UNIT-10: QUANTIFICATION AND RULES OF QUANTIFICATION

UNIT STRUCTURE

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10.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- define what is predicate logic
- define what is quantification
- explain the types of quantifiers
- discuss the rules of quantification
- discuss the procedures of proving validity of arguments

10.2 INTRODUCTION

Quantification is a twentieth century development which is chiefly credited to GottlobFrege (1848-1945), a great German logician. But the word 'quantification' was introduced by Charles S. Peirce. Quantification falls under the category of predicate logic and not propositional logic.

Predicate logic is defined as that part of symbolic logic where an internal structure of the proposition (both simple and compound) is to be analysed in order to determine the validity of a given argument. Quantification is the process of obtaining a proposition from a propositional function. A propositional function is an expression that contains an individual variable and becomes a statement when an individual constant is substituted for the individual variable. There are two ways of obtaining propositions from propositional functions. One is called Instantiation or Substitution. The other is called Quantification. There are different phrases which are employed to indicate quantity. These phrases are called quantifiers. Words that indicate quantity such as 'all', 'every', 'most' are called quantifiers. Though there are many numbers of quantifiers in English, standard logic uses only two: the maximal 'all' and the minimal 'some'. There are two types of quantifiers: Universal Quantifier and Existential Quantifier. Quantifiers state how many of the individuals or the objects have the property in question. They do not say anything about which one of the individuals have the property.

10.3 UNIVERSAL AND EXISTENTIAL QUANTIFIERS:

Universal Quantifiers- A universal quantifier can be defined as an expression that, when prefixed to a propositional function, yields a universal proposition. Examples of universal propositions are:

Everything is dynamic.

Every student is good.

All things are temporary.

Statement which contain words like 'all', 'every', 'each', 'any', 'everything', 'anything' or any word conveying the sense of 'all' are symbolized by the aid of universal quantifier. Let's take the general proposition- 'Everything is temporary'.

It can be expressed as: Given any individual thing whatever, it is temporary.

We can use 'x' (an individual variable) in place of both the pronoun 'it' and its antecedent 'thing'. We can paraphrase the proposition as: Given any x, x is temporary.

Here we have got a property, i.e. 'being temporary'. In order to symbolize this property, as it is already mentioned in the previous section, we need predicate symbol. We can symbolize the property 'being temporary' as 'Tx'. The phrase 'given any x' is called a universal quantifier. 'Given any x' is conventionally symbolized as '(x)'. The general proposition 'everything is temporary' can be symbolized as: (x) Tx

Existential Quantifiers- An existential quantifier can be defined as an expression that, when prefixed to a propositional function, yields an existential proposition. The existential quantifier '(∃x)' is used to assert that some entities (at least one) have a given property. Thus to symbolize the proposition "Something is light" or the proposition "At least one thing is light", start with the proposition from 'Lx' and prefix an existential quantifier to it. The result is the sentence (∃x)Lx, read "For some x, x is light", or "There is at least one x such that x is light", or just "Some x is light". A few more examples are:

Something is expensive. (∃x)Ex.

Something is important. (∃x)Ix.

Let us take a negative proposition:

"Some things are not temporary." It can be written as- There is at least one x, such that x is not temporary.

Here one important point is to be noted that the 'not' appears before the predicate, it does not appear in the quantity part. With the help of existential quantifier it can be translated as: (∃x)~Tx

10.4 MODERN SQUARE OF OPPOSITION:

The traditional logicians 'Square of Opposition' are criticized by the modern logicians in relation to the concept of existential import. We know that A and E propositions do not have existential import, but I and O propositions do have the existential import. The result of the acceptance of

existential import by the modern logicians led them come to the conclusion that only one relation, i.e. 'Contradictory' among the propositions is valid. Other relations contrary, sub-contrary, subaltern are invalid.

Let us first discuss the contrary relation. The traditional account of the contrary relation states that propositions 'All S is P' and 'No S is P' cannot be true together, though both of them can be false together. We know that A and E propositions do not have existential import. Because of the absence of existential import in A and E propositions, according to the modern logicians, A and E both cannot be true together. Modern logicians therefore hold the view that contrary relation is to be given up.

Again, we know that the traditional account of subcontrary relation between 'Some S is P' and 'Some S is not P' states that they may be true together, but cannot be false together. According to the Boolean interpretation, I and O propositions have existential import and because of the existential import, I and O can be false together. It follows that subcontrary relation is to be given up according to the modern logicians.

According to the Boolean interpretation in the Square of Opposition, only the relation of contradiction is valid. The following equivalences are valid in the modern square of opposition:

A = ~O

E = ~I

I = ~E

O = ~A

These equivalences can be employed to establish the equivalence of the universal and the existential quantifiers in the following way:

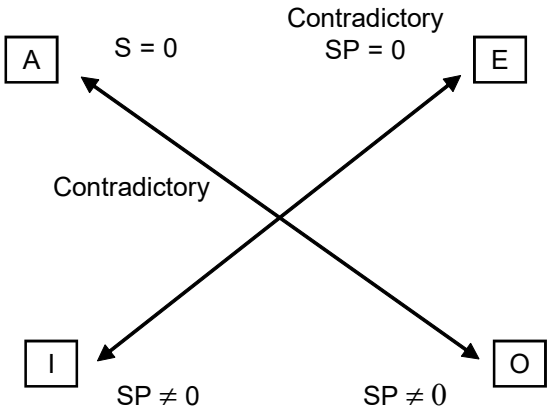
$[(x) (Sx \supset Px)] = \sim [(\exists x) (Sx \bullet \sim Px)]$

$[(x) (Sx \supset \sim Px)] = \sim [(\exists x) (Sx \bullet Px)]$

$[(\exists x) (Sx \bullet Px)] = \sim [(x) (Sx \supset \sim Px)]$

$[(\exists x) (Sx \bullet \sim Px)] = \sim [(x) (Sx \supset Px)]$

The following diagram will represent the modern square of opposition:



These four equivalences are known as Quantifier Negation (Q.N) Equivalences.

10.5 SYMBOLIZATION OF FOUR CATEGORICAL STATEMENTS WITH QUANTIFIERS (A, E, I AND O)

On the basis of quality and quantity, traditional logicians classified propositions into four types: A, E, I and O. Let us take examples for each of these propositions:

Universal affirmative(A):(All humans are mortal).

Universal negative(E): (No humans are mortal).

Particular affirmative(I): (Some humans are honest).

Particular negative(O): (Some humans are not mortal).

Each of these types is commonly referred to by its letter: the two affirmative propositions, Aand I(from the Latin 'affirmo', "I affirm"); and the two negative propositions, Eand O(from the Latin 'nego', "I deny").

Universal affirmative (A)

All humans are mortalcan be written as-

Given any individual thing whatever, if it is human then it is mortal.

Given any x, if x is human then x is mortal.

Given any x, x is human $\supset$x is mortal.

It is symbolized as- (x)(Hx$\supset$Mx)

Universal negative (E)

No humans are mortalcan be written as-

Given any individual thing whatever, if it is human, then it is not mortal.

Given any x, if x is human, then x is not mortal.

Given any x, x is human $\supset$ x is not mortal.

It is symbolized as- $(x)(Hx \supset \sim Mx)$

Particular affirmative (I)

Some humans are honestcan be written as-

There is at least one thing that is human and honest.

There is at least one thing such that it is human and it is honest.

There is at least one x such that x is human and x is honest.

There is at least one x such that x is human • x is honest.

It is symbolized as- $(\exists x)(Hx \bullet Mx)$

(In this proposition both the words 'human' and 'honest' start with the letter 'H'. Therefore in order to distinguish between these two words, we can take 'H' for the word 'human' and 'M' for the word 'honest'.

Particular negative (O)

Some humans are not mortal can be written as-

There is at least one thing that is human but not mortal.

There is at least one thing such that it is human and it is not mortal.

There is at least one x such that x is human and x is not mortal.

It is symbolized as- $(\exists x)(Hx \bullet \sim Mx)$

This is the way of symbolizing A, E, I and O propositions with the help of quantifiers.

10.6 SYMBOLIZATION OF UNIVERSAL AND EXISTENTIAL PROPOSITIONS:

Universal and existential propositions are symbolized by the process of generalization by using quantifiers. There are two quantifiers, namely, universal quantifier and existentialquantifier. The universal quantifier is symbolically expressed as '(x)' and theexistentialquantifier as '($\exists$ x)'.

Among the A,E,I and O propositions 'A' and 'E' propositions are symbolized by using the universal quantifier '(x)' and 'I' and 'O' propositions are symbolized by using the existentialquantifier '(∃x)'.

Universal proposition-

1.	Everything is movable.	Symbolic form: (x) Mx
2.	Nothing is permanent.	Symbolic form: (x) ~ Px
3.	Only teachers are invited.	Symbolic form: (x) (Ix ⊃Tx)
4.	None but citizens are eligible to vote.	Symbolic form: (x) (Ex ⊃Cx)
5.	All bananas and mangoes are sweet.	Symbolic form: (x) [(Bx v Mx) ⊃Sx]

Existential proposition-

1.	Lions exist.	Symbolic form: (∃x) Lx
2.	Not everything is mortal.	Symbolic form: (∃x) ~ Mx
3.	Not all dogs are mammals.	Symbolic form: (∃x) (Dx• ~Mx)
4.	There are elephants.	Symbolic form: (∃x) Ex
5.	Unicorns do not exist.	Symbolic form: (∃x) ~ Ux

10.7 SYMBOLIZATION OF PROPOSITIONS BY THE PROCESS OF INSTANTIATION:

Only singular propositions are symbolized by the process of instantiation. By writing an attributesymbol immediately to the left of an individual symbol, we symbolize the singular proposition affirming that the individual named has the attribute specified. Thus the singular proposition, "Socrates is human," will be symbolized simply as 'Hs'.

Let us take an example of a compound expression consisting of more than one singular proposition. For example, "Neeta is beautiful and intelligent". In this case, the first step is to symbolize the singular propositions separately following the rules of instantiation. In the second step, the logical constant, i.e. 'and' is to be symbolized. In the last step, the symbolized singular propositions are to be conjoined.

Here 'Neeta is beautiful' is a singular proposition. It is to be symbolized as 'Bn'. Again, 'Neeta is intelligent' is also a singular proposition. It is to be symbolized as 'In'. Now these two singular propositions are to be conjoined with the help of the logical constant '.' (Dot). After symbolization of the compound expression it will take the form: Bn • In

10.8 RULES OF QUANTIFICATION

We now introduce four quantifier rules governing the use of quantifiers.

Universal Instantiation (U.I.)

The rule of Universal Instantiation (U.I.) permits us to drop the universal quantifier. It permits us to move from a universally quantified proposition to a proposition which is not quantified. From a quantified formula '(x)Rx' we allowed by the rule of UI to draw the conclusion 'Ra' or 'Rb' or 'Rc' etc. Let us consider the following example:

All philosophers are honest.

Bertrand Russell is a philosopher.

Therefore, Bertrand Russell is honest.

This argument is clearly a valid argument. It can be symbolized in the following way:

1. (x) (Px⊃Hx)
2. Pb/ ∴ Hb
3. Pb⊃Hb 1 UI (Universal Instantiation)

Then we can assert the conclusion

4. Hb 2, 3 MP

We call the rule of inference that permits the leap from line 1 to line 3 Universal Instantiation, because it yields an instance, (Pb⊃Ga), of the universal generalization (x) (Px⊃Hx).

It is important to note that a universal quantifier must quantify a whole line in a proof to be dropped by UI. Thus, UI cannot be applied to the line

1. (x) Px⊃ (∃x) Hy

To obtain

2. Pa ⊃ (∃x) Hy

Because the (x) quantifier in line 1 does not quantify the whole of that line.

Similarly, UI cannot be applied to the line

1. ~ (x) (Px⊃Hx)

To obtain

2. ~ (Pa ⊃ Ha)

Because the (x) quantifier in line 1 does not quantify the negation sign, and thus does not quantify the entire line.

The general rule for UI can be stated as:

$$\frac{(x)(Mx)}{\therefore My}$$

(where 'y' is an individual symbol)

Universal Generalisation (UG)

The rule of Universal Generalisation (UG)permits us to move from a statement about an individual whether named or taken at random, to a statement about all individuals in the universe. It allows us to move from an unquantified statement to a universally quantified statement. The general rule is presented in the following way:

$$\frac{My}{\therefore (x)(Mx)}$$

(where 'y' denotes "any arbitrarily selected individual. Any arbitrarily selected individual means what is true of an individual is proved to be true of any individual".) We may also state UG as: From the substitution instance of a propositional function with respect to the name of anyarbitrarily selected individual one can validly infer the universal quantification of that propositional function.

Let us consider the following example:

No mortals are perfect.

All humans are mortal.

. ? . No humans are perfect.

1. (x) (Mx⊃ ~ Px)
2. (x) (Hx⊃Mx) / (x) (Hx⊃ ~ Px)
3. Hy⊃ My 2 UI
4. My⊃ ~ Py 1 UI
5. Hy⊃ ~ Py 3,4 H.S
6. (x) (Hx⊃ ~ Px) 5 UG

Existential Generalisation (EG)

The rule of ExistentialGeneralisation (EG) allows us to proceed from an unquantified proposition to a proposition which is existentially quantified. If you have a statement like

Ku • ~ Ru

Then by the application of the rule of EG, it is possible to get

(∃z) (Kz • ~ Rz)

It is important here to note that the statement '(∃z) (Kz • ~ Rz)' can also be obtained from the following:

Kb • ~ Rb

There is an important exception about EG. Though it is a rule of inference, it is not possible in EG to replace some or at least one variable or constant, and necessarily all.

Thegeneral rule for UI can be stated as:

(where 'y' is an individual symbol)

Let us consider the following example:

All dogs are carnivorous.

Some animals are dogs.

∴ Some animals are carnivorous.

1. (x) (Dx⊃Cx)
   (∃x) (Ax • Dx) /∴ (∃x) (Ax • Cx)
2. Aw • Dw 2 EI
3. Dw⊃Cw 1 UI
4. Aw 3 Simp.
5. Dw 3 Simp.

6. Cw 4,6 MP
7. Aw • Cw 5,7 Conj.
8. (∃x) (Ax • Cx) 8 EG

Existential Instantiation (EI)

The rule of Existential Instantiation (EI)permits us to move from an existentially quantified proposition to a proposition which is not quantified. From a quantified formula '(∃x) Rx' we are allowed by the rule of EI to draw the conclusion 'Ry'. Let us consider the following example:

1. (x) (Hx⊃Kx)
2. (∃x) (Hx)/ ∴(?x) (Kx)
3. Hy2 EI (Existential Instantiation)
4. Hy⊃Ky 1 UI
5. Ky 4, 3 M.P
6. (∃x) (Kx) 5 EG

In this proof, line 3 follows from line 2 by the inference rule calledExistential Instantiation (EI).

The general rule for EI can be stated as:

$$\frac{(\exists x)(Mx)}{\therefore My}$$

(where 'y' is an individual constant, that is a substitute for a name. However, there is an important restriction to be placed on this rule, namely, that the individual constant 'y' must not have occurred previously in the proof in which this rule is applied).

10.9 PROVING VALIDITY

The four rules of quantification can be applied to prove the validity of arguments. In order to prove validity of a given argument the following steps are to be adopted.

1. The propositions of the given argument are to be symbolically represented first. In order to represent the propositions one is to adopt the rules of quantification. For example, the proposition "All men are mortal" should not be represented as 'p', because this type

of symbolic representation is done in accordance with the rules of propositional logic. We must not forget that we are going to prove the validity of argument in predicate logic. Hence, the proposition is to be symbolized as "(x) (Mx⊃Rx)" [M-Men, R-Mortal]. In other words, here we have followed the four rules of quantification. Sometimes, a proposition in a given argument may be a singular proposition. We need to follow the rules of symbolizing a singular proposition then.

2. The quantifiers of the propositions of the given argument are to be eliminated by the application of the rules of quantification.
3. The techniques applied for the construction of formal proofs of validity in propositional logic by means of the 19 elementary valid argument forms are to be adopted here and the process will continue until we reach the conclusion.
4. Quantifier is to be added in accordance with the necessity to reach the conclusion.

Example I

All men are mortal.

All Indians are men.

∴ All Indians are mortal.

The given argument is to be symbolically represented following the rules of quantification.

(Step 1):

(x) (Mx?Rx)

(x) (Ix ?Mx)

∴ (x) (Ix ?Rx)

The validity of the argument can be shown in the following way:

1.	(x) (Mx⊃ Rx)	
2.	(x) (Ix ⊃Mx)	/ ∴(x) (Ix ⊃ Rx)
3.	My⊃ Ry	[1 UI]
4.	Iy⊃My	[2 UI]
5.	Iy⊃Ry	[4, 3 H.S]

6. (x) (Ix ⊃ Rx) [5 UG]

Here in line 3 the universal quantifier (x) is found to be dropped applying the rule U.I. to line 1 (step 2). Similarly, in line 4 the universal quantifier (x) is found to be dropped by applying the rule U.I. to line 2 (step 2). Now we have to consider line 3 and 4 as premises and line 5 is derived via Hypothetical Syllogism (H.S) from these premises (step 3). The universal quantifier (x) is found to be added in line 6 (step 4) via rule U.G. to line 5.

Example II

No athletes are bookworms.

Carol is a bookworm.

∴ Carol is not an athlete.

The given argument is to be symbolically represented by following the rules of quantification(step 1). The second premise, i.e., 'Carol is a bookworm' is a singular proposition and therefore this proposition is to be symbolized by following the rules of symbolizing a singular proposition. Same process of symbolization will have to be adopted in case of symbolization of the conclusion of the argument.

(x) (Ax ⊃ ~ Bx)

Bc

∴ Ac

The validity of the argument can be shown in the following way:

1. (x) (Ax ⊃ ~ Bx)
2. Bc /∴ Ac
3. Ac⊃ ~ Bc [1 U I]
4. ~ ~ Bc [2 D. N]
5. ~ Ac [3, 4 M. T]

In this argument, rule U I has been applied in line 1 to eliminate the universal quantifier (x). Line 3 (step 2) is derived via U I. Line 4 is derived from line 2 (step 3) via rule of D N (Double Negation). Line 5 is derived from line 3 and 4 via rule of M T (Modus Tollens). Step 4 is not applicable to this argument, since the conclusion is a singular proposition.



CHECK YOUR PROGRESS

Q.1. Objective type questions.

(a) Name the logician who is credited for the discovery of quantification.

(b) Who introduced the word 'quantification'?

(c) Identify the proposition'Socrates is honest'.

(d) How many phrases are there to indicate quantity in quantification?

Q.2. Symbolize the propositions.

(a) Nothing is permanent in life.

(b) Ram is intelligent.

(c) Shylock is not honest.

(d) Shylock is cleaver and not honest.

(e) Ram is not intelligent but honest.

(f) Hari is either intelligent or a fool.

(g) Something is not desirable.

(h) Something is worth having.

(i) Men are useful creatures.

(j) No person can escape death.

Q.3. State whether the following statements are true or false.

(a) A singular proposition is a substitution instance of a propositional function.

(b) I' and 'O' propositions are symbolized by using the existential quantifier '(∃x)'.

(c) The symbolic form of universal quantifier is '(∃x)'.

(d) Statement which contain words like 'all', 'every', 'each', 'any', 'everything', 'anything' or any word conveying the sense of 'all' are symbolized by the aid of existential quantifier.

Q.4. Answer the following questions.

(a) What are the ways of obtaining propositions from propositional function?

(b) What do you mean by Universal Quantifier?

(c) What is quantification?

(d) What do you mean by Existential Quantifier?



10.10 LET US SUM UP

- Quantification is a twentieth century development which is chiefly credited to GottlobFrege (1848-1945), a great German logician. But the word 'quantification' was introduced by Charles S. Peirce. Quantification falls under the category of predicate logic and not propositional logic.
- There are two ways of obtaining propositions from propositional functions. One is called Instantiation or Substitution. The other is called Quantification.
- There are two types of quantifiers: Universal Quantifier and Existential Quantifier.
- A universal quantifier can be defined as an expression that, when prefixed to a propositional function, yields a universal proposition. Statement which contain words like 'all', 'every', 'each', 'any', 'everything', 'anything' or any word conveying the sense of 'all' are symbolized by the aid of universal quantifier. The symbolic form of universal quantifier is '(x)'.
- An existential quantifier can be defined as an expression that, when prefixed to a propositional function, yields an existential proposition. The existential quantifier '($\exists$x)' is used to assert that some entities (at least one) have a given property.
- Among the A,E,I and O propositions 'A' and 'E' propositions are symbolized by using the universal quantifier '(x)' and 'I' and 'O' propositions are symbolized by using the existential quantifier '($\exists$x)'.
- Only singular propositions are symbolized by the process of instantiation. By writing an attributesymbol immediately to the left of an individual symbol, we symbolize the singular proposition affirming that the individual named has the attribute specified.
- There are four rules of quantification. They are-Universal Instantiation (U.I.), Universal Generalisation (UG), Existential Instantiation (EI), ExistentialGeneralisation (EG).

10.11 FURTHER READINGS

1) Sharma, B., Deka, J. (2014) A text book of Logic, 3rd Ed., Guwahati: AankBaak

2) Chakraborti, C. (2007) Logic: Informal, symbolic and Inductive, 2nd Ed., New Delhi: Prentice Hall of India Pvt. Ltd.

3) Pollock, J.L. (1969) An Introduction to Symbolic Logic, New York: Holt, Pinehart and Winston, Inc.

4) Gustason, W., Ulrich, D.E. (1973) Elementary Symbolic Logic, New York: Holt, Pinehart and Winston, Inc.

10.12 ANSWERS TO CHECK YOUR PROGRESS

Ans.1.

(a) GottlobFrege is credited for the discovery of quantification.

(b) The word 'quantification' was introduced by Charles S. Peirce.

(c) Singular proposition.

(d) There are two phrases to indicate quantity in quantification.

Ans.2.

(a) (x) ~ Px

(b) Ir

(c) ~ Hs

(d) Cs • ~ Hs

(e) ~ Ir • Hr

(f) Ih v Fh

(g) (∃x) ~ Dx

(h) (∃x) Wx

(i) (x) (Mx⊃Ux)

(j) (x) (Px⊃ ~ Ex)

Ans.3.

(a) True

(b) True

(c) False

(d) False

Ans.4.

(a) There are two ways of obtaining propositions from propositional function.

1. Instantiation and 2. Quantification

(b) A universal quantifier can be defined as an expression that, when prefixed to a propositional function, yields a universal proposition.Statement which contain words like 'all', 'every', 'each', 'any', 'everything', 'anything' or any word conveying the sense of 'all' are symbolized by the aid of universal quantifier. The symbolic form of universal quantifier is '(x)'.

(c) Quantification is the process of obtaining a proposition from a propositional function.

(d) An existential quantifier can be defined as an expression that, when prefixed to a propositional function, yields an existential proposition. The existential quantifier '(∃x)' is used to assert that some entities (at least one) have a given property.

10.13 MODEL QUESTIONS

A) Objective questions

Q 1: What is quantification?

Q 2: What are the types of quantifiers?

Q 3: What is existential quantifier?

Q 4: What is universal quantifier?

Q 5: State the symbol of universal quantifier.

Q 6: State the symbol of existential quantifier.

B) Short questions (Answer in about 100-150 words)

Q.1: Distinguish between instantiation and quantification.

Q.2: State the distinction between a propositional function and proposition.

Q 3: Explain briefly the concept of quantification

Q 4: What is universal quantifier? Explain briefly

Q 5: What is existential quantifier? Explain briefly.

Q.6: Write a short note on: Instantiation.

C) Long questions (Answer in about 300-500 words)

Q.3: Describe how to symbolize a singular proposition.

Q.4: Explain the rules of Quantification.

Q.5: Explain how traditional A,E,I,O propositions are symbolized with the help of quantifiers.

Q.7: Construct formal proof of validity for each of the following arguments.

(a) (x) (Kx⊃ Lx)

(x) [(Kx • Lx) ⊃Mx /∴(x) (Kx⊃Mx)

(b) (x) (Nx⊃ Ox)

(x) (Px⊃ Ox) /∴[(Nx v Px) ⊃ Ox]

(c) (x) (Qx⊃ Rx)

(∃x) (Qx v Rx) /∴(∃x) Rx

(d) (x) (Ax ⊃Bx)

~ Bt/ ∴At

(e) (x) (Cx⊃Dx)

(x) (Ex⊃ ~ Dx) /∴(x) (Ex⊃ ~ Cx)

(f) (x) (Fx⊃ ~ Gx)

(∃x) (Hx • Gx) /∴(∃x) (Hx• ~Fx)

(g) (x) [Sx⊃ (Tx⊃Ux)]

(x) [Ux⊃(Vx • Wx) /∴(x) [Sx⊃ (Tx⊃Vx)]

(h) (x) [(Xx vYx) ⊃ (Zx • Ax)]

(x) [(Zx v Ax) ⊃ (Xx • Yx)] /∴(x) (Xx ≡Zx)

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