PG MA S1- 01 (DSC)

TWO YEAR MASTER'S DEGREE COURSE

SEMESTER - I

Abstract Algebra

MATHEMATICS

Discipline Specific Core Course
MASTER OF SCIENCE IN MATHEMATICS

Hiranya Chandra Bhuyan School of Science and Technology

KRISHNA KANTA HANDIQUI STATE OPEN UNIVERSITY

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November, 2024

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Printed and published by Registrar on behalf of the Krishna Kanta Handiqui State Open University.

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The University acknowledges with thanks the financial support provided by the **Distance Education Council**, New Delhi, for the preparation of this study material.

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COURSE INTRODUCTION

"Abstract Algebra" is the first Discipline Specific core course of M.Sc (Mathematics) programme. Throughout this course, we will gradually build a solid foundation in abstract algebra, starting from the basic definitions and properties, and progressing to more advanced topics. Each unit will focus on a specific aspect of abstract algebra, allowing you to develop a deep understanding of the subject matter.

Unit 1: Introduction to Groups

In this unit, we will lay the foundational concepts of groups, one of the fundamental algebraic structures in mathematics.Throughout this unit, we will explore the definition of groups, basic properties, and examples to develop a solid understanding of this essential algebraic structure.

Unit 2: Lagrange's Theorem, **Normal subgroup and Quotient Groups**

We will begin by examining Lagrange's Theorem, a fundamental result that elucidates the relationship between the orders of subgroups and the order of the parent group. This theorem serves as a cornerstone in group theory and offers valuable insights into the composition of finite groups.Finally, we will explore quotient groups, which arise from the partitioning of a group by a normal subgroup. Quotient groups provide a powerful framework for understanding the structure-preserving properties of group homomorphisms and offer a deeper understanding of group theory.

Unit 3: Permutation Groups

In this unit, we will extend our exploration of groups by examining permutation groups, which play a crucial role in many areas of mathematics, particularly in combinatorics and group theory. By the end of this unit, you will have a solid understanding of permutation groups, enabling you to analyze and solve problems in abstract algebra.

Unit 4: Homomorhisms of Groups

In this unit, we will explore fundamental concepts in group theory that focus on the relationship between different groups, namely homomorphisms and isomorphisms.By the end of this unit, you will have a solid understanding of homomorphisms and isomorphisms, enabling you to analyze and compare groups with precision and clarity.

Unit 5: Direct Products of Groups and Finite Abelian Groups

In this unit, we will explore two important topics in group theory: direct products and finite abelian groups. These concepts provide valuable insights into the structure of groups and allow us to classify and understand a wide range of mathematical objects.We will begin by studying direct products of groups, which provide a systematic way of combining the elements and operations of multiple groups into a single group.Next, we will focus on finite abelian groups, which are

groups that are both finite in size and abelian, meaning that their group operation is commutative.By the end of this unit, you will have a solid understanding of direct products of groups and finite abelian groups, enabling you to analyze and classify groups.

Unit 6: Group Series and Structure

In this unit, we will delve into advanced concepts in group theory that focus on the structure and classification of groups. Next, we will delve into the structure of finite groups, focusing on important results such as the Jordan-Hölder theorem and the Sylow theorems.By the end of this unit, you will have a solid understanding of group series and the structure of finite groups.

Unit 7: Class Equation and Cauchy's Theorem

In this unit, we will explore two important concepts in group theory that provide insights into the structure and composition of groups: the class equation and Cauchy's theorem. By the end of this unit, you will have a solid understanding of the class equation, Cauchy's theorem, and their implications for the structure and composition of groups.

Unit 8: Group Actions and Sylow Theorems

In this unit, we will explore two fundamental concepts in group theory that provide deep insights into the structure and behavior of groups: group actions and Sylow theorems.By the end of this unit, you will have a solid understanding of group actions, Sylow theorems, and their implications for the structure and behavior of groups.

Unit 9: Rings, Integral Domains and Fields

In this unit, we will explore fundamental algebraic structures that generalize the concept of arithmetic operations: rings, integral domains, and fields. These structures provide the foundation for abstract algebraic reasoning and have wide-ranging applications in mathematics and beyond.By the end of this unit, you will have a solid understanding of rings, integral domains, and fields, enabling you to analyze and solve problems in abstract algebra

Unit 10: Factor Rings and Ring Homomorphism

In this unit, we will delve deeper into the study of rings, exploring important concepts such as factor rings and ring homomorphisms. These concepts play a crucial role in abstract algebra, providing powerful tools for understanding the structure and behavior of rings.By the end of this unit, you will have a solid understanding of factor rings, ring homomorphisms, and their implications for the study of rings and abstract algebra.

Unit 11: Polynomial Rings

In this unit, we will explore one of the most important algebraic structures in mathematics: polynomial rings. Polynomial rings play a central role in various areas of mathematics, including algebra, number theory, geometry, and computer science.By the end of this unit, you will have a solid understanding of polynomial rings and their implications for the study of algebra and mathematics as a whole.

Unit 12: Factorization in Integral Domains and Field Extensions

In this unit, we will explore the fundamental concepts of factorization theory within integral domains and field extensions. By the end of this unit, you will have a solid understanding of factorization theory within integral domains and field extensions, enabling you to analyze and solve problems in abstract algebra.

Unit 13: Splitting Field

In this unit, we will explore the concept of splitting fields, which is a fundamental topic in field theory and algebraic extensions. Splitting fields play a crucial role in understanding the roots of polynomials and the structure of algebraic extensions.By the end of this unit, you will have a solid understanding of splitting fields and their implications for the study of field theory and algebraic extensions.

Unit 14: Normal Extensions and Perfect Fields

In this unit, we will delve into two important concepts in field theory: normal extensions and perfect fields. These concepts play a crucial role in understanding the structure and properties of field extensions.By the end of this unit, you will have a solid understanding of normal extensions, perfect fields, and their implications for the study of field theory and algebraic structures.

Unit 15: Galois Groups and Galois Extensions

In this unit, we will explore one of the most fascinating and important topics in algebraic theory: Galois theory. Developed by Évariste Galois in the early 19th century, Galois theory revolutionized the study of field extensions and provided deep insights into the structure of polynomials and their roots.Throughout this unit, we will explore various properties and theorems related to Galois groups, including the fundamental theorem of Galois theory, which establishes a correspondence between subfields of a Galois extension and subgroups of its Galois group.By the end of this unit, you will have a solid understanding of Galois groups, Galois extensions, and their implications for the study of field theory and algebraic structures.

Each unit includes some along-side boxes to help you know some of the difficult, unseen terms. Some “EXERCISES” have been included to help you apply your own thoughts. You may find some boxes marked with: “LET US KNOW”. These boxes will provide you with some additional interesting and relevant information. Again, you will get “CHECK YOUR PROGRESS” questions. These have been designed to self-check your progress of study. It will be helpful for you if you solve the problems put in these boxes immediately after you go through the sections of the units and then match your answers with “ANSWERS TO CHECK YOUR PROGRESS” given at the end of each unit.