UNIT-12: MEMBERSHIP, INCLUSION, THE EMPTY SET

UNIT STRUCTURE

12.1. Learning objectives

12.2. Introduction

12.3. The concept of membership

12.4. The concept of inclusion

12.5. The concept of empty set

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12.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- define what is predicate logic
- define what is membership
- define what is inclusion
- what is empty set

12.2 INTRODUCTION

The concepts of membership, inclusion and empty set has occupied significant place in set theory. A set is a well-defined collection of objects having same characteristics. These objects are known as members or elements of the set. The concepts of membership, inclusion and empty set which are popularly known as set membership, set inclusion and empty set has arisen depending upon the members of sets or by depending upon those objects or elements which belongs to such sets. Whenever a certain object or element is a member of a certain set, then that object or element is known as the member of that particular set. These members belong to or

are included to such sets. Whenever a certain set includes a certain member then that member is called as a subset of that set or that particular set is known as the superset of that particular member. Hence, the idea of set inclusion has arisen from the idea of subset and superset. However, the concept of empty is quite different which contains no member at all.

12.3. THE CONCEPT OF MEMBERSHIP

The concept of membership is known as set membership. The mathematical use that an individual has a certain property is used in logical sense, that the individual is a member of a certain class. The members or elements of a set can be anything, that is, it can be numbers, people, and letters of the alphabet, other sets or any distinct or well-defined entity. Each of the unit of a set is called a member of the set. The members forming a set may be objects of any kind except the set itself. Following Bertrand Russell, it can be said that a set is never permitted to be an element of itself. But a given set may be an element of another set. Thus it is logically possible to find a set of sets. To say that an individual is a member of a certain set Greek letter '?' (Epsilon) is used between the individual letter and the class letter.

Here, while discussing about set membership there are some important points which are to be noticed like:

(A) The words like object, element and member are used synonymously.

(B) Any set is implied by capital letters like A, B, C, X, Y, Z etc.

(C) The members of a set is implied by small letters like a, b, c, x, y, z etc.

One example of set membership can be given like

Akbar was a king.

Akbar $\in$ king

a $\in$ K

Sets may be interrelated. The first interrelationship between two sets is that one set is identical with other. The principle involved here is the principle of 'intentionality of sets'. For example, two sets A and B are identical

if and only if, for the value of x, x can be called a member of A, if and only if x is a member of B. The symbolic representation of this point may be given as follows:

$A = B \leftrightarrow (x)\ [(x\ A) \leftrightarrow (x \in B)]$

For example, A = 1,2,3,4

B = 1,2,3,4 therefore, A=B

Similarly, A= Bachelor

B= Unmarried male therefore, A=B

Similarly, A= Human beings

B= Rational animal therefore, A=B

There is another two methods to imply or express a set and its members. They are like: (1) Roster or tabular form (2) Set builder form. The fundamental principle of the theory of set is the 'principle of extensionality' where the members of the set theory are specified by the way of intentional definition using a rule or semantic description. It is a way of defining a set by the characteristics property of all its elements.

(1) **Roster or tabular form:** According to this method all the members of a set are written separately by using commas (,) within brackets. For example: the set of all the vowels of English alphabet-{a,e,i,o,u}, the set of all the even numbers less than 10- {2,4,6,8}

(2) **Set builder form:**According to this method it is assumed that there is a common or general characteristic among the members of a set which is totally absent in those members which are not in the set. For example: the set of vowels of the English alphabet-{a,e,i,o,u}. There is a common characteristic among all these alphabets, i.e.; they are vowels. If this set is expressed by using the letter V, then its symbolic representation will be- V= {x: x the vowels of English alphabet}

As has been mentioned earlier the fundamental principle of the theory of set is the 'principle of extensionality'. The members of a set can be specified by the way of intentional definition using a rule or semantic description. It is a way of defining a set by the characteristics property of all

its elements. Again, it can be specified by the way of extension or by listing each member of the set. An extensional definition is donated by enclosing the list of members in brackets. Therefore, the members of the sets are called extensional. Extensional specification implies that each of the elements listed is a member of the set. For example:

A= {1, 3, 5, 7}

B={a, e, i, o, u}

Each member of a set is unique. It is so because no two members of a set are identical. If a set contains only a finite number of elements, then it is called a finite set. If an example is taken like if D is the set of the days of the week, then D is finite set. On the other hand, if a non-empty set which is not finite is called an infinite set. For example, if S is the set of the number of stars, then S is an infinite set.



LET US KNOW

Set membership means the individuals that are included in a set has a certain property i.e.; the individual is a member of a certain class.



CHECK YOUR PROGRESS

Q 1:What do you mean by set membership?

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Q 2: What are two methods to express a set and its members?

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Q 3: What is roster or tabular form?

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12.4. THE CONCEPT OF INCLUSION

The concept of inclusion is also known as set inclusion.Let A and B be two sets. If every element of A is an element of B, then A is called a subset of B. This means that "A is contained in B" i.e.; if A is a subset of B whenever a $\in$ A, then a $\in$ B.It is said as 'A is a subset of B', it means that if 'a' is an element of A implies that 'a' is an element of B. If A is a subset of B we say that B contains A or B is a superset of A. It is often convenient to use the symbol "$\rightarrow$" which means implies. With the help of this symbol we can express a subset in the following way-

A is a subset of B if a $\in$ A $\rightarrow$ a $\in$ B

This expression can be read as "A is a subset of B, if a is an element of A implies that a is also an element of B". If A is not a subset of B, then we have to draw a slash through the subset symbol. If A is a subset of B we say that B contains A or B is a superset of A which again means B includes A, or B contains A. This kind of relationship in set theory is known as set inclusion or set containment.

Again, if A is a subset of B, and if there exists at least one element of B which is not an element of A, then A is called a proper subset of B. In other words, if A is a subset of, but not equal to B, then A is called a proper subset of B. For example,

A= {1}

B= {1, 3, 5}

C= {1, 3, 5, 7}

Hence, A is a proper subset of B and C, and B is a proper subset of C.

Here, one point that has to be kept in mind that every set is a subset of itself and the empty set is subset of every set. Such types of subsets are known as improper subsets. For example, a subset A of a set B, is called a proper subset of B if, A is not equal to B and it is written as A subset B. In such case it can be said that B is a superset of A.



LET US KNOW

Set inclusion means those kind of relationship in a set theory where A is a subset of B i.e.; B contains A or B is a superset of A.



CHECK YOUR PROGRESS

Q 4:What is set inclusion?

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Q 5:What is known as subset and superset?

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ACTIVITY: 12.2

- Explain the concept of set inclusion with suitable examples.

Ans .....

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12.5. THE CONCEPT OF EMPTY SET

The concept of empty set is of utmost importance in several mathematical and logical operations. An empty set is also known as a null set, zero set or void set. The concept of empty set can be defined as that kind of set which contains no member or where no kind of member is included. This type of set is expressed by using the Greek symbol ? (phi) or ^ (lambda). For example, let 'A' the set of those stars which revolve round

the planet earth. This kind of set is called an empty set which contains no member at all. Or let 'B' be the set which contains those educated women who are the Vice-Chancellors of Gauhati University. But this is also an empty or null set as there is no member in this set. Or let 'C' be the set of people in the world who are older than 250 years. But according to the known statistics 'C' is an empty set.

Here, one important point must be mentioned that logically there can be only one null set. This is so because sets differ from one another only when they have different members. But an empty set has no member or element. Therefore, it is not possible to differentiate one null set from another null set on the basis of the difference of members. For example, the set of even prime number greater than 2 is an empty set. Hence, no null set is different from this set of even prime number greater than 2 from the point of view of the difference of members.

However, the concept of empty is a confusing concept as one may be incorrectly inclined to visualize a set as a heap of things. But, such type of visualization is not mathematically sound though it may be initially and intuitively. If a set is visualized in such a way then the mathematical idea of empty set may appear to be puzzling.

An empty set is symbolically represented in various ways. Generally by means of the Greek letter lambda (^) it is expressed that a particular set has no element of its own. The concept of empty set enables the logicians and mathematicians to perform a number of logical manipulations. For instance, it can be said that if two sets are mutually exclusive then the intersection of them produces an empty set. For example, if the set A contains the members 1, 2, 3 and 4 and the other set B contains the members 5, 6, 7, and 8 then we can say that the intersection of these two sets produces an empty set.

Again, an empty set may be a member of another set logically though nothing belongs to an empty set. Moreover, an empty is a subset of every set. For every x, x is not a member of an empty set. This is the defining property of an empty set.



LET US KNOW

The concept of empty set can be defined as that kind of set which contains no member or where no kind of member is included.



CHECK YOUR PROGRESS

Q 6: What is empty set? Give example.

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ACTIVITY: 12.3

- Explain the notion of empty set with suitable example in set theory.

Ans .....

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12.6. LET US SUM UP

- The concepts of membership, inclusion and empty set has occupied significant place in set theory. The concepts of membership, inclusion and empty set which are popularly known as set membership, set inclusion and empty set has arisen depending upon the members of sets or by depending upon those objects or elements which belongs to such sets.
- Whenever a particular person, object or element becomes a member of a particular set, then that relation is called set membership.

- The members or elements of a set can be anything, that is, it can be numbers, people, and letters of the alphabet, other sets or any distinct or well-defined entity. Each of the unit of a set is called a member of the set. The members forming a set may be objects of any kind except the set itself.
- To say that an individual is a member of a certain set Greek letter '?' (Epsilon) is used between the individual letter and the class letter.
- There is another two methods to imply or express a set and its members. They are like: (1) Roster or tabular form (2) Set builder form.
- According to the roster or tabular method all the members of a set are written separately by using commas (,) within brackets.
- According to the set builder method it is assumed that there is a common or general characteristic among the members of a set which is totally absent in those members which are not in the set.
- Each member of a set is unique. It is so because no two members of a set are identical. If a set contains only a finite number of elements, then it is called a finite set.On the other hand, if a non-empty set which is not finite is called an infinite set.
- Set inclusion means those kind of relationship in a set theory where A is a subset of B i.e.; B contains A or B is a superset of A.
- Whenever a certain set includes a certain member then that member is called as a subset of that set or that particular set is known as the superset of that particular member.
- The concept of empty set can be defined as that kind of set which contains no member or where no kind of member is included.
- An empty set is symbolically represented in various ways. Generally by means of the Greek letter lambda (^) it is expressed that a particular set has no element of its own.
- An empty set is also known as a null set, zero set or void set.
- An empty set may be a member of another set logically though nothing belongs to an empty set. Moreover, an empty is a subset of every set.

12.7. FURTHER READINGS

1. Copi, I. M. Symbolic logic, 5th edition, 1996, Prentice hall of India private limited, New Delhi
2. Mates, Benson, Elementary Logic, 1965, New York.
3. Pollck, John L. An introduction to Symbolic logic, 1969, State university of New York, Holt, Rinehart and Winston, INC.
4. Suppes, Patrick.Introduction to logic, 1957, Van Nostrand, New York.

12.8 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q.1.Set membership means that the individual that are included in a certain class has a certain property i.e.; that the individual is a member of the class.

Ansto Q.2. The two methods to express a set and its members are roster or tabular form and tabular form.

Ans to Q.3. According to the roster or tabular method all the members of a set are written separately by using commas (,) within brackets.

Ans to Q.4. Set inclusion means those kind of relationship in a set theory where A is a subset of B i.e.; B contains A or B is a superset of A.

Ans to Q.5. Whenever a certain set includes a certain member then that member is called as a subset of that set or that particular set is known as the superset of that particular member.

Ans to Q.6. The concept of empty set can be defined as that kind of set which contains no member or where no kind of member is included.

12.9. MODEL QUESTIONS

A) Very short questions:

Q 1: What does the symbol '?' (epsilon) refer to?

Q 2: What does the symbol 'λ' (lambda) refer to?

Q 3: Can an empty set be the member of another set?

Q 4: Which set is considered as the subset of any other set?

Q 5: What is set membership?

Q 6: What is set inclusion?

Q 7: What do you mean by an empty set?

Q 8: What are the two other methods to express a set and its members?

B) Short questions (answer each question in about 100-150 words)

Q 1: What is set membership?

Q 2: What do you understand by set inclusion or set containment?

Q 3: What is null or void set?

Q 4: What do you understand by subset and superset?

C) Short notes (answer each question in about 100-150 words)

Q 1: Set membership

Q 2: Set inclusion

Q 3: Null or zero set

Q 4: Subset and superset

Q 5: Roster or tabular form

D) Long questions (answer each question in about 300-500 words)

Q 1: What do you mean by set membership? Explain with examples.

Q 2: Explain set inclusion with suitable examples?

Q 3: What is null or void or empty set? Is there any logical importance of the empty set in logic? If yes then explain.

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