UNIT-13: OPERATION ON SETS, DOMAINS OF INDIVIDUALS

UNIT STRUCTURE

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13.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- defne a set operation,
- explain the concept of union of sets,
- explain the concept of intersection of sets,
- explain difference of sets,
- discuss the domain of individuals.

13.2 INTRODUCTION

Logic and set theory are very much intertwined in terms of serving as a foundation for mathematics. Mathematical logic or set theory is more foundational to mathematics. Both are equally important as being foundational, and they are intertwined. With logic, we are concerned with analyzing the justification of a statement. We logically approach the justification of a statement by studying its structure, which is composed of

premises and a conclusion. There are rules to follow that make an argument valid and therefore logical. One such important rule is that there may be several premises, but only one conclusion, and there must be a relation between the premises and the conclusion. Since set theory is the study of relations, it is very convenient to use set theory and logic together. Mathematics studies numbers, geometry, and form. Those require definitions and axioms to characterize the subjects of study. Logic is used to prove properties based on those definitions and axioms. Logic can also be used in the ordinary discourse of nonmathematical topics. Typically, the logic needed for mathematical investigations is great while that for nonmathematical investigations is small.

13.3 OPERATION ON SETS

Sets can be combined in a number of different ways to produce one or more sets. There are logical operations and set operations. If an operator defines an action, an operation is just a part of the expression. The set operator is prescribing to compute the value of an expression and assign it to a specified variable. An expression value can have a logical type; it will use logical operations to compute it. Operations on sets are some ways of constructing new sets from the given sets. Different connectives are used to define the set operations. The connectives allow us to build new sets from given ones. For this purpose, all sets are considered to be the subsets of the universe of discourse. A is a subset of B exactly when every element of A is also an element of B. There are three basic operations on sets.

1. Union of sets
2. Intersection of sets
3. Difference of sets

Inside that set are all the worlds where that variable is "true". With this representation, all set operators correspond exactly to logical operators. "and" corresponds to intersection, "or" to union, "not" to set complement.

13.3.1 Union Of Sets

The union of two sets, A and B, is the collection of points which are in A or in B (or in both). If A and B are two sets, the union of A and B is the set of all those elements which belong either to A or B or both A and B. Or the union of set A and B is the set, which consists of all the elements of A and all the elements of B, the common elements being taken only once. In order to denote the union of A and B the symbol "U" is used. Thereby, first set operation is defined using the "U"connective. Thus, the set A U B -read "A union B" or "the union of A and B"-is defined as the set that consists of all elements belonging to either set A or set B (or both). If A is the set of all animals and B is the set of all plants, then A U B is the set of all living organisms. Symbolically it can be shown in the following way:

$(x)(x \in A \cup B \leftrightarrow x \in A \text{ or } x \in B)$

It is read as for every x, x belongs to A union B, if and only if x belongs to A or x belongs to B. In other words we can say that A ? B is equal to x, such that, x belongs to A or x belongs to B. Again the union of set A and B can be represented in the following way:

$A \cup B = \{x: x \in A \text{ or } x \in B\}$

It is read as A union B is equal to x, such that x belongs to A or x belongs to B.

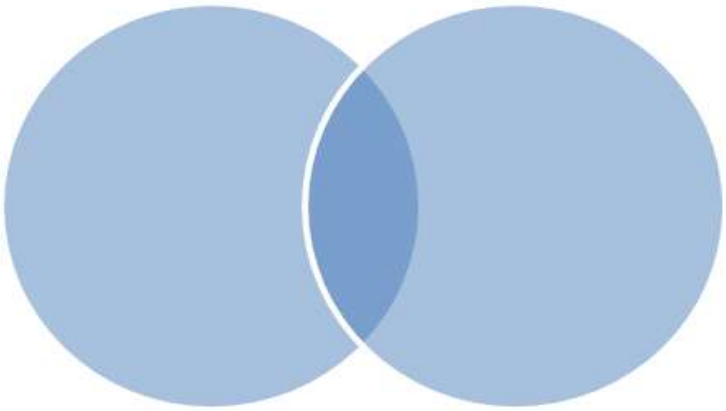
Suppose, $A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

The example of a set A, which is the set of all animals and a set B, which is the set of all plants, and its union, that is AUB will be the set of all living organisms. If we are to refer to a single element by the variable "x", then, we can say that x is a member of the union, if it is an element present in set A or in set B, or in both. The same elements cannot be used twice in the same set, or, it can be said that sets cannot have duplicate elements. Suppose we have the set A = {1, 2, 3} and the set B = {2, 3, 4}. Thus as the operation of A U B, we find that A U B = {1, 2, 3, 4}. Multiple occurrences of identical elements have no effect on the cardinality of a set or its contents.

(A U B) is called the logical sum of A and B. Therefore (A U B) can be replaced by (A + B). The operation of the union can be diagrammatically represented in the following way. A set can be visualized using Venn Diagrams In the diagram we have shaded A U B, the area of A and the area of B.



At the very outset, we can say that binary union is an associative operation. It doesn't matter in what order unions are taken. Here, the operation can be used as functional in any order and the parenthesis may be omitted without any ambiguity. Secondly, the union is commutative and, as such, the sets can be written in any order. Thirdly, one can take the union of several sets at once. Say, for example, the union of A, B, and C contains all elements of A, all elements of B, and all elements of C, and nothing else. Thus, for an element of x, x is an element of A U B U C, if and only if x is in A, or in B, or in C.

13.3. 2 Intersection of sets

The intersection is a binary operation. The intersection of sets A and B is the set of all elements which are common to both A and B. The symbol "$\cap$ " (read as an intersection), is used to denote the intersection. The intersection of two sets A and B is the set of all those elements which belong to both A and B. It can be symbolically represented in the following way:

$$A \cap B = (x: x \in A \text{ and } x \in B)$$

Suppose A = {1, 2, 3, 4, 5}

B = {4, 5, 6, 7}

A $\cap$ B = {4, 5}

Again, if A = {1, 3, 4, 6}

B = {1, 4, 10, 5}

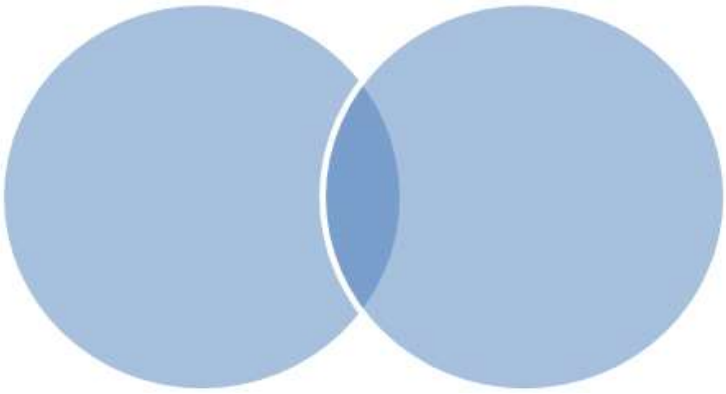
C = {2, 8, 7, 6}

Then, (A $\cap$ B $\cap$ C) = $\wedge$, since, all the three sets do not contain any common element.

Let us see some more examples that will help us to understand the operation on the intersection of sets. Let A be the set of all cricket players and B be the set of all Indian cricket players. Then the intersection of A and B will be the set of all Indian cricket players. The operation of intersection is sometimes used as a function of the product of two sets. In such circumstances, instead of using the symbol intersection, "$\cap$ ", the symbol dot " ." (A.B) or multiplication sign "×" (A×B) can be used.

From this, it is clear that the operation of intersection is presented as, $x \in A \cap B \leftrightarrow x \in A$ and $x \in B$, and it is read as x belongs to **A** intersection B if and only if x belongs to A and x belongs to B.

The operation of intersection can be diagrammatically represented in the following way. In the diagram, we have shaded A $\cap$ B, the area that is common to both A and B.



Mutually exclusive sets have nothing in common. If W = { 1, 2, 3 } and S = { a, b, c }, then

W $\cap$ S = $\wedge$

There some important properties of the operation of the intersection of sets. Firstly, the set of sets is closed under intersection, that is, intersection

always has a set value. Secondly, for all sets, intersection is associative. That is, in intersection, the result does not depend on the order of evaluation. Thirdly, for all sets, intersection is commutative. That is, the result does not depend on the order of the operands. Symbolically it can be shown as $(A \cap B) = (B \cap A)$. Lastly, one thing that can be pointed out that, each investigation has its own universal set, which includes everything under investigation, but not everything in the actual known universe. The universal set is also referred to as the set of universe (U). The universal set, U, is a left identity for intersection of sets. Symbolically it can be shown as $U \cap A = A$. Again, the universal set, U, is a right identity for intersection of sets. Symbolically it can be represented as $A \cap U = A$

13.3.3 Difference of sets

The third operation of the set is difference set. It is a binary operation. It can be understood as if A and B are two sets, the difference of the sets A and B in this order, is the set of elements which belong to A but not to B. The difference of sets A and B, $A \sim B$ or A - B, is also referred to as the relative complement of B in A. This can be symbolically represented as $(A \sim B)$ (read as A difference B). In other words, by $A \sim B$ we mean the set of all things which belong to A but not to B. Thus it can be said that for every x, $x \in A \sim B$, if and only if, $x \in A$ and $x \notin B$. It can be symbolically represented in the following way:

$(x)\ (x \in A \sim B \leftrightarrow x \in A \text{ and } x \notin B)$

It can be read as, x such that x belongs to $A \sim B$ if and only if x belongs to A but not belong to B. Say for example, if A is a set of all human beings, and B is the set of all females, then $A \sim B$ is the set of all human males. By taking some distinct elements as its members, a difference of set can be expressed in the following way:

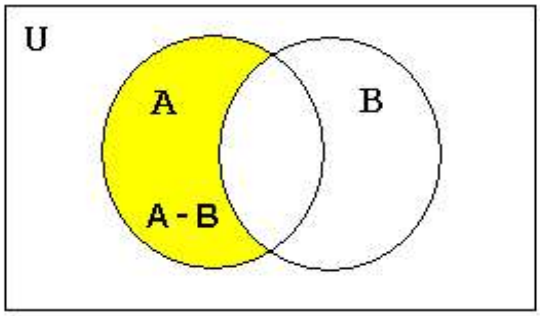
Let A = {a, b, c, d, e, f}

B = {c, d, x, y}

$(A \sim B)$ = {a, b, e, f}, since the elements a, b, e, f belong to A but not to B. Again,

(B ~ A) = {x, y}, since the elements x, y belong to B and not to A

A ~ B is sometimes symbolized as A - B. It can be diagrammatically represented in the following way. In the diagram, we have shaded A - B, the area in A which is not part of B.



It is pertinent to mention that the set of sets is closed under set difference. That is, the difference of two sets is a set. Secondly, difference of sets is not associative, that is, in general, for sets A, B, and C, (A ~ B) ~ C ≠ A ~ (B ~ C). Again, difference of sets is not commutative, that is, in general for sets A and B, A ~ B ≠ B ~ A.



ACTIVITY: 13.1

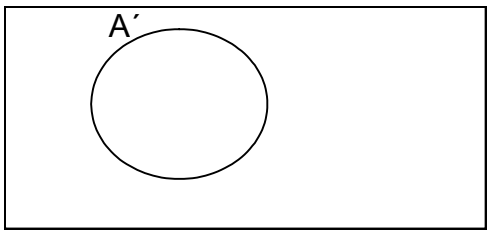
Name of operation	Symbol	Description
Union	U	union = cup
Intersection	I	intersection = cap
Difference	~	tilde operator = varies with = similar to

13.4 DOMAIN OF INDIVIDUALS

The idea of the domain of individuals or the domain of discourse is the items we want our statements to deal with. In any application of the set theory all the sets under investigation will likely be subsets of a certain set. It is known as domain of individuals or universal set or domain of discourse.

For instance, in arithmetic the domain of discourse is the "world of numbers." In daily life the domain of discourse could be apples, or persons, or even everything. The domain of discourse or universe of discourse for a system for arithmetic is (so to speak) the "world of numbers." Or, more specifically, it might be just, say, all the positive integers. Usually, the domain of discourse is not explicitly specified but assumed to be "everything." Everything, obviously, includes quite a lot. It specifically includes all concrete (as opposed to abstract) things, such as people and trees, and logic books, but it also includes all the numbers, as well as individual times and places. For example, the domain of discourse may be limited to human beings only. In another occasion, the domain of discourse may be restricted to positive whole numbers. In some cases the domain of discourse is infinitely large. When dealing with domains of this size, we cannot replace quantified sentences with their expansions.

The domain of individuals is also known as the 'domain of quantifiers'. It is a set of things that serve as possible values for its variables. The domain of a relation is the set of things that bear the relation to something else. The term 'domain' may also be employed in connection with function. Thus we may speak about the domain of a function which means the class of objects to which the function is applicable. Any function will be applicable to or is only defined over a certain class of objects. The class of objects for which a given function is defined is called the domain of the function. 'Domain of individuals' is used synonymously with the terms the 'domain of discourse'. It is the area in connection with which a particular discussion about an individual takes place. To give a precise definition of D, the following diagrammatic representation will help us widely:



The universal set may also be denoted by U: the set of all objects under the consideration. In the diagram, it is assumed that all elements are members of a fixed universe U. In set theory, the universe is considered to be the set of all possible elements in a given situation. There is the use of circles to represent sets and a rectangle to represent U. The space inside these shapes represents where elements might exist. Shading is used to represent where elements might exist after applying some set operations.

The diagram shows that any discussion about an individual belonging to 'A' will be in or with respect to a particular area. That area is represented by a rectangle in the above diagram. The rectangle, therefore, represents the domain of discourse. This domain of discourse is constituted of two classes. One is the class of A and the other is the class of Á (not A). These two classes together constitute the domain. A is the class to which the individual under consideration belongs. Á is a complimentary class. A class and its complementary class between themselves exhaust the universe or the domain of discourse. It can b expressed as A + Á = U (universe of discourse). In order to introduce a domain of individuals, one can draw a circle, put some dots in it, write some constants on the board (e.g., our "a" and "b", etc.), and then draw a line from each constant to a dot in the circle, making clear that each name goes with one and only one dot and a different name goes to each dot. The name describes nothing and the thing named is not picked out in any obvious sense by a description. It merely marks numerical difference: two names, two individuals.



ACTIVITY: 13.2

Properties of union and intersection of sets:

Associative Laws

$A \cup B \cup C = A \cup (B \cup C)$

$A \cap B \cap C = A \cap (B \cap C)$

Commutative Laws

$A \cup B = B \cup A$

$A \cap B = B \cap A$

Distributive Laws

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup B \cap C = (A \cup B) \cap (A \cup C)$

Idempotent Laws

$A \cup A = A$

$A \cap A = A.$



CHECK YOUR PROGRESS

Q 1: Each of the given propositions is false. Replace the underlined word with another word to make the proposition true.

(a) Intersection is defined using a disjunction.

(b) Set diagrams are used to illustrate set operations.



13.5. LET US SUM UP

- Set theory is nothing but the study of relations. Sets can be combined in a number of different ways to produce one or more sets. There are logical operations and set operations. Operations on sets are some ways of constructing new sets from the given sets. Different connectives

are used to define the set operations. The connectives allow us to build new sets from given ones.

- There are three operations of sets: Union of sets, Intersection of sets and Difference of sets.
- In any application of the set theory, all the sets under investigation will likely be subsets of a certain set. It is known as the domain of individuals or universal set or domain of discourse.
- The domain of individuals is also known as the 'domain of quantifiers'. It is a set of things that serve as possible values for its variables.
- The difference of set a binary operation. It can be understood as if A and B are two sets, the difference of the sets A and B in this order, is the set of elements which belong to A but not to B. The difference of sets A and B, A ~ B or A - B, is also referred to as the relative complement of B in A. This can be symbolically represented as (A ~ B) (read as A difference B).
- The intersection is a binary operation. The intersection of sets A and B is the set of all elements which are common to both A and B. The symbol "I " (read as an intersection), is used to denote the intersection.
- The union of two sets, A and B, is the collection of points which are in A or in B (or in both). If A and B are two sets, the union of A and B is the set of all those elements which belong either to A or B or both A and B. Or the union of set A and B is the set, which consists of all the elements of A and all the elements of B, the common elements being taken only once.

13.6. FURTHER READINGS

1. O'Leary, Michael L. A First Course in Mathematical Logic and Set Theory. USA: John Wiley & Sons. 2016.
2. Sharma, B. and Jahnabi Deka. A Text Book of Logic. Guwahati: Aak-Baak. 2014
3. Suppes, Patrick. Introduction to Logic. New Delhi: East-West Press. 1957.

13.7 ANSWERS TO CHECK YOUR PROGRESS

Answers to Q No 1:

a) "U"connective

b) Venn diagram

13.8. MODEL QUESTIONS

A) Objective questions

Q. 1: Give two examples of the domain of discourse.

Q. 2: What logical components are used to distinguish the operations of union and intersection?

Q 3: What is union of sets?

Q 4: What is difference of sets?

Q 5: What is intersection of sets?

B) Short questions (Answer in about 100-150 words)

Q 1: Write short notes on

a) Difference of sets.

b) Union of sets

c) Intersection of sets

C) Long questions (Answer in about 300-500 words)

Q. 1: Explain the operation of union of sets with its diagrammatical representation.

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