UNIT: 15 VENN DIAGRAM

UNIT STRUCTURE

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15.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- define a categorical proposition.
- explain the four standard form categorical propositions.
- describe different types of class relation.
- describe distribution of terms.
- illustrate Venn diagram and its uses.
- discuss the four standard form categorical propositions by means of Venn diagram

15.2 INTRODUCTION

Logic is defined as the science of reasoning. An inference or reasoning when expressed in language is called an argument. An argument is any group of propositions of which one is claimed to follow from others. An argument consists of some propositions. But, it is not a mere sum total of some propositions. Every argument has a definite structure. An argument involves the 'premise' and the 'conclusion'. The proposition which is justified or affirmed on the basis of other proposition or propositions of the argument is called the conclusion. The other proposition or the propositions which provide grounds for accepting the conclusion are called the premise or premises. Logic is chiefly concerned with the argument. Hence, proposition is considered as the basic unit of logical thinking. We need proposition to express our logical thinking. There are different ways of classifying a proposition. The standard form propositions are the typical forms of propositions recognised in Aristotelian logic. The standard form categorical propositions can be represented by using Venn diagram, named after the English mathematician and logician John Venn.

15.3 CATEGORICAL PROPOSITION AND STANDARD FORM CATEGORICAL PROPOSITION

A proposition that relates two classes is called a Categorical proposition. That is, a categorical proposition asserts something about the classes. Classes and their relationship form the content of the categorical propositions. The classes are denoted by the subject term and the predicate term. A categorical proposition asserts that either all or part of the class denoted by the subject term is included in or excluded from the class denoted by the predicate term. Thus, a proposition which asserts something about the relationships between the classes referred to by their subject and predicate terms is known as categorical proposition. Aristotelian logic is

exclusively concerned with this kind of statements and with arguments that are formed out of these propositions. Some of the examples of categorical propositions are:

1. All ducks swim
2. A fish is not a mammal
3. A few soldiers are heroes
4. All prisoners are not violent

In other words a categorical proposition is one in which the relation between the subject and the predicate is without any condition. In all the above examples it is seen that the relation between the subject and the predicate is not subject to any condition.

A categorical proposition that expresses these relations with complete clarity is called a standard-form categorical proposition. For example;

All men are animal

No man is perfect

Some students are intelligent

Some men are not wise.

Here the subject and the predicate term refer to two classes and connected by the verb 'to be'. A categorical proposition is in standard form if and only if it is a substitution instance of one of the following four forms:

All S are P

No S are P

Some S are P

Some S are not P.

Many categorical propositions, of course, are not in standard form because; they do not begin with the words 'all' 'no', 'or', 'some'. Categorical propositions have to be translated into standard form before it can be considered in logic.

The words 'all', 'no', and 'some' are called quantifiers because they specify how much of the subject class is included in or excluded from the predicate class. The letters S and P stand for the subject and the predicate

terms respectively, and the words 'are' and 'are not' are called the copula because they link the subject and the predicate term. Some of the important points about the standard form categorical proposition are:

- The 'subject term' and the 'predicate term' do not mean the same thing in logic that 'subject' and 'predicate' mean in grammar. The subject of the example statements include the quantifier words such as 'all', 'some' but the subject term does not. Similarly the predicate includes the copula 'are' but the predicate term does not.
- There are exactly three forms of quantifiers (all, no, some) and two forms of copulas (is/are, is not/are not). But in ordinary statements various forms of the verb 'to be' are considered as copula.

Standard- form categorical proposition thus represent an ideal of clarity in language.



LET US KNOW

The theory of categorical propositions originated by Aristotle has constituted one of the core topics in logic for over 2,000 years. It remains important even today.

15.4 CLASSES AND RELATION

A class or category is a group of objects having some recognizable common properties or characteristics which can be predicated about all of them. Classes can also be referred to as categories or sets. Aristotelian logic deals with the logic of classes. Aristotle defined a class as a collection of objects or entities. Thus, a category is simply a group, a set of things. For example, class of players, class of college students etc. But it is not a random collection of members. The members of a class are supposed to have a common property that can be predicated about all of them. Terms like 'class', ' sets', 'collection', 'aggregate' etc. are used synonymously.

There are various ways in which classes may be related to each other. If every member of one class is also a member of another class then the first class is said to be included in the second class. For example,

suppose we take two classes viz. 'Indians' and 'Asians'. Here, we know that all the members of the class 'Indians' are also the members of the class 'Asians'. The class 'Indians' is included in the class 'Asians. This kind of inclusion is called total class inclusion relation as the whole class 'Indians' is included in the class 'Asians'. If some, not all members of one class are also members of another, then the first class may be said to be partially included in the second class. For example, some men are poet. If some of the members of the first class are excluded from being the members of the second class, then the first class is said to be partially excluded from the second class. For example, some men are not poet. Of course, there are two classes having no members in common. For example, No triangles are circles. Such kind of relation is said to be total class exclusion relation.

15.5 THE FOUR STANDARD FORM CATEGORICAL PROPOSITIONS AND THEIR CLASS RELATIONS

Quality and quantity are attributes of categorical proposition. According to quality, propositions are divided into two-affirmative and Negative. Again according to quantity, propositions are divided into two-Universal and Particular. Both universal and particular propositions may be affirmative and negative. Hence, combing the quality and quantity principles there are four kinds of categorical propositions in Aristotelian logic. They are universal affirmative, Universal negative, particular affirmative and particular negative. The letters A, E, I and O are used as names for these four kinds of categorical propositions respectively.

The Universal affirmative proposition (A) asserts that the subject class is wholly contained in the predicate class. That is, every member of the subject class is also a member of the predicate class. Because it refers to 'every member', its quantity is universal. Since it asserts a relationship, its quality is affirmative. In other words, in a universal affirmative proposition predicate is affirmed of the whole subject. Suppose S and P are any two

classes, its schematic form is 'All S is P'. It may be illustrated by an actual example: 'All swans are white'. It expresses total class inclusion relation.

The universal negative proposition (E) asserts that there is no relation between the two classes. That is, no member of one class is a member of the other class. Its quality is negative and quantity is universal. In other words, in a universal negative proposition predicate is denied of the whole subject. Its schematic form is - 'No S is P'. It may be illustrated by an actual example: 'No children are cruel'. Here, the first class is wholly excluded from the second class. Thus a universal negative proposition expresses the total class exclusion relation.

Particular affirmative proposition (I) affirms that only some, not all members of one class are also members of another class. Its quality is affirmative and quantity is particular. In other words, in a particular negative proposition predicate is affirmed of the part of the subject. For example: Some flowers are yellow. Its schematic form is 'Some S is P'. This kind of proposition expresses partial class inclusion relation.

Particular negative proposition (O) asserts that some members of a class are not members of another class. Its quality is particular and quantity is negative. Hence, in a particular negative proposition predicate is denied of the part of the subject. For example: 'Some flowers are not yellow'. Its schematic form is 'Some S is not P'. Here, quality is negative and quantity is particular.

Thus, the four categorical propositions, named A, E, I and O stand for universal affirmative, universal negative, particular affirmative, particular negative, respectively are known as standard-form categorical propositions. Standard-form categorical propositions refer to two classes.

15.6 DISTRIBUTION OF TERMS

Like quality and quantity, which are the attributes of propositions, distribution is an attribute of the terms of propositions. A proposition may refer to all members of a class or some of the members of a class designated by its subject and predicate term.

The term 'distribution' refers to the ways in which the terms can occur in the categorical propositions. In other words, by the 'distribution' of a term, it is meant that 'taking it universally or referring to all or parts of it'. A term is said to be distributed if the proposition makes an assertion about every member of the class denoted by the term; otherwise it is undistributed. That means, in a proposition a term is distributed if it refers to all the members of a class designated by the term. Any 'A' proposition of the form 'All S is P' refers to all members of the class designated by its subject term 'S' but does not refer to all the members of the class designated by its predicate term 'P'. Hence, 'A' proposition distributes its subject term but not the predicate term.

The form of universal negative proposition is 'No S is P'. The universal negative proposition i.e. 'E' proposition refers to all the members of the class designated by its subject term as well as by its predicate term. Thus 'E' proposition distributes both the subject and the predicate term.

The form of particular affirmative proposition is 'Some S is P'. The particular affirmative proposition i.e. 'I' proposition refers to some members of the class designated by its subject term and some members of the class designated by its predicate term. Thus 'I' proposition distributes neither the subject nor the predicate.

The form of particular negative proposition is 'Some S is not P'. The particular negative proposition i.e. 'O' proposition distributes the predicate term but not its subject term.

Thus, 'A' proposition distributes its subject term but not the predicate term.

'E' proposition distributes both the subject and the predicate term.

'I' proposition distributes neither the subject nor the predicate.

'O' proposition distributes the predicate term but not the subject term.

The various relations between classes such as class inclusion or class exclusion either affirmed or denied by the four standard form categorical propositions can be represented by using the Venn diagram.



LET US KNOW

Category: A predicate or a fundamental class of things

Class: A collection of objects or entities with a common characteristic.

Some: In logic, 'some' means at least one.



CHECK YOUR PROGRESS

Q 1:Answer briefly

a) What is categorical proposition?

.....

b) Give an example of categorical proposition?

.....

c) What do you mean by a class?

.....

d) What are the four different standard form categorical propositions?

.....

Q 2: Define

a) Universal affirmative proposition

.....

b) Particular negative proposition

.....

Q 3: State whether the following statements are true (T) or False (F)

a) 'A' proposition distributes subject. (True/False)

b) 'E' proposition distribute predicate.(True/False)

Q 4: Fill in the blanks:

a) Standard-form categorical propositions refer to …… classes.

b) The words 'are' and 'are not' are called the ……..because they link the subject and the predicate term.



ACTIVITY: 15.3

- Do you find any distinction between a categorical proposition and a standard form categorical proposition?

Ans .....

.....

.....

.....

15.7 VENN DIAGRAM

A Venn diagram is an illustration of the relationships between sets and among sets, indicated by the arrangement of the circles. Venn diagram uses circles to represent sets, as for example by drawing one circle within another to indicate that the first set is a subset of a second set. Venn diagram is thus a method of representing classes and propositions. Copi has defined Venn as an "Iconic representations of categorical propositions, and of arguments to display their logical forms using overlapping circles". The Venn diagram is named after its inventor, British mathematician and logician John Venn (1834-1923).

15.8 USE OF VENN DIAGRAM

The Venn diagram is used in many fields such as in statistics, computer science, linguistics, logic, and probability. Fundamentally they are used to teach elementary set theory, as well as illustrate simple set relationships in probability, statistics, and computer science. The uses of Venn diagram in formal logic can be brought out in the following way.

- Venn diagram is used to represent the information expressed by the four kinds of categorical propositions. That is, the relationship between the classes designated by the subject and the predicate term of categorical proposition is represented by means of Venn diagram.

- The technique of Venn diagram is used to determine the validity or invalidity of syllogistic argument. Thus it enables us to distinguish valid syllogistic argument from invalid syllogistic argument.
- Venn diagrams are used to explain how the two statements relate to each other.
- The logical relations between sets and operations on sets can be represented by using Venn diagrams.

15.9 HISTORY

The history of the Venn diagram, which is otherwise known as the set diagram goes back to 1880, when British logician and philosopher John Venn introduced it in his research paper "On the Diagrammatic and Mechanical Representation of Propositions and Reasonings", which was published in the 'Philosophical Magazine and Journal of Science'. Venn himself did not use the term "Venn diagram" and referred to his invention as "Eulerian Circles". The first to use the term "Venn diagram" was Clarence Irving Lewis in 1918, in his book "A Survey of Symbolic Logic". At that time, John Venn referred to the Venn diagram as the Eulerian circle, because it resembles Euler diagrams, invented by Leonhard Euler in the 18th century. Venn diagrams differ from the use of Euler circles in a few important ways.

15.10 ARISTOTELIAN AND BOOLEAN STANDPOINT

Before going to discuss about the diagrammatic representation of the standard form categorical propositions through Venn diagram, we are to concentrate at the meaning of universal affirmative (A) and universal negative propositions (E) from Aristotelian and Boolean standpoints.

The Aristotelian standpoint differs from the Boolean standpoint only with regard to universal (A and E) propositions. The two standpoints are identical with regard to particular (I and O) propositions.

Aristotle held that universal propositions have existential import. On the other hand, the nineteenth-century English mathematician and logician, George Boole held that no universal propositions have existential import.

George Boole is one of the founders of modern symbolic logic. Boole is now chiefly remembered for Boolean Algebra, named after him. He applied algebraic techniques to traditional Aristotelian logic. Boole proposed a calculus for syllogism. It involved translating each syllogism in arithmetical notation. This idea led to invent propositional calculus and Boolean algebra. His chief works were: The Mathematical Analysis of Logic and An Investigation of the Laws of Thought. The four standard form categorical propositions have the following meaning from the Boolean standpoint.

A-All S are P= No members of S are outside P

E- No S are P= No members of S are inside P

I-Some S are P=A least one S exists, and that S is a P

O-Some S are not P=At least one S exists, and that S is not a P.

John Venn has adopted this interpretation of categorical propositions, while he developed the diagrammatic technique for representing the information expressed by categorical propositions. In Venn diagram, there is an arrangement of overlapping circles in which each circle represents the class denoted by a term in a categorical proposition.

The Boolean square of opposition or the modern square of opposition

Drawing on some ideas of Boole we can build a square of opposition. In Boolean symbolic interpretation, the interrelation among the four standard form categorical propositions appears very clearly. It is obvious that A and O propositions are contradictories. Analogously, E and I propositions contradict each other. This relationship of mutually contradictory pairs of propositions is represented in a diagram called Modern Square of opposition or Boolean square of opposition. The diagram can be represented by the following figure-

*First, we notice that both I and O propositions have existential import, because both assert the existence of at least one entity. On the other hand, A and E propositions both do not have existential import.

*Secondly, if two propositions are related by the contradictory relation, they necessarily have opposite truth value. Thus, if a certain A proposition is given as true, the corresponding O proposition must be false. Similarly, if a certain I proposition is given as false, the corresponding E proposition must be true. But no other inferences are possible. In particular, given the truth value of an A or O proposition, nothing can be determined about the truth value of the corresponding E or I propositions. These propositions are said to have logically undetermined truth value. Similarly, given the truth value of an E or I proposition, nothing can be determined about the truth value of the corresponding A or O propositions. They, too, are said to have logically undetermined truth value.



LET US KNOW

Category: A predicate or a fundamental class of things

Class: A collection of objects or entities with a common characteristic.

Some: In logic, 'some' means at least one.

15.11 SYMBOLISM AND DIAGRAMS FOR STANDARD FORM CATEGORICAL PROPOSITIONS.

The symbolisation of the standard form categorical propositions involves set notation. The symbols used for set notation are: ?, =, ∧, ?, I The Boolean interpretation of categorical propositions depends upon the

notion of an empty set. The symbols that represent empty class are ? (Lambda) or O (zero). For example, when we say that "the class 'S' is empty", it means that "the class 'S' does not have any member". We symbolise it as S= $\wedge$ or S= O. On the other hand, to say that the class designated by 'S' is not empty is to say the class 'S' have members. We symbolise the denial by drawing line through the equality sign. For example, S $\neq \wedge$ or S $\neq$ O. It means that the class S is not empty.

The concept of complementary class is necessary to explain in this connection. A complementary class is the collection of all things that do not belong to the original class. For example, if S designates the class of 'all students', then its complementary class would be "all things that are not students" which is to be symbolised as ?. It means not S.

If two classes suppose S and P, have some members in common, this common membership will be called the product or intersection of the two classes; it is symbolised as SP or SI P and it is read as S intersection P. The symbol '?' is known as 'intersection. Thus the statement that claims that the product of the two classes is empty is symbolised as SP= $\wedge$ or SP=O or SI P = $\wedge$

On the other hand the statement which expresses that the product of two classes is not empty is symbolised as SP$\neq\wedge$ or SP$\neq$ O or SI ? P = $\wedge$.

SYMBOLISATION: The Standard form categorical propositions are symbolised in set notation as:

A- All S is P = SP=O or SP=I .

The A proposition asserts that the part of the S circle that lies outside the P circle is empty.

E- No S is P= SP =O or SP= I .

The E proposition asserts that the area where the two circles overlap is

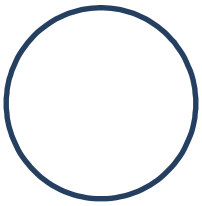
empty.

I-Some S is P= SP $\neq$ O or SP $\neq \mathrm{I}$.

The I proposition asserts that the area where the two circles overlap is not empty.

O-Some S is not P= SP $\neq$ O or SP= $\wedge$. The O proposition asserts that the part of the S circle that lies outside the P circle is not empty.

VENN DIAGRAM: After discussing the symbolization procedure, we still need to learn how to diagram a standard form categorical proposition. In Venn diagram, we represent a class by a circle labelled with the term that designates the class. Thus, the class 'S' is diagrammed as shown below:



S **Figure I**

It represents the class 'S' but says nothing about it. Hence to represent the information by the class, two kinds of marks are used: (1) shading an area and (2) Placing an X in an area. Shading an area means that the shaded area is empty and placing an X in an area means that at least one thing exists in that area. Thus, we shade the interior of the circle to indicate that the class 'S' has no members. And we place an X any where in the interior of the circle to represent that there is at least one member of the class 'S'. This is represented by the following diagram:

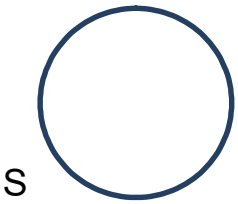


Figure II

S=O

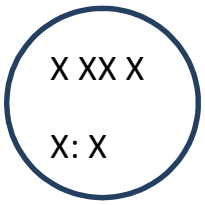
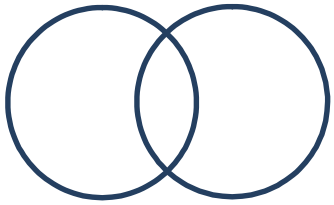


Figure III

S¹O

Standard form categorical propositions have exactly two terms. Therefore, to diagram the standard form categorical propositions two overlapping circles are required. In all the A, E, I and O propositions the subject and the predicate terms that represent classes are abbreviated by

S and P. Usually, the left hand circle represents the subject term (S) and the right hand circle represents the predicate (P) term. Such a diagram looks like this:



S **P** **Figure iv**

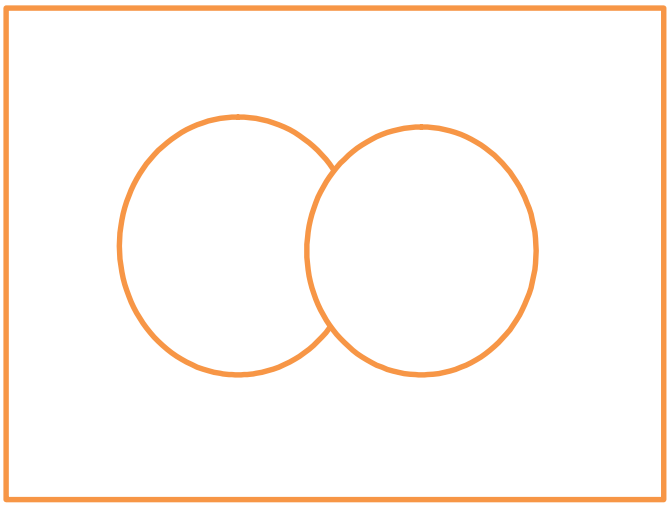
This figure diagrams only the two classes but does not give any information about their class relationship. The above diagram of the two overlapping circles represents different types of class relationship expressed by the four kinds of categorical propositions.

Firstly, the area that designates those members of S that are not members of P is symbolised as SP. The symbol P (P-bar) has been used to indicate the regions which are not P

Secondly, the area that designates those members of P that are not members of S is symbolised as SP. The symbol S (S-bar) has been used to indicate the regions which are not S.

Thirdly the area designating members of S that are at the same time members of P is symbolised as SP. This part represents the intersection of the class S and P or the product of S and P i.e., all things that belong to both S and P.

Finally, the area that designates where no members of either S or P can be found is symbolised as SP. Diagrammatically, we may represent this as follows:

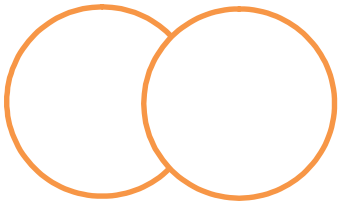


Thus, this diagram that represents various classes is known as Venn diagram. Universe of discourse: The rectangle outlining the diagram represents the universe of discourse (U.D) which means the context or everything that is assumed or believed for the discussion. There are many kinds of things in the world, such as people, plants, animals, physical objects etc. The collection of things that we are talking about on a given occasion constitutes the universe of discourse for that occasion.

Diagrams for standard form categorical propositions: Now we are in position to learn how to complete the diagrams for our four standard form categorical propositions. So, let us first diagram the Universal affirmative (A) proposition.

The 'A' proposition asserts that no members of S are outside P. In other words, all members of the class S are also the members of class P. Its schematic form is - "All S is P". It is symbolised as SP=O or SP= I . It means the part of the circle S which is not in P is empty. This is represented by shading the part of the S circle that lies outside the P circle. The diagram for the 'A' proposition is given below:

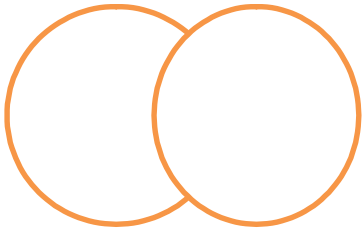
S **P**



A: All S is P

SP=O

E Proposition: 'E' Proposition is the universal negative proposition. Its schematic form is -"No S is P". It asserts that no members of S are inside P. In other words, the class S is totally excluded from the class P. It is symbolised as SP= I or SP=O. It means the common area of the circle S and P is empty. This is represented by shading the part of the S circle that lies inside the P circle. The diagram for the 'E' proposition is given below:

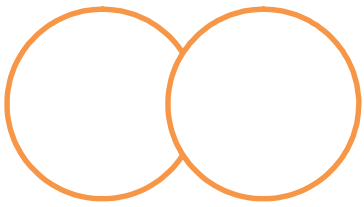


E=No S is P

SP=O

I Proposition: 'I' proposition is the particular affirmative proposition. Its schematic form is - "Some S is P". It asserts that there exists at least one member in the class S and that is also a member of the class P. In other words, the class of S is partly included in the class P. It implies the product of the class S and P is not empty. It is symbolised as SP≠O or SP≠I . This is represented by placing an X in the area where the S and P circles overlap. The diagram for the 'I' proposition is given below:

S



P

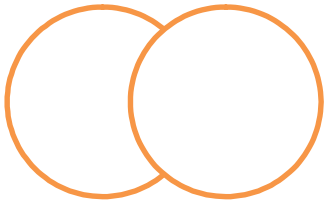
I: Some S is P

SP ≠ O

O Proposition: 'O' proposition is the particular negative proposition. Its schematic form is- "Some S is not P". The 'O' proposition asserts that at

least one S exists, and that S is not a P. It is symbolised as SP≠O or SP≠I . . The 'O' proposition is asserts that at least one S exists, and that S is not a P. This is represented by placing an X in the part of the S circle that lies outside the P circle. This X represents an existing thing that is an S but not a P. The diagram for the 'O' proposition is given below:

S **P**



O: Some S is not P

SP ≠ O



CHECK YOUR PROGRESS

Q 5: Give brief answer

a) State the symbol which represents the empty class.

.....

b) tate the symbol which represents the intersection of the two classes.

.....

c) Symbolise universal negative proposition.

.....

Q 6) What is Venn diagram?

.....

Q 7) Mention two uses of Venn diagram.

.....

Q 8) Draw Venn diagram for universal negative proposition.

.....



15.12 LET US SUM UP

- Proposition is the basic unit of logical thinking. There are different ways of classifying a proposition. A proposition that relates two classes or categories is called a categorical proposition. The classes are denoted respectively by the subject term and the predicate term. For example, 'All students are honest', 'grass is green'.
- The subject and the predicate term express different types of class relations such as total class inclusion, total class exclusion, partial class inclusion and partial class exclusion relations. These relations are expressed by the four standard form categorical propositions.
- The four standard form categorical propositions are: A-Universal affirmative (All S is P), E-universal negative (No S is P), I-particular affirmative (some S is P) and O-particular negative (some S is not P). Thus, these four propositions are referred to as standard form categorical propositions.
- The nineteenth century English logician and mathematician John Venn developed a system of diagrams to represent the categorical propositions. These diagrams have come to be known as Venn Diagrams



15.13 FURTER READINGS

1) Chakraborti. C., Logic-Informal, Symbolic & Inductive
2) Hurley. J. P., Introduction to Logic
3) Baronett. S. and Sen. M., Logic
4) Copi. I.M. & Cohen. C., Introduction to Logic
5) Sharma. B. and Deka J., A text Book of logic
6) (https://www.lucidchart.com/blog/2013/01/17/a-history-of-the-venn-diagram)

15.14 ANSWER TO CHECK YOUR PROGRESS

Ans to Q No 1.

a) A proposition that relates two classes or categories is called a Categorical proposition. For example, All ducks swim.

b) Some students are honest.

c) A class or category is a group of objects having some recognizable common properties or characteristics which can be predicated about all of them.

d) The four standard form categorical propositions are universal affirmative Universal negative, particular affirmative, particular negative. The letters A, E I and O are used as names for these four kinds of categorical propositions respectively.

Ans to Q No 2:

a) The Universal affirmative proposition is one which asserts that the subject class is wholly contained in the predicate class. In other words, in a universal affirmative proposition predicate is affirmed of the whole subject.

b) Particular negative proposition is one which asserts that some members of a class are not members of another class. Hence, in a particular negative proposition predicate is denied of the part of the subject.

Ans to Q No 3:

a) True

b) False.

Ans to Q No 4:

a) Two

b) quantifiers

Ans to Q No 5:

a) $\wedge$

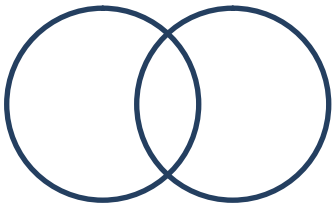
b) I

C) A-All S is P= SP=O

Ans to Q No 6: A Venn diagram is an illustration of the relationships between sets and among sets, indicated by the arrangement of the circles.

Ans to Q No 7: I) Venn diagram is used to represent the four standard form categorical propositions. II) The technique of Venn diagram is used to determine the validity or invalidity of syllogistic argument.

Ans to Q No 8:



S

E: No S is P

SP=O

15.15 MODEL QUESTIONS

A) Very short questions

Q 1: Define categorical propositions.

Q 2: What is standard form categorical proposition?

Q 3: Give an example of categorical proposition

Q 4: What is venn diagram?

Q 5: What do you mean by total class inclusion relation.

B) Short questions (Answer each question in about 150 words)

Q 1: Write short notes on:

a) Class and relation

b) Uses of Venn diagram

c) Symbolisation of A, E, I and O propositions in Boolean interpretation.

Q 2: Briefly explain the concept of categorical proposition.

Q 3: What is standard form categorical proposition? Briefly explain

Q 4: What is Boolean square of opposition? Briefly explain

C) Long questions (Answer each question in about 300-500 words)

Q 1: What is categorical proposition? Explain

Q 2: Explain the four standard form categorical propositions with examples.

Q 3: What is Venn diagram? Explain how to represent the four standard form categorical propositions by Venn diagram.

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