and numbers close to a base (such as 10, 100, 1000, etc.). This makes them especially useful in mental mathematics, competitive examinations, and day-to-day numerical problem solving.

13.3.1 ADDITION

In mathematics, there are four fundamental operation in which addition is most frequently used to combine different values. In addition, one has to remember carry forward values at several places but inadvertently ignored which gives a wrong result.

To overcome this difficulty, there is a Vedic mathematical sutra “Ekadhiken Poorvena” which means “by one more than the one before”. In this method, whenever the value greater than nine is obtained, carry forward value is marked by the point (.) above the left side digit and on the stage at which points are added, one is added in the resultant number.

Example: Add 1234 and 1972

Solution:

$$\begin{array}{cccc} & \dot{1} & \dot{2} & 3 & 4 \\ + & 1 & 9 & 7 & 2 \\ \hline & 3 & 2 & 0 & 6 \end{array}$$

Here $\begin{cases} 4+2=6 \\ 3+7=10 \\ 9+2+\dot{1}=12 \\ 1+1+\dot{1}=3 \end{cases}$

Example: Add 17329, 36097 and 68379

Solution:

$$\begin{array}{cccccc} & \cdot & & \cdot & \cdot & \\ & 1 & 7 & 3 & 2 & 9 \\ & \cdot & & \cdot & \cdot & \\ & 3 & 6 & 0 & 9 & 7 \\ + & 6 & 8 & 3 & 7 & 9 \\ \hline & 12 & 1 & 8 & 0 & 5 \end{array}$$

Here $\begin{cases} 9+7+9=25 \\ 2+9+7+\underline{1}+\underline{1}=20 \\ 3+3+\underline{1}+\underline{1}=8 \\ 7+6+8=21 \\ 6+3+1+\underline{1}+\underline{1}=12 \end{cases}$

The underlined '1' is value of a point.

Verification of answer

To verify the answer, the sum of digits of given numbers must be equal to the sum of digits of resultant number. In finding sum of digits, repeat the process until it is a number.

In Example 2:

$$(1+7+3+2+9)+(3+6+0+9+0+7)+(6+8+3+7+9)$$

$$\begin{aligned} &= 22+25+33 \\ &= (2+2)+(2+5)+(3+3) \\ &= 4+7+6 = 17 = 1+7 = \mathbf{8} \end{aligned}$$

And the resultant is $121805 = 1+2+1+8+0+5 = 17 = 1+7 = \mathbf{8}$

Since, the sum of digits of given numbers is equal to the sum of digits of resultant number, therefore the answer is correct.

*Sum of digits of number is called **Bijank**of that number.

Addition using Zero ending numbers

The numbers 10, 50, 1000, etc. which are ended with zero are called zero ending numbers. In the method of addition using zero ending numbers, a number ended with zero is taken which is nearest to one of given number. Then, another number is added by subtracting the difference of zero ending number and first given number from another number. For example, to add 187 with 23, we take 190 which is nearest zero ending number to 187, then we add 23-(190-187) =23-3=20 with 190. Thus, the result is 20+190=210.

13.3.2 SUBTRACTION

Subtraction is a fundamental operation in Mathematics, often termed as inverse of addition. In the process of subtraction using Vedic method, sutras 'Ekadhikena Poorvena', 'EkanyunenaPoorvena' or 'NikhilumNavatahaCharmamDashtaha' can be used for numbers.

Subtraction using 'Ekadhikena Poorvena'

In this method, to subtract a number (subtrahand) from another number, subtract the digits from right, if the digit of subtrahend is greater, then put a dot (.) on the next to the left and subtract that digit from (10+number). Now, to subtract a dotted digit, that should be increased by one and this process continued up to an end.

Example: Subtract 16889 from 36875

Solution:

$$\begin{array}{cccccc} & 3 & 6 & 8 & 7 & 5 \\ & & \dot{} & \dot{} & \dot{} & \\ - & 1 & 6 & 8 & 8 & 9 \\ \hline & 1 & 9 & 9 & 8 & 6 \end{array}$$

Since, we cannot subtract 9 from 5, so put a dot (.) next to the left (i.e. 8) of a subtrahend and subtract 9 from 15 i.e. 15-9=6. Now, to subtract a dotted 8, subtract 8+1=9 from 7, which is again impossible, so repeat the process and subtract 9 from 17 i.e. 17-9=8.

Following same way,

$$18 - 9 = 9$$

$$16 - 7 = 9$$

$$3 - 2 = 1$$

Thus, $36875 - 16889 = 19986$.

Subtraction using ‘EkanyunenaPoorvena’

In this method, for getting the difference, the value is borrowed, the dot (.) is put below the left side digit of upper value (minuend) and when the difference is taken at that stage first point mark value is reduced.

Example: Subtract 6788 from 7397

Solution:

	7	3	9	7
		.	.	
-	6	7	8	8
	0	6	0	9

To subtract 8 from 7, put a dot (.) below the left side digit of minuend and subtract 8 from 17. Now, 17-8=9.

To subtract 8 from 9, subtract 8 from 8, i.e. 8-8=0

Similarly, to subtract 7 from 3, 13-7=6 and to subtract 6 from7, 6-6=0. Thus, 7397-6788=609.

Subtraction using ‘NikhilumNavatahaCharmamDashtaha’

Nikhilum sutra is used for subtraction but this is particularly useful if minuend is a multiple of 100. The meaning of the Vedic sutra ‘NikhilumNavatahaCharmamDashtaha’ is ‘All from nine and the last from ten’. So, right side is subtracted from 10 and remaining from 9.

Example: Subtract 8732 from 10000

Solution:

1	0	0	0	0
-	8	7	3	2

1 2 6 8

Subtract 2 from 10 i.e., 10-2=8 and rest from 9, i.e., 9-3=6, 9-7=2, 9-8=1. Thus, 10000-8732=1268.

13.3.3 MULTIPLICATION

Multiplication is another fundamental operation in Mathematics. There is a traditional way to multiply the numbers but it is easy to do with the help of the vedic sutra."NikhilumNavathaCharmamDashtaha" and "UrdhvaTiryak" are used to get an easiest way of calculation.

Multiplication Using "Nikhilum" Sutra

"NikhilumNavatahaCharmamDashtaha" is used to simply the process of multiplication. The meaning of this sutra is "ail from nine and last from ten". With the help of this sutra we get the complement of a number which is used for finding the product.

We can classify the use of this sutra in the following classes.

(i) When both the numbers are less than the nearest working base

When both the numbers are less than nearest working base, then first find the complements of multiplicand and multiplier, that should be in digits equal to multiplicand and multipliers i.e, for the number 97, complement is 3 but it should be written in the form 03. Also, put a negative sign before complement which shows that the number is how much less than the working base. Write corresponding complements against multiplicand and multipliers respectively.

Now, find the product of these complements, the result is right side of resultant. Also, the sum of cross numbers is the left side of resultant. In this process we should be very careful about the carryovers. In the product of single digits number right side will with be single digit and for the product of two digits numbers right side will with be two digits and so on, the remaining carryover will be with left side digit (by adding).

Example: Find the product of 5 and 6.

Solution: Complements of 5 and 6 are respectively 5 and 4

Therefore,

5 -5

6		-4
1	/	20

Now, complements are put against the multiplicand and multiplier respectively with negative sign. Find the product of (-5) and (-4) which is 20 and cross sum is 5-4or6-5(=1). Thus, 20 is left side and 1 is right side of resultant. But left side should be in one digit. So, 2 is added to 1. So, left side is 0 and right side is now 3.

Hence, 5 x 6 = 30

Example: Find the product of 995 and 997.

Solution: Complements of 995 and 997 are 005 and 003 respectively.

995		-005
997		-003
992	/	015

Complements are put against numbers with negative sign. Find the product of (-005) and (-003) which is 15 but it should be of three digits, so put one zero before 15. Also, find the cross sum which is (997-005) ie, 992.

Thus, 995 x 997 =992 015.

(ii) When both the numbers are greater than the nearest working base

In those cases, in which numbers are greater than the nearest working base. Everything is same but the complements of multiplicand and multiplier are put against corresponding numbers with positive sign.

Example: Multiply 13 and 16.

Solution Since the complements of 13 and 16 are 03 and 06 respectively.

13		03
16		06
19	/	18

Now, complements are put against multiplicand and multiplier with positive sign. Find the product (03) and (06) which is 18. Also, cross sum is 16+03-19

Thus, 13x16=19/18=208

Example: Solve 1013 x 1006

Solution Complements of 1013 and 1006 are 13 and 6.

1013		013
1006		006
1019	/	078

Put Complements against numbers and find the product 13 and 6 which is 78.

And cross sum =1006+13=1019.

So, left side of number is 1019 and right side is 078.

Thus, 1013x1006 = 1019078

(iii) When one number is less than working base and other is greater than working base

In this case greater number (than working base) has a complement against that with positive sign and other smaller number has a complement against that with negative sign. These signs are used in cross sum and put a bar on the product for right side. All other rules regarding digit surplus, digit deficit etc, will be exactly the same as before.

Example: Solve 1013x986

Solution Complements of 1013 and 986 are respectively 13 and-14.

1013		13
986		-14
999	/	182

Here, left side=986+13 or 1013-14-999 and right side = 13 x 14 = (10 + 3)(10 + 4)= 10(10 + 3 + 4) + (3 x 4) =10 (17)+12= 170+12=182. Thus, 1013x986=999/(182) = 999 / (1000 – 182) =999818.

(iv) When the numbers are not nearer to working base

When the numbers are not nearer to working base 10, or multiple of 10. Take a base near the numbers but cross sum (left side of resultant) is multiplied by the number (which is obtained by dividing assumed base by last working base). Here it is noted that this number is an integer.

All other rules will be exactly the same as before.

Example: Find the product of 65 and 57.

Solution: Let base is 60 which is 6 times to the last working base 10.

Complements of 65 and 57 are +05 and-03 respectively.

65		+05
57		-03
62	/	15
=65-03		Or 57+5

Left side =62x6=372

and right side=$\bar{1}5$.

Thus, 65x57=372/$\bar{1}5$ = 370/ (20 – 15) =370/5=3705.

13.3.4 DIVISION

The Vedic methods of division are based on the Sutras "Nikhilam Navataha Charmam Dashtaha", "Paravartya Yojayet" and "Urdhva Triyagbhyam". The other methods have limitation that they are applicable to a particular class of problems on the other hand, "Urdhva Triyagbhyam" is very general method and quicker compared to our conventional method, therefore this sutra is often called the "crowing gem of all the sutras".

Division Using "Nikhilam" Sutra

The process of division of a number (dividends) by another number (divisor) comprises of the following steps.

Step 1

First separate extreme right digit or a block of digits of a dividend by drawing a slash equal to number of digits in divisor. This block is known as remainder block and left block is called quotient block.

Step 2

Put the sum of digits of first left block, below the right digits block.

Step 3

Find the sum of digits of left block except last digit of left block towards right and put this sum below the quotient column and continue the same process upto extreme left digit.

Step 4

Find the sum of numbers of left and right block, the value so obtained in the left part is quotient and rest right part is remainder.

Step 5

If the remainder is greater than the divisor, continue the same process in remainder block, up to remainder become less than divisor.

Example Solve 4657 ÷ 9.

Solution

9	465 50	7 15	
	515	2	2 2
		2	4
	517		4

Therefore, Quotient=517 and remainder =4

Division Using "ParavartyaYojayet"

In last sutra it is clear that the application of Nikhilum sutra is suitable when divisor-digits are big, yet this sutra is not useful when the divisor consists of small digits. So, we need a sutra which covers other cases. We have the sutra "Paravartya Yojayet" which means "Transpose and Apply". This method is suitable if divisor is nearer to the 10 or multiple of 10. In this method we follow the following steps.

Step I

Signs of all digits of divisor except the first digit of divisor towards left are changed.

Step II

One vertical line is drawn separating the digits of dividend in such a way that total digits in right block remain equal to the number of those digits of the divisor whose signs are changed.

Step III

The rest of the process is very much as before.

Example: Solve 256÷11

Solution:

$$\begin{array}{ccc} 11 & 25 & 6 \\ \bar{1} & \bar{2} & \bar{3} \\ & 23 & 3 \end{array}$$

Therefore, quotient is 23 and remainder is 3.

Division Using "Urdhva Triyagbhyam"

The "Urdhva Triyagbhyam" method is very general method and quicker compared to our conventional method. To apply the method, we follow the following steps.

Step I

Write the divisor in two parts, separating the parts by a slant line. The right-hand part is called as Dhwajanka or the flag digit.

Step II

Write the dividend, in front of the divisor as in the conventional method, but keep the digits well-spaced so that we may write the next two lines under them in a more legible way. Divide this number also into two parts by a slant line so that the number of digits on the right-hand part are same in both the divisor and the dividend.

Step III

Set up the next two lines for writing the intermediate gross dividend and intermediate actual dividend. Set the space above the dividend for writing the answer (the quotient) as usual. Now, write the starting two digits of the divisor on the "actual dividend" line.

Step IV

Divide this number, by the first digit of the divisor. Write the quotient at its usual place, at the top. Write the remainder, in the "gross dividend" line.

Step V

Take down the next digit of the main dividend and write it to the right of the remainder of step IV. These two digits together make the next gross dividend.

Step VI

The actual dividend, however, would be this gross dividend minus the product of the last quotient and the flag digit. Write it down on the "actual dividend" line, exactly below the gross dividend.

Step VII

Divide above difference again by the first digit of the divisor, write the quotient on the top line and remainder on the "gross dividend" line. Pull the next digit of the main dividend to the right of this remainder. Find the actual dividend which is written at the appropriate place.

Step VIII

Continue this procedure until all the digits of the main dividend are exhausted. The number obtained in the top most line is the quotient and in the actual dividend line after the slant line is the remainder.

Example: Solve $38985 \div 73$.

Solution:

By following the steps in the above method.

	534
7/3	3 89 8/5
Gross dividend	39 38/13
Actual dividend	3 824 29/3

Therefore, quotient is 534 and remainder is 3.

13.4 APPLICATIONS IN POLYNOMIAL AND HIGHER CALCULATIONS

Addition of Algebraic Expressions

This method is applicable in single variable polynomials. Here we have to keep in mind that the terms containing negative sign, that sign must be kept in the form of bar over the coefficient.

Polynomials	**Coefficients of variable**
$7x+2$	7 2
$3x^2+5x+1$	3 5 1
x^3+x+1	1 01 1
x^4-7x^3+3x-5	1 $\bar{7}$ 0 3 $\bar{5}$

Now we add coefficients in vertical position.

			7	2
		3	5	1
	1	0	1	1
1	$\bar{7}$	0	3	$\bar{5}$
1	$\bar{6}$	**3**	**16**	$\bar{1}$

Therefore, resultant polynomial is $x^4-6x^3+3x^2+16x-1$

Subtraction of Polynomials

The Vedic sutra, 'Paravartya Yojayet' which means 'Transpose and apply' is used to subtract polynomial one from another. First change the sign of digits in each term of the subtrahend and add that to the corresponding term of the minuend.

Example given below explains the method clearly.

Example: Subtract $x^4+3x^3+6x^2+7x+7$ from $x^5-x^3+8x^2+3$

Solution:

x^5	x^4	x^3	x^2	x	Constant
1	0	$\bar{1}$	8	0	3
$\bar{0}$	$\bar{1}$	$\bar{3}$	$\bar{6}$	$\bar{7}$	$\bar{7}$
1	$\bar{1}$	$\bar{4}$	2	$\bar{7}$	$\bar{4}$

Thus, required difference is $x^5-x^4+2x^3+2x^2-7x-4$

Multiplication of Polynomials using 'Urdhva Triyagbhyam'

The product of two polynomial can be done with the help of the vedic sutra 'Urdhva Triyagbhyam'. We have the following example,

Example: Solve $(2x+3)(2x+9)$

Solution:

Polynomial	x-base	Numbers
2x+3	2	3
2x+9	2	9

Term involving x^2 is $\begin{matrix}2\\2\end{matrix}\uparrow = 2\times 2 = 4$

Term involving x is 2x9+2x3=24

And term independent of x =3x9=27

Thus, $(2x+3)(2x+9)=4x^2+24x+27$

Division of Polynomials

In the procedure of division of a polynomial by another, the vedic sutras "Paravartya Yojayet" and "Urdhva Triyagbhya" are used. The meaning of "Paravartya Yojayet" is "transpose and apply'. Also, the meaning of "Urdhva Triyagbhyam" is vertical and crosswise. The application of "Urdhva Triyagbhyam" sutra is explained in the following example.

Example: Divide x^2+4x+5 by $x+2$.

Solution:

Here, x^2 and x are the first term of dividend and divisor, so first term of the quotient is $\frac{x^2}{x}=x$. Now, multiply x by 2 to get $2x$ but we need $4x$ which we can get by multiplying again x by 2 and then adding $2x$ + $2x$ = $4x$ so second term of quotient should be 2. Now, to get the value of remainder, multiply second term of quotient with 2, it is 4, so remainder = 1.

13.5CHECK YOUR PROGRESS

(1) Add 299 and 31 using zero ending numbers.

(2) Add $3x^3+5x^2+9, 4x^4+2x+3$ and $2x^2+x+8$.

(3) Subtract 268289 from 653901 using 'Ekanyunena Poorvena'.

(4) Subtract 876 from 100000 using 'Nikhilum Navataha Charmam Dashtaha'.

(5) Find the product of 91 and 93 using 'Nikhilum' sutra.

(6) Find the product $(9x^2+6x+5)$ and $(4x^2+6x+5)$.

(7) Solve $14268\div867$ using 'Nikhilum' sutra.

(8) Solve $23689\div112$ using 'Paravartya Yojayet'.

(9) Solve $56378\div49$ using 'Urdhva Triyagbhyam'.

13.6LET US SUM UP

- The unit sets out to help students understand how Vedic sutras can be used as practical tools for faster and simpler mathematical calculations.
- It introduces the idea of Vedic Mathematics as a system of mental calculation based on simple rules (sutras) that make problem-solving easier and quicker.
- Vedic sutras can be applied to addition, subtraction, multiplication, and division to reduce lengthy steps and save time.
- By using Vedic methods, calculations become more systematic, improving both speed and accuracy compared to conventional techniques.
- The sutras are also applied to algebraic expressions and polynomials, helping in expansion, factorization, and solving higher-order calculations in a simple and efficient way.

13.7 ANSWERS TO CHECK YOUR PROGRESS

Answer to Q1: 300

Answer to Q2: $4x^4 + 3x^3 + 7x^2 + 3x + 20$

Answer to Q3: 385612

Answer to Q4: 99124

Answer to Q5: 8463

Answer to Q6: $36x^4 + 78x^3 + 101x^2 + 60x + 25$

Answer to Q7: Quotient=16 and remainder=396

Answer to Q8: Quotient=211 and remainder=57

Answer to Q9: Quotient=1150 and remainder=28

13.8FURTHER READING

1. Tirthaji, B. K. (1965). *Vedic Mathematics*. Motilal Banarsidass Publishers.
2. Oakridge International School. (2024). *A detailed exploration of Vedic mathematics: Understanding sutras, sub-sutras, and their practical applications*. *International Education and Research Journal*, 10(10).
3. Devi, S. (2020). *Applications of Vedic mathematics in algebra*. *International Research Journal on Advanced Science Hub*, *2*(11S).
4. Sharma, V. (2019). *A study on Vedic mathematics: An ancient method*. *Mathematical Sciences Journal*, 1(1).
5. Kumari, R. (2022). *An overview of the ancient Indian Vedic mathematics techniques*. *IJRASET Journal*

13.8 MODEL QUESTIONS

Q1. Add using “Ekadhikena Poorvena”

a) 763 and 169

b) 847 and 328

c) 543789 and 268289

Q2. Add using zero ending numbers and also verify your answer

a) 187 and 35
b) 479 and 224
c) 99982 and 88

Q3. Add the following polynomials using vedic method

a) $x^3 + x + 1 \, and \, x^2 + x - 1$

b) $x^5 - 6x^2, x^4 + x^3 + 6x^2 + 8x + 2 \, and \, x^2 - 7x + 9$

c) $x^4 - 3x^3 + 8x^2 + 7x + 9 \, and \, x^3 + 6x^2 + 2x + 7$

Q4. Subtract the first number from the second number using 'Ekadhikena Poorvena'

a) 632, 931
b) 25602, 36499
c) 32369060, 39403230

Q5. Subtract the first number from the second number using 'Ekanyuneva Poorvena'

a) 1690, 1930
b) 169001,326400
c) 21694, 36942

Q6. Subtract the first number from the second number using 'Nikhilum' sutra

a) 19568, 30125
b) 1690, 1783
c) 905265, 1878642

Q7. Subtract the first polynomial from the second polynomial using 'Paravartya Yojayet'

a) $16x^2 + 15x + 4, x^2 - 1$

b) $x^3 + 2x^2 + 3x + 9, 4x^3 + 4x^2 + 9x + 2$

c) $x^4 - 1, x^5 + x^4 + x^2 + x - 1$

Q8. Solve the following using 'Nikhilum' sutra

a) 113x998
b) 996x1008
c) 9651x8993

Q9. Solve the following using 'Urdhva Triyagbhyam'

a) 99963x99872
b) 334x307
c) 99x104

Q10. Find the product of the polynomials

a) $(4x^2+2x+1)\,and\,(8x^2+2x+5)$
b) $(ax^2+bx+c)\,and\,(cx^2+dx+e)$
c) $(x^3+1)\,and\,(x^2+1)$

Q11. Solve the following using 'Nikhilum method'

a) $1555\div 893$
b) $13103\div 8907$
c) $19234\div 83$

Q12. Solve the following using 'Paravartya method'

a) $10453\div 102$
b) $281118\div 1003$
c) $13981\div 1363$

Q13. Solve the following using 'Urdhva Triyagbhyam method'

a) $3300\div 34$
b) $62032\div 67$
c) $8888\div 72$

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UNIT 14: RELEVANCE OF VEDIC KNOWLEDGE IN MODERN MATHEMATICS

UNIT STRUCTURE

14.1 Learning Objectives
14.2 Introduction
14.3Comparison between Vedic and Western Approaches
14.3.1 Arithmetic
14.3.2 Algebra
14.3.3 Geometry
14.4 Applications of Vedic Principles to Modern Problem Solving
14.5 Let Us Sum Up
14.6 Answers to Check Your Progress
14.7 Further Reading
14.8 Model Questions

14.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- Understand the key differences between Vedic and Western methods in arithmetic, algebra, and geometry.
- Recognize how Vedic sutras simplify calculations compared to traditional Western steps.
- Apply Vedic principles to solve modern mathematical problems quickly and accurately.
- Appreciate the relevance of Vedic knowledge in everyday computations and advanced studies.

14.2 INTRODUCTION

Vedic mathematics offers a system of calculation based on 16 main sutras and 13 sub-sutras derived from ancient Indian texts. These formulas provide shortcuts for complex operations that Western mathematics often handles through lengthy procedures. This unit explores how Vedic approaches differ from Western ones in core areas and shows their practical use today. Students and professionals benefit from faster mental math, reducing errors in exams and real-world tasks. The methods emphasize pattern recognition and intuition, making math accessible.

14.3 COMPARISON BETWEEN VEDIC AND WESTERN APPROACHES

Vedic mathematics focuses on mental computation using word formulas called sutras, while Western methods rely on step-by-step algorithms taught in schools. Vedic techniques often solve problems in one line, whereas Western ones use multiple steps with carrying, borrowing, and long multiplication tables.

14.3.1ARITHMETIC

In arithmetic, Vedic sutras like "NikhilamNavatascaramamDasatah" (All from 9 and the last from 10) handle multiplication near powers of 10 quickly. For example, to multiply 98 by 97, Western long multiplication involves writing partial products and adding: $98 \times 90 = 8820, 98 \times 7 = 686$, total 9506 after carrying. Vedic base method uses base 100: deficiencies -2 and -3, cross-subtract $98 - 3 = 95$, multiply $-2 \times -3 = 6$, answer 9506 in seconds mentally.

Subtraction also differs. Western borrowing for $1000 - 456$ needs adjusting digits step-by-step. Vedic "All from 9, last from 10" gives 543 directly: $9 - 4 = 5, 9 - 5 = 4, 10 - 6 = 4$. Addition uses pairs summing to 10 for grouping, unlike Western column addition.

Division by 9 or 11 uses flags and vinculum (bar notation for negatives), converting$78 \div 9$ to vinculum 12 ÷ 9 = 8 remainder 6, faster than repeated subtraction.

Nikhilam Multiplication for 98×97

$$\begin{array}{l} 98 \mid -02 \\ 97 \mid -03 \\ \hline 95 \mid \quad 06 \end{array}$$

14.3.2ALGEBRA

Algebraic factorization in Western methods tests factors systematically. For$x^2 - 7x + 12 = 0$, trial gives $(x - 3)(x - 4)$. Vedic "Sunyam Samuccaye" (sum is zero) spots coefficient sum zero for factors.

Quadratics like $2x^2 + 5x - 3 = 0$ use "Paravartya Yojayet" (transpose and adjust). Western quadratic formula is lengthy: $x = \frac{[-b \pm \sqrt{b^2 - 4ac}]}{2a}$. Vedic vertical-crosswise solves in steps mimicking arithmetic.

Polynomial division, say $x^3 + 5x^2 + 4x - 20 \div x + 2$, Western long division takes lines. Vedic "Nikhilam" flags divisor, divides in fewer steps.

Simultaneous equations like $3x + 4y = 25, 2x + 3y = 18$ use Vedic proportion for quick elimination.

14.3.3GEOMETRY

Geometry in Vedas links to altar construction (Sulba Sutras), approximating $\sqrt{2}$ as 1.4142135 via Pythagoras-like triples. Western Euclidean proofs use axioms step-by-step.

Vedic "UrdhvaTiryagbhyam" (vertically and crosswise) computes areas mentally. For rectangle 23×14, pattern gives 322 in one line vs. Western $23 \times 10 = 230$, $23 \times 4 = 92$, total 322.

Pythagoras theorem proofs in Sulba Sutras rearrange areas simply, similar to Western but intuitive. Coordinate geometry uses sutras for distances.

FVertically-Crosswise for Area 23×14

$$2\ 3$$
$$1\ 4$$
$$-\ -\ -\ -\ -$$
$$3|2|2$$

(vertical $3 \times 4 = 12$ carry 1; cross $3 \times 1 + 2 \times 4 = 11 + 1 = 12$ carry 1; vertical$2 \times 1 + 1 = 3$)

CHECK YOUR PROGRESS

Q1: How does Vedic Nikhilam differ from Western multiplication for 99×98?

Q2: Give a Vedic sutra for recurring decimals like $1/19$.

Q3: Compare steps for factoring $x^2 + 5x + 6$ in both systems.

Q4: What base does Vedic use for 89×92?

14.4 APPLICATIONS OF VEDIC PRINCIPLES TO MODERN PROBLEM SOLVING

Vedic methods apply to computing, competitive exams, and engineering. "UrdhvaTiryagbhyam" speeds digital multipliers in VLSI chips, used in processors for parallel computation.

In exams like CAT or JEE, base method squares 997^2: base 1000, -3 twice, cross$997 - 3 = 994$, $(-3)^2 = 9$, answer 994009 mentally. Western calculator or long squaring takes longer.

Cube roots like $\sqrt[3]{373248}$use "Anurupyena" approximation, faster than Western trial-error.

Calendars use "ZrdhvaTiryagbhyam" for day calculation: year flags, mental addition.

Finance: percentages via "Urdhva" for $12\% \, of \, 450 = 54$. Trigonometry tables via sutras aid navigation.

CHECK YOUR PROGRESS

Q5: Compute 98^2 using Vedic base.

Q6: Divide $123456 \div 9$ using Vedic.

Q7: Square root of 104976 using Vedic.

Q8: Why is Vedic useful in competitive exams?

14.5 LET US SUM UP

- Vedic sutras simplify arithmetic via bases and patterns, unlike Western multi-step processes.
- Algebra and geometry gain speed through intuitive formulas like vertically-crosswise.
- Modern applications include fast exam solving, computing, and mental math for professionals.
- Overall, Vedic knowledge enhances accuracy and efficiency in daily and advanced math.

14.6 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q1: Vedic: base $100, -1/-2$, cross $99 - 2 = 97$, $(-1)(-2) = 2$, answer 9702 (one line). Western: long multiplication with partials and adds (multiple steps).

Ans to Q2: Ekadhikena Purvena; generates 0.052631578947368421 repeating mentally.

Ans to Q3: Left as an exercise

Ans to Q4: Left as an exercise

Ans to Q5: Base $100, -2$, cross $98 - 2 = 96$, $(-2)^2 = 4,9604$.

Ans to Q6: Vinculum $123456 = 12345|6$, divide by $9 = 13717$ remainder 3.

Ans to Q7: 324 (duplex method).

Ans to Q8: Mental speed, accuracy for time-bound questions like multiplications, squares.

14.7 FURTHER READING

1. Bharati Krishna Tirtha, Vedic Mathematics, Motilal Banarsidass, Delhi.
2. State Council of Educational Research and Training, Fundamentals and Applications of Vedic Mathematics, New Delhi.
3. Kenneth R. Williams, Discover Vedic Mathematics, Vedic Mathematics Research Group.
4. VedicMaths.Org, “Practise and Free Books on Vedic Mathematics.”
5. Moonpreneur, “Vedic Math Sutra 1: Ekadhikena Purvena.”

14.8 MODEL QUESTIONS

Q1. Explain Nikhilam sutra with 95×93 example vs Western.
Q2. Compare Vedic and Western quadratic solving.
Q3. How does UrdhvaTiryagbhyam apply to geometry?
Q4. List modern uses of Vedic math in computing.
Q5. Compute 999×997 using base method.
Q6. Factor $x^2 - 9x + 20$ Vedically.
Q7. Why fewer steps in Vedic division?
Q8. Square 112 using Vedic.
Q9. Describe vinculum in subtraction.
Q10. Relevance of Sulba Sutras to geometry.

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