PG MA S1- 02 (DSC)

TWO YEAR MASTER'S DEGREE COURSE

SEMESTER - I

Differential Equations

MATHEMATICS

Discipline Specific Core Course
MASTER OF SCIENCE IN MATHEMATICS

Hiranya Chandra Bhuyan School of Science and Technology

KRISHNA KANTA HANDIQUI STATE OPEN UNIVERSITY

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November, 2024

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Printed and published by Registrar on behalf of the Krishna Kanta Handiqui State Open University.

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The University acknowledges with thanks the financial support provided by the **Distance Education Council**, New Delhi, for the preparation of this study material.

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COURSE INTRODUCTION

"Differential Equations "is the second Discipline specific core course of MSc Mathematics Programme.Differential equations are fundamental mathematical tools used to model and analyze various phenomena in science, engineering, economics, and many other fields. This course is designed to introduce you to the theory and applications of differential equations, covering both ordinary and partial differential equations.

Unit 1: Introduction to differential Equations

In this unit, we'll start by defining differential equations and understanding their classification based on order and degree. We'll explore initial value problems (IVPs) and boundary value problems (BVPs), setting the stage for further exploration into solving these types of equations.

Unit 2: Differential Equations of First order and First Degree

Here, we'll dive into differential equations of the first order and first degree. We'll cover techniques such as the variable separable method, homogeneous and linear differential equations, and Bernoulli's equation, which are fundamental for solving various types of differential equations.

Unit 3: Exact Differential Equation of first order and first degree

This unit focuses on exact differential equations of the first order and first degree. We'll learn about exact equations and methods for reducing equations to exact form, providing us with powerful tools for solving differential equations.

Unit 4: Existence and Uniqueness

In this unit, we will delve into fundamental concepts regarding the existence and uniqueness of solutions to ordinary differential equations (ODEs). Furthermore, we will discuss the fundamental theorem of existence and uniqueness, which provides conditions under which solutions to initial value problems for ordinary differential equations exist and are unique.

Unit 5: Linear Homogeneous Differential Equation of Higher Order

In this unit, we will extend our study of differential equations beyond first-order equations to explore higher order linear differential equations. We will explore different types of higher order linear differential equations, including homogeneous and non-homogeneous equations, constant coefficient equations, and variable coefficient equations.

Unit 6: Orthogonality and Independence in ODEs

In this unit, we will explore important concepts related to the orthogonality and independence of solutions to linear homogeneous ODEs. We will begin by defining what it means for solutions to a differential equation to be orthogonal or independent. Orthogonality of solutions refers to the property that their inner product is zero, while independence of solutions means that no solution can be expressed as a linear combination of the others.

Unit 7: First order Linear partial Differential Equations

In this unit, we will explore a fundamental class of partial differential equations (PDEs) known as first-order linear PDEs. First-order linear PDEs involve partial derivatives of the dependent variable(s) with respect to one or more independent variables, and they exhibit linearity in both the dependent variable(s) and their derivatives.

Unit 8: Cononical forms, Monge's Method and Method of Separation of Variables

In this unit, we will explore a fundamental class of partial differential equations (PDEs) where the coefficients are constants and the highest-order derivatives involve second derivatives.

Unit 9: Second and Higher Order PDE with Constant Coefficients

In this unit, we will extend our study of partial differential equations (PDEs) beyond second-order equations to explore higher order linear PDEs.

Unit 10: First order Nonlinear PDE-I

n this unit, we will explore a fundamental class of partial differential equations (PDEs) known as first-order nonlinear PDEs. First-order nonlinear PDEs arise in various fields of science and engineering, describing phenomena with complex dynamics, such as fluid flow, reaction-diffusion systems, and nonlinear optics.

Unit 11: First Order Nonlinear PDE-II

In this unit, we will continue our exploration of first-order nonlinear partial differential equations (PDEs), focusing on more advanced topics and techniques for analyzing and solving these equations. Nonlinear PDEs arise in various fields of science and engineering, describing phenomena with complex and nonlinear dynamics.

Unit 12: Elliptic Differential Equations

In this unit, we will explore a class of partial differential equations (PDEs) known as elliptic differential equations. Elliptic equations are widely encountered in various fields of science and engineering, including physics, mathematics, and engineering.

Unit 13: Parabolic Differential Equations

In this unit, we will explore a class of partial differential equations (PDEs) known as parabolic differential equations.Parabolic equations are characterized by their second-order derivatives with respect to time and first-order derivatives with respect to space.

Unit 14: Hyperbolic Differential Equations

In this unit, we will explore a class of partial differential equations (PDEs) known as hyperbolic differential equations.Hyperbolic equations are characterized by their second-order derivatives with respect to both time and space, and they often arise in problems involving waves and oscillations.

Each unit includes some along-side boxes to help you know some of the difficult, unseen terms. Some "EXERCISES" have been included to help you apply your own thoughts. You may find some boxes marked with: "LET US KNOW". These boxes will provide you with some additional interesting and relevant information. Again, you will get "CHECK YOUR PROGRESS" questions. These have been designed to self-check your progress of study. It will be helpful for you if you solve the problems put in these boxes immediately after you go through the sections of the units and then match your answers with "ANSWERS TO CHECK YOUR PROGRESS" given at the end of each unit.