# CERTIFICATE PROGRAMME IN VEDIC MATHEMATICS

(CBCS in Accordance with NEP 2020)

## VEDIC ARITHMETIC

Hiranya Chandra Bhuyan School of Science and Technology

## KRISHNA KANTA HANDIQUI STATE OPEN UNIVERSITY

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# COURSE INTRODUCTION

Vedic Arithmetic is an ancient yet powerful system of computation derived from Vedic Sutras, emphasizing speed, simplicity, and mental efficiency. This course introduces learners to fundamental arithmetic operations using Vedic techniques, enabling faster calculations and deeper numerical understanding. Through systematic study of addition, subtraction, multiplication, division, fractions, decimals, and advanced topics like LCM, HCF, squares, cubes, and Sulba Sutra arithmetic, the course blends traditional wisdom with modern mathematical application.

## Unit 1: Basic Arithmetic through Vedic Sutras

This unit introduces the concept of Vedic arithmetic and its historical background. It focuses on important Vedic Sutras used in arithmetic operations, with special emphasis on *Ekadhikena Purvena*.

## Unit 2: Addition

This unit explains Vedic techniques for addition that simplify calculations. It introduces Sutras applied in addition and methods for rapid mental addition.

## Unit 3: Subtraction

This unit discusses efficient Vedic methods for subtraction. Special attention is given to the *Nikhilam Sutra* for simplifying subtraction problems.

## Unit 4: Multiplication

This unit focuses on the *Urdhva-Tiryak Sutra* for multiplication. It includes practical applications in compound multiplication, along with practice involving proportion.

## Unit 5: Division

This unit presents Vedic methods of division such as the *Nikhilam* and *Paravartya* methods. It also introduces concepts like the remainder theorem and argumental division.

## Unit 6: Divisibility Rules

This unit explains Sutras used for quick divisibility checks. It covers divisibility rules for both single-digit and two-digit numbers.

## Unit 7: Vedic Methods for LCM

This unit introduces basic definitions related to LCM. It focuses on simple and effective Vedic techniques for finding the Least Common Multiple.

## Unit 8: Vedic Methods for HCF

This unit discusses fundamental concepts of HCF. It presents Vedic methods for finding HCF and solving combined LCM–HCF problems.

## Unit 9: Techniques for Square and Square-roots

This unit introduces techniques such as *Dvandva-Yoga* and the *Yavadunam Sutra*. It also covers methods for squares and square-roots, including special cases and basic principles.

## Unit 10: Techniques for Cube and Cube-roots

This unit explains Vedic methods for finding cubes and cube-roots. It includes the *Anurupya* and *Yavadunam Sutras* and methods based on inspection.

## Unit 11: Fractions in Vedic Mathematics

This unit introduces fractions from the Vedic perspective. It explains the concept of *Vinculum* and its use, along with the algebra of fractions.

## Unit 12: Decimals in Vedic Mathematics

This unit deals with decimal numbers using Vedic techniques. It covers numbers ending in nine, other endings, and multiples of basic fractions.

## Unit 13: Arithmetic in the Sulba Sutras

This unit explores the role of arithmetic in geometric constructions described in the Sulba Sutras. It focuses on ratios, proportions, scaling, and approximate values used in altar measurements.

## Unit 14: Vedic Ratio and Proportion

This unit introduces the concepts of ratio and proportion in Vedic arithmetic. It emphasizes their relevance and explains the crosswise method (*Urdhva-Tiryak Sutra*) for solving proportion problems.

Each unit includes some alongside boxes to help you know some of the difficult, unseen terms. Some "EXERCISES" have been included to help you apply your own thoughts. You may find some boxes marked with: "LET US KNOW". These boxes will provide you with some additional interesting and relevant information. Again, you will get "CHECK YOUR PROGRESS" questions. These have been designed to self-check your progress of study. It will be helpful for you if you solve the problems put in these boxes immediately after you go through the sections of the units and then match your answers with "ANSWERS TO CHECK YOUR PROGRESS" given at the end of each unit.

# Unit 1: BASIC ARITHMETIC THROUGH VEDIC SUTRAS

## UNIT STRUCTURE

1.1 Learning Objectives
1.2 Introduction
1.3Introduction to Vedic Arithmetic
1.4 Important Vedic Sutras for Arithmetic
1.5Ekadhikena Purvena
1.6Comparing Vedic and Conventional Methods
1.7 Let Us Sum Up
1.8 Answers to Check Your Progress
1.9 Further Reading
1.10 Model Questions

## 1.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- understand the idea of Vedic mathematics and its place in basic arithmetic.
- identify and describe important Vedic sutras that are useful for the four basic operations.
- apply the sutra Ekadhikena Purvena for fast multiplication and squaring of suitable numbers.
- compare Vedic methods with conventional methods and check answers using simple reasoning.

## 1.2 INTRODUCTION

Arithmetic is the foundation of all later mathematics, and includes the operations of addition, subtraction, multiplication and division.Vedic mathematics offers compact verbal formulas, called sutras, that give mental methods for many arithmetic tasks in one or two steps, instead of long written procedures. These sutras come from a tradition popularised by Jagadguru Swami Bharati Krishna Tirtha, who systematised sixteen main sutras and thirteen sub–sutras that can be applied across arithmetic, algebra and other branches. In this unit the focus is on using a few of these sutras to handle basic arithmetic more quickly and with better insight, especially through the sutra Ekadhikena Purvena.

## 1.3 INTRODUCTION TO VEDIC ARITHMETIC

Vedic arithmetic refers to ways of doing calculations that follow certain Sanskrit sutras, rather than the stepwise school methods usually taught today.Each sutra is a short phrase, but it

suggests a complete method; for example, one sutra tells how to adjust from a base like 10 or 100, while another tells how to work vertically and crosswise with digits.Because the rules are simple and pattern-based, many additions, subtractions and multiplications can be done mentally, which is helpful in everyday work and in time-bound examinations.Vedic arithmetic does not replace standard methods, but gives an alternative view that shows structure in numbers and encourages flexible thinking.When used along with the usual algorithms, these sutras can help learners estimate better, detect errors more easily, and gain confidence in handling numbers.

### Some Basic Terms:

Before using the sutras, it is useful to recall a few number ideas that appear often in Vedic methods. A base is a number like 10, 100 or 1000 whose digits are 1 followed by zeros; numbers close to such a base often allow shorter methods for addition, subtraction and multiplication. The complement of a digit with respect to 10 is the number that must be added to reach 10, so the complement of 3 is 7 and of 8 is 2. Another helpful idea is the digit sum, which is the sum of all digits of a number, reduced to a single digit if needed by adding again; for example, the digit sum of 687 is 6+8+7=21, then 2+1=3. Digit sums can be used as a quick check on answers, since addition or multiplication usually preserve certain properties modulo 9.

## CHECK YOUR PROGRESS

Q1) What is meant by a “base” in Vedic arithmetic, and give two examples.
Q2) Define the complement of a digit (with respect to 10) and give complements of 2, 6 and 9.
Q3) Find the digit sum of 538 and of 9042.

## 1.4 IMPORTANT VEDIC SUTRAS FOR ARITHMETIC

Although there are sixteen main sutras, only a few are needed to begin work with basic arithmetic.

Four sutras that are especially useful at this level are:

- Ekadhikena Purvena (By one more than the previous one)
- NikhilamNavatashcaramamDashatah (All from 9 and the last from 10)
- Urdhva-Tiryagbhyam (Vertically and crosswise)
- ParavartyaYojayet (Transpose and apply)

Ekadhikena Purvena is mainly used in this unit for multiplying certain numbers and for squaring numbers ending in 5. NikhilamNavatashcaramamDashatah is useful for subtraction from bases like 10, 100 or 1000 and also for multiplication of numbers close to such bases. Urdhva-Tiryagbhyam gives a general pattern for multiplication of multi-digit numbers, working with digits vertically and crosswise. ParavartyaYojayet appears more in division and in algebra, but the basic idea of adjusting and combining terms can already be sensed in simple fraction work.

### 1.4.1 Nikhilam and Simple Subtraction from a Base:

Consider subtracting a number from 1000; the sutra “All from 9 and the last from 10” provides a direct way to do this.

To find 1000 − 784, subtract each digit of 784 from 9 except the last digit, which is subtracted from 10, giving 216 as the answer.

The same pattern works with 100, 10 000 and other pure powers of 10, as long as the smaller number has been written with leading zeros to match the number of digits.

For example, 10 000 − 68 is treated as 10 000 − 0068, and gives 9932 by taking 0→9, 0→9, 6→3 and 8→2 (last from 10).

## CHECK YOUR PROGRESS

Q4)Use “All from 9 and the last from 10” to find:

a. $1000 - 329$

b. $10000 - 4075$

Q5)Write 59 as a number near the base 100 and then find 100 − 59 using the same sutra.

## 1.5EKADHIKENA PURVENA

The sutra Ekadhikena Purvena is usually translated as “By one more than the previous one. Here, “previous one” refers to a certain digit in the problem, and “one more” gives the number by which we multiply or otherwise operate to obtain the answer.In the original text this sutra appears in many contexts, including the expansion of certain recurring decimals and the handling of some algebraic forms. At the basic arithmetic level, the sutra is especially famous for two applications: squaring numbers ending in 5, and multiplying numbers of a special pattern.

### 1.5.1 Squaring Numbers Ending in 5:

If a number ends with digit 5, its square can be written directly using Ekadhikena Purvena. Write the number as a single block of digits before 5, followed by 5; for example, 25 has block 2, 75 has block 7, and 125 has block 12.

The method has two simple steps:

- multiply the block by “one more than itself”;
- write 25 at the end.

For 25²: the block is 2, one more is 3, and $2 \times 3 = 6, so\ 25^2 = 625$.

For 75²: the block is 7, one more is 8, and 7 × 8 = 56, so $75^2 = 5625$.

For a three-digit example, 125²: the block is 12, one more is 13, and 12 × 13 = 156, so $125^2 = 15625$.

In each case the last two digits “25” are fixed, and only the product “block × (block + 1)” needs to be found.

This works because $10n \cdot a + 5$, when squared, gives $10^2 n \cdot a(a+1) + 25$, so the main work is in the product $a(a+1)$.

### 1.5.2 Multiplication by Numbers of the Form$10\ldots-1$:

Ekadhikena Purvena can also be linked with multiplication by numbers like 9, 99, 999 and so on, which are one less than a power of 10.
A common result used in teaching is that when a number is multiplied by a string of 9s having as many digits as the number itself, the answer can be obtained by reducing the number by 1 and then appending the complement of each digit.
For example, $765 \times 999$ can be written as $764\,235$: here $765 - 1 = 764$, and each digit of 765 has complement 2, 3, 5, giving 235.
This procedure combines the idea of “one less than the previous number” with Nikhilam (using complements), and is often presented in Vedic arithmetic manuals as a single compact technique.
A similar pattern holds when there are more 9s than digits of the number, after mentally adding leading zeros to the number to match lengths. These methods can be helpful for mental calculation and for building understanding of how numbers behave near a base.

## CHECK YOUR PROGRESS

Q6) Use Ekadhikena Purvena to find the squares of:
a 35
b 95
c 205
Q7) Explain in your own words why the ending “25” always appears when squaring a number that ends in 5.

## 1.6COMPARING VEDIC AND CONVENTIONAL METHODS

Conventional school methods for squaring, such as column multiplication, require setting out the number twice and writing intermediate products, which can be slow and prone to slips. In contrast, Ekadhikena Purvena gives a single mental step for numbers ending in 5, which frees attention for checking and estimation.
Similarly, subtraction from powers of 10 can be done digit by digit using Nikhilam, rather than borrowing across columns, which many learners find confusing at first.
While these Vedic methods are not limited to mental work, their main strength lies in showing clear numeric patterns, which can strengthen conceptual learning in arithmetic.

## CHECK YOUR PROGRESS

Q8) Square 85 by the usual long multiplication, and also by Ekadhikena Purvena; compare the steps in each method.
Q9) Use “All from 9 and the last from 10” to find 1000 − 456, then check using ordinary subtraction.

Q10) Describe one situation (for example in shopping, banking, or exams) where a mental Vedic method could save time.

## 1.7 LET US SUM UP

- Vedic mathematics uses short sutras to give fast mental methods for basic arithmetic instead of long written procedures.
- The idea of base numbers like 10, 100 and 1000, along with complements and digit sums, helps to simplify many calculations and to check answers quickly.
- Important sutras for arithmetic at this level include Ekadhikena Purvena, NikhilamNavatashcaramam Dashatah, Urdhva-Tiryagbhyam and Paravartya Yojayet.
- Ekadhikena Purvena is especially useful for squaring numbers ending in 5 and for certain multiplications near powers of 10.
- Vedic methods do not replace standard algorithms but add speed, flexibility and pattern-recognition, making arithmetic more intuitive and less mechanical.

## 1.8 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q1: A base is a number of the form 1 followed by zeros, such as 10, 100, 1000 and so on, used as a reference point for nearby numbers.
Examples include 10 and 100.
Ans to Q2: The complement of a digit with respect to 10 is the number that must be added to it to make 10.
Complements are: of 2 is 8, of 6 is 4, and of 9 is 1.
Ans to Q3:
Digit sum of $538$: $5+3+8 = 16$, then 1+6 = 7.2.pdf
Digit sum of $9042$: $9+0+4+2 = 15$, then 1+5 = 6.
Ans to Q4:
a $1000 - 329$: write 329, subtract $3 \to 6, 2 \to 7, 9 \to 1$ (last from 10), giving 671.
b $10000 - 4075$: treat as $10000 - 4075 = 5925$ via $4 \to 5, 0 \to 9, 7 \to 2, 5 \to 5$.
Ans to Q5: 59 is near base 100; 100 − 59 can be done as 41 using 5→4 and 9→1, so

$$59 + 41 = 100$$

Ans to Q6:
a 35²: block 3, next 4, product $3 \times 4 = 12$, so $35^2 = 1225$.
b 95²: block 9, next 10, product $9 \times 10 = 90$, so $95^2 = 9025$.
c 205²: block 20, next 21, product $20 \times 21 = 420$, $so\ 205^2 = 42025$.
Ans to Q7: Any number ending in 5 can be written as 10a + 5; its square is
$100a^2 + 100a + 25$, which is $100a(a+1) + 25$, so the last two digits must be 25.
Ans to Q8: By long multiplication, 85 × 85 gives 7225 after writing and adding partial products; by Ekadhikena Purvena, block is 8, next 9, product 72, then write 25 to get 7225.
Ans to Q9: Using the sutra, $1000 - 456$ is 544 by $4 \to 5, 5 \to 4, 6 \to 4$; ordinary subtraction also yields 544.

Ans to Q10: In a timed test, quickly squaring numbers like 35, 45 or 95 using Ekadhikena Purvena reduces working time and lowers the chance of copying errors.

## 1.9 FURTHER READING


1. Bharati Krishna Tirtha, Vedic Mathematics, Motilal Banarsidass, Delhi.
2. State Council of Educational Research and Training, Fundamentals and Applications of Vedic Mathematics, New Delhi.
3. Kenneth R. Williams, Discover Vedic Mathematics, Vedic Mathematics Research Group.
4. VedicMaths.Org, “Practise and Free Books on Vedic Mathematics.”
5. Moonpreneur, “Vedic Math Sutra 1: Ekadhikena Purvena.”


## 1.10 MODEL QUESTIONS

Q1. Define Vedic arithmetic and explain how it differs from conventional arithmetic methods.
Q2. State the sutras Nikhilam Navatashcaramam Dashatah and Ekadhikena Purvena, and describe their roles in basic arithmetic.
Q3. Show how to subtract 10000 − 7398 using the Nikhilam sutra, explaining each digit step.
Q4. Use Ekadhikena Purvena to square 135 and 215, and verify the answers using ordinary multiplication.
Q5. Discuss how digit sums can be used to check results obtained by Vedic methods, with suitable examples.

# UNIT 2: ADDITION

## UNIT STRUCTURE

2.1 Learning Objectives
2.2 Introduction
2.3Vedic Addition Techniques
2.3.1 Place Value Addition (Column-wise)
2.3.2Ekadhika (Dot Method)
2.3.3 Addition Using Base Values
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2.4.2 Puranapuranabhyam
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2.4.4 Yavadunam
2.5 Rapid Mental Addition Methods
2.5.1 Rounding and Adjusting Method
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2.6 Let us know more
2.7 Check Your Progress
2.8 Let us sum up
2.9 Answers to Check Your Progress
2.10 Further Reading
2.11 Model Questions

## 2.1 LEARNING OBJECTIVES

After studying this chapter, learners will be able to:

- Understand the concept of addition in Vedic Mathematics as a logical and mental process
- Apply various Vedic addition techniques such as place value, base method, grouping, and left-to-right addition
- Use Vedic Sutras like *Sankalana–Vyavakalanabhyam*, *Puranapuranabhyam*, *Anurupyena*, and *Yavadunam* in addition
- Perform rapid mental addition accurately without written steps
- Select suitable base values (10, 100, 1000) to simplify calculations

- Estimate results and verify answers for accuracy
- Develop speed, confidence, and number sense in everyday calculations
- Solve real-life and examination-oriented problems using Vedic methods

## 2.2 INTRODUCTION

Addition is the most fundamental operation in mathematics, forming the basis for all higher numerical processes. In Vedic Mathematics, addition is not merely a mechanical process of carrying digits from right to left; rather, it is a logical, flexible, and mentally efficient activity. Vedic addition techniques encourage clarity of thought, number sense, and speed, making calculations both accurate and enjoyable. This chapter presents a systematic study of Vedic addition through techniques, Sutras, and rapid mental methods.

## 2.3 VEDIC ADDITION TECHNIQUES

Vedic addition is based on simplicity and mental ease. The techniques rely on recognizing number patterns, using base values, and adjusting numbers to reduce calculation effort.

### 2.3.1 PLACE VALUE ADDITION (Column-wise)

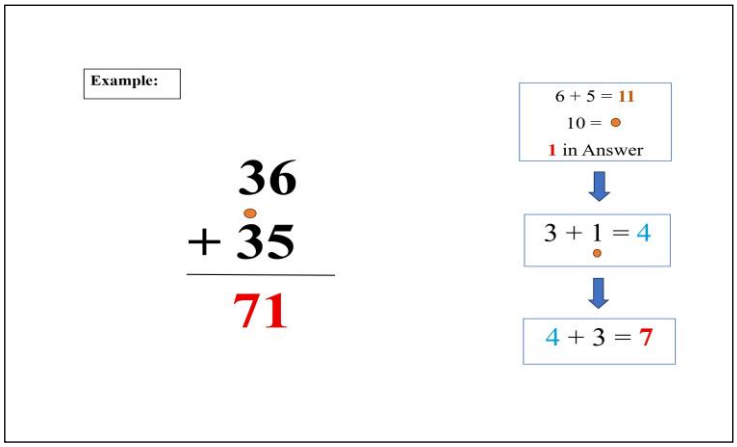
Add digits by their place value (hundreds, tens, ones) and then sum the results.

**Example**: For 423 + 222, add 4+2=6 (hundreds), 2+2=4 (tens), 3+2=5 (ones) for 600+40+5=645.

### 2.3.2 EKADHIKA (Dot Method)

Use a dot above a digit to indicate a carry-over.

- When a column sum exceeds 9 (e.g., 7+4=11), put a dot above the first digit (7) and carry the 1 to the next column.
- When adding the next column, include the dot (e.g., 6+1=7) before proceeding with the column's digits.



## 2.3.3 ADDITION USING BASE VALUES

One of the most powerful ideas in Vedic Mathematics is the use of bases such as 10, 100, 1000, and so on. When numbers are close to a base, addition becomes extremely simple.

**Example 1:**98 + 96
Both numbers are close to the base 100.

- 98 is 2 less than 100
- 96 is 4 less than 100
- Total deficit = 2 + 4 = 6
- Subtract from base: 100 − 6 = **194**

**Example 2:**102 + 105

- Excess over 100: 2 and 5
- Total excess = 7
- Result = 100 + 7 = **207**

This method eliminates long written steps and is ideal for mental calculation.

## 2.3.4 LEFT-TO-RIGHT ADDITION

Traditional arithmetic emphasizes right-to-left addition, but Vedic mathematics prefers left-to-right processing, which aligns naturally with human thinking.

**Example:**567 + 248

- Hundreds: 500 + 200 = 700
- Tens: 60 + 40 = 100 → total 800
- Units: 7 + 8 = 15
- Final answer: **815**

This method allows the mind to process large numbers smoothly without waiting for carry-over from the units place.

## 2.3.5 BREAKING NUMBERS INTO CONVENIENT PARTS

This technique involves splitting numbers into easily manageable components.

**Example:**384 + 629

- 384 + 600 = 984
- 984 + 29 = **1013**

By adding the larger part first, mental strain is reduced and accuracy improves.

## 2.3.6GROUPING FOR MULTIPLE ADDITIONS

When adding many numbers, grouping helps simplify calculations.
**Example:**
Add: 25, 75, 48, 52

- (25 + 75) = 100
- (48 + 52) = 100
- Total = **200**

This approach is especially useful in rapid calculations and competitive examinations.

## 2.4 SUTRAS APPLIED IN ADDITION

Vedic Mathematics is governed by 16 Sutras. While some Sutras are primarily associated with multiplication or division, several play a vital role in addition as well.

### 2.4.1 SANKALANA-VYAVAKALANABHYAM

**Meaning:***By Addition and by Subtraction*

This is the fundamental Sutra for addition. It encourages balancing numbers by making small adjustments to simplify calculations.

**Example:**599 + 487

- Increase 599 by 1 → 600
- Decrease 487 by 1 → 486
- 600 + 486 = **1086**

The Sutra highlights that addition and subtraction can be used together to simplify mental arithmetic.

### 2.4.2 PURANAPURANABHYAM

**Meaning:***By Completion or Non-completion*

This Sutra is useful when numbers are close to a round figure.

**Example:**89 + 27

- 89 needs 11 to reach 100
- Subtract 11 from 27 → 16
- Result = 100 + 16 = **116**

This Sutra strengthens the idea of completing a base before proceeding.

### 2.4.3 ANURUPYENA

**Meaning:***Proportionately*

This Sutra supports proportional adjustment, particularly when working with related numbers.

**Example:**49 + 51

- Both numbers are equally distant from 50
- Their sum = 100

Such proportional thinking enhances number sense and speed.

### 2.4.4 YAVADUNAM

**Meaning:***Whatever the deficiency*

Although commonly used in multiplication, this Sutra also assists addition near a base.

**Example:**97 + 95

- Deficiencies: 3 and 5
- Total deficiency = 8

Result = 100 − 8 = **192**

## 2.5 RAPID MENTAL ADDITION METHODS

Rapid mental addition is one of the most practical benefits of Vedic Mathematics. With regular practice, calculations that once required pen and paper can be performed instantly.

### 2.5.1 ROUNDING AND ADJUSTING METHOD

Numbers are rounded to convenient values and then adjusted.

**Example:**347 + 658

- Round 347 to 350 (+3)
- Subtract 3 from 658 → 655
- 350 + 655 = **1005**

This method is extremely effective for large numbers.

## 2.5.2 COMPLEMENTARY NUMBER METHOD

Complements of 10, 100, or 1000 are used to simplify addition.

**Example:**68 + 47

- Complement of 47 (to 100) = (100 − 47) = 53
- 68 + 53 = 121
- Subtract 100 → **115**

This technique trains the mind to work efficiently with complements.

## 2.5.3 CONTINUOUS TOTAL METHOD

Used when adding several numbers mentally by maintaining a running total.

**Example:**Add: 32, 18, 45, 25

- 32 + 18 = 50
- 50 + 45 = 95
- 95 + 25 = **120**

With practice, the running total becomes automatic.

## 2.5.4 FAST COLUMN ADDITION

Instead of writing numbers in columns, the mind processes them sequentially.

**Example:**Add: 245, 369, 186

- 245 + 369 = 614
- 614 + 186 = **800**

This method improves speed and concentration.

### 2.5.5 ESTIMATION AND VERIFICATION

Vedic mathematics emphasizes estimation as a tool for accuracy.

**Example:**498 + 503 ≈ 500 + 500 = 1000
Actual answer = **1001**

This quick check ensures reliability in mental calculations.

## 2.6 LET US KNOW MORE

Vedic Mathematics originates from the ancient Indian system of knowledge derived from the **Atharva Veda**. The techniques were rediscovered and systematized in modern times by **Sri Bharati Krishna Tirthaji Maharaj**. Unlike conventional arithmetic, which depends heavily on written algorithms, Vedic Mathematics emphasizes **mental agility, logical reasoning, and pattern recognition**.

Addition in Vedic Mathematics is not treated as an isolated operation. Instead, it is deeply connected with **subtraction, complements, proportionality, and base systems**. The Sutras provide flexible ways of thinking, allowing the same problem to be solved by multiple methods. This flexibility strengthens understanding rather than rote learning.

Practicing Vedic addition enhances **both hemispheres of the brain**—the logical side through structure and the creative side through visualization and mental adjustment. Over time, learners develop an intuitive sense of numbers, enabling them to estimate, calculate, and verify results quickly.

Vedic addition also lays the foundation for advanced operations such as **multiplication, subtraction, division, and algebraic thinking**, making it an essential starting point in the study of Vedic Mathematics.

## 2.7 CHECK YOUR PROGRESS

Q1) Add the following numbers using suitable base values (10, 100, or 1000):

a) 98 + 97
b) 96 + 89
c) 102 + 105
d) 995 + 988

Q2) Add the following numbers using the left-to-right method:

a) 456 + 273
b) 684 + 219
c) 739 + 158
d) 527 + 468

Q3) Apply the appropriate Sutra (Sankalana–Vyavakalanabhyam or Puranapuranabhyam) to solve the following:

a) 599 + 378
b) 89 + 27
c) 699 + 248
d) 84 + 19
e) 999 + 467

Q4) Solve the following mentally without writing intermediate steps:

a) 347 + 658
b) 468 + 532
c) 675 + 325
d) 749 + 251

Q5) Add the following sets of numbers using grouping or continuous total method:

a) 25, 75, 48, 52
b) 34, 66, 27, 73
c) 18, 42, 58, 82
d) 125, 375, 225, 275

Q6) Fill in the blanks:

a) 98 + ___ = 200
b) 1000 − 7 + ___ = 1000
c) 349 + ___ = 1000

## 2.8 LET US SUM UP

- Addition is the **foundation of all arithmetic operations**
- Vedic addition focuses on **mental clarity, speed, and accuracy**
- Techniques like **base method, grouping, left-to-right addition, and breaking numbers** simplify calculations
- Vedic Sutras guide the logical application of addition through **balancing and adjustment**
- Rapid mental methods reduce dependence on written calculations

- Estimation acts as a powerful tool for **verification and confidence**
- Regular practice transforms addition into an **effortless mental skill**
- Mastery of Vedic addition prepares learners for **advanced Vedic Mathematics and real-life applications**

## 2.9 ANSWERS TO CHECK YOUR PROGRESS

**Answer to Q1:**a) 195, b) 185, c) 207, d) 1983

**Answer to Q2:**a) 729, b) 903, c) 897, d) 995

**Answer to Q3**: a) 977, b) 116, c) 947, d) 103, e) 1466

**Answer to Q4:**a) 1005, b) 1000, c) 1000, d) 1000

**Answer to Q5:**a) 200, b) 200, c) 200, d) 1000

**Answer to Q6:**a) 102, b) 7, c) 651

## 2.10 FURTHER READING

1. Vedic Mathematics, Jagadguru Swami Sri Bharati Krishna Tirthaji
    a. Maharaja.
2. The History of Ancient Indian Mathematics, C.N. Srinivasiengar, The
    a. World Press, Calcutta, 1967.
3. The History of Hindu Mathematics, Asia Publishers House, Bombay,
    a. 1962.

## 2.11 MODEL QUESTIONS

Q1. Add 999, 998, and 997 mentally.
Q2. Find the sum of the first five multiples of 25.
Q3. Add 125 + 375 + 625 + 875 mentally.
Q4. Without writing, find: 888 + 222 + 111
Q5. Add all numbers from 45 to 55.
Q6. Fill in the blanks:
a) 495 + ___ = 1000; b) 84 + ___ = 100
Q7. Estimate the sum first, then find the exact answer:
a) 498 + 503, b) 721 + 289

# UNIT 3: SUBTRACTION

## UNIT STRUCTURE

3.1 Learning Objectives
3.2 Introduction
3.3Vedic Methods of Subtraction
3.3.1 Subtraction by Adjustment Method
3.3.2Left-to-Right Subtraction
3.3.3 Decomposition Method
3.3.4 Complementary Subtraction Method
3.3.5 Subtraction Near a Base
3.4The Nikhilam Sutra
3.4.1 Nikhilam Sutra for Two-Digit Numbers
3.4.2 Nikhilam Sutra for Three-Digit Numbers
3.4.3 Subtracting Any Number Using Nikhilam
3.4.4 Subtraction Using Complement Addition
3.5 Rapid Mental Subtraction Using Nikhilam
3.5.1 Fast Subtraction from Round Numbers
3.5.2 Subtraction from Non-Base Numbers
3.5.3 Continuous Subtraction Method
3.5.4 Verification Using Estimation
3.6 Let us know more
3.7 Check Your Progress
3.8 Let us sum up
3.9 Answers to Check Your Progress
3.10 Further Reading
3.11 Model Questions

## 3.1 LEARNING OBJECTIVES

After studying this chapter, the learner will be able to:

- Understand subtraction as a fundamental arithmetic operation through the lens of Vedic Mathematics
- Explain the principles of complementary thinking and base values used in Vedic subtraction
- Apply various Vedic methods of subtraction such as adjustment, left-to-right subtraction, decomposition, and complements
- Use the **Nikhilam Sutra** effectively to subtract numbers from powers of 10
- Perform rapid and accurate mental subtraction without relying on traditional borrowing
- Verify subtraction results using estimation and logical reasoning

- Develop speed, accuracy, and confidence in arithmetic calculations useful for daily life and competitive examinations

## 3.2 INTRODUCTION

Subtraction is an essential arithmetic operation that develops number sense, accuracy, and mental agility. In conventional mathematics, subtraction is often viewed as more difficult than addition because of borrowing and multiple written steps. Vedic Mathematics transforms subtraction into a simple, logical, and mentally manageable process by using complements, base values, and powerful Sutras. This chapter explores Vedic methods of subtraction with special emphasis on the **Nikhilam Sutra**, which forms the backbone of rapid subtraction techniques.

## 3.3 VEDIC METHODS OF SUBTRACTION

Vedic subtraction is based on clarity, balance, and mental ease. Instead of borrowing repeatedly, Vedic methods rely on complementary thinking and adjustment strategies.

### 3.3.1 SUBTRACTION BY ADJUSTMENT METHOD

In this method, the subtrahend is adjusted to a convenient number, and the same adjustment is made to the minuend.

**Example:**654 − 297

- Adjust 297 to 300 by adding 3
- Add the same 3 to 654 → 657
- Now subtract: 657 − 300 = **357**

This method eliminates borrowing and speeds up mental calculation.

### 3.3.2 LEFT-TO-RIGHT SUBTRACTION

Vedic subtraction often proceeds from left to right, allowing the mind to process higher place values first.

**Example:**784 − 263

- Hundreds: 700 − 200 = 500
- Tens: 80 − 60 = 20
- Units: 4 − 3 = 1
- Final answer: **521**

This method is particularly effective when the digits of the minuend are larger than those of the subtrahend.

## 3.3.3 DECOMPOSITION METHOD

Numbers are broken into simpler parts to make subtraction easier.
**Example:**523 − 178

- Subtract 100: 523 − 100 = 423
- Subtract 78: 423 − 78 = 345
- Result: **345**

This method improves flexibility in mental calculations.

## 3.3.4 COMPLEMENTARY SUBTRACTION METHOD

Instead of subtracting directly, the complement of the subtrahend is added.

**Example:**1000 − 368

- Complement of 368 (to 1000) = 632
- Therefore, 1000 − 368 = **632**

This method forms the foundation for the Nikhilam Sutra.

## 3.3.5 SUBTRACTION NEAR A BASE

When numbers are close to a base like 10, 100, or 1000, subtraction becomes effortless.

**Example:**100 − 97 = **3**
1000 − 998 = **2**

The concept of deficiency plays a crucial role in such calculations.

## 3.4 THE NIKHILAM SUTRA

TheSutra **“NikhilamNavataścaramam Daśataḥ”** translates to **“All from 9 and the last from 10.”**

This Sutra is one of the most powerful tools in Vedic Mathematics and is extensively used in subtraction, multiplication, and division. In subtraction, it enables quick computation using complements.

## 3.4.1 NIKHILAM SUTRA FOR TWO-DIGIT NUMBERS

To subtract a two-digit number from 100, subtract each digit from 9 and the last digit from 10.

**Example:**100 − 47

- First digit: 9 − 4 = 5
- Last digit: 10 − 7 = 3
- Result: **53**

## 3.4.2 NIKHILAM SUTRA FOR THREE-DIGIT NUMBERS

To subtract a three-digit number from 1000, subtract each digit from 9 and the last digit from 10.

**Example:**1000 − 386

- 9 − 3 = 6
- 9 − 8 = 1
- 10 − 6 = 4
- Result: **614**

## 3.4.3 SUBTRACTING ANY NUMBER USING NIKHILAM

The Sutra can be extended to subtract numbers from any power of 10.

**Example:**10,000 − 4,729

- 9 − 4 = 5
- 9 − 7 = 2
- 9 − 2 = 7
- 10 − 9 = 1
- Result: **5,271**

## 3.4.4 SUBTRACTION USING COMPLEMENT ADDITION

Vedic subtraction replaces borrowing with addition of complements.

**Example:**734 − 486

Step 1: Find complement of 486 to 1000 = 514
Step 2: Add: 734 + 514 = 1248
Step 3: Drop the base (1000) → **248**

This method is extremely fast and accurate.

## 3.5 RAPID MENTAL SUBTRACTION USING NIKHILAM

Vedic subtraction excels in mental calculation, especially in competitive exams.

## 3..5.2 FAST SUBTRACTION FROM ROUND NUMBERS

**Example:**5000 − 297

- Complement of 297 = 703

Answer = **4703**

## 3.5.2 SUBTRACTION FROM NON-BASE NUMBERS

**Example:**832 − 298

- Adjust 298 to 300
- 832 − 300 = 532
- Add back 2 → **534**

## 3.5.3CONTINUOUS SUBTRACTION METHOD

Used when subtracting multiple numbers mentally.

**Example:**1000 − 125 − 375

- 1000 − 125 = 875
- 875 − 375 = **500**

## 3.5.4VERIFICATION USING ESTIMATION

**Example:**803 − 398 ≈ 800 − 400 = 400
Actual answer = **405**

Estimation ensures correctness.

## 3.6 LET US KNOW MORE

Vedic Mathematics is an ancient system derived from the Vedas, designed to make calculations simple, fast, and mentally engaging. Unlike conventional subtraction methods that depend heavily on borrowing and written steps, Vedic subtraction emphasizes **mental clarity**, **balance**, and **logical flow**.

The concept of **complements** plays a central role in Vedic subtraction. By understanding how numbers relate to base values such as 10, 100, or 1000, learners can transform difficult subtractions into easy additions. The **Nikhilam Sutra**, meaning *"All from 9 and the last from 10,"* is especially powerful, as it allows instant subtraction from round numbers.

These techniques are widely used in mental math, abacus training, and competitive exams, helping learners improve concentration, memory, and numerical intelligence. With regular practice, Vedic subtraction becomes intuitive and enjoyable rather than mechanical.

## 3.7 CHECK YOUR PROGRESS

Q7) Solve using adjustment (adding or subtracting convenient values):

a) $654 - 297$
b) $832 - 498$
c) $745 - 199$
d) $912 - 398$

Q8) Subtract using the left-to-right method:

a) $784 - 263$
b) $965 - 432$
c) $853 - 241$
d) $726 - 314$

Q9) Find the answers using complementary thinking:

a) $100 - 68$
b) $100 - 47$
c) $1000 - 365$
d) $1000 - 829$

Q10) Apply the Nikhilam Sutra (All from 9 and the last from 10):

a) $100 - 23$
b) $100 - 58$
c) $1000 - 486$
d) $1000 - 709$

Q11) Solve mentally without writing intermediate steps:

a) $803 - 398$
b) $1000 - 248$
c) $725 - 199$
d) $900 - 487$

## 3.8 LET US SUM UP

- Subtraction is an essential arithmetic operation that builds number sense and mental agility.
- Vedic Mathematics simplifies subtraction by replacing borrowing with adjustment, decomposition, and complementary thinking.
- Methods such as Adjustment, Left-to-Right Subtraction, and Decomposition make calculations faster and more flexible.
- The Nikhilam Sutra forms the backbone of rapid subtraction, especially when subtracting from powers of 10.
- Complement addition enables quick and accurate subtraction even with large numbers.
- Vedic subtraction techniques are highly effective for mental calculations and competitive examinations.
- Mastery of these methods strengthens overall arithmetic skills and prepares learners for advanced mathematical concepts.

## 3.9 ANSWERS TO CHECK YOUR PROGRESS

**Answer to Q1:**a) 357, b) 334, c) 546, d) 514

**Answer to Q2:**a) 521, b) 533, c) 612, d) 412

**Answer to Q3**: a) 32, b) 53, c) 635, d) 171

**Answer to Q4:**a) 77, b) 42, c) 514, d) 291

**Answer to Q5:**a) 405, b) 752, c) 526, d) 413

## 3.10 FURTHER READING

1. Vedic Mathematics, Jagadguru Swami Sri Bharati Krishna Tirthaji
    a. Maharaja.
2. The History of Ancient Indian Mathematics, C.N. Srinivasiengar, The
    a. World Press, Calcutta, 1967.
3. The History of Hindu Mathematics, Asia Publishers House, Bombay,
    a. 1962.

## 3.11 MODEL QUESTIONS

Q1. Subtract 999 from 2,500.
Q2. Find the difference between 10,000 and 3,999.
Q3. Subtract 375 from 1,000 using complements.
Q4. Find 888 − 489 mentally.
Q5. Subtract the sum of 248 and 352 from 1,000.

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# UNIT 4: MULTIPLICATION

## UNIT STRUCTURE

4.1 Learning Objectives
4.2 Introduction
4.3Urdhva–Tiryagbhyam Sutra
4.4Geometrical view of the sutra
4.5 Practical application in compound multiplication
4.6 Practice in compound multiplication
4.7 Proportion in compound multiplication
4.8 Let us sum up
4.9 Answers to Check Your Progress
4.10 Further Reading
4.11 Model Questions

## 4.1 LEARNING OBJECTIVES

After studying this chapter, the learner will be able to:

- understand the meaning of the Urdhva–Tiryagbhyam sutra and its place in Vedic multiplication.
- apply the vertically and crosswise pattern for multiplication of two–, three– and higher–digit numbers.
- use Urdhva–Tiryagbhyam for compound multiplication involving several factors.
- develop speed and accuracy in compound multiplication through systematic practice.
- apply ideas of proportion to plan and check compound multiplications in practical situations.

## 4.2 INTRODUCTION

Multiplication is one of the central operations in arithmetic, and many day–to–day computations, such as calculating areas, costs and products of several factors, depend on it. Traditional long multiplication is reliable, but it can become lengthy and error–prone when the numbers are large or when several factors have to be multiplied one after another.

Vedic mathematics provides a compact way to handle multiplication based on certain sutras, or guiding formulae, from which Urdhva–Tiryagbhyam is the most general for multiplication. This unit explains the sutra and shows how it leads to a systematic method for simple and compound multiplication, with emphasis on clear patterns, careful layout and plenty of practice.

## 4.3 URDHVA–TIRYAGBHYAM SUTRA

The Sanskrit phrase “Urdhva–Tiryagbhyam” translates as “vertically and crosswise.” It is a general method for multiplication that can be applied to numbers of any size, and in algebra it also works for products of polynomials. The method builds the product digit by digit by combining pairs of digits vertically (same place) and crosswise (diagonally) and adding the partial products with proper carries.

At the heart of the sutra lies the distributive property of multiplication. When two multi–digit numbers are written in expanded place–value form, every digit of one number multiplies every digit of the other, and Urdhva–Tiryagbhyam simply arranges these products in a vertical and diagonal pattern. This pattern can be visualised geometrically as a network of line segments or as a grid of rectangles, which helps the learner see why the method works and how each partial product is placed.

### Two–digit multiplication:

Consider the multiplication $AB \times CD$, where $A$ and $C$ are tens digits and $B$ and $D$ are units digits. The Urdhva–Tiryagbhyam pattern for two–digit numbers has three steps:

- Step 1 (right vertical): Multiply the units digits $B \times D$. The result gives the units place of the product, with any carry sent to the next step.
- Step 2 (crosswise): Multiply crosswise and add: $A \times D + B \times C$. The sum, plus any carry from step 1, gives the tens place digit and a further carry.
- Step 3 (left vertical): Multiply the tens digits $A \times C$ and add any carry from step 2 to get the remaining left part of the answer.

This can be represented by a simple geometrical figure: two vertical segments for the vertical products and two diagonal segments crossing each other for the crosswise products. When learners draw the two vertical line pairs and the crossing diagonals, they can label each intersection with the corresponding digit product and see how the crosswise addition collects the contributions for the middle place.

**Example 1:** $23 \times 14$

Let $a = 2, b = 3$ and $c = 1, d = 4$, so the numbers are $23$ and $14$.

- Step 1 (right vertical):
  Units: $3 \times 4 = 12$.
  Write 2 in the units place, carry 1.
- Step 2 (crosswise):
  Crosswise products: $2 \times 4 = 8$ and $3 \times 1 = 3$.
  Sum: $8 + 3 = 11$.

Add carry from step 1: $11 + 1 = 12$.
Write 2 in the tens place, carry 1.

- Step 3 (left vertical):
  Tens: $2 \times 1 = 2$.
  Add carry: $2 + 1 = 3$.

So the product is $322$.
Geometrically, you can think of:

- Right vertical segment for $3 \times 4$,
- Two crossing diagonals for $2 \times 4$ and $3 \times 1$,
- Left vertical segment for $2 \times 1$.

$$\begin{array}{r} 2\ 3 \\ \times\ \ 1\ 4 \\ \hline 2 \mid 2 \mid 2 \\ (2\times 1)\ (2\times 4\ +\ 3\times 1)\ (3\times 4) \\ \downarrow\downarrow \\ 3\ 2\ 2 \end{array}$$

**Example 2:** $62 \times 48$

Let $a = 6, b = 2$ and $c = 4, d = 8$.

- Step 1 (right vertical):
  Units: $2 \times 8 = 16$.
  Write 6, carry 1.

- Step 2 (crosswise):
  Crosswise products: $6 \times 8 = 48$ and $2 \times 4 = 8$.
  Sum: $48 + 8 = 56$.
  Add carry: $56 + 1 = 57$.
  Write 7, carry 5.

- Step 3 (left vertical):
  Tens: $6 \times 4 = 24$.
  Add carry: $24 + 5 = 29$.

So the product is $2976$.

## 4.4 GEOMETRICAL VIEW OF THE SUTRA

A helpful way to understand Urdhva–Tiryagbhyam is through rectangles on a grid. For instance, when two two–digit numbers $10A + B$ and $10C + D$ are multiplied, their product expands as[5] $(10A + B)(10C + D) = 100AC + 10(AD + BC) + BD$.
Each term corresponds to an area of a rectangle formed by placing a segment of length $10A$ next to $B$ and a segment of length $10C$ next to $D$.

If these segments are drawn along two perpendicular axes, the whole product becomes the total area of four rectangles: one large rectangle for $100AC$, two medium rectangles for $10AD$ and $10BC$, and one small rectangle for $BD$. Placing these rectangles in a grid visually confirms that the units area comes from the small rectangle, the middle area from the two medium rectangles together, and the left part from the large rectangle, matching exactly the vertical and crosswise steps of the sutra.

For three–digit numbers, a similar picture has a $3 \times 3$ grid of rectangles. The units place gets contributions from the bottom–right small rectangle, the next place from the two rectangles that touch this corner, the next from three rectangles meeting at the next diagonal, and so on, forming diagonal “bands” across the grid. This diagonal grouping is exactly the crosswise rule of Urdhva–Tiryagbhyam extended to three digits.

## 4.5 RAPID MENTAL SUBTRACTION USING NIKHILAM

In many practical situations, we multiply more than two numbers, for example when calculating total cost as quantity multiplied by rate and then further multiplied by another factor like number of days or number of units in a pack. This is called compound multiplication because the final answer is a product of several factors, and Urdhva–Tiryagbhyam can be used at each stage to keep the work systematic and compact.

A common way to handle compound multiplication is to proceed step by step. First multiply two numbers using Urdhva–Tiryagbhyam and record the intermediate product; then multiply this result by the next number, again with the same pattern, and continue until all factors are included. When every intermediate step uses a clear vertical and crosswise layout, the risk of omitting a partial product or misplacing a digit is reduced, which is important when the factors are large or when mental calculation is attempted.

## CHECK YOUR PROGRESS

Q1. What does the phrase “Urdhva–Tiryagbhyam” mean in English, and what main idea of multiplication does it reflect?

Q2. In two–digit multiplication $AB \times CD$, which step of Urdhva–Tiryagbhyam gives the middle (tens) digit, and how is it computed?

Q3. How can a rectangle or grid diagram help in understanding the vertical and crosswise pattern of the sutra?

Q4. Why is Urdhva–Tiryagbhyam useful in compound multiplication involving several factors?

## 4.6 PRACTICE IN COMPOUND MULTIPLICATION

To gain confidence with this method, consistent practice is essential. Learners can begin with pairs of two–digit numbers, then move on to three–digit by two–digit and three–digit by three–digit products, followed by sequences where the result of one multiplication becomes the starting point for the next.

While practising, it is important to keep the carries under control. A clear written layout uses separate small spaces above each column to hold the carry digits so that they can be added at the next step without confusion. After the Vedic method gives a result, the traditional long multiplication method may be used occasionally as a check, especially for early exercises, until confidence in the new pattern is formed.

**Worked pattern for compound multiplication**

Suppose a student needs the product of three numbers, such as $a \times b \times c$, where each is a two– or three–digit number. The steps can be organised as:

- First stage: compute $P_1 = a \times b$ by the Urdhva–Tiryagbhyam method, carefully recording all partial products and carries.
- Second stage: compute $P_2 = P_1 \times c$ again by the same rule, treating $P_1$ as a multi–digit number and applying the vertical and crosswise pattern with the digits of $c$.[1]

If more factors are involved, the process is repeated, always using the latest product and the next factor. In teaching practice, it is helpful to ask learners to highlight or box each intermediate result, so that the structure of the entire compound multiplication is visible at a glance and can be checked step by step.

## CHECK YOUR PROGRESS

Q5. When multiplying three numbers using Urdhva–Tiryagbhyam, how is the calculation usually broken into stages?

Q6. What simple habit in layout helps prevent errors in carries during practice?

Q7. How can the traditional long multiplication method support practice in the Vedic approach?

## 4.7 PROPORTION IN COMPOUND MULTIPLICATION

In many practical problems, numbers that appear in a compound multiplication are related by proportion, such as doubling a quantity, scaling a recipe, or adjusting the length and breadth of a rectangle while keeping the area in mind. Understanding proportion allows the learner to simplify multiplications before applying the sutra or to check whether obtained answers are reasonable.

For example, if one factor is doubled and another is halved, their product remains unchanged, which means that the compound multiplication can sometimes be rearranged to easier factors without altering the result. Similarly, in rate problems, if quantity and rate change in opposite proportion so that the product should stay roughly the same, a sudden large increase or decrease in the computed product is a warning sign that a multiplication or carry has been mishandled.

**Geometrical sense of proportion:**

Proportional changes in compound multiplication can also be visualised geometrically. When the area of a rectangle is calculated as length multiplied by breadth, changing one side and adjusting the other in inverse proportion corresponds to stretching the rectangle in one direction and squeezing it in the other so that the area remains constant. Drawing pairs of rectangles with the same area but different side lengths helps the learner connect multiplication and proportion in a concrete visual way.

For products of three numbers, the same idea extends to a box or cuboid, where length, breadth and height can vary, but the volume, the product of the three, expresses the combined effect. Viewing compound multiplication as finding area or volume strengthens the learner's intuition about how increasing or decreasing one factor affects the overall product and how proportional adjustments in other factors can compensate.

## 4.8 LET US SUM UP

- Urdhva–Tiryagbhyam is a general multiplication sutra that builds the product using vertical and crosswise combinations of digits, and it works for numbers of any size.
- The method has a clear geometrical interpretation through grids or rectangles, where diagonal groupings of small rectangles correspond to the crosswise additions.
- Compound multiplication with several factors can be organised as a sequence of Urdhva–Tiryagbhyam steps, using intermediate products and clear notation for carries.
- Regular practice with small and medium–sized numbers, checked occasionally by traditional methods, helps develop speed and accuracy with this sutra.

- Understanding proportion and viewing products as areas or volumes support simplification, estimation and checking of compound multiplications.

## 4.9 ANSWERS TO CHECK YOUR PROGRESS

**Ans to Q1**: The phrase “Urdhva–Tiryagbhyam” means “vertically and crosswise,” and it reflects the idea that digits are multiplied in vertical and diagonal pairs to form each place of the product.

**Ans to Q2**: The middle digit (tens place) in $AB \times CD$ comes from the crosswise step, where $A \times D$ and $B \times C$ are computed and then added, together with any carry from the units’ multiplication.

**Ans to Q3**: A rectangle or grid diagram breaks each number into segments along perpendicular axes; each digit product is represented by a small rectangle, and grouping rectangles along diagonals shows how vertical and crosswise products combine into successive place values.

**Ans to Q4**: The sutra is useful in compound multiplication because it gives a compact, repeatable pattern for each pairwise product, allowing long strings of factors to be handled systematically with less written work and fewer chances for missing a partial product.

**Ans to Q5**: The calculation is usually broken into stages by first finding the product of two numbers using Urdhva–Tiryagbhyam, then multiplying that intermediate result by the third number, and repeating similar steps if more factors are present.

**Ans to Q6**: A simple habit is to reserve small spaces above each digit column for carries and to write each carry immediately when it is formed, so that it is clearly available for the next addition and not forgotten.

**Ans to Q7**: The traditional long multiplication method can be used as an independent check on answers obtained by Urdhva–Tiryagbhyam, particularly in the early stages of learning, which reassures learners that the new pattern is reliable.

## 4.10 FURTHER READING


- Vedic Mathematics, Jagadguru Swami Sri Bharati Krishna Tirthaji Maharaja.
- The History of Ancient Indian Mathematics, C.N. Srinivasiengar, The World Press, Calcutta, 1967.
- The History of Hindu Mathematics, Asia Publishers House, Bombay, 1962.


## 4.11 MODEL QUESTIONS

Q1. Explain the meaning of the Urdhva–Tiryagbhyam sutra and describe, in your own words, how it can be used to multiply two two–digit numbers.

Q2. Describe the geometrical interpretation of Urdhva–Tiryagbhyam for the product of two three–digit numbers, using a grid or rectangular diagram.

Q3. Discuss in detail how Urdhva–Tiryagbhyam can be applied to perform compound multiplication of three or more factors. Illustrate with a numerical example.

Q4. What role does practice play in mastering compound multiplication with the Urdhva–Tiryagbhyam sutra? Suggest a graded plan of exercises for students.

Q5. Explain how ideas of proportion can be used to simplify or check compound multiplication problems. Provide at least two practical contexts where this is helpful.

Q6. Compare the Urdhva–Tiryagbhyam method with the traditional long multiplication method in terms of steps, layout and suitability for mental calculation.

Q7. How can geometrical visualisation (areas and volumes) deepen a learner's understanding of multiplication, especially in the context of compound products?

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# UNIT 5: DIVISION

## UNIT STRUCTURE

5.1 Learning Objectives
5.2 Introduction
5.3Nikhilaṁmethod
5.4 Parāvartya method
5.5The remainder theorem
5.6 Argumentaldivision
5.7 Let Us Sum Up
5.8 Answers to Check Your Progress
5.9 Further Reading
5.10 Model Questions

## 5.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- identify and explain the main Vedic methods of division:*Nikhilaṁ*, *Parāvartya*, and argumental method of division.
- choose a suitable Vedic method for different divisors.
- apply the *Parāvartya*method for polynomial division and recognize when *Nikhilaṁ* is unsuitable;
- learn about the remainder theorem.
- learn about the argumental division.

## INTRODUCTION

In this unit, we will discuss the important methods of division in Vedic Mathematics and their appropriate applications. We will explain the key ideas behind the *Nikhilaṁ*, *Parāvartya*, and argumental method of divisionhighlighting the situations in which each method is most effective.The unit also familiarizes students with the remainder theorem and *Parāvartya*method for polynomial division.

## 5.3 NIKHILAṀ METHOD

In Vedic mathematics there are different methods of calculations for different types of numbers. Likewise in division also there are different methods for divisors being close to the bases and those for random numbers. In this section we will deal with divisors containing large-digits using the sutra "*NikhilaṁNavataścaramaṁDaśatah"* which literally means "all from 9 and the last from 10". In this method of division, we first split the dividend by a vertical line in two parts, the left-hand part for the quotient and the right-hand part for the remainder.

Let us first divide a dividend by the number 9. The same method can be applied for the division by bigger numbers close to the bases.

Let us understand the method with the help of a few examples.

**Example 1:** Divide 13 by 9.

**Solution:** In this example, the divisor is 9, the base is 10, and the difference between the divisor and the base is 1.

Step 1: Since the base 10 has one zero in it, we divide the dividend in such a way that the right-hand part has one digit. We write the number as shown below:

$$9) \quad 1/3$$

Step 2: We bring down the first digit of the dividend as thequotient and place it below the line as shown below:

$$\begin{array}{l} 9) \quad 1/3 \\ \quad\quad\downarrow \\ \underline{\quad\quad} \\ \quad\quad 1 \end{array}$$

Step 3: We now multiply this quotient digit with the difference 1 and place it below the second digit of the dividend in the remainder side and add the digits as shown below:

$$\begin{array}{l} 9) \quad 1/3 \\ \underline{\quad\quad} \ /1 \\ \quad\quad 1/4 \end{array}$$

Thus, when 13 is divided by 9, the quotient is 1 and the remainder is 4.

Note: We observe that the first digit of the dividend becomes the quotient and sum of the two digits becomes the remainder.

**Example 2:** Divide 123 by 9.

**Solution:** Step 1:Proceeding like Example 1, we write the dividend as shown:

$$9) \quad 12 \ / \ 3$$

Step 2: We bring down the first digit of the dividend as the first digit of the quotient and place it below the line as shown below:

$$\begin{array}{l} 9) \quad 12 \ / \ 3 \\ \quad\quad\downarrow \\ \underline{\quad\quad} \\ \quad\quad 1 \end{array}$$

Step 3: We now multiply this quotient digit with the difference 1 and place it below the second digit of the dividend. Then add up the digits in the second column and write the sum as the second digit of the quotient as shown below:

$$\begin{array}{l} 9) \quad 12 \ / \ 3 \\ \quad\quad 1 \\ \underline{\quad\quad} \\ \quad 13 \end{array}$$

Step 4: We now multiply the new quotient digit with the difference 1 and place it below the third digit of the dividend and we add the digits in the third column as shown below:

$$\begin{array}{rl} 9) & 12 \,/\, 3 \\ & \underline{\quad 1 \,/\, 3} \\ & 13 \,/\, 6 \end{array}$$

Thus, when 123 is divided by 9, the quotient is 13 and the remainder is 6.

**Example 3:** Divide 237 by 9.
**Solution:** Proceeding like Example 2, we get

$$\begin{array}{rl} 9) & 23 \,/\, 7 \\ & \underline{\quad 2 \,/\, 5} \\ & 25 \,/\, 12 \end{array}$$

In this case, we observe that the remainder is greater than the divisor, which is not permissible. Thus, we redivide the remainder 12 by 9 as shown below:

$$\begin{array}{rl} 9) & 1/2 \\ & \underline{\quad /1} \\ & 1/3 \end{array}$$

The quotient 1 is then carried over to the original quotient 25 and the remainder 3 becomes the final remainder as follows:

$$\begin{array}{rl} 9) & 23 \,/\, 7 \\ & \underline{\quad 2 \,/\, 5} \\ & \underline{\quad 25 \,/\, 12} \\ & 26 \,/\, 3 \end{array}$$

Thus, when 237 is divided by 9, the quotient is 26 and the remainder is 3.

**Example 4:** Divide 21451 by 9.
**Solution:** Proceeding like before, we get

$$\begin{array}{rl} 9) & 2\,1\,4\;\;5 \;/\, 1 \\ & \underline{\quad 2\,3\;\;7 \;/} \\ & 2\,3\,7_{1}2 \;/ \end{array}$$

But 12 is a double-digit number so for the quotient part we place 2 and carry over 1 to the previous digit of quotient i.e. 7 so that the quotient becomes 2382. However, we place 12 itself under 1(remainder part) and get 13 in the remainder part as follows:

$$\begin{array}{rl} 9) & 2\,1\,4\,5 \,/\, 1 \\ & \underline{\quad 2\,3\,7 \,/\, 12} \\ & 2\,3\,8\,2 \,/\, 13 \end{array}$$

Since the remainder 13 is greater than the divisor 9, so we further divide 13 by 9 and get 1 and 4 as quotient and remainder, respectively. The quotient 1 is then carried over to the original quotient 2382 and the remainder 4 becomes the final remainder.

Thus, when 21451 is divided by 9, the quotient is 2383 and the remainder is 4.

Next, we check the next lower numbers 8, 7, etc as divisors and note the following results:

8) 1/3 /2 → 1/5

7) 1 /3 /3 → 1 /6

6) 1/3 /4 → 1/7

In the case of 8, we have to multiply the quotient digit by 2, in the case of 7, by 3 and in the case of 6, by 3 and so on. In other words, it is clear that we have to multiply the quotient digit by the divisor's complement from 10. This implies that the *Nikhilaṁ* rule about the subtraction of all from 9 and of the last from 10 is applicable here. Next, we extend the division process to numbers greater than 9, especially numbers close to the bases. Let us consider some examples to understand the working:

**Example 5**: Divide 124 by 89.
**Solution:** Here, the divisor is 89, the base is 100, and the difference between the divisor and the base is 11.
Since the base 100 has two zeros in it, we divide the dividend in such a way that the right-hand part has two digits. We write the number as shown below:

89) 1 / 24
11

Now we bring down the first digit of the dividend as the first digit of the quotient as done in the previous divisions.

89) 1 / 24
11 ↓
1 /

Next, we multiply the first digit of the quotient by 11 and place it below the next digits of the dividend. Then, add up the digits in both the columns starting from the left as shown below:

89) 1 / 24
11 ↓ / 11
1 / 35

Thus, when 124 is divided by 89, the quotient is 1 and the remainder is 35.

**Example 6:** Divide 10102 by 899.
**Solution:** Here, the divisor is 899, the base is 1000, and the difference between the divisor and the base is 101.
Since the base 1000 has three zeros in it, we divide the dividend in such a way that the right-hand part has three digits. We write the number as shown below:

899) 10 / 102
101

Now, we bring down the first digit of the dividend as the first digit of the quotient as shown below.

$$\begin{array}{l} ____899) \quad 10 \,/\, 102 \\ 101 \quad \downarrow \\ \hline 1 \end{array}$$

Next, we multiply the first digit of the quotient by the difference 101 and place it below the next digits of the dividend as shown

$$\begin{array}{l} ____899) \quad 10 \,/\, 102 \\ 101 \quad \downarrow 1 \,/\, 01 \\ \hline 1 \end{array}$$

We have written digits only under 2 digits of the remainder side and not all three. That means we are still not in the remainder side. So, we will just add the digits in one column as shown below

$$\begin{array}{l} ____899) \quad 10 \,/\, 102 \\ 101 \quad \downarrow 1 \,/\, 01 \\ \hline 1\ 1 \end{array}$$

We now multiply the new digit of the quotient with the difference101 and place them below the next digits in the dividend starting from the third digit as shown.

$$\begin{array}{l} ____899) \quad 10 \,/\, 102 \\ 101 \quad \downarrow 1 \,/\, 01 \\ \qquad\qquad\ \ 101 \\ \hline 1\ 1 \end{array}$$

Now since we are in the remainder side and we have written at least one digit below all the digits in the dividend, we will add all the digits column wise starting from the right as shown:

$$\begin{array}{l} \quad 899) \quad 10 \,/\, 102 \\ ____101 \quad \downarrow 1 \,/\, 01 \\ \qquad\qquad\ \ /\, 101 \\ \quad 1\ 1 \,/\, 213 \end{array}$$

Thus, when 10102 is divided by 101, the quotient is 11 and the remainder is 213.

**Example 7:** Divide 12345 by 7999.
**Solution:**

$$\begin{array}{l} ____7999) \quad 1 \ / \ 2345 \\ 2001 \quad \downarrow \ / \ 2001 \\ \hline 1 \ / \ 4346 \end{array}$$

Thus, when 12345 is divided by 7999, the quotient is 1 and the remainder is 4346.

**Example 8:** Divide 200165 by 8987.
**Solution:**

$$____8987) \quad 2\,0 \ / \ 0165$$

$$1013 \quad \downarrow 2 \;/\; 026$$
$$\quad\;\; /\; 2026$$
$$2\,2 \;/\; 2451$$

Thus, when 200165 is divided by 8987, the quotient is 22 and the remainder is 2451.

This method (*Nikhilaṁ*) of division is particularly useful when the divisor contains big digits. The reason for this is that the bigger the digits, the smaller will be the required complement from 9 or 10, which makes the multiplication task easier. Moreover, we have to perform only small single-digit multiplications. Another advantage of this method is there is no subtraction to be done at all. In other words, in *Nikhilaṁ*method,we don't need to do any subtraction and any division at all, and still obtain the required quotient and the required remainder.

The limitation of *Nikhilaṁ*method is that in case of divisors containing small-digits, the process becomes too long and cumbersome. In this case, another division method named *Parāvartya*method is used which we will explain in details in the next section.

## CHECK YOUR PROGRESS

Q. 1: Divide 11001 by 88.
Q. 2: Divide 101020 by 8888.
Q. 3: Divide 1030007 by 98987.

# 5.4 PARĀVARTYA METHOD

The Vedic sutra *"Parāvartya Yojayet"* means "Transpose and apply". *Parāvartya*method is suitable for divisors having small-digits. This method is somewhat similar to the *Nikhilaṁ* method but with a little difference. Here when the divisor is greater than the base, we will have a negative complement. Let us understand the process with the help of a few examples.

**Example 9**: Divide 1234 by 112.
**Solution:** The divisor 112 is 12 more than the base 100. As the base is 100, we will have 2 digits in the remainder. We write the number as below and bring down the first digit of the dividend 1 as the first digit of the quotient:

$$\begin{array}{l} 112\,) \quad 1 \quad 2 \;/\; 3 \quad 4 \\ -1-2 \\ \hline 1 \end{array}$$

We then multiply 1 with the difference $-12$ and write $-1$ and $-2$ below the next digits of the dividend 2 and 3 as shown below:

$$\begin{array}{l} 112\,) \quad 1 \quad 2 \;/\; 3 \quad 4 \\ -1-2 \quad\;\; -1 \quad -2 \\ \hline 1 \end{array}$$

Next in the second column, we add the two digits and bring down 1. We multiply 1 with the difference $-12$ and write $-1$ and $-2$ below the last two digits of the dividend. Adding the digits in the last two columns we get 11 as the quotient and 2 as the remainder as shown below:

$$\begin{array}{rl} 112\,) & 1\quad 2\quad /\quad 3\quad 4 \\ -1-2 & \quad -1\quad -2 \\ \hline & \quad\quad\quad -1-2 \\ & 1\quad 1\quad /\quad 0\quad 2 \end{array}$$

**Example 10:**Divide 10569 by 103.
**Solution:** The divisor 103 is 3 more than the base 100 but we will write it as 03 as the base 100 has 2 zeros in it. Also, we will have 2 digits in the remainder. So, the calculation will look like below:

$$\begin{array}{rl} 103\,) & 1\quad 0\quad 5\quad /\quad 6\quad 9 \\ -0-3 & \quad -0\ -3 \\ & \quad\quad -0\quad -0 \\ \hline & \quad\quad\quad\quad -0-6 \\ & 1\quad 0\quad 2\quad /\quad 6\quad 3 \end{array}$$

Thus, when 10569 is divided by 103, the quotient is 102 and the remainder is 63.

Sometimes in this type of division a negative digit can occur in the remainder side or in the quotient side. In that case, let us understand the method to be followed with the help of the following examples.

**Example 11:**Divide 123456 by 1123.
**Solution:** The divisor 1123 is 123 more than the base 1000. As the base is 1000, we will have 3 digits in the remainder. We write the number as shown and bring down the first digit of the dividend 1 as the first digit of the quotient.

$$\begin{array}{rl} 1123\,) & 1\quad 2\quad 3\quad /\quad 4\quad 5\quad 6 \\ -1-2-3 & \\ \hline & \\ & 1 \end{array}$$

Now, $1\times(-123) = -123$, we place $-1$, $-2$ and $-3$ in the next three columns as shown. Then, we add the digits in the second column.

$$\begin{array}{rl} 1123\,) & 1\quad 2\quad 3\quad /\quad 4\quad 5\quad 6 \\ -1-2-3 & \quad\quad -1-2\quad -3 \\ \hline & 1\quad 1 \end{array}$$

Similar to Step 2, we place $-1$, $-2$ and $-3$ in the next three columns and add the digits in the third column.

$$\begin{array}{rl} 1123\,) & 1\quad 2\quad 3\quad /\quad 4\quad 5\quad 6 \\ -1-2-3 & \quad\quad -1-2\quad -3 \\ & \quad\quad\quad\quad -1\quad -2-3 \\ \hline & 1\quad 1\quad 0 \end{array}$$

$0 \times (-123) = 000$, so we place $-0, -0, -0$ in the next three columns. Since no column is left we add the digits in each column starting from the right most column as shown below.

$$\begin{array}{rr} 1\,1\,2\,3\,) & 1\ \ 2\ \ 3\ /\ 4\ \ 5\ \ 6 \\ -1-2-3 & -1-2\ -3 \\ & \quad -1\ -2\ -3 \\ & \quad\quad -0\ -0-0 \\ \hline & 1\ \ 1\ \ 0\ /-1\ \ 2\ \ 6 \end{array}$$

The digits of the remainder are $-1$, 2 and 6. When we take their place values, we have the remainder $-100 + 20 + 6 = -74$, which is negative. In that case we carry over 1 from the quotient side (i.e., 1123 in this example) and subtract the negative remainder from this. Thus, the final remainder is 1123 – 74 = 1049, and the quotient is 110 – 1 = 109.

**Example 12:**Divide 438603 by 1121.

**Solution:** The divisor 1121 is 121 more than the base 1000. As the base is 1000, we will have 3 digits in the remainder. We write the number as shown and bring down the first digit of the dividend 4 as the first digit of the quotient.

$$\begin{array}{rr} 1\,1\,2\,1\,) & 4\ \ 3\ \ 8\ /\ 6\ \ 0\ \ 3 \\ -1-2-1 & \\ \hline & 4 \end{array}$$

Now, $4\times(-121) = -484$, we place $-4$, $-8$ and $-4$ in the next three columns as shown. Then, we add the digits in the second column.

$$\begin{array}{rr} 1\,1\,2\,1\,) & 4\ \ 3\ \ 8\ /\ 6\ \ 0\ \ 3 \\ -1-2-1 & \quad -4\ -8\ -4 \\ \hline & 4\ -1 \end{array}$$

Now, $-1\times(-121) = +121$, we place $+1$, $+2$, and $+1$ in the next three columns and add the digits of the third column.

$$\begin{array}{rr} 1\,1\,2\,1\,) & 4\ \ 3\ \ 8\ /\ 6\ \ 0\ \ 3 \\ -1-2-1 & \quad -4\ -8\ -4 \\ & \quad\quad +1\ +2\ +1 \\ \hline & 4\ -1\ \ 1\ / \end{array}$$

In the next step, multiplying 1 with $-121$ we get $-121$, so we place $-1$, $-2$ and $-1$ in the next three columns and since there are no columns left, we add the digits in the columns starting from the right.

$$\begin{array}{rr} 1\,1\,2\,1\,) & 4\ \ 3\ \ 8\ /\ 6\ \ 0\ \ 3 \\ -1-2-1 & \quad -4\ -8\ -4 \\ & \quad\quad +1\ +2\ +1 \\ & \quad\quad\quad -1\ -2-1 \\ \hline & 4\ -1\ \ 1\ /\ 3\ -1\ \ 2 \end{array}$$

The digits of the remainder are 3, $-1$ and 2. Taking their place values, we have the remainder $300 - 10 + 2 = 292$

In the quotient we have the digits 4, −1 and 1. Taking their place values, we have the quotient 400 – 10 + 1 = 391.

**Example 13:**Divide 13905 by 113.
***Solution:***

$$\begin{array}{r|l} 113 & 1\ \ 3\ \ 9\ \ 0\ \ 5 \\ -1-3 & \ \ -1-3 \\ & \ \ \ \ -2\ \ -6 \\ & \ \ \ \ \ \ -4-12 \\ \hline & 1\ \ 2\ \ 4\ -10\ -7 \end{array}$$

Here, we get the remainder −100 – 7 = − 107, which is negative. Thus, carrying over 1 from the quotient we get the final remainder 113 – 107 = 6, and the quotient 124 – 1 = 123.

## CHECK YOUR PROGRESS

Q. 4: Divide 12349 by 1133.
Q. 5: Divide 1356 by 182.
Q. 6: Divide 21841 by 117.

## 5.5 THE REMAINDER THEOREM

If $E, D, Q$ and $R$ be the dividend, the divisor, the quotient and the remainder, respectively in a case of division, their relationship can be expressed algebraically as:

$$E = D.Q + R$$

If the divisor $D = x - p$, the expression becomes

$$E = Q(x - p) + R$$

And if we put $x = p$, the relationship will give $E = R$. That is, the dividend $E$ automatically becomes the remainder $R$ if $p$ is substituted for $x$. In general terms, if $E$ be $ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + \cdots$ and if $D = x - p$, then the remainder $R$ is $ap^n + bp^{n-1} + cp^{n-2} + dp^{n-3} + \cdots$ .This is the remainder theorem.

Applying Heron's process of Synthetic Division and *Parāvartya* method, we can find the quotient and remainder as shown in the following examples:

**Example 14:** Divide $12x^2 - 8x - 32$ by $x - 2$.
**Solution:** We put the divisor $x - 2$ on the left, just below it put down the $-2$ with its sign changed and do the multiplication work exactly we did in the previous sections.

$$\begin{array}{r|l} x-2 & 12x^2 - 8x\ \ -32 \\ 2 & \ \ \ \ \ \ 24\ \ +32 \\ \hline & 12x + 16\ \ \ \ 0 \end{array}$$

Thus, when $12x^2 - 8x - 32$ is divided by $x - 2$, the quotient is $12x + 16$ and the remainder is 0.

**Example 15:** Divide $x^4 - x^3 + x^2 + 3x + 5$ by $x^2 - x - 1$.

**Solution:** We will follow the same process as in Example 12. The only difference is that we have to reverse the signs of the coefficients in all the other terms except the first term of the divisor.

$$\begin{array}{rrl} x^2-x-1 & x^4-x^3+x^2 & +3x+5 \\ \hline 1+1 & 1\ \ +1 & \\ & 0\ \ \ +0 & \\ & \hline & 2\ \ +2 \\ & x^2\ +0\ +2 & +5x+7 \end{array}$$

Thus, when $x^4-x^3+x^2+3x+5$ is divided by $x^2-x-1$, the quotient is $x^2+2$ and the remainder is $5x+7$.

**Example 16:** Divide $x^5+x^3-7x^2+9$ by $x^3-x^2+2x-3$.
**Solution:**

$$\begin{array}{rrl} x^3-x^2+2x-3 & x^5+0+x^3 & -7x^2+0\ \ +9 \\ \hline 1-2+3 & 1-2 & +3 \\ & 1 & -2\ \ +3 \\ & \hline & 0\ \ +0\ \ +0 \\ & x^2+x+0 & -6x^2+3x+9 \end{array}$$

Thus, when $x^5+x^3-7x^2+9$ is divided by $x^3-x^2+2x-3$, the quotient is $x^2+x$ and the remainder is $-6x^2+3x+9$.
CHECK YOUR PROGRESS:

Q. 7: Divide $7x^2+5x+3$ by $x+1$.
Q. 8: Divide $x^3-3x^2+3x-1$ by $x^2-2x+1$.

## 5.6 ARGUMENTALDIVISION

Apart from the *Nikhilam*method and the *Parāvarty*amethod, there is a third method of division which is one of simple argumentation based on the *ŪrdhvaTiryak Sutra.* The following examples will explain and illustrate it:

**Example 17:** Divide $12x^2-8x-32$ by $x-2$.
**Solution:** $12x^2$ divided by $x$ gives us $12x$, which is therefore the first term of the quotient. Thus, $Q=12x+\cdots$.
Now, $12x\times(-2)=-24x$, but we have $-8x$ in the dividend. That is, we must get the remaining $16x$. This must result from multiplying the $x$ of the divisor by $16$. Therefore, the absolute term in the divisor must be $16$. Thus, $Q=12x+16$. Since $-2\times16=32$, we get $R=0$.

**Example 18:** Divide $x^3+7x^2+6x+5$ by $x-2$.
**Solution:**$x^3$ divided by $x$ gives us $x^2$, which is therefore the first term of the quotient. Thus, $Q=x^2+\cdots$.
Now, $x^2\times(-2)=-2x^2$, but we have $7x^2$ in the dividend. That means we must get $9x^2$ more. This means we have to multiply the $x$ of the divisor by $9x$. Thus, $Q=x^2+9x+\cdots$.

For the third term of the quotient, we already have $9x \times (-2) = -18x$, but we have $6x$ in the dividend. Thus, we have to multiply $x$ of the divisor by $24$. This is the third term of the quotient. Thus, $Q = x^2 + 9x + 24$.
Now, this last term of the quotient multiplied by $-2$ gives $-48$. But the absolute term in the dividend is $5$. Thus, we have to get an additional 53. But there is no further term left in the dividend. This means that the $53$ will remain as the remainder.
Thus, when $x^3 + 7x^2 + 6x + 5$ is divided by $x - 2$, the quotient is $x^2 + 9x + 24$ and the remainder is $53$.

## CHECK YOUR PROGRESS

Q. 9: Divide $3x^2 - x - 5$ by $3x - 7$.
Q. 10: Divide $x^3 + 2x^2 + 3x + 5$ by $x^2 - x - 1$.

## 5.7 LET US SUM UP

- The three commonly discussed methods for division are the *Nikhilaṁ*, *Parāvartya*, and argumental methods of division.
- In arithmetic division, *Nikhilaṁ* works well only when the divisor digits are large (6, 7, 8, 9), and is less helpful for small divisors.
- The *Parāvartya*method is more flexible and applicable when divisor digits are small.
- The *Nikhilaṁ* method is generally unsuitable for polynomial division**.** In algebraic division, the *Parāvartya*method is almost invariably more effective.
- Apart from these two methods there is another argumentation method based on the *ŪrdhvaTiryak Sutra.*

## 5.8 ANSWERS TO CHECK YOUR PROGRESS

**Answer to Q. 1:**$Q = 125, R = 1.$

**Answer to Q. 2:**$Q = 11, R = 3252.$

**Answer to Q. 3**: $Q = 10, R = 40137.$

**Answer to Q. 4:**$Q = 10, R = 1019.$

**Answer to Q. 5:**$Q = 7, R = 82.$

**Answer to Q. 6:**$Q = 186, R = 79.$

**Answer to Q. 7**: $Q = 7x - 2, R = 5.$

**Answer to Q. 8**:$Q = x - 1, R = 0.$

**Answer to Q. 9:** $Q = x + 2, R = 9.$

**Answer to Q. 10:** $Q = x + 3, R = 7x + 8.$

## 5.9 FURTHER READING


1. Vedic Mathematics, Jagadguru Swami Sri Bharati Krishna TirthajiMaharaja.
2. The History of Ancient Indian Mathematics, C.N. Srinivasiengar, TheWorld Press, Calcutta, 1967.
3. The History of Hindu Mathematics, Asia Publishers House, Bombay,1962.


## 5.10 MODEL QUESTIONS

Q. 1: Find the quotient and the remainder using *Nikhilam*method:

i. Divide 1211 by 779.
ii. Divide 1248 by 987.
iii. Divide 112131 by 8798.
iv. Divide 214121 by 9978.
v. Divide 101008 by 898.

Q. 2: Find the quotient and the remainder using *Parāvartya*method:

i. Divide 15649 by 12.
ii. Divide 23284 by 1103.
iii. Divide 10269 by 103.
iv. Divide 12349 by 1133.
v. Divide 104135 by 1121.

Q. 3: Find the quotient and the remainder of the following:

i. Divide $3x^2 - x - 5$ by $x - 5$ by $3x - 7$.
ii. Divide $6x^4 + 13x^3 + 39x^2 + 37x + 45$ by $x^2 - 2x - 9$.
iii. Divide $x^5 + x^3 - 7x^2 + 9$ by $x^3 - 2x^2 + 2x - 3$.

# UNIT 6: DIVISIBILITY RULES

## UNIT STRUCTURE

6.1 Learning Objectives
6.2 Introduction
6.3Sutras for Quick Divisibility Check
6.4 Divisibility by Single Digit Numbers
6.5Divisibility by Two Digit Numbers
6.6 Let Us Sum Up
6.7 Answers to Check Your Progress
6.8 Further Reading
6.9 Model Questions

## 6.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- explain the idea of divisibility and why quick divisibility tests are useful in arithmetic and algebra.
- state Vedic sutras that are used for mental divisibility checks.
- apply divisibility rules and sutra-based shortcuts for single digit divisors such as 2, 3, 4, 5, 6, 8, 9 and 11.
- extend the same ideas to some simple two-digit divisors using suitable Vedic methods.
- use digit sum and related checks to confirm whether a calculated quotient is likely to be correct.

## 6.2 INTRODUCTION

Divisibility is a basic idea in arithmetic. When one number is divisible by another, it means that the first number can be written as a whole number multiple of the second number, with no remainder. For example, 42 is divisible by 7 because 42 equals 6 times 7, and there is no leftover part. In many problems we only need to know whether such exact division is possible, not the actual quotient. Quick divisibility rules help in such situations.

In the usual school approach, divisibility rules are often given as separate facts to memorise. For example, learners are told that a number is divisible by 5 if it ends in 0 or 5, or by 4 if its last two digits form a multiple of 4. In Vedic mathematics, such rules are connected to sutras, or verbal formulas, that describe general patterns. This way divisibility tests become part of a larger system rather than isolated tricks.

Vedic mathematics introduces digit sum methods, osculators and other tools for checking divisibility by single and double digit numbers. These tools rely on simple operations like adding digits, subtracting related multiples or shifting and combining parts of the number. Once the logic of these actions is understood, learners can carry out divisibility checks quickly in their heads. This is particularly helpful when simplifying fractions, factoring expressions or checking answers in longer calculations.

## 6.3 SUTRAS FOR QUICK DIVISIBILITY CHECK

In Vedic mathematics, divisibility rules can be seen as applications of a few important sutras. One useful idea is that of casting out nines, which flows from the sutra that states that when the sum on both sides is the same, it may be taken as zero. This idea lies behind the digit sum test for divisibility by 3 and 9. When the digits of a number are added, the result gives information about the remainder when dividing by 9. If the digit sum is a multiple of 3, then the original number is also a multiple of 3. If the digit sum is a multiple of 9, then the number is divisible by 9.

Another principle is related to osculators. An osculator is a special small number derived from the divisor, which is used to reduce the original number step by step. For example, for some divisors ending in 9 or 1, we can find an osculator by using a shortened form of the divisor. The Vedic sutra often cited for this process is associated with transposing and adjusting. It tells us how to shift part of the number, multiply by the osculator, and then add or subtract to obtain a smaller number that has the same divisibility property. Repeating this process until a small, manageable number is obtained allows us to decide about divisibility without performing full long division.

Digit based divisibility checks can also be interpreted using the sutra stating that the sum of products of coefficients behaves in a simple way. In the context of divisibility, each digit plays the role of a coefficient of a power of 10. Since powers of 10 behave regularly when reduced modulo a small divisor, we can replace the original number by a weighted sum of its digits, where the weights repeat in a pattern. The well known test for divisibility by 11, which uses alternating sums of digits, is an example of this. In that test, the alternating sum of digits has the same remainder when divided by 11 as the original number.

These sutra based ideas do more than supply quick rules. They show that divisibility checks are consequences of deep regularities in our base ten system. This viewpoint helps learners understand why such tests work. When the logic is clear, students can even devise their own rules for less common divisors by combining the same principles of digit sums, complements and osculators.

## CHECK YOUR PROGRESS

Q1. How does the idea of digit sum help in devising divisibility tests for numbers like 3 and 9.
Q2. What is an osculator, in the context of Vedic divisibility methods.
Q3. Explain in simple words why alternating sums of digits can be used in a divisibility test for 11.

## 6.4 DIVISIBILITY BY SINGLE DIGIT NUMBERS

Divisibility by 2, 5 and 10 is very simple and relies only on the last digit. A number is divisible by 2 if its last digit is 0, 2, 4, 6 or 8, because each of these digits stands for an even number of ones. A number is divisible by 5 if its last digit is 0 or 5, since 10 and 5 are multiples of 5 and every power of 10 is divisible by 5. Divisibility by 10 requires that the last digit be 0, because 10, 100, 1000 and similar numbers all end in 0 and are multiples of 10.

Divisibility by 3 and 9, as mentioned earlier, can be tested using the digit sum. For divisibility by 3, add all the digits of the number. If the sum is a multiple of 3, then the whole number is divisible by 3. For divisibility by 9, the same process is used but the final sum must be a multiple of 9. This is because 10 is 1 more than 9, and each power of 10 leaves a remainder of 1 when divided by 9, so the remainder of the full number is just the remainder of the sum of its digits.

Divisibility by 4 depends on the last two digits, because 100 is divisible by 4 and all higher powers of 10 are also divisible by 4. This means that the part of the number beyond the last two digits contributes a multiple of 100, which in turn is a multiple of 4. Therefore, to test divisibility by 4, it is enough to look at the last two digits and see if they form a number divisible by 4. For example, 312 is divisible by 4 because 12 is divisible by 4.

For divisibility by 8, a similar but slightly larger test is used. Since 1000 is divisible by 8, any power of 10 from that point onwards is a multiple of 8. Hence, the rest of the number beyond the last three digits contributes a multiple of 1000, which is divisible by 8. We only need to check

whether the number formed by the last three digits is divisible by 8. For example, 7 , 4 and 8 as last three digits signal whether the full number is divisible by 8.

The test for divisibility by 6 combines the tests for 2 and 3, because 6 equals 2 times 3. A number is divisible by 6 if it is even and if the sum of its digits is divisible by 3. Similarly, a test for 12 would combine the conditions for 3 and 4. These combined tests show how simple single digit rules can be joined to give rules for composite numbers.

Divisibility by 7 is more subtle, and several mental methods exist. One common pattern, which fits with Vedic reasoning, is to take the last digit, double it and subtract it from the remaining leading part of the number. If the result is a small known multiple of 7 or zero, or if repeating the process leads to such a number, then the original number is divisible by 7. For example, to test 203, take 20 and 3. Double 3 to get 6 and subtract from 20 to get 14, which is divisible by 7, so 203 is divisible by 7. This method makes use of the special behaviour of 10 modulo 7 and can be seen as a simple osculator method.

Divisibility by 11 uses the alternating sum rule. Starting from the rightmost digit, form an alternating sum where the signs change between plus and minus. If this alternating sum is a multiple of 11, including zero, then the original number is divisible by 11. For example, for 308, compute 8 minus 0 plus 3, which gives 11, so 308 is divisible by 11. This rule arises because powers of 10 alternate between having remainder 1 and remainder minus 1 when divided by 11.

These single digit rules, once understood, allow students to avoid long division in many routine situations. They also prepare the ground for the more advanced Vedic methods used for divisibility by two digit numbers, which usually combine digit reasoning with osculators or adjusted bases.

## CHECK YOUR PROGRESS

Q4. State and explain the rule for testing divisibility by 4.

Q5. How can you check quickly whether a number is divisible by 8.

Q6. What two conditions must be satisfied for a number to be divisible by 6.

Q7. Describe the step by step procedure for testing divisibility by 7 using the method of doubling the last digit.

## 6.5DIVISIBILITY BY TWO DIGIT NUMBERS

Divisibility by two digit numbers can often be reduced to divisibility by simpler numbers through factorisation. For example, 12 is 3 times 4, so a number divisible by 12 must be divisible by both 3 and 4. This leads to a combined test: the sum of the digits must be a multiple of 3 and the last two digits must form a multiple of 4. Similarly, for 15, which is 3 times 5, the last digit must be 0 or 5 and the digit sum must be a multiple of 3. Such combined tests are direct consequences of the prime factorisation of the divisor.

For some two digit numbers, especially those ending in 9 or 1, Vedic methods introduce osculators to give compact mental tests. Consider divisibility by 19. We look for a small number that can play the role of an osculator. One approach uses the fact that 20 is close to 19. Since 20 is 1 more than 19, subtracting the last digit from double the rest of the number can serve as a test, though other equivalent forms are also used. By repeating a suitable operation, the original number is replaced by a shorter number that preserves the property of being divisible by 19 or not.

Another example is divisibility by 13. Since 1001 equals 7 times 11 times 13, any three digit block repeated in a number of a certain form can hint at divisibility by 13. In practice, a more straightforward method uses a fixed small multiplier. One such rule multiplies the last digit by 9 and adds it to the rest of the number. If repeating this process yields a number that is a clear multiple of 13, the original number is divisible by 13. This is again an application of seeing how 10 behaves modulo 13 and turning that into a simple digit rule.

In many textbooks that draw on Vedic mathematics, divisibility by numbers like 7, 11 and 13 is presented together. This is not accidental, because these three numbers are factors of 1001. When a number with four or more digits is written, it can be broken into blocks of three digits from the right. If the alternating sum of these three digit blocks is a multiple of 7, 11 or 13, suitable conclusions can be drawn about divisibility. These combined rules show how thinking in terms of blocks, rather than single digits, may be more convenient for some divisors.

For divisors like 25 or 50, the rules again rely mainly on the last two digits. Since 100 is divisible by 25 and 50, any number whose last two digits form a multiple of 25 will be divisible by 25, and any number whose last two digits are 00 or 50 will be divisible by 50. These are very useful in money related contexts, where amounts are often expressed in rupees and paise or similar units.

The important point is that Vedic methods do not treat each two digit divisor as a completely new case. Instead, they use factorisation, digit sums, block sums and osculators in a systematic way. Once these building blocks are understood, learners can often develop or recall the test for a new divisor on the spot, instead of memorising a long list of separate rules.

## CHECK YOUR PROGRESS

Q8) How can factorisation be used to test divisibility by two digit numbers such as 12 or 15? Explain with one example.
Q9) What is the role of osculators in testing divisibility by two digit numbers like 13 or 19? Explain the method briefly.

## 6.6 LET US SUM UP

- Vedic mathematics explains divisibility through sutras and digit-based methods, showing that divisibility rules arise from general properties of the base-ten number system.
- Digit sum methods, used for testing divisibility by 3 and 9, come from expressing a number as the sum of its digits multiplied by powers of 10, all giving the same remainder.
- Simple rules for divisibility by 2, 4, 5, 6, 8, 10 and 11 mainly depend on the last digit or last few digits of a number.
- The divisibility test for 7, using repeated doubling of the last digit and subtraction, demonstrates how mental methods can replace long division.
- These single-digit tests also help in forming rules for composite numbers.
- Divisibility by two-digit numbers is handled using factorisation, combined tests, osculators, and block-wise reasoning.
- Methods for divisors such as 12, 15, 13 and 19 show that divisibility rules reveal the structure of numbers, not just memorised tricks.

## 6.7 ANSWERS TO CHECK YOUR PROGRESS

**Ans to Q1:**Digit sums help because any number can be written as a sum of its digits times powers of 10, and each power of 10 leaves remainder 1 when divided by 9 or 3. This means the remainder on division by 3 or 9 is the same as the remainder of the digit sum, so checking whether the digit sum is a multiple of 3 or 9 gives a correct divisibility test.

**Ans to Q2:**An osculator is a smaller helper number associated with a divisor, used to reduce a large number step by step while preserving its divisibility property. In practice, we use the

osculator to multiply the last digit or a part of the number, then add or subtract this from the remaining part, creating a shorter number that is easier to test.

**Ans to Q3:**Alternating sums of digits work for 11 because powers of 10 leave remainders that alternate between 1 and minus 1 when divided by 11. When we form an alternating sum of digits, we are actually computing the remainder of the original number modulo 11. If this alternating sum is a multiple of 11, then the original number is also divisible by 11.

**Ans to Q4:**To test divisibility by 4, look only at the last two digits of the number. If those two digits form a number divisible by 4, then the whole number is divisible by 4, because the part of the number beyond the last two digits is a multiple of 100, and 100 is divisible by 4.

**Ans to Q5**: To check divisibility by 8, examine the last three digits of the number. If the three digit number formed by these digits is divisible by 8, then the entire number is divisible by 8, since the remaining part of the number contributes a multiple of 1000, and 1000 is divisible by 8.

**Ans to Q6:**For a number to be divisible by 6, it must satisfy two conditions at the same time. First, it must be even, so its last digit must be 0, 2, 4, 6 or 8. Second, the sum of its digits must be divisible by 3.

**Ans to Q7:**To test divisibility by 7 with the doubling method, separate the last digit from the rest of the number. Double the last digit, then subtract this double from the remaining leading part of the number. If the result is a small multiple of 7, zero, or if repeating the process a few times leads to such a number, then the original number is divisible by 7.

**Ans to Q8:**Factorisation helps in testing divisibility by two digit numbers by breaking the divisor into its prime factors. For example, 12 can be written as 3 × 4. A number is divisible by 12 only if it is divisible by both 3 and 4. This means the sum of its digits must be divisible by 3 and its last two digits must form a number divisible by 4. Similarly, for 15 = 3 × 5, a number must be divisible by both 3 and 5.

**Ans to Q9:**Osculators are small helper numbers used in Vedic mathematics to test divisibility by certain two digit numbers such as 13 or 19. The method reduces a large number step by step by multiplying the last digit by the osculator and adding or subtracting it from the remaining part of the number. This produces a smaller number with the same divisibility property, making it easier to decide whether the original number is divisible by the given divisor.

## 6.8 FURTHER READING

1. Bharati Krishna Tirtha, Vedic Mathematics, Motilal Banarsidass, Delhi.
2. State Council of Educational Research and Training, Fundamentals and Applications of Vedic Mathematics, New Delhi.
3. Kenneth R. Williams, Discover Vedic Mathematics, Vedic Mathematics Research Group.
4. VedicMaths.Org, “Practise and Free Books on Vedic Mathematics.”
5. Moonpreneur, “Vedic Math Sutra 1: Ekadhikena Purvena.”

## 6.9 MODEL QUESTIONS

**Q1.** Explain how digit sums are used to test divisibility by 3 and 9. Illustrate with three examples for each divisor.

**Q2.** State the divisibility rules for 2, 4, 5, 8 and 10, and give one example in each case.

**Q3**. Describe in detail the Vedic method for testing divisibility by 7 using repeated operations on the last digit. Work through two complete examples.

**Q4.** Show how the test for divisibility by 11 arises from the behaviour of powers of 10 modulo 11, and apply it to at least three different numbers.

**Q5.** Explain how combined tests for 12 and 15 can be built using the rules for their prime factors. Give examples of numbers that satisfy and numbers that fail these tests.

**Q6.** Discuss how osculators can be used to create divisibility tests for certain two digit numbers such as 13 or 19. Provide at least one worked example for a number of your choice.

**Q7.** Explain the digit sum method used in Vedic mathematics for checking divisibility. Illustrate your answer with simple examples.

# UNIT 7: VEDIC METHODS FOR LCM

## UNIT STRUCTURE

7.1 Learning Objectives
7.2 Introduction
7.3Basic Definitions
7.3.1 Factors and Multiples
7.3.2 Common Factors and HCF
7.3.3 Common Multiples and LCM
7.3.4 Conventional Methods for LCM
7.4 Vedic View of LCM
7.5Vedic Method for Finding LCM
7.5.1 Basic Vedic Procedure
7.5.2 Examples with Explanation
7.5.3 Combining with HCF
7.6 Let Us Sum Up
7.7 Answers to Check Your Progress
7.8 Further Reading
7.9 Model Questions

## 7.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- recall and use the basic ideas of factors, common factors, multiples and least common multiple.
- find the LCM of two or more numbers using usual prime factor and division methods.
- explain the logic of the Vedic approach to LCM based on factor patterns and divisibility ideas.
- apply simple Vedic style procedures to obtain the LCM in fewer steps for suitable sets of numbers.

## 7.2 INTRODUCTION

In many practical situations it is necessary to find a number that is a common multiple of two or more given numbers.Examples include deciding the time when events with different time periods coincide, choosing a size of packet that can be filled with items from different groups, or selecting a cycle for maintenance when machines have different service intervals.The smallest positive number that is a multiple of all the given numbers is called their least common

multiple.School methods usually find the LCM by prime factorisation or by repeated division.Vedic mathematics looks at the same task through patterns of factors and divisibility and tries to cut down repeated work.The aim is not to replace the known procedures, but to give quicker mental routes when the numbers have a structure that can be used.

## 7.3 BASIC DEFINITIONS

In this section, the basic terms needed for LCM work are revised and illustrated through simple examples.These notions are common to both conventional and Vedic treatments and help in understanding why the Vedic method works.

### 7.3.1 FACTORS AND MULTIPLES

A factor of a natural number is a positive integer that divides it exactly, leaving no remainder.
For example, the factors of 12 are 1, 2, 3, 4, 6 and 12, since each of these divides 12 without leaving any remainder.

A multiple of a number is obtained by multiplying it by a natural number.
Thus 12, 24, 36 and 48 are multiples of 12, and every multiple of 12 is also divisible by 3 and 4 because 12 itself is divisible by 3 and 4.

If one number divides another, the first number is called a factor and the second number is called a multiple in that relation.This simple idea lies behind all methods for LCM and HCF.

### 7.3.2 COMMON FACTORS AND HCF

When two or more numbers share a factor, that factor is called a common factor.
For example, 12 and 18 have common factors 1, 2, 3 and 6.

The highest common factor, or HCF, is the largest of the common factors.
In the example above, the HCF of 12 and 18 is 6.

There is a close connection between HCF and LCM: the product of two positive integers equals the product of their HCF and their LCM.
This relation is often written as

$$number\ 1 \times number\ 2 = HCF \times LCM.$$

## 7.3.3 COMMON MULTIPLES AND LCM

A common multiple of two or more numbers is any number that is a multiple of each of them. For example, 12 is a common multiple of 3 and 4, and 24 is a common multiple of 4, 6 and 8.

Among all common multiples, the least common multiple is the smallest positive one. Thus the LCM of 3 and 4 is 12, and the LCM of 4, 6 and 8 is 24.

For two numbers it is often easy to list a few multiples and find the first common one. For larger numbers or for three or more numbers this becomes slow, which is why systematic methods are used.

## 7.3.4 CONVENTIONAL METHODS FOR LCM

Two main school methods are commonly used.The first is the prime factorisation method, in which each number is written as a product of prime powers.

To find the LCM, the highest power of each prime that appears in any of the numbers is chosen, and these are multiplied together.
For example, $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$, so the LCM has $2^2$ and $3^2$, which gives 36.

The second method is the repeated division or ladder method.
The numbers are written in a row, and each step divides them by a common prime, writing the quotients in the next row until all become 1; the product of all divisors used is the LCM.

Both methods are reliable, but for many exam problems they may take more time than necessary.Vedic ideas aim to reduce work by recognising special patterns and by using divisibility rules more actively.

## CHECK YOUR PROGRESS

Q1. List all factors of 18 and 24 and find their HCF.
Q2. Write the first five multiples of 7 and 9, then identify the least common multiple from your list.
Q3. Use prime factorisation to find the LCM of $20, 30\ and\ 45$.

## 7.4 VEDIC VIEW OF LCM

Vedic mathematics treats LCM and HCF as two sides of the same coin and uses divisibility patterns to shorten calculations.Instead of always going down to prime factors, it often looks for

composite common factors and for overlaps among the given numbers.An important idea used in Vedic treatments is that numbers can be grouped or split so that known factors appear clearly.If one of the numbers is already a factor of another, then it need not contribute anything extra to the LCM.

The sutra “By addition and by subtraction” encourages flexible use of sums and differences to detect divisibility.The sutras on remainders, and on using the last digit, can also help in quick tests whether a number divides another.

In Vedic style teaching, the LCM of a set of numbers is often built step by step.
At each step the new number is tested against the LCM so far, and only the missing factors are added.

### Linking HCF and LCM:

A useful Vedic style approach is to first find the HCF of the numbers, and then use the relation between product, HCF and LCM.
For two numbers $a\ and\ b$ this relation is

$$a \times b = HCF(a,b) \times LCM(a,b)$$

If the HCF can be identified quickly, this formula gives the LCM with only one division.
For example, if HCF of 24 and 36 is seen to be 12, then LCM is

$$(24 \times 36) \div 12 = 72$$

When more than two numbers are involved, the idea can be used iteratively.
The LCM of three numbers can be seen as the LCM of one number with the LCM of the other two.

This way of thinking reduces the problem to a sequence of two number problems, which is often easier to manage mentally.Vedic methods emphasise such stepwise simplifications.

## 7.5VEDIC METHOD FOR FINDING LCM

There is no single fixed “Vedic LCM algorithm” to replace all others.Instead, several sutras guide a flexible procedure that depends on the structure of the given numbers.In this unit, a simple and practical Vedic style procedure is described that can be applied to small sets of numbers.It uses ideas of cancellation, common factors and quick divisibility checks to minimise work.

## 7.5.1BASIC VEDIC PROCEDURE

The idea is easiest to see for three numbers, say $a, b\ and\ c$. They are arranged in a row and then progressively simplified by dividing out common factors wherever possible.A typical strategy goes as follows:
First, look for a common factor between any pair, say between a and b, and divide both by that factor.If the factor also divides the third number c, divide it as well. Note down the common factor separately since it is part of the LCM.

Next, repeat the search for common factors among the simplified numbers. At each stage, the aim is to remove as much common content as possible with a single factor, rather than splitting it into primes immediately. When no more non trivial common factor is present among any pair, the remaining numbers are pairwise co prime. The LCM is then the product of all the common factors collected and the remaining co prime parts.

In practice this process is often written in a short ladder form similar to the standard division method, but the choice of divisors is more flexible. Instead of insisting on primes, any composite factor that divides at least two of the numbers can be used.This matches the spirit of Vedic mathematics, where methods encourage the student to notice convenient factors and to think creatively, rather than to follow a fixed narrow path.

## 7.5.2EXAMPLES WITH EXPLANATION

Consider the LCM of $12, 18\ and\ 30$.Observe that 6 divides all three numbers, so divide each by 6 to get $2, 3\ and\ 5$ and record the factor 6.

Now 2, 3 and 5 have no common factor greater than 1, and are pairwise co prime.
The LCM is therefore $6 \times 2 \times 3 \times 5 = 180$.

Without going to prime factors of each number separately, a single step with 6 has removed much of the common content.This shortens the working, especially when done mentally.

As another example, take 16, 24 and 40.
Here 8 divides all three numbers, so divide each by 8 to get 2, 3 and 5.

Again the remaining numbers are co-prime, and the LCM is $8 \times 2 \times 3 \times 5 = 240$.
The prime factor method would have written each number as a product of prime powers first; the Vedic style approach reaches the answer more directly.

When the numbers are not all divisible by a single common factor, the procedure can still be applied in stages.The aim is always to use the largest helpful factor at each stage, chosen by quick inspection.

## 7.5.3COMBINING WITH HCF

The relation between HCF and LCM can be used alongside the factor based procedure.For pairs of numbers where the HCF is obvious, it may be quicker to use the formula for LCM rather than to carry out further splitting.

For example, if the triple is $12, 15\ and\ 20$, one possible route is to first find the LCM of 12 and 20 and then combine with 15.
Since $HCF(12, 20)$ is 4, $LCM(12, 20)\ is\ (12 \times 20) \div 4 = 60$.

Now the problem reduces to finding LCM of 60 and 15.As 15 divides 60 exactly, the LCM of 60 and 15 is 60 itself, so the LCM of 12, 15 and 20 is 60.

This example illustrates a useful pattern: whenever one of the numbers divides another exactly, the smaller number does not change the LCM.Recognising such patterns is central to Vedic style simplification.For larger sets, similar reasoning can be applied.
Numbers can often be rearranged so that those with clear divisibility relations are handled together.

## CHECK YOUR PROGRESS

Q4. Use the Vedic procedure described above to find the LCM of 18, 27 and 30. Show the intermediate simplified triples.
Q5. Find the LCM of 14, 20 and 35 by first removing a common factor from any pair, then proceeding as in the method.

## 7.6 LET US SUM UP

- This unit began by revisiting the basic ideas of factors, multiples, common factors and common multiples, and by defining the least common multiple.
- Two common school methods for LCM, prime factorisation and repeated division, were recalled and placed in context.
- The Vedic view treats LCM and HCF as closely linked and uses sutra-based thinking to look for factor patterns and divisibility shortcuts.

- A simple Vedic style procedure was described in which the given numbers are reduced step by step by dividing out common factors, and the LCM is formed as the product of all collected factors and the remaining co prime parts.
- The relation between product, HCF and LCM was also highlighted as a useful tool within the Vedic framework.
- By combining quick recognition of HCF with the factor-based simplification, students can handle a wide variety of LCM problems with both speed and clarity.

## 7.7 ANSWERS TO CHECK YOUR PROGRESS

**Ans to Q1:**Factors of 18 are $1, 2, 3, 6, 9\ and\ 18$.

Factors of 24 are $1, 2, 3, 4, 6, 8, 12\ and\ 24$.

Common factors are $1, 2, 3\ and\ 6$, so the HCF is 6.

**Ans to Q2:**First five multiples of 7 are $7, 14, 21, 28\ and\ 35$.

First five multiples of 9 are $9, 18, 27, 36\ and\ 45$.

The least common multiple in these lists is 63, which appears later as $7 \times 9$; so the LCM of 7 and 9 is 63.

**Ans to Q3:**Prime factorisation:

$$20 = 2^2 \times 5,$$
$$30 = 2 \times 3 \times 5,$$
$$45 = 3^2 \times 5.$$

The LCM has $2^2, 3^2\ and\ 5$, so $LCM = 4 \times 9 \times 5 = 180$.

**Ans to Q4:** Given numbers are$18, 27, 30$.

First, 9 divides all three; dividing by 9 gives 2, 3 and 10, and record factor 9.

Now there is no common factor for all three, but 2 divides 10; treat 2 and 10: 2 divides 10, so 10 already contains 2.

Remaining numbers are 2, 3, 10 seen as co-prime in pairs except for 2 inside 10.

$$LCM = 9 \times 2 \times 3 \times 5 = 270.$$

**Ans to Q5:** Given numbers are$14, 20, 35$.

First, 7 divides 14 and 35; dividing by 7 gives 2 and 5 and leaves 20 unchanged, record factor 7.

Now numbers are 2, 20 and 5; observe that 2 divides 20 and 5 is separate.

Prime factorisation of 20 is $2^2 \times 5$, so combined set is $7, 2^2\ and\ 5$.

$$LCM = 7 \times 4 \times 5 = 140.$$

## 7.8 FURTHER READING

1. Bharati Krishna Tirtha, Vedic Mathematics, Motilal Banarsidass, Delhi.
2. State Council of Educational Research and Training, Fundamentals and Applications of Vedic Mathematics, New Delhi.
3. Kenneth R. Williams, Discover Vedic Mathematics, Vedic Mathematics Research Group.
4. VedicMaths.Org, “Practise and Free Books on Vedic Mathematics.”
5. Moonpreneur, “Vedic Math Sutra 1: Ekadhikena Purvena.”

## 7.9 MODEL QUESTIONS

**Q1.** Define factor, multiple, common factor and least common multiple, each with an example.

**Q2.** Explain how the HCF of two numbers can be used to find their LCM, and illustrate with a numerical example.

**Q3**. Using prime factorisation, find the LCM of 24, 32 and 60.

**Q4.** Apply the Vedic style factor-based procedure to find the LCM of 16, 24 and 36, showing each simplification step.

**Q5.** For the numbers 12, 18, 30 and 45, suggest an efficient order in which to group them to apply the combined HCF and LCM approach, and compute the LCM.

# UNIT 8: VEDIC METHODS FOR HCF

## UNIT STRUCTURE

8.1 Learning Objectives
8.2 Introduction
8.3Basic Definitions
8.4 Vedic Method for Finding HCF
8.4.1 Using Repeated Differences
8.4.2 Anurupyena and Working with Proportions
8.5Combined Problems of LCM and HCF
8.6 Let Us Sum Up
8.7 Answers to Check Your Progress
8.8 Further Reading
8.9 Model Questions

## LEARNING OBJECTIVES

After going through this unit, you will be able to:

- you will be able to explain the meaning of common terms such as factors, multiples, highest common factor and least common multiple.
- you will be able to find the HCF of two or three numbers using both conventional methods and compact Vedic ideas.
- you will be able to handle problems where HCF and LCM are used together, and interpret statements about them in simple word problems.

## 8.2 INTRODUCTION

In earlier units, attention was mainly on operations like addition, subtraction, multiplication and division, and on faster ways of doing these with the help of Vedic sutras.Another important theme in arithmetic is the study of factors and multiples of whole numbers, which leads to the ideas of highest common factor and least common multiple.These ideas are not only theoretical, but appear directly in problems involving sharing, repeated events and simplification of fractions.

A common factor shows how two or more numbers share a common “building block” in their structure, while the highest common factor shows the largest such building block.On the other hand, common multiples show how different cycles or quantities can be matched, and the least common multiple shows the earliest point where they come together.Vedic mathematics gives

short mental approaches that help to see these structures more clearly and to compute HCF and LCM without long division in many practical cases.

## 8.3 BASIC DEFINITIONS

A factor of a whole number is another whole number that divides it exactly, leaving remainder zero.For example, 3 is a factor of 18 because 18 divided by 3 is 6 with no remainder, while 5 is not a factor of 18 because 18 divided by 5 leaves remainder 3.
Every whole number has at least two factors, namely 1 and the number itself, and numbers that have exactly these two factors are called prime numbers.

A multiple of a number is a quantity obtained by multiplying that number by another whole number.For instance, 18, 36 and 54 are multiples of 18 because they are 18×1, 18×2 and 18×3 respectively.A number can have infinitely many multiples, because the multiplying whole number can grow without limit.

The highest common factor, also called greatest common divisor, of two or more numbers is the largest number that divides each of them exactly.If we list all factors of two numbers and pick the largest factor common to both lists, we obtain their HCF.
For example, the factors of 12 are $1, 2, 3, 4, 6\ and\ 12$, while the factors of 18 are 1, 2, 3, 6, 9 and 18, so the highest common factor of 12 and 18 is 6.

The least common multiple of two or more numbers is the smallest non zero number that is a multiple of each of them.If we write some multiples of each number and look for the first common entry, that number is the LCM.
For example, multiples of 4 are $4, 8, 12, 16, 20$ and so on, and multiples of 6 are 6, 12, 18, 24 and so on, so the least common multiple of 4 and 6 is 12.

For any pair of positive integers, there is a useful relation that connects HCF and LCM with the product of the numbers.If two numbers are $a\ and\ b$, and their HCF is H while their LCM is $L$, then $a \times b$ equals $H \times L$.
This relation is often used in problems where either HCF or LCM is missing and can be found from the other.

**Co-prime Numbers and Common Mistakes:**

Two numbers are called co prime if their highest common factor is 1, even though each number may itself be composite.
For example, 8 and 15 are co-prime because they have no common factor other than 1, even though both are not prime.Being co-prime means that the numbers do not share any prime factor, which is a useful idea when simplifying fractions or finding LCM.

A frequent mistake made by learners is to think that if two numbers are co-prime, their LCM is their sum, which is not correct.In fact, for co-prime numbers the LCM is the product of the two numbers, because there is no factor that can be shared.Another common error is to mix up HCF and LCM as “largest” and “smallest” numbers, without checking if they are factors or multiples.

## CHECK YOUR PROGRESS

Q1 What is meant by a factor of a number? Give one example.
Q2 Define HCF of two numbers in your own words.
Q3 List all factors of 18 and 24, and hence find their HCF.
Q4 Explain what is meant by co prime numbers, and give one pair of co-prime numbers.

## 8.4 VEDIC METHOD FOR FINDING HCF

The conventional method for finding HCF uses repeated division.Given two numbers, we divide the larger by the smaller, then divide the previous divisor by the remainder, and so on until the remainder becomes zero.The last non zero remainder is the HCF.
While this is systematic, it often leads to several steps and is usually carried out on paper.Vedic mathematics encourages a more flexible view.The guiding idea is that common factors do not change if we replace numbers by simpler numbers which differ from them by multiples of each other.
In simple language, subtracting one number from another without changing the common factors allows us to bring the numbers down to smaller and easier forms.

### 8.4.1 USING REPEATED DIFFERENCES

One way to see the Vedic spirit in HCF is to use repeated subtraction instead of long division.
Suppose we want the HCF of 84 and 30.
We can subtract 30 from 84 to get 54, then again subtract 30 from 54 to get 24, and now look at the pair 30 and 24.
Next, subtract 24 from 30 to get 6, and then note that 24 is a multiple of 6, so the HCF is 6.

The same process can be made quicker by using the idea of multiplying or scaling.
Instead of subtracting 30 two times from 84, we can see directly that 84 minus 2×30 is 24, since subtracting a multiple is the same as repeated subtraction.
The important point is that subtracting or adding any multiple of one number from another does not change the common factors.

For three numbers, say 84, 30 and 18, we can first find the HCF of 84 and 30, which is 6, and then find the HCF of this result with 18.
Here, 18 is already a multiple of 6, so the overall HCF of the three numbers is also 6.
This stepwise reduction fits well with mental computation and avoids writing long columns.

## 8.4.2 ANURUPYENA AND WORKING WITH PROPORTIONS

The sub sutra “Anurupyena” emphasises working proportionately, which can simplify HCF tasks where numbers have a common visible factor.
If all numbers in a problem are even, for example, we can divide each by 2 and find the HCF of the reduced set, then multiply the final result by 2 at the end.
This reduces the size of numbers and brings them closer to mental handling.

Take the HCF of 132 and 180.
Both are divisible by 4, so we first consider 33 and 45.
Now the HCF of 33 and 45 can be found by differences: 45 minus 33 is 12, then 33 minus 12 is 21, and 21 minus 12 is 9.
Now 12 minus 9 is 3, and 9 is a multiple of 3, so the HCF of 33 and 45 is 3, and therefore the HCF of 132 and 180 is 3×4 which is 12.

This proportional reduction is especially helpful for combined HCF and LCM questions.
By peeling away obvious common factors using simple mental observation, the heart of the problem becomes much lighter.
It also trains the eye to recognise common patterns such as divisibility by 2, 3, 5 or 9.

## CHECK YOUR PROGRESS

Q5 Use repeated differences to find the HCF of 96 and 60.
Q6 Use a proportional reduction to find the HCF of 210 and 168.
Q7 Find the HCF of 42, 70 and 98 by first simplifying all three numbers with a common factor if possible.

## 8.5COMBINED PROBLEMS OF LCM AND HCF

Many word problems in arithmetic involve both HCF and LCM. Such problems usually talk about largest possible equal groups, smallest time after which events coincide, or conditions on the product of numbers. Having a clear picture of what HCF and LCM mean helps to decide which one to use in each situation.

When a problem refers to dividing things into equal parts with nothing left over, the HCF often appears as the size of each largest equal part. For example, if a rope of length 84 metres and another of length 126 metres must be cut into equal pieces without any waste, the length of each piece will be the HCF of 84 and 126. On the other hand, if a problem mentions cycles, such as bells ringing at different intervals and asking when they will ring together again, the LCM is the natural tool.

In some questions, both HCF and LCM are given and one or more of the original numbers are unknown.
Here the relation “product equals HCF times LCM” helps to connect the given data.
For example, if two numbers have HCF 8 and LCM 240, and one of the numbers is 48, then the other number must be found from $48 \times x = 8 \times 240$.

**Example 1:**Consider two numbers whose HCF is 6 and LCM is 72.
If one of the numbers is 18, then the other number can be found from 18×x = 6×72.
Here 6×72 is 432, and dividing 432 by 18 gives 24, so the other number is 24.
This simple algebraic step uses the HCF–LCM relation directly.

Now consider a sharing problem, where there are 72 red beads and 108 blue beads to be arranged into necklaces, with each necklace having the same number of beads and only one colour.
The largest number of necklaces that can be made is given by the HCF of 72 and 108, because each necklace will contain that number of beads.
After finding the HCF as 36, we see that there will be two red necklaces and three blue necklaces, each with 36 beads.

**Example 2**: As an LCM example, imagine three lights that flash every 12 seconds, 18 seconds and 30 seconds respectively.
To find when they will all flash together next, we look for the least common multiple of 12, 18

and 30.

By factorising or using proportional tricks, we can write 12 as $2^2 \times 3$, 18 as $2 \times 3^2$ and 30 as $2 \times 3 \times 5$, so the LCM is $2^2 \times 3^2 \times 5 = 180$ seconds = 3 minutes.

In many school textbooks, such problems are followed by checks using digit sum methods to avoid careless mistakes.

Although such checks do not prove correctness in every case, they give a quick indication and fit well with the mental style of Vedic arithmetic.

Learners are encouraged to use estimation alongside these checks to keep answers reasonable.

## 8.6 LET US SUM UP

- Basic definitions related to factors and multiples were revisited, with special attention to highest common factor and least common multiple.
- The meaning of co-prime numbers was also clarified, along with common misunderstandings that often cause confusion between HCF and LCM.
- A simple Vedic view on HCF emphasises the use of repeated differences and proportional reduction instead of full long division.
- The central relation connecting HCF, LCM and the product of two numbers was highlighted as a key tool for solving such questions.
- The Vedic approach does not discard standard methods, but offers mental shortcuts and patterns that make the structure of numbers more visible. With practice, these methods can make HCF and LCM problems quicker, more intuitive and less dependent on lengthy written work.

## 8.7 ANSWERS TO CHECK YOUR PROGRESS

**Ans to Q1**: A factor of a number is a whole number that divides it exactly without leaving any remainder.

For example, 4 is a factor of 20 because 20 divided by 4 is 5.

**Ans to Q2**: The HCF of two numbers is the largest whole number that divides both numbers exactly.

It is the greatest among all their common factors.

**Ans to Q3**:

Factors of 18 are 1, 2, 3, 6, 9 and 18.

Factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.

The common factors are 1, 2, 3 and 6, so the HCF is 6.

**Ans to Q4**: Co-prime numbers are numbers whose highest common factor is 1, even though each number may itself be composite.

An example is 8 and 15, which share no common factor other than 1.

**Ans to Q5:** To find HCF of 96 and 60, subtract 60 from 96 to get 36, then find HCF of 60 and 36.

Subtract 36 from 60 to get 24, and then find HCF of 36 and 24.

Now 36 minus 24 is 12, and 24 is a multiple of 12, so the HCF is 12.

**Ans to Q6**: For 210 and 168, both are divisible by 2, 3 and 7, so first divide both by 6, for example, to get 35 and 28.

The HCF of 35 and 28 can be found by subtraction: 35 minus 28 is 7, and 28 is a multiple of 7, so their HCF is 7.

Multiplying back by 6 gives an overall HCF of 42 for 210 and 168.

**Ans to Q7**: For 42, 70 and 98, note that all are divisible by 14, so first divide to get 3, 5 and 7.

The HCF of 3, 5 and 7 is 1, so there is no further common factor beyond 14 in the original numbers.

Hence the HCF of 42, 70 and 98 is 14.

## 8.8 FURTHER READING


1. Bharati Krishna Tirtha, Vedic Mathematics, Motilal Banarsidass, Delhi.
2. State Council of Educational Research and Training, Fundamentals and Applications of Vedic Mathematics, New Delhi.
3. Kenneth R. Williams, Discover Vedic Mathematics, Vedic Mathematics Research Group.
4. VedicMaths.Org, “Practise and Free Books on Vedic Mathematics.”
5. Moonpreneur, “Vedic Math Sutra 1: Ekadhikena Purvena.”


## 8.9 MODEL QUESTIONS

Q1. Define factor, multiple, HCF and LCM, with two examples of each.

Q2. Explain with an example how repeated subtraction can be used to find the HCF of two numbers.

Q3. Use a suitable Vedic idea to find the HCF of 264 and 198, showing intermediate steps.

Q4. The HCF of two numbers is 9 and their LCM is 180. If one of the numbers is 45, find the other number.

Q5. Two bells ring at intervals of 12 minutes and 18 minutes. If they ring together at 9 am, at what time will they ring together again?

Q6. There are 90 red balloons and 126 blue balloons. What is the largest number of identical packs that can be made without any balloon left over, and how many of each colour will each pack contain?

Q7. Give two examples of pairs of co-prime numbers and explain why their LCM equals their product.

Q8. A teacher says “the HCF of two numbers is always smaller than their LCM”. Discuss with examples and say whether this is always true.

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# UNIT 9: TECHNIQUES FOR SQUARE AND SQUARE-ROOTS

## UNIT STRUCTURE

9.1 Learning Objectives
9.2 Introduction
9.3 The Dvandva-Yoga (duplex combination process)
9.4 The Yavadunam Sutra
9.5 Simpler Method
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9.7 Well known first principles
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9.9 Modus Operandi for Vargamula (Square Root)
9.10 Let Us Sum Up
9.11 Further Reading
9.12 Answer to Check Your Progress
9.13 Model Questions

## 9.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- Understand the concept of squaring and square roots using Vedic mathematical approaches.
- Apply the Dvandva-Yoga (duplex combination process) for efficient squaring of numbers.
- Use the Yavadunam Sutra to quickly find squares of numbers close to a given base.
- Extract square roots systematically using the Vargamula (square-root) method.

## 9.2 INTRODUCTION

This unit introduces efficient techniques for squaring numbers and extracting square roots, with particular emphasis on methods drawn from Vedic Mathematics. It aims to develop both conceptual understanding and computational skill by presenting systematic procedures such as the Dvandva-Yoga (duplex method), the Yavadunam Sutra, and the simpler method, alongside well-known first principles of square and square-root calculations. By integrating logical reasoning, pattern recognition, and place-value concepts, the unit demonstrates how complex

arithmetic operations can be performed accurately and swiftly, often mentally. These techniques not only enhance speed and accuracy in numerical computation but also deepen appreciation of the mathematical insight embedded in ancient Indian methods, making the study of squares and square roots both practical and intellectually enriching.

## 9.3 THE DVANDVA-YOGA (DUPLEX COMBINATION PROCESS)

The term "Dvandva Yoga," or Duplex, is used in two distinct contexts. The first one is done by squaring, and the second one is done by cross-multiplication. In this case, it is used in both ways.For a single central digit, the square$(a^2)$ is used, and for an even number of digits, say $a$ and $b$, evenly spaced from either end, the double of the cross product$(2ab)$ is used. Several examples will clarify the procedure. The duplex is represented by the symbol $D$, and we can see that

**Ex. 1:** For $3, D = 3^2 = 9$

**Ex. 2:** For $6, D = 6^2 = 36$

**Ex. 3:** For $35, D = 2 \times (3 \times 5) = 30$

**Ex. 4:** For $104, D = 2 \times (1 \times 4) + 0^2 = 8$

**Ex. 5:** For $375, D = 2 \times (3 \times 5) + 7^2 = 79$

**Ex. 6:** For $081, D = 2 \times (0 \times 1) + 8^2 = 64$

**Ex. 7:** For $1483, D = 2 \times (1 \times 3) + 2 \times (4 \times 8) = 70$

**Ex. 8:** For $2590, D = 2 \times (2 \times 0) + 2 \times (5 \times 9) = 90$

**Ex. 9:** For $32574, D = 2 \times (3 \times 4) + 2 \times (2 \times 7) + 5^2 = 77$

**Ex. 10:** For $159378, D = 2 \times (1 \times 8) + 2 \times (5 \times 7) + 2 \times (9 \times 3) = 77$

Now, we see the method of squaring in the following examples.

**Ex 11.** $305^2 = D \text{ for } 3/ \quad D \text{ for } 30/ \quad D \text{ for } 305/ \quad D \text{ for } 05/ \quad D \text{ for } 5$

$$= 3^2/ \quad 2 \times (3 \times 0)/ \quad 2 \times (3 \times 5) + 0^2/ \quad 2 \times (0 \times 5)/ \quad 5^2$$

$$= 9/ \quad 0/ \quad 30/ \quad 0/ \quad 25$$

$$= 9/ \quad 0 + 3/ \quad 0/ \quad 0 + 2/ \quad 5$$

$$= 93025$$

If you understand the duplex method and its use in squaring, you can obtain the answer in a single line. For example, $305^2 = 93_3 02_2 5 = 93025$

**Explanation:**

**Step 1:** Duplex of $5$ is $5^2 = 25$. Put the unit digit $(5)$ of duplex in answer line and carry over the other $(2)$.

**Step 2:** $2 \times (0 \times 5) + 2$ (carried)$= 2$; write it down at 2nd position.

**Step 3:** $2 \times (3 \times 5) + 0^2 = 30$; write it down 0 and carry over $3$.

**Step 4:** $2 \times (3 \times 0) + 3$ (carried)$= 3$; write it down at 4th position.

**Step 5:** $3^2 = 9$; write it down.

**Ex 12.** $896^2 = 80_{16}2_{18}8_{11}1_{3}6 = 802816$

**Explanation:**

**Step 1:** Duplex of $6$ is $6^2 = 36$. Put the unit digit $(6)$ of duplex in answer line and carry over the other $(3)$.

**Step 2:** $2 \times (9 \times 6) + 3$ (carried)$= 111$; write it down 1 and carry over the other $11$.

**Step 3:** $2 \times (8 \times 6) + 9^2 + 11\ (\text{carried}) = 188$; write it down 8 and carry over $18$.

**Step 4:** $2 \times (8 \times 9) + 18$ (carried)$= 162$; write it down 2 and carry over $16$.

**Step 5:** $8^2 + 16\ (\text{carried}) = 80$; write it down.

**Ex 13.** $2367^2 = 5_{1}6_{4}0_{7}2_{8}6_{8}8_{4}9 = 5602689$

**Explanation:**

**Step 1:** Duplex of $7$ is $7^2 = 49$. Put the unit digit $(9)$ of duplex in answer line and carry over the other $(4)$.

**Step 2:** $2 \times (6 \times 7) + 4$ (carried)$= 88$; write it down 8 and carry over the other $8$.

**Step 3:** $2 \times (3 \times 7) + 6^2 + 8\ (\text{carried}) = 86$; write it down 6 and carry over $8$.

**Step 4:** $2 \times (2 \times 7) + 2 \times (3 \times 6) + 8$ (carried)$= 72$; write it down 2 and carry over $7$.

**Step 5:** $2 \times (2 \times 6) + 3^2 + 7\ (\text{carried}) = 40$; write it down 0 and carry over $4$.

**Step 6:** $2 \times (2 \times 3) + 4$ (carried)$= 16$; write it down 6 and carry over the other $1$.

**Step 7:** $2^2 + 1\ (\text{carried}) = 5$; write it down.

**Ex 14.** $73214^2 = 53_{4}6_{4}0_{3}2_{6}8_{2}9_{1}79_{1}6 = 5360289796$

**Ex 15.** $5432819^2 = 29_{4}5_{5}1_{5}5_{11}5_{10}2_{16}2_{12}2_{12}8_{6}6_{14}7_{2}6_{8}1 = 5360289796$

## 9.4 THE YAVADUNAM SUTRA

The Yavadunam Sutra in Vedic Mathematics provides a shortcut for squaring numbers close to a base (like 10, 100, 1000) by using the number's deficiency (how much less it is than the base); you find the result in two parts: the number minus the deficiency (or number plus the excess if it's above the base) on the left, and the square of the deficiency/excess on the right, ensuring the right side has the correct number of digits (matching the base's zeros) with proper carry-overs.The Yavadunam Sutra is especially efficient for mental calculation when numbers are near a power of 10.

**Steps to Apply Yavadunam sutra for Squaring**

**Step 1: Choose a Base-** Select the nearest power of 10 (e.g., for 97, base is 100; for 8, base is 10).
**Step 2: Find the Deficiency (D)-** Subtract the number from the base (e.g., for 97, $D = 100 - 97 = 3$).
**Step 3: Calculate the Left Side-** Subtract the deficiency from the original number (Number - D). (e.g., $97 - 3 = 94$).
**Step 4: Calculate the Right Side-** Square the deficiency ($D^2$). (e.g., $3^2 = 9$).
**Step 5: Combine with Carry-Over-** Place the left part and the right part together, ensuring the right part has the same number of digits as there are zeros in the base (adding leading zeros if needed), and handle any carry-overs from the right side to the left.

**Ex 16.** $97^2$ (Base 100, D=3):

Left: $97 - 3 = 94$

Right: $3^2 = 09$ (two digits for base 100)

Result: 9409.

**Ex 17.**$996^2$ (Base 1000, D=4):

Left: $996 - 4 = 992$

Right:$4^2 = 016$ (needs to be 016 for base 1000)

Result: 992016.

**Ex 18.**For Numbers Above the Base (Yavadunam-Tavadunikritya): Use an excess instead of deficiency.

$103^2$ (Base 100, Excess=3):

Left: $103 + 3 = 106$

Right: $3^2 = 09$

Result: 10609

Similarly, Trick to square number using Yavadunam which is closer to 100/1000 (Base 100/1000) is:

**Ex 19.** $93^2 = (93 - 7)/7^2 = 86/49 = 8649$

**Ex 20.** $89^2 = (89 - 11)/11^2 = 78/121 = 7921$

**Ex 21.** $113^2 = (113 + 13)/13^2 = 126/169 = 12769$

**Ex 22.** $1002^2 = (1002 + 2)/2^2 = 1004/004 = 1004004$

**Ex 23.** $9996^2 = (9996 - 4)/4^2 = 9992/0016 = 99920016$

**Ex 24.** $99991^2 = (99991 - 9)/9^2 = 99982/00081 = 9998200081$

**Ex 25.** $10004^2 = (10004 + 4)/4^2 = 10008/0016 = 100080016$

**Ex 26.** $999978^2 = (999978 - 22)/22^2 = 999956/000484 = 999956000484$

**Ex 27.** $1000012^2 = (1000012 + 12)/12^2 = 1000024/000144 = 1000024000144$

## CHECK YOUR PROGRESS 1

**1.** Find the squares of the following numbers by using duplex method:

**(i)** 835 **(ii)** 8432 **(iii)** 45321 **(iv)** 530026 **(v)** 73010932

**2.** Find the squares of the following numbers by using Yavadunam sutra:

**(i)** 10004 **(ii)** 997 **(iii)** 100014 **(iv)** 999985 **(v)**9999987

## 9.5 SIMPLER METHOD

Above result can also be achieved by a simpler method and easier method which does not necessitate the “mixed” fractions, the transforming of them into the improper-fraction-shape etc, but gives us the answer immediately.

It will be observed that, in all the examples dealt with above,

Since

$$D^2 + N^2 = (N + 1)^2$$

$$\therefore D^2 = (N+1)^2 - N^2 = 2N + 1 = N + (N+1)$$

In other words, the square of the given number is the sum of two consecutive integers at the exact middle. For instance, if 7 be the given number, its square=49 which can be split up into the two consecutive integers 24 and 25.

$$\therefore 7^2 + 24^2 = 25^2$$

Similarly,

**Ex 28.** If $a = 9$, its square $81 = 40 + 41$, therefore $9^2 + 40^2 = 41^2$

**Ex 29.** If $a = 35$, its square $1225 = 612 + 613$, therefore $35^2 + 612^2 = 613^2$

**Ex 30.** If $a = 57$, its square $3249 = 1624 + 1625$, therefore $57^2 + 1624^2 = 1625^2$

**Ex 31.** If $a = 141$, its square $19881 = 9940 + 9941$, therefore $141^2 + 9940^2 = 9941^2$

## 9.6 THE CASE OF EVEN NUMBERS

The square of an odd number is necessarily odd and can be split up into two consecutive integers. But what about even numbers whose squares will always be even and cannot be split up into two consecutive numbers. The answer is that such cases should be divided off by 2 and other powers of 2 until an odd number is reached and then the final result should be multiplied proportionately.

For example, if $a = 52$, we divide it by 4 and get the odd number 13. Its square 169=84+85.

$$\therefore 13^2 + 84^2 = 85^2$$

Multiplying all the terms by $4^2$, we say

$$52^2 + 336^2 = 340^2$$

There are many other simple and easy methods by which we can tackle the problem ($a^2 + b^2 = c^2$) by means of clues and conclusions deducible from $3^2 + 4^2 = 5^2$, $5^2 + 12^2 = 13^2$, $8^2 + 15^2 = 17^2$ etc.

## 9.7 WELL KNOWN FIRST PRINCIPLES

The well-known first principles of square roots contain the definition of a square root, its properties, and the concept of the principal square root.The foundationalprinciples that regulate the computation of square roots are outlined below.

(i) The given number is first arranged in two-digit groups from right to left; and a single digit if any left over at the left-hand- end is counted as a simple group by itself.

(ii) The number of digits in the square root will be the same as the number of digits-group in the given number itself including a single digit if any such there be. Thus 16 will count as one group, 144 as two groups and 1024 as two.

(iii) If the square root contains n digits, the square must consist of 2n or 2n-1 digits.

(iv) Conversely, if the given number has n digits, the square root will contain $\frac{n}{2}$ or $\frac{n+1}{2}$ digits.

(v) In case of pure decimals, the number of digits in the square is always double that in the square root.

(vi) The square of the first nine natural numbers are 1,4,9,16,25,36,49,64 and 81. This means

- An exact square cannot end in 2,3, 7 or 8.
- A complete square ending in 1 must have either 1 or 9 mutual complements from 10 as the last digits of its square root,
- A square can end in 4, only if the square root ends in 2 or 8 complements
- The ending of a square in 5 or 0 means that its square root too ends in 5 or 0 respectively
- A square ending in 6 must have 4 or 6 complements as the last digit in its square root
- The termination of an exact square in 9 is possible only if the square root ends in 3 or 7 complements.

In other words, this may be more briefly formulated thus,

- 1,5,6 and 0 at end of a number reproduce themselves as the last digits in its square.
- Squares of complements from ten have the same last digit. Thus, $1^2$ and $9^2$;$2^2$ and $8^2$; $3^2$ and $7^2$; $4^2$ and $6^2$; $5^2$ and $5^2$;$\text{and}0^2$ and $10^2$ have same ending, namely 1,4,9,6,5 and 0 respectively.
- 2,3,7 and 8 are out of court altogether, as the final digit of a perfect square.

## 9.8 READILY AVAILABLE FIRST DATA

Thus, before we begin the straight extracting of a square root by “straight division” method, we start with previous knowledge of

- The number of digits in the square root
- The first digit thereof.

**Ex 32.** 74562814, N=8

∴ N in square root= $\frac{N}{2} = 4$ and the first digit thereof is 8.

**Ex 33.** 963106713, N=9

∴ N in square root= $\frac{N+1}{2} = 5$ and the first digit thereof is 3.

**Ex 34.** $(.7104)^2$ must contain 8 decimal digits.

**Ex 35.** $\sqrt{.16} = .4$

**Ex 36.** $\sqrt{.0064} = .08$

**Ex 37.** $\sqrt{.000049} = .007$

**Ex 38.** $\sqrt{.00007(0)} = .008$

**Ex 39.** $\sqrt{.00000007} = .0002$

**Ex 40.** $\sqrt{.09} = .3$

**Ex 41.** $\sqrt{.9} = \sqrt{.90} = .9$

## 9.9 MODUS OPERANDI FOR VARGAMULA (SQUARE ROOT)

The Vedic approach to square roots is founded on two fundamental mathematical observations:

- Digits of a perfect square occur in pairs, starting from the rightmost digit.
- The square root of a number can be constructed digit by digit, beginning with the highest place value.

This method may be viewed as the inverse operation of squaring, carried out through structured subtraction and controlled multiplication. It aligns with the Vedic sutra “Vilokanam” (By Observation), wherein estimation and logical reasoning play a central role.

**Steps to find Square Root**

**Step 1:**Grouping of Digits (Period Formation)-The first step in the Vedic square root method is the formation of periods.Starting from the rightmost digit, the given number is divided into groups of two digits each.

Example:

$$20736 \Rightarrow 2 \mid 07 \mid 36$$

Each group corresponds to one digit of the square root.

**Step 2:** Determination of the First Digit- Identify the largest perfect square less than or equal to the leftmost group.The square root of this perfect square becomes the first digit of the required square root.

Example:For the group 2,

$$1^2 = 1 \leq 2$$

Hence, the first digit of the square root is 1.

**Step 3:**Subtraction and Carry Forward- After determining the first digit, Subtract the square of the obtained digit from the leftmost group.Bring down the next group to the right of the remainder.This forms the new working dividend.

This step mirrors the natural logic of undoing the squaring process.

**Step 4:** Formation of the Trial Divisor- To obtain the next digit of the square root, Double the root obtained so far.Append a zero to this value.The resulting number acts as the trial divisor.

This step ensures proportional growth of the square root in accordance with place value principles.

**Step 5:** Determination of the Next Digit- A suitable digit is now selected such that:

$$(\text{Trial Divisor} + X) \times X \leq \text{Current Dividend}$$

The largest possible digit satisfying this condition is chosen and appended to the square root.

**Step 6:** Repetition of the Process

The above steps—subtraction, digit grouping, trial divisor formation, and digit selection—are repeated until all digit groups are exhausted. The resulting number is the exact square root of the given perfect square.

**Ex 42:** Find the square root of 20736

**Step 1:** Grouping of digits

$$2 \mid 07 \mid 36$$

**Step 2**: First digit

$$1^2 = 1 \leq 2$$

Root = 1

Remainder = 2−1=1

Bring down $07 \rightarrow 107$

**Step 3:** Trial divisor

$$2 \times 1 = 2 \Rightarrow 20$$

Find x such that:

$$(20 + x)x \leq 107$$

Testing values:

$$x = 4 \Rightarrow 24 \times 4 = 96$$

$x = 5 \Rightarrow 25 \times 5 = 125$ (exceeds)

Thus, x=4

Root = 14

Remainder = 107−96=11

Bring down 36 → 1136

**Step 4:** Next trial divisor

$$2 \times 14 = 28 \Rightarrow 280$$

Find x such that:

$$(280 + x)x \leq 1136$$

$$x = 4 \Rightarrow 284 \times 4 = 1136$$

So, next digit = 4

Hence,$\sqrt{20736} = 144$

**Ex 43:** Find the square root of119716.

**Step 1:** Group the digits into pairs (from right to left)

$$119716 \Rightarrow 11 \mid 97 \mid 16$$

Each pair will give one digit of the square root.

**Step 2:** Find the first digit of the square root

Consider the leftmost group: 11. Largest perfect square $\leq 11$ is:

$$3^2 = 9$$

So,first digit of the square root = 3

Subtract:11−9=2

Bring down the next pair 97

New dividend = 297

**Step 3:** Find the second digit

Double the root obtained so far:$2 \times 3 = 6$

Write it as 60 (trial divisor)

Now find a digit x such that:

$$(60 + x) \times x \leq 297$$

Try values:

$$x = 4 \Rightarrow 64 \times 4 = 256$$

$x = 5 \Rightarrow 65 \times 5 = 325$ (Too large)

So,Second digit = 4

Root becomes 34

Subtract:297−256=41

Bring down the next pair 16

New dividend = 4116

**Step 4:** Find the third digit

Double the root obtained so far:2×34=68

Write it as 680

Now find digit x such that:

$$(680 + x) \times x \leq 4116$$

Try values:

$$x = 6 \Rightarrow 686 \times 6 = 4116$$

So, Third digit = 6

Root becomes 346

Hence,$\sqrt{119716} = 346$

## CHECK YOUR PROGRESS 2

**1.**Find the square root of the following number

**(i)** 16384 **(ii)** 552049 **(iii)** 69906321 **(iv)** 7457531449

## 9.10 LET US SUM UP

A summary of the lessons we learnt from this unit is provided below.

- Squaring and square-root extraction can be performed efficiently using structured methods from Vedic Mathematics.

- The Dvandva-Yoga (duplex method) provides a systematic and compact approach for squaring numbers of any length.
- The Yavadunam Sutra offers a rapid technique for squaring numbers near a chosen base by using deficiency or excess.
- The simpler method explains squaring through logical decomposition, especially effective for odd numbers and adaptable to even numbers.
- Well-known first principles help in identifying perfect squares, determining possible last digits, and estimating square roots.
- The Vargamula method enables step-by-step extraction of square roots through period formation, trial divisors, and controlled subtraction.
- These techniques improve computational speed, accuracy, and mathematical insight while strengthening mental calculation skills.

## 9.11 ANSWERS TO CHECK YOUR PROGRESS

**Check your progress 1**

**Ans no 1**

**(i)** 697225 **(ii)** 71098624 **(iii)** 2053993041 **(iv)** 280927560676

**(v)** 5330596191508624

**Ans no 2**

**(i)**10008/0016 **(ii)** 994/009 **(iii)**100028/00196 **(iv)** 999970/000225

**(v)** 9999974/0000169

**Check your progress 2**

**Ans no 1**

**(i)**128 **(ii)**743 **(iii)** 8361 **(iv)** 86357

## 9.12 FURTHER READING

1. Vedic Mathematics, Jagadguru Swami Sri Bharati Krishna Tirthaji Maharaja

2. The History of Ancient Indian Mathematics, C.N. Srinivasiengar, The World Press, Calcutta, 1967.
3. The History of Hindu Mathematics, Asia Publishers House, Bombay, 1962

## 9.13 MODEL QUESTIONS

**1.**Explain the Dvandva-Yoga (duplex combination process) and illustrate it with suitable examples.

**2.** Find the squares of the following numbers by using duplex method:

**(i)** 965 **(ii)** 3639 **(iii)** 82325 **(iv)** 830221 **(v)** 46060938

**3.** State the Yavadunam Sutra and describe the steps involved in applying it for squaring numbers near a base.

**4.** Find the squares of the following numbers by using Yavadunam sutra:

**(i)** 10002 **(ii)** 994 **(iii)** 100017 **(iv)** 999982 **(v)**9999988

**5.** List and explain the well-known first principles related to perfect squares and square roots.

**6.** Explain the Vargamula (square-root) method step by step.

**7**. Find the square root of the following number

**(i)**21609 **(ii)**687241 **(iii)**40564161 **(iv)**9280239556

# UNIT 10: TECHNIQUES FOR CUBE AND CUBE-ROOTS

## UNIT STRUCTURE

10.1 Learning Objectives
10.2 Introduction
10.3The Anurupya Sutra
10.4The Yavadunam sutra
10.5Cube root by method of inspection
10.6 The Algebraic Method of Extracting Cube Roots
10.7 First Principle
10.8 Let Us Sum Up
10.9 Further Reading
10.10 Answer to Check Your Progress
10.11 Model Questions

## 10.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- Understand the concept of cubes and cube roots and their significance in mathematical computations.
- Apply the Anurupya Sutra to compute cubes of numbers efficiently.
- Use the Yavadunam Sutra to find cubes of numbers near a convenient base.
- Determine cube roots of exact cubes by the method of inspection.
- Explain the algebraic expansion underlying the cube-root extraction process.
- Extract cube roots of large numbers using the algebraic method based on first principles.

## 10.2 INTRODUCTION

The study of cubes and cube roots occupies an important place in elementary and higher mathematics, as it forms the foundation for understanding algebraic expansions, number structure, and higher computational techniques. In this unit, the extraction of cube roots is

approached not merely as a mechanical process, but as a logically ordered method based on clear algebraic principles. Drawing from Vedic Mathematics, the chapter introduces efficient sutra-based techniques such as the Anurupya Sutra, the Yavadunam Sutra, and the method of inspection, before developing the algebraic method of cube-root extraction from first principles. By systematically analysing how each digit of the cube root is governed by specific terms in the expansion of a cube, this chapter enables learners to extract cube roots of large numbers accurately and confidently, while deepening their conceptual understanding of the underlying mathematics.

## 10.3 THE ANURUPYA SUTRA

Anurupya Sutra is one of the sixteen principal sutras of Vedic Mathematics. The Sanskrit term Anurūpya means “proportionate” or “in proportion”, and the sutra is commonly translated as:

“AnurūpyeŚūnyam Anyat”

When one is in proportion, the other is zero.

The Anurupya Sutra is applied when numbers or algebraic expressions are not close to a convenient base (such as 10, 100, 1000) but are instead in proportion to a sub-base or working base. It simplifies calculations by exploiting proportional relationships.

In its process the cube values of the “first ten natural numbers” ie, 1 to 10, are used. To calculate the cube value of two-digit numbers, we proceed like this:

**Step 1:** The first thing we have to do is to put down the cube of the first digit in a row of 4 figures. The other three numbers in the row of answer should be written in a geometrical ratio in the exact proportion which is there between the digits of the given number.

**Step 2:** The second step is to put down, under the second and third numbers, just two times of second and third number. Then add up the two rows.

**Ex 1.** $11^3$

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| | 2 | 2 | |
| 1 | 3 | 3 | 1 |

**Ex 2.** $13^3$

| | | | |
|---|---|---|---|
| 1 | 3 | 9 | 27 |
| | 6 | 18 | |
| 2 | 1 | 9 | 7 |

**Ex 3.** $19^3$

| | | | |
|---|---|---|---|
| 1 | 9 | 81 | 729 |
| | 18 | 162 | |
| 6 | 8 | 5 | 9 |

**Ex 4.** $32^3$

| | | | |
|---|---|---|---|
| 27 | 18 | 12 | 8 |
| | 36 | 24 | |
| 32 | 7 | 6 | 8 |

**Ex 5.** $97^3$

| | | | |
|---|---|---|---|
| 729 | 567 | 441 | 343 |
| | 1134 | 882 | |
| 912 | 6 | 7 | 3 |

## 10.4 THE YAVADUNAM SUTRA

The Yavadunam Sutra in Vedic Mathematicsoffers a shortcut for cubing numbers close to a power of 10 (like 10, 100, 1000) by focusing on their deficiency (or excess), breaking the answer into three parts: (Number ± Deficiency) / (Deficiency²) / (Deficiency³) (adjusted for base), making complex cubing fast, especially for exams.

**Ex 6:** $104^3$

**Step 1:**Select the nearest power of 10, in this case it will be 100

**Step 2:**Surplus from 100 is 4, we take twice the surplus which is 8. We then increase 104 still further by twice the surplus i.e8 set $(104 + 8) = 112$ as left-side portion of the answer

112/

**Step 3:** We then put down new surplus multiplied by original surplus $(12 \times 4 = 48)$ as middle portion of the product

$$1018/48/$$

**Step 4:** On the right-hand side, we put down the cube of the original surplus $4^3 = 64$. Thus

$$104^3 = 112/48/64 = 1124864$$

**Ex 7:** $1006^3$

**Step 1:**Select the nearest power of 10, in this case it will be 1000

**Step 2:**Surplus from 1000 is 6, we take twice the surplus which is 12. We then increase 1006 still further by twice the surplus i.e 12 set $(1006 + 12) = 1018$ as left-side portion of the answer

$$1018/$$

**Step 3:** We then put down new surplus multiplied by original surplus $(18 \times 6 = 108)$ as middle portion of the product

$$1018/108/$$

**Step 4:** On the right-hand side, we put down the cube of the original surplus $6^3 = 216$. Thus

$$1006^3 = 1018/108/216 = 1018108216$$

**Ex 8:** $95^3$

**Step 1:**Select the nearest power of 10, in this case it will be 100

**Step 2:**Deficiency of 95 from 100 is 5, we take twice the deficit which is 10. We then decrease 95 still further by twice the deficiti.e10 set $(95 - 10) = 85$ as left-side portion of the answer

$$85/$$

**Step 3:** We then put down new deficit multiplied by original surplus $(15 \times 5 = 75)$ as middle portion of the product

$$85/75/$$

**Step 4:** On the right-hand side, we put down the cube of the original deficit$(-5)^3 = -125$. Thus

$$95^3 = 85/75/_{-1}\overline{25} = 857375$$

**Ex 9:** $997^3$

**Step 1:**Select the nearest power of 10, in this case it will be 1000

**Step 2:**Deficiency of 997 from 1000 is 3, we take twice the deficit which is 6. We then decrease 997 still further by twice the deficiti.e6 set $(997 - 6) = 991$ as left-side portion of the answer

$$991/$$

**Step 3:** We then put down new deficit multiplied by original surplus $(9 \times 3 = 27)$ as middle portion of the product

$$991/027/$$

**Step 4:** On the right-hand side, we put down the cube of the original deficit$(-3)^3 = -27$. Thus

$$997^3 = 991/027/\overline{027} = 991026973$$

**Ex 10.**$10005^3 = 10015/0075/0125 = 1001500750125$

**Ex 11.**$100012^3 = 100036/00432/01728 = 1000360043201728$

**Ex 12.**$1000007^3 = 1000021/000147/000343 = 1000021000147000343$

**Ex 13.**$9991^3 = 9973/0243/\overline{0729} = 997302429271$

**Ex 14.**$99998^3 = 99994/00012/\overline{00008} = 999940001199992$

## CHECK YOUR PROGRESS 1

**1.**Find the cubes of the following numbers by using Anurupya Sutra

**(i)** 16 **(ii)** 29 **(iii)** 47 **(iv)**78 **(v)** 93

**2.**Find the cubes of the following numbers by using Yavadunam Sutra

**(i)** 96 **(ii)** 92 **(iii)** 994 **(iv)**108 **(v)**1009 **(vi)** 9997

**(vii)** 10004

## 10.5 CUBE ROOT BY METHOD OF INSPECTION

(i) The lowest cubes, i.e., the cubes of the first nine natural numbers are $1,8,27,64,125,216,343,512$ and 729

(ii) Thus, they all have their own distinct endings; and there is no possibility of overlapping or doubt as in the case of square.

(iii) Therefore, the last digit of the cube root of an exact cube is obvious

- Cube ends in 1; so, cube root ends in 1
- Cube ends in 2; so, cube root ends in 8
- Cube ends in 3; so, cube root ends in 7
- Cube ends in 4; so, cube root ends in 4
- Cube ends in 5; so, cube root ends in 5
- Cube ends in 6; so, cube root ends in 6
- Cube ends in 7; so, cube root ends in 3
- Cube ends in 8; so, cube root ends in 2 and
- Cube ends in 9; so, cube root ends in 9

(iv) In other words,

- 1,4,5,6,9 and 0 repeat themselves in the cube-ending and
- 2,3,7 and 8 have an inter-play of complements from 10.

(v) The number of digits in a cube root is the same as the number of 3-digit groups in the original cube including a single digit or a double-digit group if there is any.
(vi) The first digit of the cube-root will always be obvious from the first group in the cube.
(vii) Thus, the number of digits, the first digit and the last digit of the cube root of an exact cube are the data with which we start, when we begin the work of extracting the cube root of an exact cube.

Let F, L and n be the symbols for the first digit, the last digit and the number of digits in the cube root of an exact cube.

**Ex 15.** For 226,981
Last digit = 1
Cube ends in 1; so, cube root ends in 1
Therefore, L = 1

Grouping: $226 \mid 981$
Leftmost group = 226
Nearby cubes:$6^3 = 216 \leq 226$
$7^3 = 343$ (Too large)
So, cube root starts with 6. Therefore, F = 6

$226|981 \rightarrow 2$ groups $\rightarrow n = 2$
Hence, F=6, L=1 and n=2

**Ex 16.** For 4,269,813
Last digit = 3
Cube ends in 3; so, cube root ends in 7
Therefore, L = 7

Grouping: $4|269|813$
Leftmost group = 4
Nearby cubes:$1^3 = 1 \leq 4$
So, cube root starts with 1. Therefore, F = 1

$4|269|813 \rightarrow 3$ groups $\rightarrow n = 3$
Hence, F=1, L=7 and n=3

**Ex 17.** For 248,858,189
Last digit = 9
Cube ends in 9; so, cube root ends in 9
Therefore, L = 9

Grouping: $248|858|189$
Leftmost group = 248
Nearby cubes:$6^3 = 216 \leq 248$
$7^3 = 343$ (Too large)
So, cube root starts with 6. Therefore, F = 6

$248|858|189 \rightarrow 3$groups $\rightarrow n = 3$
Hence, F=6, L=9 and n=3

**Ex 18.** For 83,453,453, F=4, L=7 and n=3

**Ex 19.** For 1,548,816,893, F=1, L=7 and n=4

**Ex 20.** For 6,700,108,456,013, F=1, L=7 and n=5

**Ex 21.** For 91,010,000,000,468, F=4, L=2 and n=5

## 10.6 THE ALGEBRAIC METHOD OF EXTRACTING CUBE ROOTS

The cube root is extracted digit by digit, starting from the last digit and moving leftwards. This is possible because the cube of a number can be expanded algebraically, and each term of the expansion controls a specific place value (units, tens, hundreds, thousands, etc.).

Let the cube root be a three-digit number:

$$N = 100J + 10K + L$$

Then,

$$(100J + 10K + L)^3$$
$$= 1{,}000{,}000J^3 + 300{,}000J^2K + 30{,}000J^2L + 30{,}000JK^2 + 6{,}000JKL + 1000K^3$$
$$+ 300JL^2 + 300K^2L + 30KL^2 + L^3$$

Removing the powers of ten and putting the result in algebraical form, we note the following

The units' place is determined by $L^3$

The tens place is determined by $3KL^2$

The hundreds' place is determined by $3K^2L + 3JL^2$

The thousands' place is determined by $K^3 + 6JKL$

and so on…

**Ex 22.**Extract the cube root of33,076,161

**Step 1:** Preliminary Identification

Grouping: $33 \mid 076 \mid 161$

Number of digits in cube root: $n = 3$

From inspection:First digit $F = 3$ and Last digit $L = 1$

**Step 2:** Eliminate Units Digit

$$L^3 = 1^3 = 1$$

Subtract:$33{,}076{,}161 - 1 = 33{,}076{,}160$

Unit's digit eliminated.

**Step 3:** Find $K$ (Tens Digit)

Now consider:$3KL^2 = 3K$

We look at the tens digit (6).

So:$3K \equiv 6 (\mathrm{mod}\ 10) \quad \Rightarrow K = 2$

Subtract:$3 \times 2 = 6$

**Step 4:** Find J (Hundreds Digit)

Now compute:$3K^2L + 3JL^2 = 12 + 3J$

The hundreds digit ends in 1, so:

$$3J + 12 \equiv 1 (\mathrm{mod}\ 10) \quad \Rightarrow 3J \equiv 9 \quad \Rightarrow J = 3$$

**Step 5:** Final Result

Digits obtained:J=3, K=2, L=1

Hence, $\sqrt[3]{33{,}076{,}161} = 321$

**Ex 23.**Extract the cube root of 76,928,302,277

**Step 1:** Preliminary Identification

Grouping: $76|928|302|277$

Number of digits in cube root: $n = 4$

From inspection:First digit $F = 4$ and Last digit $L = 3$

Let the cube root be:

$$N = 1000M + 100J + 10K + L$$

From above, we have $M = 4, L = 3$

**Step 2:** Eliminate Units Digit

$$L^3 = 3^3 = 27$$

Subtract:$76{,}928{,}302{,}277 - 27 = 76{,}928{,}302{,}250$

Unit's digit eliminated.

**Step 3:** Find $K$ (Tens Digit)

Now consider:$3KL^2 = 27K$

We look at the tens digit (5).

So:$27K \equiv 5 (\mathrm{mod}\ 10) \quad \Rightarrow K = 5$ [$\because 27 \times 5 = 135$ ends in 5]

Subtract:$27 \times 5 = 135$(its contribution to tens place)

Subtract:$76928302250 - 1350 = 76928300900$

**Step 4:** Find $J$ (Hundreds Digit)

Now compute:$3K^2L + 3JL^2 = 225 + 27J$

The hundreds digit ends in 9, so:

$$27J + 225 \equiv 9 (\mathrm{mod}\ 10) \quad \Rightarrow 27J \equiv 4 (\mathrm{mod}\ 10) \Rightarrow J = 2$$

**Step 5:** Final Result

Digits obtained: M=4, J=2, K=5, L=3

Hence, $\sqrt[3]{76{,}928{,}302{,}277} = 4253$

**Ex 24.**Extract the cube root of 355,045,312,441.

**Step 1:** Preliminary Identification

Grouping: $355|045|312|441$

Number of digits in cube root: $n = 4$

From inspection:First digit $F = 7$ and Last digit $L = 1$

Let the cube root be:

$$N = 1000M + 100J + 10K + L$$

From above, we have $M = 7, L = 1$

**Step 2:** Eliminate Units Digit

$$L^3 = 1^3 = 1$$

Subtract:$355{,}045{,}312{,}441 - 1 = 355{,}045{,}312{,}440$

Unit's digit eliminated.

**Step 3:** Find $K$ (Tens Digit)

Now consider:$3KL^2 = 3K$

We look at the tens digit (4).

So:$3K \equiv 4 (\mathrm{mod}\ 10) \quad \Rightarrow K = 8$ [$\because 3 \times 8 = 24$ ends in 4]

Subtract:$3 \times 8 = 24$(its contribution to tens place)

Subtract:$355{,}045{,}312{,}440 - 240 = 355{,}045{,}312{,}200$

**Step 4:** Find $J$ (Hundreds Digit)

Now compute:$3K^2L + 3JL^2 = 192 + 3J$

The hundreds digit ends in 2, so:

$$3J + 192 \equiv 2(\text{mod } 10) \quad \Rightarrow 3J \equiv 0(\text{mod } 10) \Rightarrow J = 0$$

**Step 5:** Final Result

Digits obtained:$M = 7, J = 0, K = 8, L = 1$

Hence, $\sqrt[3]{355{,}045{,}312{,}441} = 7081$

## 10.7 FIRST PRINCIPLE

The cube-root process is governed entirely by the algebraic expansion of a sum of digits. Let the cube root of a given number be:

$$x = a + b + c + d + \cdots$$

Then,

$$x^3 = (a + b + c + d + \cdots)^3$$

This cube expands into ordered groups of terms, and each group determines one digit of the cube root.

The method works by:

- Identifying digits one by one, and
- Removing (subtracting) the corresponding algebraic cube contributions at each stage.

**Sequence of the Various Digits**

Each “place” (or digit position) in the cube root is governed by a specific algebraic term.

| **Place** | **Controlled by** |
|---|---|
| 1st | $a^3$ |
| 2nd | $3a^2b$ |
| 3rd | $3ab^2 + 3a^2c$ |
| 4th | $6abc + b^3$ |
| 5th | $3ac^2 + 3b^2c$ |
| 6th | $3bc^2$ |
| 7th | $c^3$ |

and so on.

This sequence dictates what must be subtracted at each stage.

Dividends, Quotients, and Remainders

The working proceeds using three quantities at every stage:

Dividend (D) – the number currently being worked on.

Quotient (Q) – the next digit of the cube root

Remainder (R) – what remains after subtraction

Rules stated in the text:

- First $D, Q, R$ are obtained by Vilokanam (inspection).
- From the second dividend, no deduction is made.
- From the third, subtract $3a^2b$
- From the fourth, subtract $6abc + b^3$
- From the fifth, subtract $3ac^2 + 3b^2c$
- From the sixth, subtract $3bc^2$
- From the seventh, subtract $c^3$and so on.

**Practical Modus Operandi (Step-by-Step)**

**Step 1:** Grouping

Divide the given number into groups of three digits from the right.

Each group gives one digit of the cube root.

**Step 2:** First Digit (Inspection)

Take the leftmost group.Find the largest digit a such that:

$$a^3 \leq \text{first group}$$

Write a as the first quotient digit.

Subtract$a^3$to get the first remainder.This step requires no division.

**Step 3:** Second Digit

Bring down the next group to the remainder → second gross dividend.Do not subtract anything.Divide by:$3a^2$

The quotient gives the second digit $b$.

**Step 4:** Third Digit

Bring down the next group → third gross dividend.

Subtract:3ab2

Divide the result by $3a^2$

Quotient gives the third digit c.

**Step 5:** Fourth Digit

Bring down the next group.

Subtract:$6abc + b^3$

Divide by $3a^2$

Quotient gives the next digit.

**Step 6:** Further Digits

Continue similarly using the prescribed subtraction terms in sequence:

| Stage | Subtraction |
|---|---|
| 5th | $3ac^2 + 3b^2c$ |
| 6th | $3bc^2$ |
| 7th | $c^3$ |

Stop when no further quotient digit is possible.

**Ex 25.**Find the cube root of$258474853$.

**Step 1:** Grouping the Number

The given number is divided into groups of three digits from the right:

$$258 \mid 474 \mid 853$$

Each group will contribute one digit of the cube root.

**Step 2:** Finding a

Gross Dividend:258

Inspection (Vilokanam):

$$6^3 = 216 \leq 258 < 343 = 7^3$$

So, first quotient digit: $a = 6$

First remainder:$258 - 216 = 42$

Result so far:$Q_1 = 6,\ \ R_1 = 42$

**Step 3:**Finding b

Second Gross Dividend:$424$

Divisor:$3a^2 = 3 \times 6^2 = 108$

Division:$424 \div 108 = 3$ remainder $100$

So, $b = 3$

Result so far:$Q_2 = 3,\ R_2 = 100$

**Step 4:** Finding c

Third Gross Dividend:1007

Subtract known cube term:$3ab^2 = 3 \times 6 \times 3^2 = 162$

Actual Working Dividend:$1007 - 162 = 845$

Division:$845 \div 108 = 7$ remainder $89$

So, $c = 7$

Result so far:$Q_3 = 7,\ R_3 = 89$

**Step 5:**

Fourth Gross Dividend:894

Subtract cube corrections:$6abc + b^3 = (6 \times 6 \times 3 \times 7) + 27 = 756 + 27 = 783$

Actual Working Dividend:894−783=111

Division:111÷108=0 remainder 111

Result so far:$Q_4 = 0,\ R_4 = 111$

**Step 6:**

Gross Dividend:1118

Subtract:$3ac^2 + 3b^2c = 3(6)(7^2) + 3(3^2)(7) = 882 + 189 = 1071$

Actual Dividend:$1118 - 1071 = 47$

Division:$47 \div 108 = 0$ remainder 47

Result so far:$Q_5 = 0,\ R_5 = 47$

**Step 7:**

Gross Dividend:475

Subtract:$3bc^2 = 3 \times 3 \times 7^2 = 441$

Actual Dividend:$475 - 441 = 34$

Division:$34 \div 108 = 0$ remainder 34

Result so far:$Q_6 = 0,\ \ R_6 = 34$

**Step 8:**

Gross Dividend: 343

Subtract:$c^3 = 7^3 = 343$

Actual Dividend:$343 - 343 = 0$

Result so far:$Q_7 = 0,\ \ R_7 = 0$

This means that the given number is a perfect cube, that the work of extracting its cube root is over and that cube root is 637.

**Ex 26.**Find the cube root of33076161.

**Step 1:** Group the Number

Separate the digits into groups of three from the right:

$$33 \mid 076 \mid 161$$

**Step 1:** First Digit a

First group:$33$

By inspection:$3^3 = 27 \leq 33 < 64 = 4^3$

So,$a = 3$

Remainder$= 33 - 27 = 6$

Result so far:$Q_1 = 3,\ \ R_1 = 6$

**Step 3:** Finding b

Second Gross Dividend:$60$

Divisor:$3a^2 = 3 \times 3^2 = 27$

Division:$60 \div 27 = 2$ remainder $6$

So, $b = 2$

Result so far:$Q_2 = 2,\ \ R_2 = 6$

**Step 4:** Finding c

Third Gross Dividend: 27

Subtract known cube term:$3ab^2 = 3 \times 3 \times 2^2 = 36$

Actual Working Dividend:$67 - 36 = 31$

Division:$31 \div 27 = 1$ remainder $4$

So:$c = 1$

Result so far:$Q_3 = 1,\ R_3 = 4$

**Step 5:**

Fourth Gross Dividend:46

Subtract cube corrections:$6abc + b^3 = (6 \times 3 \times 2 \times 1) + 2^3 = 36 + 8 = 44$

Actual Working Dividend: 46−44=2

Division:$2 \div 27 = 0$ remainder 2

Result so far:$Q_4 = 0,\ R_4 = 2$

**Step 6:**

Gross Dividend: 21

Subtract:$3ac^2 + 3b^2c = 3(3)(1^2) + 3(2^2)(1) = 9 + 12 = 21$

Actual Dividend:$21 - 21 = 0$

Result so far:$Q_5 = 0,\ R_5 = 0$

**Step 7:**

Gross Dividend: 6

Subtract:$3bc^2 = 3 \times 2 \times 1^2 = 6$

Actual Dividend:$6 - 6 = 0$

Result so far:$Q_6 = 0,\ R_6 = 0$

**Step 8:**

Gross Dividend: 1

Subtract:$c^3 = 1^3 = 1$

Actual Dividend:$1 - 1 = 0$

Result so far:$Q_7 = 0,\ \ R_7 = 0$

This means that the given number is a perfect cube, that the work of extracting its cube root is over and that cube root is 321.

## CHECK YOUR PROGRESS 2

**1.**Applying the algebraic method to findthe cube roots of the following numbers

**(i)**83453453 **(ii)**143055667 **(iii)**11345123223 **(iv)**2840362499528

**2.**Applying the first principleto findthe cube roots of the following numbers

**(i)** 29791 **(ii)**105823817 **(iii)**248858189**(iv)**12278428443

**(v)**76928302277 4253

## 10.8 LET US SUM UP

A summary of the lessons we learnt from this unit is provided below.

- Cubes and cube roots are fundamental concepts that help in understanding number structure and algebraic expansions.
- Vedic Mathematics provides efficient sutra-based techniques, such as the Anurupya Sutra and the Yavadunam Sutra, for rapid computation of cubes.
- The method of inspection allows quick identification of the first and last digits of the cube root of an exact cube.
- The algebraic method of extracting cube roots is based on the expansion of $(a + b + c + \cdots)^3$.
- The given number is divided into groups of three digits, with each group contributing one digit to the cube root.
- The cube-root extraction proceeds digit by digit through the use of gross dividends, quotients, and remainders.

## 10.9 ANSWERS TO CHECK YOUR PROGRESS

**Check your progress 1**

**Ans no 1**

(i) 4096 (ii) 24389 (iii) 103823 (iv) 474552 (v) 804357

**Ans no 1**

(i)884736 (ii) 778688 (iii)982107784 (iv)1259712 (v)1027243729

(vi) 999100269973 (vii) 1001200480064

**Check your progress 2**

**Ans no 1**

**(i)**437 **(ii)** 523 **(iii)**2247 **(iv)**14162

**Ans no 2**

**(i)** 31 **(ii)** 473 **(iii)**629 **(iv)**2307 **(v)** 4253

## 10.10 FURTHER READING


1. Vedic Mathematics, Jagadguru Swami Sri Bharati Krishna Tirthaji Maharaja
2. The History of Ancient Indian Mathematics, C.N. Srinivasiengar, The World Press, Calcutta, 1967.
3. The History of Hindu Mathematics, Asia Publishers House, Bombay, 1962


## 10.11 MODEL QUESTIONS

**1.**What is meant by Vilokanam in the extraction of cube roots?

**2.** Describe the role of inspection in determining the first digit of a cube root.

**3.** Explain the First Principle governing the extraction of cube roots. Illustrate the principle with suitable algebraic expressions.

**4.** Distinguish between the method of inspection and the algebraic method of extracting cube roots.

**5.**Find the cubes of the following numbers by using Anurupya Sutra

**(i)** 23 **(ii)** 27 **(iii)** 53 **(iv)**72 **(v)** 95

**6.**Find the cubes of the following numbers by using Yavadunam Sutra

**(i)** 91 **(ii)** 98 **(iii)** 996 **(iv)**1011 **(v)**10003 **(vi)** 99994

**(vii)** 100009

**7.**Applying the algebraic method to findthe cube roots of the following numbers

**(i)**83453453 **(ii)**143055667 **(iii)**43725816667 **(iv)**4512574703125

**(v)**210358244856817

**8.**Using first principles, extract the cube roots of:

**(i)**331373888 **(ii)**4508479808 **(iii)**95569357248 **(iv)**850085506817

**(v)**918330048

# UNIT 11: FRACTIONS IN VEDIC MATHEMATICS

## UNIT STRUCTURE

11.1 Learning Objectives
11.2 Introduction
11.3Introduction to Fractions
11.4 Vinculum and Its Use in Fractions
11.5Algebra of Fractions
11.6 Let Us Sum Up
11.7 Answers to Check Your Progress
11.8 Further Reading
11.9 Model Questions

## 11.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- understand the meaning and types of fractions used in elementary arithmetic and algebra.
- convert between mixed numbers, improper fractions and decimal forms comfortably.
- explain the idea of vinculum and convert ordinary numbers to vinculum form and back.
- apply vinculum to simplify operations with fractions, especially addition, subtraction and division.
- use basic Vedic ideas in handling algebraic fractions, including simplification and partial fractions.

## 11.2 INTRODUCTION

Fractions arise whenever a whole quantity is divided into equal parts, for example sharing food, measuring lengths or expressing probabilities. In school mathematics, fractions are often learned through procedures such as finding common denominators, converting to decimals and simplifying ratios. Many learners feel that these procedures are long and hard to remember, especially when numbers become large.

Vedic Mathematics gives a fresh way to see fractions as patterns that repeat and as companions of decimals, instead of as a separate difficult topic. The use of sutras like “Ekadhikena Purvena”, “NikhilamNavatashcaramamDashatah” and “ParavartyaYojayet” connects fractions to recurring decimals, complements from 10 and quick division. The device of

vinculum, where large digits are replaced by small “bar digits”, further reduces the mental load in fraction operations.

This unit first reviews fractions in a simple language, then explains the vinculum idea and shows how to use it for fraction work, and finally develops the algebra of fractions with Vedic insights. The aim is not to replace standard school methods, but to add quicker and more intuitive methods alongside them. With practice, these techniques help you carry out many fraction calculations mentally or in very few written steps.

## 11.3 INTRODUCTION TO FRACTIONS

A fraction is a number that represents a part of a whole, written as a/b where a and b are integers and b is not zero. The number a is called the numerator, and it tells how many equal parts are taken. The number b is called the denominator, and it tells into how many equal parts the whole has been divided. Thus $\frac{3}{4}$ means that a whole is divided into four equal parts and three such parts are taken.

Fractions can be proper, improper or mixed. A proper fraction has numerator smaller than denominator, such as $\frac{2}{5}\ or \frac{7}{12}$. An improper fraction has numerator greater than or equal to denominator, such as $\frac{9}{4}\ or \frac{5}{5}$. A mixed number combines a whole number and a proper fraction, such as $2\frac{1}{3}$, which really means $2\ +\frac{1}{3}\ or \frac{7}{3}$. It is often useful to convert mixed numbers to improper fractions when doing operations, and to convert back at the end if the answer is to be expressed in mixed form.

Equivalent fractions are different-looking fractions with the same value. For example, $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}\ and \frac{50}{100}$ all represent the same quantity. They are obtained by multiplying or dividing numerator and denominator by the same non-zero integer. Simplification of a fraction means dividing numerator and denominator by their greatest common divisor so that no further common factor is left. For instance, $\frac{18}{24}$ can be simplified by dividing by 6, giving $\frac{3}{4}$.

To compare two fractions with different denominators, one common method is to convert them to a common denominator. For $\frac{2}{3}\ and \frac{3}{5}$, a common denominator is 15, and we write $\frac{2}{3}\ = \frac{10}{15}$ and $\frac{3}{5}\ = \frac{9}{15}$, so $\frac{2}{3} > \frac{3}{5}$. Another compact method is cross-multiplication: for $\frac{a}{b}$ and $\frac{c}{d}$, compare

$ad\ and\ bc$; the fraction with larger product in the numerator position is larger. This cross-multiplication pattern will appear again in the Vedic treatment of fractions.

Operations with fractions follow simple rules. To add or subtract, convert to a common denominator, then add or subtract numerators and keep the denominator. To multiply, multiply numerators together and denominators together. To divide by a fraction, multiply by its reciprocal, which is obtained by interchanging numerator and denominator. These rules remain the same whether or not we use Vedic methods; the Vedic ideas mainly help in doing the arithmetic within these rules more quickly.

At this stage, it is also useful to see the link between fractions and decimals. Any fraction with denominator equal to a power of 10 can be written directly as a decimal, for example $\frac{37}{100} = 0.37$. Denominators containing only factors 2 and 5 also give terminating decimals, such as $\frac{1}{8} = 0.125$ and $\frac{3}{20} = 0.15$. When the denominator has factors like 3, 7, 11 and so on, the decimal expansion becomes recurring, like

$\frac{1}{3} = 0.333\ldots$ or $\frac{1}{7} = 0.142857142857\ldots$. Vedic methods focus strongly on these recurring patterns and show how to obtain them in one-line ways.

## CHECK YOUR PROGRESS

Q1. Distinguish between a proper and an improper fraction with an example of each.

Q2. What does it mean for two fractions to be equivalent? Give one pair of equivalent fractions.

Q3. Explain how you would compare $\frac{3}{4}\ and\ \frac{5}{7}$ without converting them to decimals.

Q4. Write the reciprocal of $\frac{5}{8}$ and state in which operation with fractions reciprocals are used.

## 11.4 VINCULUM AND ITS USE IN FRACTIONS

Vinculum is a method of writing numbers so that the digits are as small as possible, usually at most 5, by treating some digits as negative. A digit with a bar on top is read as negative; for example, 7 can be thought of as 10 − 3 and written as 1 and a bar 3 (one ten and negative three). Similarly, 9 can be written as 1 and a bar 1, and 8 as 1 and a bar 2. The total value of the number remains the same, but the digits become smaller and easier to handle mentally.

To convert a normal positive number to vinculum form, a convenient rule is to scan from right to left and whenever a digit is 6 or larger, replace it by its complement from 10 with a bar, and add 1 to the digit on its left. For instance, to convert 78, look at 8; its 10's complement is 2, so 8 becomes bar 2 and we add 1 to 7 making 8; now 8 is again large, so its complement is 2 bar and we add 1 to a new digit on the left, giving $1\,\bar{2}\,\bar{2}$, which still represents 78. In practice, one usually stops after the first step and writes 78 as $1\bar{2}2$ only when it truly simplifies a calculation; for teaching, simpler examples like 76 = $1\bar{2}4$ (since $100 - 24 = 76$) are used.

The key idea is that a block like $100 - 24$ can be written as 1 with bar 2 and bar 4, because 1 hundred plus negative twenty plus negative four equals 76. In decimal notation, we do not normally write the negative signs explicitly as minus signs; instead, we use a bar over the digit to remind us that it should be subtracted, not added, when we compute the value. This representation is especially helpful when numbers lie close to powers of 10, 100 or 1000, since base methods in Vedic Mathematics make heavy use of such nearness.

In the context of fractions, vinculum helps whenever numerators or denominators contain digits 6, 7, 8 or 9 and we are using sutras like "NikhilamNavatashcaramamDashatah" or "ParavartyaYojayet". For example, in dividing by 9 or 99 or 999, deficits from 10 or 100 or 1000 appear naturally, such as −1 or −2, and it is convenient to represent them as bar 1 or bar 2. When we write 18 as $2\bar{2}$, we are really saying 20 − 2, which fits very well with a base 10 or base 100 method.

Vinculum is also helpful in multi-step fraction operations. Suppose we are adding $\frac{7}{9}$ and $\frac{5}{9}$. The ordinary sum is $\frac{12}{9}$, which simplifies to $\frac{4}{3}$. In a base 10 viewpoint, 7 is $1\bar{3}$ and 5 is 5, so $7 + 5 = 1\bar{3} + 5 = 8 - 2 = 6$, and the overshoot of a full 9 can be tracked through complement logic. For more complex examples, where several fractions share a common denominator near a power of 10, vinculum makes the mental tracking of carries and borrows more uniform.

### Vinculum in Fraction to Decimal Conversion

The most striking use of vinculum for fractions is in converting vulgar fractions (ordinary fractions) to recurring decimals. For denominators ending in 9 or numbers close to 10, 100, 1000 and so on, the Ekadhikena Purvena sutra guides us to multiply or divide repeatedly by "one more than the previous digit". Remainders in such division can be written in vinculum form to keep digits small.

Take $\frac{1}{19}$ as a classic example. The conventional long division $1 \div 19$ gives the recurring decimal $0.052631578947368421\ldots$, which has 18 digits before the pattern repeats. The Vedic method notices that the penultimate digit of the denominator 19 is 1, so the Ekadhika is 2. Instead of performing long division, one can start from the last decimal digit 1 and repeatedly multiply by 2, carrying extra digits to the left. In this process, numbers like 16, 18 and so on appear as intermediate results, and treating them in vinculum form (for example 16 as $2\bar{4}$, 18 as $2\bar{2}$) keeps the work simple and helps recognise when the sequence has completed.

A similar method works for $\frac{1}{29}$, where the Ekadhika is 3. Starting from 1 and repeatedly multiplying by 3 from right to left produces the 28-digit recurring block of the decimal for $\frac{1}{29}$. Again, vinculum is helpful for intermediate two-digit products such as 21 or 27, which can be treated as $3\bar{9}$ or $2\bar{3}$ in mental steps, though the final written digits remain ordinary positive digits. In both cases, the vinculum acts behind the scenes as a mental aid rather than as part of the final decimal.

The vinculum approach to recurring decimals also links nicely with the idea of complements from 9. In many denominators, the digits of the recurring block can be split into two halves whose corresponding digits add up to 9. When that happens, once one half is found by a simple rule, the other half follows mechanically by writing complements. Vinculum makes this complement relation very natural, since bar digits represent “what is missing” from a base.

## CHECK YOUR PROGRESS

Q5. In your own words, describe what a vinculum digit is.

Q6. How would you represent the number 76 in vinculum form using the base 100 idea?

Q7. Why is vinculum especially useful when working with denominators that are close to powers of 10?

Q8. For the fraction $\frac{1}{19}$, what is the Ekadhika used in the Vedic recurring decimal method?

## 11.5ALGEBRA OF FRACTIONS

Algebra of fractions extends the rules of number fractions to expressions containing variables such $as\ x, y$ and so on. A simple algebraic fraction is of the form $\frac{ax+b}{cx+d}$ or (polynomial)/(polynomial). The same basic rules apply: to add or subtract, convert to a common

denominator; to multiply, multiply numerators and denominators; to divide, multiply by the reciprocal. What changes is the arithmetic, which now involves symbolic manipulation instead of only numbers.

Vedic Mathematics contributes two main ideas here. One is the use of "ParavartyaYojayet", which means "transpose and apply", for handling denominators of the form $(x - a), (x - b))$ and similar linear factors. The other is the use of partial fractions in a quick mental way. Partial fraction decomposition expresses a rational function like $\frac{3x^2+2x-1}{(x-1)(x-2)(x-3)}$ as a sum $\frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

Instead of solving simultaneous equations in A, B and C by the standard method, the Vedic approach often evaluates the expression at $x = 1, 2 \; and \; 3$ directly, treating each value as a quick "paravartya" step.

For example, suppose we wish to write $3x^2 + 2x - 1$ over $(x - 1)(x - 2)(x - 3)$ as a sum of simple fractions. In the standard approach, we set

$$3x^2 + 2x - 1 = A(x - 2)(x - 3) + B(x - 1)(x - 3) + C(x - 1)(x - 2)$$

and then match coefficients. In the Vedic method, we put x = 1 at once, which kills the terms with factors $(x - 1)$ and gives $3(1)^2 + 2(1) - 1 = A(1 - 2)(1 - 3)$, so A can be found directly by simple numeric arithmetic. Repeating this at $x = 2$ and $x = 3$ gives $B \; and \; C$. This is a direct and mental way of extracting coefficients without messy expansions.

Simplifying algebraic fractions with Vedic help also appears in solving equations. Take an equation like $\frac{1}{x-1} + \frac{1}{x-2} = 1$. Multiplying through by $(x - 1)(x - 2)$ and simplifying leads to a quadratic, but a Vedic observer might use the idea of Samuccaya (the sum of numerator and denominator patterns) to spot that values $x = 2 \; and \; x = 3$ are natural trial values. Vedic practice encourages such pattern-based guessing, coupled with quick checks using complements and sutras.

Another rich area is the link between algebraic fractions and calculus. Integration of rational functions often uses partial fractions. Once an expression is decomposed into sums like $\frac{A}{x-a}$, the integral becomes $A \; log|x - a|$ plus a constant. The faster we can split a rational function

into pieces, the more quickly we can integrate. The Paravartya approach gives almost “one-line” decompositions in many textbook cases, making integration problems feel much lighter.

Even when only school-level algebra is involved, fraction simplification benefits from Vedic multiplication and division. When multiplying expressions like $(x + \frac{5}{7})(x - \frac{3}{7})$, base methods with 7 as a “local base” and viewing 5/7 and 3/7 as complements can speed up the work. Similarly, expressions like $\frac{1-x}{1+x}$ or $\frac{1+x}{1-x}$ appear in trigonometry and coordinate geometry, and Vedic-style cross-multiplication and ratio handling makes transformations between such forms quick and reliable.

## CHECK YOUR PROGRESS

Q9. What is meant by a partial fraction decomposition of a rational function?

Q10. How does writing an algebraic fraction as a sum of simpler fractions help in integration?

Q11. Explain briefly how “ParavartyaYojayet” is used in working with algebraic fractions.

## 11.6 LET US SUM UP

- Fractions describe parts of a whole and include proper, improper, and mixed forms, along with equivalent fractions and simplification.
- Fractions and decimals are closely connected; recurring decimals represent rational numbers in another form.
- Vinculum notation uses negative digits to keep numbers small and is helpful in Vedic arithmetic.
- Vinculum is especially useful for operations involving complements of 10, 100, and 1000, including division and recurring decimals.
- Algebraic operations on fractions follow standard rules, but Vedic sutras make them faster and more mental.
- Sutras like *ParavartyaYojayet* simplify partial fractions, cross-multiplication, and rational expressions.

## 11.7 ANSWERS TO CHECK YOUR PROGRESS

**Ans to Q1**: A proper fraction has numerator smaller than denominator, such as $\frac{3}{7}$. An improper fraction has numerator greater than or equal to denominator, such as $\frac{9}{4}$.

**Ans to Q2**: Two fractions are equivalent if they represent the same value even though their numerators and denominators differ, for example $\frac{1}{2}$ and $\frac{3}{6}$.

**Ans to Q3**: To compare $\frac{3}{4}$ and $\frac{5}{7}$, cross-multiply: $3 \times 7 = 21$ and $4 \times 5 = 20$. Since $21 > 20$, we have $\frac{3}{4} > \frac{5}{7}$.

**Ans to Q4**: The reciprocal of $\frac{5}{8}$ is $\frac{8}{5}$. Reciprocals are used when dividing by a fraction; we multiply by the reciprocal instead.

**Ans to Q5**: A vinculum digit is a digit written with a bar over it to indicate that it is to be subtracted rather than added, acting like a negative digit in positional notation.

**Ans to Q6**: Using base 100, 76 can be thought of as $100 - 24$, so it can be written as 1 with bar 2 and bar 4, representing $1 \times 100 - 20 - 4$.

**Ans to Q7:** When denominators are close to powers of 10, deficits such as $1, 2\ or\ 3$ from those bases occur repeatedly; writing them as small bar digits makes the repeated operations in division and complements easy to handle mentally.

**Ans to Q8:** For $\frac{1}{19}$, the Ekadhika is 2, since the penultimate digit of 19 is 1 and "one more than the previous one" gives 2.

**Ans to Q9**: Partial fraction decomposition is the process of expressing a rational function as a sum of simpler rational functions whose denominators are typically linear or quadratic factors.

**Ans to Q10**: Once a rational function is written as a sum of terms like $\frac{A}{x-a}$, each term can be integrated directly as $A\ log|x - a|$, turning a difficult integral into a sum of simple ones.

**Ans to Q11**: "ParavartyaYojayet" guides us to transpose factors from denominator to numerator by substituting special values (such as roots of denominator factors) and applying simple arithmetic to obtain the coefficients of the decomposed fractions.

## 11.8 FURTHER READING


1. Bharati Krishna Tirtha, Vedic Mathematics, Motilal Banarsidass, Delhi.
2. State Council of Educational Research and Training, Fundamentals and Applications of Vedic Mathematics, New Delhi.
3. Kenneth R. Williams, Discover Vedic Mathematics, Vedic Mathematics Research Group.
4. VedicMaths.Org, "Practise and Free Books on Vedic Mathematics."
5. Moonpreneur, "Vedic Math Sutra 1: Ekadhikena Purvena."

## 11.9 MODEL QUESTIONS

Q1. Define a mixed fraction and explain how to convert it into an improper fraction with an example.

Q2. Explain the steps for converting a fraction like 3/8 into a decimal and state why its decimal terminates.

Q3. Describe, with a simple example, how vinculum can simplify subtraction involving large digits.

Q4. Explain the role of Ekadhikena Purvena in obtaining the recurring decimal of 1/19.

Q5. Write the number 89 in a vinculum form based on 100 and show that the value remains unchanged.

Q6. State and solve a simple problem where fraction addition becomes easier if the denominators are viewed as close to a power of 10.

Q7. Give an example of an integration problem in which partial fractions are helpful, and outline the steps without doing full algebraic detail.

Q8. Discuss how Vedic ideas such as vinculum and Paravartya can reduce the number of steps in solving a problem involving algebraic fractions.

# UNIT 12: DECIMALS IN VEDIC MATHEMATICS

## UNIT STRUCTURE

12.1 Learning Objectives
12.2 Introduction
12.3Preliminary
12.4 Numbers ending in nine
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12.5Other Endings
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12.6 Multiples of the basic fractions
12.7 Let Us Sum Up
12.8 Answers to Check Your Progress
12.9 Further Reading
12.10 Model Questions

## 12.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- Understand how Vedic Mathematics treats decimals and recurring decimals as a natural extension of fraction methods.
- Convert fractions into terminating, recurring and partly–recurring decimals using simple sutras.
- Apply special techniques for fractions with denominators ending in 9 and explore how these give recurring decimals.
- Handle decimals for other endings such as 3 and 7 by suitable “auxiliary” adjustments.
- Use Vedic approaches to find decimal equivalents of multiples of basic fractions quickly and accurately.

## 12.2 INTRODUCTION

Decimals play an important role in numerical calculations, and Vedic Mathematics offers simple and elegant techniques to handle them quickly and accurately. This unit introduces the Vedic approach to decimals by building on basic ideas and gradually moving towards faster mental methods.

We begin with the **preliminary concepts**, revising the meaning of decimal representation and its connection with fractions. Special attention is given to **numbers ending in nine**, where Vedic methods use complements to the base (such as 10 or 100) to simplify calculations. The unit then considers **numbers with other endings**, showing how similar principles can be extended beyond the special case of nine.

Finally, the study of **multiples of basic fractions** highlights how recurring and terminating decimals can be generated efficiently using Vedic techniques. Through these topics, the unit demonstrates how decimals, which often appear complicated, can be treated systematically and mentally using Vedic Mathematics.

## 12.3 PRELIMINARY

In school mathematics, decimals usually appear first as another way of writing fractions, for example $\frac{1}{4} = 0.25$ or $\frac{3}{5} = 0.6$. In Vedic Mathematics, decimals are also closely tied to fractions, but there is a strong emphasis on recurring patterns and very fast mental methods to obtain those patterns. This unit builds on that idea and shows how certain sutras make decimals simple and systematic instead of tedious or mysterious.

A useful starting observation is that denominators containing only factors 2 and 5 always produce terminating decimals when the numerator is 1. For example, the decimal forms of $\frac{1}{2}, \frac{1}{5}, \frac{1}{4}, \frac{1}{8}, \frac{1}{25}$ and similar fractions all end after a finite number of digits, and each factor 2 or 5 contributes one decimal digit in the result. When the denominator contains other prime factors, such as $3, 7, 11$ or higher primes, the decimal form becomes recurring or partly recurring, and this is where Vedic techniques show their full strength.

It is also important to notice a simple digit–based rule which the original text stresses repeatedly. When the numerator is 1, if the fraction gives a terminating decimal, then the last digit of the denominator multiplied by the last digit of the decimal answer ends in 0, as in $\frac{1}{25} = 0.04$ where 5 and 4 give 20. When the decimal is recurring, that same product always ends in 9, as in

$\frac{1}{19} = 0.\overline{052631578947368421}$ where 9 and 1 give 9, or $\frac{1}{7} = 0.\overline{142857}$ where 7 and 7 give 49. This simple observation lets you predict the last digit of a recurring decimal in advance and becomes a guiding idea in the methods that follow.

## 12.4 NUMBERS ENDING IN NINE

Fractions whose denominators end in 9 form one of the most important families in Vedic treatment of decimals. When a denominator such as 19, 29, 39 and so on appears, a standard sutra “Ekadhikena Purvena” (by one more than the previous one) is used to generate the recurring decimal digits almost mechanically. The “previous one” here is the penultimate digit of the denominator, so for 19 it is 1, and “one more” gives 2, which becomes the key multiplier or divisor in the method.

### 12.4.1 EKADHIKENA PURVENA: MULTIPLICATION APPROACH

Take $\frac{1}{19}$. The conventional long division to 18 recurring places needs 18 distinct steps of division, each with trial digits and subtraction. Using Ekadhikena Purvena, you first note that the last digit of the denominator is 9, so the last digit of the recurring decimal must be 1, because their product should end in 9. Start from the rightmost end with 1, then move leftwards repeatedly multiplying by 2 (the Ekadhika of the penultimate digit 1) and handling carries as in ordinary multiplication.

Working backwards, one writes 1 as the last digit, then multiplies by 2 to get 2, then 4, then 8, then 16 (write 6, carry 1), and continues this pattern until the entire recurring cycle appears. The digits build up as 052631578947368421 when written in the usual left–to–right order, so $\frac{1}{19} = 0.\overline{052631578947368421}$. The striking feature is that, instead of repeated division by 19, there is only a single repetitive multiplication by 2, with each new product feeding the next step until the digits start repeating.

This multiplication version of the sutra works just as well for other denominators ending in 9, with the multiplier changing to the Ekadhika of the penultimate digit. For denominators like 29 or 39, the Ekadhika will be 3 or 4 respectively, so the basic step is repeated multiplication by 3 or 4, and the decimal digits emerge directly. Since all the work is with small single–digit multipliers, the computation remains light even when the recurring block is long, making it suitable for mental calculation once the pattern is understood.

### 12.4.2 EKADHIKENA PURVENA: DIVISION APPROACH

The same sutra supports a second method based on division, in which the recurring digits are obtained by repeated division with the Ekadhika as divisor. For $\frac{1}{19}$, the divisor is again 2, but now the work runs from left to right, beginning with an initial quotient digit 0 and treating the remainder carefully. One standard procedure is to place 0 as the first quotient digit, 1 as the first remainder and then successively “prefix” each remainder to the previous quotient digit to form the next dividend.

In this manner, dividing 10 by 2 gives 5, remainder 0; then prefixing 0 to 5 gives 5 as the next dividend; division gives 2 remainder 1, and prefixing 1 to 2 gives 12 as the next dividend. Repeating this rule (divide, record quotient digit, prefix remainder to that digit to form the next dividend) produces the same sequence of digits 052631578947368421 that appeared in the multiplication method. It is a division process, yet the divisor stays fixed and small (2) throughout, and there are no trial digits or large subtractions, only this systematic dividend formation.

The division variant has an advantage when one wishes to write the decimal from left to right and stop at a chosen number of places. It is also especially useful for understanding the internal structure of remainders, because the sequence of remainders exhibits regular patterns, such as each remainder, with a suitable adjustment, being a fixed multiple of the next. Those patterns are later exploited in more advanced analysis of recurring decimals in the original text.

## CHECK YOUR PROGRESS

Q1. Use Ekadhikena Purvena (multiplication approach) to write the first 12 digits of the recurring decimal for $\frac{1}{29}$.

Q2. For $\frac{1}{19}$, explain why 1 must be the last digit of the recurring block before you begin the Ekadhika process.

Q3. In the division approach for $\frac{1}{19}$, describe how the next dividend is formed from the previous remainder and quotient digit.

Q4. For a fraction whose denominator ends in 9 and numerator 1, explain the rule that connects the last digit of the denominator with the last digit of the recurring decimal.

## 12.5OTHER ENDINGS

Denominators ending in digits other than 9 can often be brought into a convenient form so that the methods of the previous section become applicable. Vedic Mathematics achieves this using two main ideas: multiplying the denominator to get a number ending in 9 (or 99, 999, and so on), and using “auxiliary fractions” where numerator and denominator are adjusted together in a simple way. In this way, decimals for denominators ending in 3, 7 or 1 can also be worked out in a uniform framework.

## 12.5.1DENOMINATORS ENDING IN THREE

For fractions with denominators ending in 3, a common step is to multiply the denominator by 3 to obtain a new denominator ending in 9. For example, a denominator such as 13 can be turned into 39 by multiplying by 3, and the Ekadhika method can then be used with the new denominator. The same multiplier must act on the numerator to keep the value of the fraction unchanged, so the original fraction is rewritten with an equivalent but more convenient denominator.

The text notes that in such cases the last digit of the recurring decimal becomes 3, consistent with the rule that the product of the last decimal digit and the last digit of the denominator ends in 9. Once a suitable multiple ending in 9 is found, the rest proceeds almost exactly as with denominators like 19 or 29: one finds the Ekadhika, then uses repeated multiplication or division by that number to generate the recurring block. However, some denominators ending in 3 show short recurring cycles, while others display full cycles whose length is one less than the denominator or a divisor of that number.

Short examples mentioned in the source include fractions where the recurring part has only two decimal places, such as those equivalent to $\frac{1}{30}$—like structures with numerator 1. In other instances, the cycle length can be larger but still derived from the prime factor structure of the denominator once it has been transformed into a number ending in 9. The important pedagogic message here is that students should look at the last digit and factor pattern of the denominator before they begin any detailed computation, as those features signal both method and expected cycle length.

## 12.5.2DENOMINATORS ENDING IN SEVEN

Denominators ending in 7 are treated in a parallel fashion, with some additional rich structure in the remainders. A standard example is $\frac{1}{7} = 0.\overline{142857}$, where the recurring block has six digits

and each remainder in the usual division process follows a regular pattern when viewed as a geometric progression under multiplication by 3 modulo 7. The text generalises this by noting that fractions with denominators 7, 17, 27, 37 and so on can be handled by finding an Ekadhika based on multiplying the denominator by 7 to get a form ending in 9.

For denominators like 7, the last digit of the decimal is 7 so that 7 times 7 ends in 9, and for larger denominators ending with 7, the same product rule holds for the last digits. Once again, either a multiplication–based or division–based repeated step, using the appropriate Ekadhika, will generate the recurring digits without trial and error. In more advanced discussion, the author shows that the sequence of remainders for several such denominators fits simple algebraic patterns of the form $a, d, a-2d, 2a-5d$ and so on, where the coefficients are directly related to how far the penultimate digit falls short of 9.

This level of pattern recognition does more than shorten calculation; it also encourages students to see recurring decimals as structured objects with internal regularity rather than random strings. For teaching at school level, it is often enough to give a few guided examples, such as $\frac{1}{7}$ and $\frac{1}{17}$, and to let learners observe that the recurring cycles of different denominators often have lengths equal to or dividing the denominator minus one. These observations naturally lead into number–theoretic ideas while keeping the arithmetic process simple and concrete.

## 12.6MULTIPLES OF THE BASIC FRACTIONS

Once the recurring decimal for a basic fraction with numerator 1 is known, it is easy to obtain the decimals for multiples of that fraction by simple digit manipulations. For instance, starting from $\frac{1}{7} = 0.\overline{142857}$, one can find $\frac{2}{7}, \frac{3}{7}$ and so on by multiplying the recurring block by 2, 3 and so forth, remembering to handle carries cyclically within the six–digit pattern. This method allows many decimals to be written down without fresh division, and it strengthens the understanding that all such fractions share the same recurring cycle, only shifted.

To see this concretely, consider again the decimal of $\frac{1}{7}$. Doubling 142857 gives 285714, which is the recurring block for $\frac{2}{7}$, and tripling gives 428571 for $\frac{3}{7}$. Each multiple simply rotates the same set of digits, so that every non–zero numerator less than 7 corresponds to one cyclic shift of the basic pattern. In class, this is a powerful illustration that recurring decimals for a fixed prime denominator form a family governed by rotation, which can be explored experimentally with learners.

The same principle applies to more complicated denominators like 19, where the recurring block has 18 digits. Once $\frac{1}{19}$ is available as $0.\overline{052631578947368421}$, higher numerators such as 2, 3 and 7 give their decimals by multiplying that long block accordingly and managing carries across the whole sequence. Although the block is longer, the work still avoids repeated division, and for smaller numerators mental handling is quite feasible because the carries are sparse and the pattern is highly regular.

Vedic techniques also generalise this idea using the sutra “Anurupyena” (proportionately), which reminds the learner that proportionally related fractions have proportionally related decimal expansions. For example, if a denominator is scaled by a small factor, the decimal can sometimes be obtained by suitably grouping or shifting digits in the known expansion of the original fraction. This becomes particularly useful in competition settings, where quick recognition of such proportional relationships can save significant time.

## CHECK YOUR PROGRESS

Q5. Using multiples of basic fractions, write the decimal form of $\frac{5}{9}$.

Q6. Find the decimal expansion of $\frac{3}{7}$ using the idea of *multiples of basic fractions*.

## 12.7 LET US SUM UP

- Vedic methods provide quick mental techniques for working with decimals through simple preliminary rules.
- Special shortcuts are available for handling numbers ending in nine, making calculations faster and easier.
- Numbers with other digit endings can also be managed efficiently using Vedic decimal techniques.
- Decimals related to basic fractions and their multiples follow regular patterns that simplify conversion and calculation.
- Overall, Vedic Mathematics reduces lengthy decimal operations to short, systematic mental steps.

## 12.8 ANSWERS TO CHECK YOUR PROGRESS

**Ans to Q1.** For $\frac{1}{29}$, the last digit of the recurring decimal must be 1 so that the product with 9 ends in 9, and the Ekadhika of the penultimate digit 2 is 3. Working from the right with repeated multiplication by 3, and then arranging the digits in the usual order, the first 12 digits of the

recurring decimal are 034482758620, so $\frac{1}{29} = 0.\overline{034482758620689655172413793 1}$ and the initial requested part is $0.034482758620$.

**Ans to Q2.** Since the numerator is 1 and the decimal is recurring, the rule states that the product of the last digit of the denominator and the last digit of the decimal must end in 9. The last digit of 19 is 9, and 9 times 1 is 9, so 1 is the only possible last digit for the recurring block, which is therefore fixed before any computation.

**Ans to Q3.** In the division approach with divisor 2, write 0 as the first quotient digit and treat 1 as the first remainder. At each step, the next dividend is formed by prefixing the current remainder to the most recent quotient digit (for example, remainder 1 and quotient digit 0 give dividend 10, then remainder 0 and quotient digit 5 give dividend 5, and so on), and this dividend is then divided by 2 to produce the next quotient digit and remainder.

**Ans to Q4.** For a fraction with numerator 1 and denominator ending in 9, the Vedic digit rule asserts that the last digit of the denominator multiplied by the last digit of the recurring decimal must end in 9. Since the last digit of the denominator is fixed as 9, the last digit of the decimal block must be chosen so that this product ends in 9, which restricts it to digits like 1, 3 or 7 depending on the particular denominator.

**Ans to Q5:** Since,$\mathbf{\frac{1}{9}} = 0.\overline{1}$

Multiplying by 5, we get, $\frac{5}{9} = 0.\overline{5}$

**Ans to Q6:** Since, $\mathbf{\frac{1}{7}} = 0.\overline{142857}$

multiplying by 3 gives$\frac{3}{7} = 0.\overline{428571}$

This is a cyclic shift of the same repeating digits.

## 12.9 FURTHER READING


1. Bharati Krishna Tirtha, Vedic Mathematics, Motilal Banarsidass, Delhi.
2. State Council of Educational Research and Training, Fundamentals and Applications of Vedic Mathematics, New Delhi.
3. Kenneth R. Williams, Discover Vedic Mathematics, Vedic Mathematics Research Group.
4. VedicMaths.Org, “Practise and Free Books on Vedic Mathematics.”
5. Moonpreneur, “Vedic Math Sutra 1: Ekadhikena Purvena.”


## MODEL QUESTIONS

**Q1**. Explain how the presence or absence of prime factors 2 and 5 in the denominator affects whether the decimal expansion of $\frac{1}{n}$ is terminating or recurring, illustrating with two examples of each type.

**Q2**. Describe in your own words the multiplication–based Ekadhikena Purvena method for denominators ending in 9, and apply it to find at least eight recurring digits of $\frac{1}{39}$.

**Q3**. For denominators ending in 7, outline how the Vedic approach uses patterns in remainders to understand the recurring decimal of $\frac{1}{7}$ and its multiples.

**Q4**. Define an auxiliary fraction in the context of Vedic treatment of decimals and explain how such fractions help in handling denominators that do not naturally end in 9.

**Q5**. Given the recurring decimal for $\frac{1}{13}$, explain a method for obtaining the decimals of $\frac{2}{13}$ and $\frac{3}{13}$ without performing fresh long division.

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# UNIT 13: ARITHMETIC IN THE SULBA SUTRAS

## UNIT STRUCTURE

13.1 Learning Objectives
13.2 Introduction
13.3Use of Arithmetic for Geometric Constructions
13.4 Ratios, Proportions, and Scaling
13.5Approximate Values Used in Altar Measurements
13.6 Let Us Sum Up
13.7 Answers to Check Your Progress
13.8 Further Reading
13.9 Model Questions

## 13.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- understand why arithmetic was essential for constructing Vedic fire altars described in the Sulba Sutras.
- explain how ratios and proportions guided the shapes and sizes of different altar designs.
- describe how ancient priests used scaling to change the size of an altar while keeping its area fixed.
- interpret some approximate numerical values, such as the value of the diagonal of a square, used in altar measurements.

## 13.2 INTRODUCTION

The Sulba Sutras are ancient Sanskrit texts that give rules for constructing Vedic fire altars. They form a part of the larger Kalpa literature and were written to guide priests in building ritual spaces with exact shapes and areas. The word “sulba” refers to a cord or measuring rope, and these texts describe geometric constructions using such cords on the ground. Although the ritual aim was religious, the work itself is full of geometry and arithmetic.

In these constructions, the priest was not simply drawing figures for study but actually marking out earthwork altars with strict conditions. The area of the altar had to match the ritual prescription, while the shape could vary between square, rectangular, circular, or more elaborate bird and turtle forms. This needed clear rules about lengths, areas, and angle relations. Arithmetic appears throughout, behind the scenes of these geometric instructions.

The Sulba Sutras therefore provide an early example of applied mathematics. They show how practical needs can push people to create and use numerical methods long before formal algebraic notation exists. In this unit, arithmetic in the Sulba Sutras will be seen through three lenses: how it supports geometric constructions, how ratios and proportions control the design, and how approximate values are used when exact numbers are hard to obtain.

## 13.3 USE OF ARITHMETIC FOR GEOMETRIC CONSTRUCTIONS

The first and perhaps most important role of arithmetic in the Sulba Sutras is to support geometric constructions with cords. When a text instructs the builder to draw a square of a given area, or to transform a square altar into a rectangular or circular one with the same area, numbers are always in the background. To preserve the required area, one must know how to change the side lengths correctly.

Altar construction starts by laying out basic reference lines. Typically, an east–west line is first fixed as a central axis. A perpendicular north–south line is then drawn to create a coordinate-like framework on the ground. To draw the perpendicular, the texts describe right triangle constructions that, in modern terms, rely on Pythagorean triples. For example, a triangle with sides in ratios like 3, 4, and 5 gives a right angle at the corner opposite the longest side. Though expressed verbally, the underlying idea is the same as the Pythagorean theorem.

Addition and subtraction appear in the way lengths are built up from units. Traditional units such as angula (finger breadth) and purusha (a larger body-based unit) are combined to get the required side lengths of bricks and altars. A typical instruction might say that an altar is to have a certain number of purushas in one direction and another number in the other direction, or that a brick is obtained by taking a half, a quarter, or an eighth of another standard piece. In practice, this means repeated use of fractions like one half or one quarter, along with simple multiplication and division.

To convert one shape into another without changing the area, the Sulba Sutras give stepwise geometric procedures that translate, in modern terms, into area equations. For instance, if a square altar must be transformed into a rectangle of a given ratio of sides but equal area, the construction effectively solves the relation that the product of the new sides equals the original

square's area. Each step of the construction uses simple arithmetic with lengths or fractions of lengths: cutting off a part, adding another, or rearranging pieces.

A particularly striking use of arithmetic is found in the construction of diagonals. The diagonal of a square with side one unit has length equal to the square root of two. The Sulba Sutras give a practical approximation for this value. To obtain it, they suggest a procedure that could be described in terms of fractions added one after another: increase a unit length by one third, then again by one fourth of that third, and then reduce slightly by a very small part. When translated into modern notation, this layered description gives a numerical value close to the true square root of two.

In all these cases, arithmetic is not written down as equations but is embedded into verbal instructions about cords and lines. Yet the builder had to understand and apply the operations correctly. Slight errors would change the shape or area of the altar, which was not acceptable in a ritual that demanded precision. Thus, the Sulba Sutras show arithmetic working hand in hand with geometry to achieve practical goals.

## CHECK YOUR PROGRESS

Q1. Why was it important for the builder to preserve the area of an altar when changing its shape?

Q2. How does a right triangle help in drawing perpendicular lines in Sulba Sutra constructions?

Q3. Name two basic arithmetic operations that appear in the use of traditional length units.

Q4. What kind of quantity is the diagonal of a unit square, and why did the Sulba Sutras need an approximation for it?

## 13.4 RATIOS, PROPORTIONS, AND SCALING

The second main role of arithmetic in the Sulba Sutras lies in ratios and proportions. Altars were often specified not only by their total area but also by the shape, such as the ratio between length and breadth. In many constructions, the side lengths stand in simple proportional relationships like 5 to 4 or 3 to 2. These ratios ensure a standard shape even as sizes change.

For example, consider a rectangular altar that must always keep its sides in the ratio 5 to 4. If the total area must be increased while preserving the same shape, both length and breadth must grow in the same proportion. In modern language, if the original sides are 5a and 4a, then the new sides might be 5ka and 4ka, where k is a scaling factor. The Sulba Sutras express such

ideas through instructions that tell the builder how to measure off multiples or fractions of given lengths using a cord.

Scaling is also used when the texts speak about constructing many different altars that all have the same area, commonly a fixed number of square purushas, but different symbolic shapes. One design might be a simple square, another might be shaped like a bird, and a third might have more complex boundaries. The arithmetic behind these designs ensures that the combined area of all pieces adds up to the same total. When bricks of different sizes are specified, their dimensions are chosen proportionally so that they fit together without leaving gaps or overlaps.

Ratios also appear in the internal divisions of an altar. A side may be split into equal parts, say four or eight segments, in order to mark positions of smaller internal structures or to shape curves with piecewise straight segments. To perform such splits in practice, the builder must apply division and sometimes understand equivalent fractions. If a side is divided into eight equal parts and three of them are needed, the length three eighths of the side must be located accurately with a cord.

In some Sulba Sutra problems, the text spells out numerical recipes for modifying an existing dimension. One may be told to increase a measure by its third or its fifth, then reduce by a small fraction of that increment. Translated to modern arithmetic, this becomes a composition of proportional steps. It shows a clear awareness that linear scaling changes area in a predictable way, and that careful choice of factors can make the new figure match a prescribed size.

Altars for special rituals sometimes needed to increase in area gradually across a series of years, while still keeping the same general shape. This requirement leads to sequences of scaling factors. Though the Sulba Sutras do not present these as algebraic sequences, the instructions effectively tell the builder how to compute a new set of side lengths each time. Ratios and scaling thus help to organise a whole family of altars that are bigger or smaller versions of a basic pattern.

## 13.5APPROXIMATE VALUES USED IN ALTAR MEASUREMENTS

The third theme in this unit is the use of approximate numerical values. The Sulba Sutras were concerned with actual construction on the ground, where some lengths cannot be expressed as

simple whole numbers or rational fractions. Instead of leaving them undefined, the texts suggest approximations that are accurate enough for ritual purposes.

The most famous such value is the approximation to the square root of two, which appears in the rule for the diagonal of a square. If a square has side one unit, its diagonal is the length needed to form a new square whose area is twice that of the original. To double an altar's area while keeping it square, the builder must lengthen the side by a factor equal to the square root of two. The Sulba Sutras give a stepwise verbal formula that, in modern form, yields a decimal value close to 1.414. This shows a careful balance between simplicity and accuracy.

Approximations also arise when the texts deal with converting a circle into a square or a square into a circle. For example, one rule says that to draw a circle equal in area to a given square, a certain fraction of the diagonal must be taken and then slightly adjusted. Conversely, to square the circle, the diameter is divided into several parts, and then a few of those parts are removed along with small fractions of what remains. What is encoded here is a rough value for the number pi, hidden inside a geometrical recipe that an altar builder can follow with a cord and pegs.

The units themselves carry approximate relationships. A purusha is a body-based unit, and an angula is a smaller measure based on finger breadth. The conversion between them, such as 120 angulas making one purusha, is exact by definition. However, when these are compared with modern metric measures, their values in centimetres become approximate. In the original context, what mattered was internal consistency. As long as the same unit and conversion were used throughout a construction, the ratios and areas preserved their intended values.

One can also see approximate thinking in the way bricks are counted and sized. If the altar should cover a fixed area using roughly 200 bricks, it may not be possible to get a perfect match with one single brick size. The Sulba Sutras solve this by introducing bricks that are halves or quarters of a base size, and by allowing small adjustments in shape. The arithmetic behind this is the search for combinations of bricks whose total area matches the target as closely as required. Although the texts do not speak of error bounds, the practice clearly accepts tiny deviations while trying to make them negligible.

These approximate values and constructions highlight a practical side of arithmetic. The goal was not to obtain pure numerical exactness in modern algebraic form, but to ensure that actual measured altars satisfied the ritual conditions with sufficient precision. The Sulba Sutras show

that ancient builders were able to handle complicated shapes using only cords, pegs, and cleverly chosen numerical recipes. Even when exact values could not be expressed in simple fractions, useful approximations were found and encoded verbally.

## CHECK YOUR PROGRESS

Q5. What does it mean to keep the sides of a rectangle in the ratio 5 to 4 while changing its size?

Q6. How does scaling affect both the shape and the size of an altar?

Q7. Why are fractions such as one half or one eighth important in dividing a side into equal parts?

Q8. In what way do the Sulba Sutras describe stepwise changes to a length using ratios?

## 13.6 LET US SUM UP

- The Sulba Sutras reveal that arithmetic forms the underlying basis of geometric rules used in the construction of Vedic fire altars, linking numerical operations directly with ritual practice.
- Whole numbers and fractions are systematically used to measure lengths, combine segments, preserve areas, and construct perpendiculars and diagonals, anticipating ideas like the Pythagorean theorem.
- Ratios and proportions play a central role in fixing altar shapes and creating similar altars of different sizes, showing a clear understanding of scaling and fractional division.
- Carefully chosen approximations, such as for $\sqrt{2}$ and for squaring the circle, demonstrate a practical approach to numerical accuracy suited to ritual requirements.
- Overall, the Sulba Sutras illustrate early applied arithmetic, where mathematical ideas emerge naturally from concrete construction needs while foreshadowing later abstract mathematics.

## 13.7 ANSWERS TO CHECK YOUR PROGRESS

**Ans to Q1:** Preserving the area of an altar was important because ritual rules specified the exact area that the altar must cover. Changing the shape without keeping the area fixed would violate these prescriptions and make the construction unsuitable for the intended sacrifice.

**Ans to Q2**: A right triangle provides a simple way to set out a right angle. By measuring sides that satisfy a fixed ratio, such as three, four, and five parts, and stretching a cord to match these, the builder can ensure that the angle between the shorter sides is a true right angle, which is then used to draw perpendicular lines.

**Ans to Q3:** Addition and multiplication appear when units are combined to reach larger lengths, for example adding several angulas to make a purusha, while division and subtraction are used when a length is split into halves, quarters, or smaller fractions and when parts are cut off to adjust a dimension.

**Ans to Q4:** The diagonal of a unit square is an irrational quantity, meaning that it cannot be expressed exactly as a simple fraction of whole numbers. The Sulba Sutras needed an approximation for it so that builders could practically mark this diagonal on the ground and use it to construct squares with double the area of a given square.

**Ans to Q5**: Keeping the sides of a rectangle in the ratio 5 to 4 means that, whatever the actual measurements, the longer side is always five parts and the shorter side is four parts of the same basic unit. When the size changes, both sides are multiplied by the same scaling factor so that their ratio remains 5 to 4.

**Ans to Q6:** Scaling multiplies all linear dimensions by a common factor, so the shape of the altar stays similar while its size changes. The angles remain the same and the ratios between corresponding sides are preserved, but the area grows or shrinks according to the square of the scaling factor.

**Ans to Q7**: Fractions such as one half or one eighth are important because they allow a side to be divided into equal segments. For instance, dividing a side into eight equal parts ensures that each segment has exactly one eighth of the total length, which is essential when marking positions or constructing smaller internal figures.

**Ans to Q8:** The Sulba Sutras describe stepwise changes by instructing the builder to increase a length by a fraction of itself, such as its third or its fourth, and sometimes then to decrease it by a smaller fraction of that increment. This layered procedure effectively applies ratios one after another to reach a carefully adjusted new length.

## 13.8 FURTHER READING


1. Bharati Krishna Tirtha, Vedic Mathematics, Motilal Banarsidass, Delhi.
2. State Council of Educational Research and Training, Fundamentals and Applications of Vedic Mathematics, New Delhi.
3. Kenneth R. Williams, Discover Vedic Mathematics, Vedic Mathematics Research Group.
4. VedicMaths.Org, “Practise and Free Books on Vedic Mathematics.”
5. Moonpreneur, “Vedic Math Sutra 1: Ekadhikena Purvena.”

## 13.9 MODEL QUESTIONS

**Q1.**Describe how the Sulba Sutras use basic arithmetic operations in setting out altar constructions on the ground.

**Q2.**Explain with an example how ratios and proportions help in preserving the shape of an altar while changing its size.

**Q3**. Discuss the importance of the approximation to the square root of two in the context of doubling the area of a square altar.

**Q4.**How do the Sulba Sutras address the problem of converting a square altar into a circular one of equal area, and what does this tell us about their use of approximate values?

**Q5.**In what ways do the arithmetic ideas in the Sulba Sutras show a connection between practical craftsmanship and abstract mathematical thinking?

# UNIT 14: VEDIC RATIO AND PROPORTION

## UNIT STRUCTURE

14.1 Learning Objectives
14.2 Introduction
14.3 Concept of Ratio
14.4 Relevance of Ratios in Vedic arithmetic
14.5 Concept of Proportion
7.5.1 Direct proportion
7.5.2 Inverse proportion
14.6 Crosswise Method (UrdhvaTiryak Sutra) for Proportion
14.7 Let Us Sum Up
14.8 Answers to Check Your Progress
14.9 Further Reading
14.10 Model Questions

## 14.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- Explain the meaning of a ratio using simple numerical examples.
- Interpret ratios in daily life and in basic Vedic arithmetic problems.
- Describe the idea of proportion and identify proportional relationships in numbers.
- Use the UrdhvaTiryak Sutra to solve simple problems involving proportion.

## 14.2 INTRODUCTION

In earlier units, the focus was on basic operations such as addition, subtraction, multiplication and division using Vedic methods. In this unit, the attention moves to comparison of quantities rather than only to their totals or differences. Ratios and proportions help to compare numbers in a compact way. In Vedic arithmetic, these ideas blend naturally with sutras like UrdhvaTiryak, which allows quick mental handling of proportional relationships. This approach helps students see ratio and proportion not as dry rules but as patterns that can be observed, extended and computed with ease.

## 14.3 CONCEPT OF RATIO

A ratio compares two quantities of the same kind by division. When we write the ratio of 3 to 5, we usually write it in the form $3:5\ or\ \frac{3}{5}$. The two numbers being compared are called the terms

of the ratio, with the first term known as the antecedent and the second term as the consequent. If three apples cost 15 rupees, the ratio of cost to number of apples is $15:3$, which can be simplified to $5:1$ by dividing both terms by 3.

Ratios are usually expressed in their simplest form. To simplify a ratio, both terms are divided by their common factor, just as in fractions. For example, the ratio $18:24$ can be reduced to $3:4$ by dividing both terms by 6. When two ratios are equal after simplification, they are said to be equivalent ratios, such as $2:3$ and $10:15$.

Ratios have no unit, because the units in numerator and denominator are the same and cancel out. For instance, if a length of 8 cm is compared with a length of 20 cm, the ratio $8:20$ simplifies to $2:5$, and this is written without any unit. This makes ratios useful for comparing lengths, weights, times or other quantities without worrying about the specific units.

Ratios can compare more than two quantities as well. If three brothers share 600 rupees in the ratio 2:3:5, it means that for every 2 parts received by the first, the second receives 3 parts and the third receives 5 parts. The total number of parts is $2+3+5=10$, and each part is worth $\frac{600}{10}=60$ rupees. The shares are then 120, 180 and 300 rupees respectively.

**Ratios in daily situations:**

Many daily situations can be understood clearly by expressing them as ratios. If a class has 12 girls and 18 boys, the ratio of girls to boys is $12:18$, which simplifies to $2:3$. If a drink is prepared by mixing 1 part of syrup with 4 parts of water, the ratio of syrup to water is $1:4$, while the ratio of syrup to total drink is $1:5$. When a recipe uses 2 cups of flour and 1 cup of sugar, the sweetness of the mixture depends on the ratio $2:1$, and any preparation that keeps this ratio will taste similar.

Ratios are also used to describe speed, density and other practical measures. If a car travels 60 kilometres in 1 hour, the ratio of distance to time is $60:1$, which is usually written as 60 km per hour. In a map, a scale might be written as $1:1{,}00{,}000$, which means that 1 cm on the map represents 1,00,000 cm in the real world. By reading ratios in this way, students begin to see them as compressed descriptions of real relationships.

## 14.4 RELEVANCE OF RATIOS IN VEDIC ARITHMETIC

In Vedic mathematics, many techniques rely on seeing patterns and proportional relationships in numbers rather than only performing mechanical steps. When a problem is expressed in terms of ratios, it often reveals a structure that can be attacked with an appropriate sutra. For instance, the sutra “Samuccaya” is applied when combined terms show a certain balance, and such balance can often be expressed using ratios. Ratios thus act as a bridge between the language of a word problem and the compact numerical form needed for a sutra based solution.

Ratios also help in checking the reasonableness of answers obtained by fast Vedic methods. If the ratio of two quantities in the question is known, the same ratio or a related one should often appear in the answer. For example, in a simple sharing problem where the ratio is $2:3$, any final

shares must preserve this ratio after simplification. This consistency acts as a mental “digit sum” kind of check at the level of proportional relationships.

Moreover, ratios prepare the ground for later topics such as percentages, simple interest and profit and loss, where the core idea is still comparison. If a quantity is described as “increased by one fourth”, it means that the new quantity is to the old quantity in the ratio $5:4$. Handling these changes becomes easier when a student is comfortable thinking in ratios and can move between ratio form, fraction form and verbal descriptions without confusion.

## 14.5CONCEPT OF PROPORTION

A proportion is a statement that two ratios are equal. For example, the statement $\frac{2}{3}=\frac{4}{6}$ expresses that $2:3$ is in proportion to $4:6$. In words, one says “2 is to 3 as 4 is to 6”. The four numbers 2, 3, 4 and 6 are called the terms of the proportion, with 2 and 6 known as the extremes and 3 and 4 as the means. In any true proportion, the product of the extremes equals the product of the means.

The equality of cross products is the basic test for proportion. If $\frac{a}{b}=\frac{c}{d}$, then $a\times d=b\times c$. For instance, to check whether $5:7$ and $15:21$ are in proportion, compute the cross products: $5\times 21=105$ and $7\times 15=105$. As the products match, the two ratios form a proportion. This simple test allows one to verify or construct proportions even when the numbers are large.

Proportion is especially useful when one quantity changes and another must change in a matching way. If 3 notebooks cost 45 rupees, the cost per notebook is 15 rupees, and the proportion $\frac{3}{45}=\frac{7}{x}$ can be used to find the cost of 7 notebooks. Setting up the proportion correctly and then solving it reveals how the second quantity scales. In this example, the value of $x$ must satisfy $3x=45\times 7$, so $x=105$.

There are two broad types of proportion commonly used: direct proportion and inverse proportion. In direct proportion, as one quantity increases, the other quantity also increases in the same ratio. Inverse proportion behaves in the opposite way: as one quantity increases, the other decreases so that their product remains constant. For example, the time taken to complete a fixed amount of work by a group of workers is inversely proportional to the number of workers, assuming all work at the same rate.

### 14.5.1DIRECT PROPORTION

Two quantities are in direct proportion if the ratio of their corresponding values stays constant. Suppose 4 pens cost 40 rupees and 6 pens cost 60 rupees. The ratios of number of pens to cost are $4:40$ and $6:60$, which both simplify to $1:10$. If the number of pens is doubled, the cost is also doubled; if the number of pens is tripled, the cost becomes three times. This constant pattern of change is the hallmark of direct proportion.

Direct proportion problems can often be written in the form $\frac{x_1}{y_1}=\frac{x_2}{y_2}$, where $x$ and $y$ represent the two related quantities. Once three of these values are known, the fourth can be found by cross

multiplication. In class situations, many such problems occur: distance and time at constant speed, number of items bought and total cost, or length and breadth of similar figures, all follow direct proportion.

## 14.5.2INVERSE PROPORTION

In inverse proportion, the product of two quantities is constant, so that when one increases, the other decreases. If 4 workers take 9 days to finish a job, the total work can be expressed as $4 \times 9 = 36$ worker days. If 6 workers are employed instead, and they work at the same rate, the time needed will be such that $6 \times time = 36$, giving $time = 6\,days$. Here, the ratios of number of workers to time are 4:9 and 6:6, which do not match, but the products $4 \times 9\,and\,6 \times 6$ are equal.

To handle inverse proportion in symbolic form, one can use the relation $x_1y_1 = x_2y_2$. If any three of these values are known, the fourth can be obtained by simple division. For example, if a pump can empty a tank in 8 hours and a second identical pump is added, the time taken becomes 4 hours, since $1 \times 8 = 2 \times 4$. Understanding this pattern is important when using Vedic methods, because the form of the proportion decides which steps will be rapid and safe.

## CHECK YOUR PROGRESS

Q1. Write any two equivalent forms of the ratio $18:24$ and explain why they are equivalent.
Q2. A mixture contains water and milk in the ratio $3:5$. If the total quantity of the mixture is 32 litres, find the quantities of water and milk.
Q3. Check whether the ratios $7:9\,and\,21:27$ are in proportion. Show the cross products.
Q4. Four workers take 15 days to build a wall. Assuming the same type of work and workers, in how many days will 10 workers build the same wall, if time varies inversely with number of workers?

## 14.5CROSSWISE METHOD (URDHVA TIRYAK SUTRA)FOR PROPORTION

The UrdhvaTiryak Sutra, often translated as “vertically and crosswise”, is mainly known as a fast multiplication method. Its crosswise pattern, however, can also be applied to problems of proportion. In a simple proportion involving four numbers, such as

$a:b = c:d$, the classic rule of cross multiplication already matches the spirit of Urdhva Tiryak. The product of the extremes $a \times d$ equals the product of the means $b \times c$, and this equality can be visualised as crossing lines between the terms.

Consider a proportion where one term is unknown, such as $\frac{a}{b} = \frac{c}{x}$. By the crosswise idea, one writes $a \times x = b \times c$, and then solves for $x$ as $x = \frac{b \times c}{a}$. The crosswise multiplication $ac$ and $bx$ uses the same mental movement as Urdhva Tiryak multiplication, which combines vertical and diagonal products. Once this habit is formed, solving simple proportion problems becomes a single mental step instead of several written steps.

In Vedic style, one often writes the known ratio and the desired ratio in a neat pair and then immediately visualises the cross products. If a map is drawn with scale 1:50,000, and a distance of 3 cm on the map corresponds to some distance $x$ km on the ground, the proportion $\frac{1}{50{,}000} = \frac{3}{x}$ can be written. Applying crosswise thinking gives

$1 \times x = 3 \times 50{,}000$, so $x = 1{,}50{,}000$ cm, or 1.5 km. The steps are the same as conventional cross multiplication, but the emphasis is on quick mental pairing.

**Using UrdhvaTiryak pattern in word problems:**

Word problems on ratio and proportion often require setting up one or more proportions before any computation is done. The UrdhvaTiryak way of thinking encourages students to keep all four terms of the proportion in view at once and to strike the necessary cross products mentally. For example, if 8 metres of cloth cost 560 rupees, and the cost of 15 metres is required, the proportion is $\frac{8}{560} = \frac{15}{x}$. Crosswise multiplication gives $8x = 560 \times 15$. Instead of writing all intermediate steps, a student can compute the cross product $560 \times 15$ using a suitable Vedic multiplication method and then divide by 8.

In some problems, one of the ratios may be presented as a fraction and the other as a ratio, but the crosswise method still applies. Suppose a recipe uses sugar and flour in the ratio 2:5, and a cook has 300 grams of flour. The question asks how much sugar is needed for the same taste. The proportion $\frac{2}{5} = \frac{x}{300}$ is written, and crosswise multiplication gives $5x = 600$, so $x = 120$ grams. By training to arrange the numbers clearly and apply the crosswise pattern, students reduce errors and gain speed.

The UrdhvaTiryak pattern becomes even more helpful when several proportional steps are combined. For instance, if a worker's wage is directly proportional to the number of hours worked, and also proportional to the number of units produced per hour, two proportions may need to be blended. In such cases, each little proportion is handled by the same crosswise structure, so the learner does not have to remember separate "formulas" for every situation. Vedic arithmetic aims at this kind of unity of method.

## CHECK YOUR PROGRESS

Q5. If 7 kg of rice cost 280 rupees, use the crosswise method to find the cost of 11 kg of rice.
Q6. A map has a scale of $1:2{,}00{,}000$. If two cities are 350 km apart in reality, find the distance between them on the map, using the idea of proportion.
Q7. The ratio of sugar to flour in a cake recipe is $3:8$. If 520 grams of flour are used, find the amount of sugar needed so that the same proportion is maintained.

## 14.7 LET US SUM UP

- A ratio compares two quantities of the same kind by division, expressed in simplest form without units, and can extend to more than two quantities for sharing problems like dividing money among brothers.
- A proportion states two ratios are equal, verified by equal cross products of extremes and means; it includes direct proportion (constant ratio as quantities increase together) and inverse proportion (constant product, like workers and time).
- Ratios appear in everyday scenarios such as class demographics, recipes, speeds, and maps; in Vedic arithmetic, they bridge word problems to sutras and enable quick checks on solutions.
- The crosswise method from UrdhvaTiryak Sutra solves proportions by mentally computing cross products, applicable to word problems like costs, maps, and recipes for rapid mental arithmetic.

## 14.8 ANSWERS TO CHECK YOUR PROGRESS

**Ans to Q1:** The ratio $18:24$ can be written as $3:4$ by dividing both terms by 6, and as 9:12 by dividing each term by 2. All these are equivalent because each pair of numbers has the same simplified form $3:4$.

**Ans to Q2:** The total number of parts in the ratio $3:5\ is\ 3\ +\ 5\ =\ 8$. Each part is worth $\frac{32}{8} = 4$ litres. Water corresponds to 3 parts, so its quantity is $3\ \times\ 4\ =\ 12$ litres. Milk corresponds to 5 parts, so its quantity is $5\ \times\ 4\ =\ 20$ litres.

**Ans to Q3**: Compute the cross products: $7 \times 27 = 189$ and $9 \times 21 = 189$. As the products of extremes and means are equal, $7:9\ and\ 21:27$ form a true proportion.

**Ans to Q4**: Let the required number of days be $x$. In inverse proportion, number of workers × time is constant, so $4\ \times\ 15\ =\ 10\ \times\ x$. This gives $60\ =\ 10x, so\ x = 6\ days$.

**Ans to Q5:** Let the cost of 11 kg be $x$. The proportion is $\frac{7}{280} = \frac{11}{x}$. Cross multiplication gives $7x = 280 \times 11$. Thus $7x = 3080$, so $x = \frac{3080}{7} = 440$ rupees.

**Ans to Q6**: Let the map distance be $x$ cm. The scale $\frac{1}{2,00,000} = \frac{x}{\text{actual distance in cm}}$ is used. Convert 350 km to cm: 350 km = 35,000,000 cm. Then $\frac{1}{2,00,000} = \frac{x}{35,000,000}$. Crosswise, $1 \times 35,000,000 = 2,00,000 \times x$, so $x = \frac{35,000,000}{2,00,000} = 175$ cm.

**Ans to Q7**: Let the required sugar be $x$ grams. The proportion is $\frac{3}{8} = \frac{x}{520}$. Crosswise multiplication gives $3 \times 520 = 8x$, so $1560 = 8x$, hence $x = 195$ grams.

## 14.9 FURTHER READING

1. Bharati Krishna Tirtha, Vedic Mathematics, Motilal Banarsidass, Delhi.

2. State Council of Educational Research and Training, Fundamentals and Applications of Vedic Mathematics, New Delhi.
3. Kenneth R. Williams, Discover Vedic Mathematics, Vedic Mathematics Research Group.
4. VedicMaths.Org, “Practise and Free Books on Vedic Mathematics.”
5. Moonpreneur, “Vedic Math Sutra 1: Ekadhikena Purvena.”

## 14.10 MODEL QUESTIONS

**Q1.** Define ratio and write three different examples from daily life where ratios are used for comparison.

**Q2.** Explain how a ratio with three terms, such as 2:3:5, can be used to share a sum of money. Illustrate with a numerical example.

**Q3.** What is a proportion? State the property of extremes and means in a proportion, and verify it for the numbers 6, 8, 9 and 12.

**Q4.** Distinguish between direct and inverse proportion with suitable examples from real life.

**Q5.** A certain paint is prepared by mixing red and white colours in the ratio 3:7. If 2.5 litres of red colour are used, find the quantity of white colour needed to keep the same ratio.

**Q6.** Using the idea of proportion, solve: If 12 metres of cloth cost 780 rupees, how much will 19 metres of the same cloth cost?

**Q7.** Show how the UrdhvaTiryak idea of “vertically and crosswise” is reflected in the usual method of solving $\frac{a}{b} = \frac{c}{x}$ for $x$.

**Q8.** A recipe for fruit juice uses 4 parts of concentrate and 9 parts of water. If a caterer wants to prepare 26 litres of juice with the same taste, find the quantities of concentrate and water.