UNIT 3 TRUTH FUNCTIONS, INTERDEFINITIONS OF LOGICAL CONSTANTS

UNIT STRUCTURE

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3.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- explain the meaning of Truth Function
- explain the need of Truth Functions in symbolic logic
- discuss various Truth Functions through truth table
- describe the Inter-definitions of various Logical Constants

3.2 INTRODUCTION

This unit introduces to you the concept of truth functionality. Truth function is said to be the basis of propositional logic. The concept of truth function plays a pivotal role in propositional logic, because truth function helps determining the truth values of compound statements. The basic logical constants, namely, '~', '•', 'v', '⊃', '≡' are called truth functional operators. These constants are also known as truth functional connectives. These connectives yield truth functional compounds. To put it in other words, by means of the application of these connectives various truth functional compound expressions are generated. A detailed account of the meaning of these connectives is dealt with in the next chapter.

Let us try to know the background of the concept of truth function. Symbolic logic deals with proposition. A proposition is said to have two truth values: Truth (T) and falsity (F). For example, proposition 'p' may be either true or false. A simple proposition is true if it corresponds to a fact. Again, a simple proposition is false if it does not correspond to a fact. Let us take an example, 'it is raining' is a simple proposition and the truth of this proposition will depend on whether it is actually raining or not. Now one question may crop up here: is a compound proposition true if it corresponds to a fact and false if it does not correspond to a fact? The answer will be that the truth value of a compound proposition depends on the truth value of the component parts of the compound proposition. Here, component parts mean the simple propositions that constitute the compound proposition. What is the truth value of the proposition 'if it rains then Nistha will get wet' if 'it rains' is true (T) and 'Nistha will get wet' is false (F). If we look at the proposition carefully we shall see that here we are not concerned with the reasons why Nistha will not get wet, with whether she carried an umbrella or travelled on a car or whatever. We even do not bother about whether the parts of the compound proposition are really true or false. Our sole aim here is to see how the truth value of the whole proposition is dependent on the truth value of its components. In order to determine the truth value of the proposition, i.e., 'if it rains, then Nistha will get wet' we must know the rules of computation for each of five logical constants [detailed

meaning of logical constant is discussed in UNIT 4]. The symbolic representation of the proposition 'if it rains then Nistha will get wet' will be: 'p ⊃ q'. Here, 'p' stands for 'it rains', 'q' stands for 'Nistha will get wet' and ' ' stands for 'if-then'. By following the computation table for implication we can compute the truth value of the proposition: 'p ⊃ q', i.e., for an implicative function if 'p' is 'T' and 'q' is 'F', then the truth value of the compound proposition will be 'F'.

3.3 DEFINITION OF TRUTH FUNCTION

The concept of truth function in propositional logic is mathematical in origin. The notion of function is found in elementary mathematics. An expression is said to be a function of a given variable or variables, if the value of the expression is uniquely determined when the variable or variables take a determinate value. Let us take the following example:

Y = 5x + 4

In the above example y is said to be a function of x, because the value of y is dependent on the value of x. Thus if x takes the value 6, the value of y will be 34. Again, if x takes the value 7, then the value of y will be 39. Since the value of the variable y is dependent on the value of the variable x, therefore y is regarded to be the function of x.

The mathematical idea of function has been extended into logic and the idea of function has come to be known as truth function in propositional logic. Logicians prefer truth function to function, because they are concerned with the truth values of expressions. Symbolic logic, instead of dealing with numerical value, deals with truth values, i.e., 'T' and 'F'. Hence, it is called a bi-valued system.

According to the idea of truth function, the truth or falsity of a compound proposition is uniquely determined by the truth or falsity of the constituent simple propositions. The truth value of the compound proposition can be calculated by knowing the truth value of the constituent propositions and also by keeping in mind the definitions of the logical constants or logical connectives. The five basic logical connectives are: '~', 'v', '•', '⊃', '≡'. These

logical connectives yield truth functional compounds. **A truth function can be defined as any compound expression whose truth value is completely determined by the truth value of its constituent simple propositions.** The logical constants and their symbolic representations are given below:

Symbol	Name Logical	function	Meaning
~	Tilde	Negation	Not, it is not the case that
•	Dot	Conjunction	and, also, moreover
v	Wedge	Disjunction	or, unless
⊃	Horseshoe	Implication	if-then, only-if
≡	Triple bar	Equivalence	if and only if

The following is an example of truth function:

Saul Kripke is a logician and Bertrand Russell is a mathematician. The above compound expression can be symbolically represented in the following way: p • q.

'p • q' is said to be a truth function, because the truth value of the expression is determined by the value of the propositions 'p' and 'q' taken separately. Generally small letters of English alphabet such as p, q, r, s, t are [illegible] compound expression. These letters are known as p[illegible]ables.



CHECK YOUR PROGRESS

Q 1. State whether the following statements are True or False:

a) Truth function is a compound expression. (True/ False)

b) 'And' ('o') is a propositional variable. (True/ False)

c) There are five basic truth functional operators. (True/ False)

Q 2. What is truth function?

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Q 2. What do you mean by truth or falsity of a proposition?

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3.4 PROPOSITIONAL VARIABLE

Variable, in algebra, is a symbol (usually a letter) standing in for an unknown numerical value in an equation. Variable is a kind of symbol that can stand for any one of a given range of values. For example, ab=24. Here, 'a' and 'b' can stand for any value within a definite range of 1, 2, 3, 4, 5.….24. If the value of 'a' is '6' and the value of 'b' is '4', then the value of ab = 24.

The lower case letters of the English alphabet 'p', 'q', 'r', 's', 't' etc. in a compound proposition stand for simple propositions. These letters are known as propositional variables, because a variable can stand for any proposition whatsoever.

The meaning of a variable varies from place to place, from time to time. It does not have a definite meaning. Therefore a variable may be given for any range of value. For example:

If p then q

p

Therefore q

Here, p and q are variables. These variables can be substituted by any proposition. Let us take the following example to clarify the meaning of a variable.

Example 1

If you read hard (p), then you will pass in the examination (q).

You read hard (p)

Therefore you will pass in the examination (q).

The above argument can be symbolically represented by the variables, namely, 'p' and 'q' in the following way:

$p \supset q$

p

$\therefore q$

Let us cite another argument

Example 2

If you take the medicine (p), then you will recover speedily (q).

You take the medicine (p)

Therefore you will recover speedily (q).

The above argument can also be symbolically represented by the variables, 'p and 'q' in the following way:

$p \supset q$

p

$\therefore q$

We can see that in example 1 'p' stands for 'you read hard' and in example 2 'p' stands for 'you take the medicine'. Again, in example no. 1 'q' stands for 'you will pass in the examination' and in example no. 2 'q' stands for 'you will recover speedily'. This shows that the variables remain same although the propositions standing for the variables vary.

3.5 BASIC TRUTH FUNCTIONS

The following are the basic truth functions with their symbolic representations

	Truth function	**Symbolic representation**
a)	Negative or Contradictory function	$\sim p$
b)	Conjunctive function	$p \bullet q$
c)	Disjunctive function	$p \vee q$
d)	Implicative function	$p \supset q$
e)	Equivalence or biconditional function	$p \equiv q$

Apart from these five basic truth functions there is another truth function, namely, Alternative
truth function.

3.5.1 Negative or Contradictory function

The function of 'not' is referred to as negation. The negation of a proposition is another proposition whose truth value depends on the original proposition. For example, if the statement 'he is playing' is true, then the

negation of the statement 'he is not playing' would be false. Symbolically, it can be put in this way: "if 'p' is true, then 'not p' is false". Again, if the statement 'he is not playing' is true, then its contradictory statement 'he is playing' would be false. Symbolically the statement can be put in this way: "if 'p' is false, then 'not p' is true". The following truth table for negation will show how any statement having the form of a negation (~ p) is determined by the truth value of the statement that is negated (p). The following is the truth table for negation:

P	**~P**
T	F
F	T

The value of ~ p depends on the value of the variable p, hence ~ p is the truth function of p

3.5.2 Conjunctive function

A truth functionally compound expression composed of two simple propositions by the connective 'and' (o) is called conjunctive function. For example, 'Plato founded the Academy and Aristotle founded the Lyceum'- is a compound proposition. The logical connective 'and' (o) joins two simple propositions, namely, 'Plato founded the Academy' and 'Aristotle founded the Lyceum' and thereby it forms a compound proposition. The symbolic formulation for this conjunctive function is 'p o q'. (Here, 'p' stands for 'Plato founded the Academy' and 'q' stands for 'Aristotle founded the Lyceum').

The constituents of a conjunctive proposition 'p' and 'q' are called conjuncts. Variable 'p' is called 'left conjunct' and variable 'q' is called 'right conjunct'. The truth value of a conjunctive function is calculated according to the following rule covering all possible cases:

1. If the left conjunct is true and the right conjunct is true, then the conjunctive truth function as a whole is true.

2. If the left conjunct is true and the right conjunct is false, then the conjunction as a whole is false.
3. If the left conjunct is false and the right conjunct is true, then the conjunction as a whole is false.
4. If the left conjunct is false and the right conjunct is false, then the conjunction as a whole is false.

From the above conditions the following general rule may be extracted:

A conjunctive function is true when all its conjuncts are true. A conjunctive function is false if one of the conjuncts is false.

By following the definitions of truth operators or logical connectives we can construct a truth table for conjunctive truth function:

p	q	p • q
T	T	T
T	F	F
F	T	F
F	F	F

The truth values under the logical connective 'o' exhibit all the possible truth values of the compound structure 'p o q'.

Conjunctive function has got another important characteristic which is expressible in terms of some 'mathematical jargon'. Conjunctive function can be commutative, associative and idempotent. These three properties can be symbolically represented in the following way:

$(p \bullet q) \equiv (q \bullet p)$ Commutative property

$p \bullet (q \bullet r) \equiv (p \bullet q) \bullet r$ Associative property

$(p \bullet p) \equiv p$ Idempotent property

3.5.3 Disjunctive function

A truth functionally compound expression composed of two simple propositions by the connective 'or' (v) [in the sense of inclusive disjunction]

both the disjuncts can be true, while exclusive disjunction rules out this possibility and declares that only one or the other, not both of the disjuncts is true. For example, 'Bertrand Russell is either a logician or he is a mathematician' is a disjunctive proposition. The symbolic representation of this proposition will be: 'p v q' (Here, 'p' stands for 'Bertrand Russell is a logician' and 'q' stands for 'Bertrand Russell is a mathematician').

The constituents of a disjunctive proposition 'p' and 'q' are called disjuncts. Variable 'p' is called 'left conjunct' and variable 'q' is called 'right conjunct'. The truth value of a conjunctive function is calculated according to the following rule covering all possible cases:

1. If the left disjunct is true and the right disjunct is true, then the disjunction as a whole is true.
2. If the left disjunct is true and the right disjunct is false, then the disjunction as a whole is true.
3. If the left disjunct is false and the right disjunct is true, then the disjunction as a whole is true.
4. If both left disjunct and right disjunct are false, then the disjunction as a whole is false.

From the above conditions the following general rule may be extracted:

A disjunctive function is true when at least one of the disjuncts is true. When all the disjuncts are false, the disjunctive function is false.

By following the definitions of truth operators or logical connectives we can construct a truth table for disjunctive truth function:

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

The truth values under the logical connective 'v' contain all the possible truth values of the compound structure 'p v q'.

Just as conjunctive function, disjunctive function also has certain mathematical properties. Disjunctive function has commutative, associative and idempotent properties.

These properties can be symbolically represented in the following way:

$(p \vee q) \equiv (q \vee p)$	Commutative property
$p \vee (q \vee r) \equiv (p \vee q) \vee r$	Associative property
$(p \vee p) \equiv p$	Idempotent property

3.5.4 Implicative or Conditional function

A truth functionally compound expression composed of two simple propositions by the connective 'if-then' ($\supset$) is called Implicative or conditional function. For example, 'if you work hard, then you will be able to reach your goal.' The logical connective 'if-then' ($\supset$) joins two simple propositions, namely, 'you work hard' and 'you will be able to reach your goal' and thereby it forms a compound proposition. The symbolic formulation for this implicative function is '$p \supset q$'. (Here, 'p' stands for 'you work hard' and 'q' stands for 'you will be able to reach your goal.')

In this type of compound the component between the 'if' and the 'then' is called the antecedent. Here, 'you work hard' is an antecedent. The component proposition that follows the 'then' is called consequent. The truth value of a conjunctive function is calculated according to the following rule covering all possible cases:

1. If the antecedent is true and the consequent is true, then the conditional function as a whole is true.
2. If the antecedent is true and the consequent is false, then the conditional function as a whole is false.
3. If the antecedent is false and the consequent is true, then the conditional function as a whole is true.

4. If the antecedent is false and the consequent is false, then the conditional function as a whole is true.

From the above conditions the following general rule may be extracted:

1. A material conditional will be true whenever its consequent is true, irrespective of what value its antecedent has -true or false.
2. A material conditional will be true whenever its antecedent is false, irrespective of what value its consequent has -true or false.

By following the definitions of truth operator or logical connective '⊃' we can construct a truth table for conditional truth function:

P	q	p ⊃ q
T	T	T
T	F	F
F	T	T
F	F	T

3.5.5 Equivalence or Biconditional function

A truth functionally compound expression composed of two simple propositions by the connective 'if and only if' (≡) is called equivalence or biconditional function. For example, 'One can apply for the post of Assistant Professor if and only if one qualifies NET '-is a compound proposition. The logical connective 'if and only if' (≡) joins two simple propositions, namely, 'One can apply for the post of Assistant Professor' and 'one qualifies NET'; and thereby it forms a compound proposition. The symbolic formulation for this biconditional function is 'p ≡ q'. (Here, 'p' stands for 'One can apply for the post of Assistant Professor' and 'q' stands for 'one qualifies NET')

The truth value of a biconditional function is calculated according to the following rules covering all possible cases:

1. If both the constituents are true, then the biconditional function as a whole is true.

2. If the left constituent is true and the right constituent is false, then the biconditional function as a whole is false.
3. If the left constituent is false and the right constituent is true, then the biconditional function as a whole is false.
4. If both the constituents are false, then the biconditional function as a whole is true.

From the above conditions the following general rule may be extracted:

A biconditional function is true when both the constituent propositions have the same truth value. A biconditional function is false when one constituent proposition is true and the other constituent proposition is false.

By following the definitions of truth operators or logical connectives we can construct a truth table for biconditional truth function:

p	q	p = q
T	T	T
T	F	F
F	T	F
F	F	T



CHECK YOUR PROGRESS

Q 3. State whether the following statement is True or False.

a) There are five basic truth functions. (True/ False)

b) Propositional variables can stand for any proposition whatsoever. (True/ False)

c) The lower case letters of English alphabet are taken as propositional variables. (True/ False)

Q 4. When is a conjunctive function false?

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Q 5. When is a disjunctive function false?

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Q 6. When is a conditional function false?

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Q 7. When is a biconditional function true?

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ACTIVITY: 3.2

- Prepare a chart for the basic truth functions with suitable examples

Ans .....

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3.6 ALTERNATIVE FUNCTION

A truth functionally compound expression composed of two simple propositions by the connective 'or' (v) [in exclusive sense] is called alternative truth function. For example, 'Dodere is either literate or illiterate'-is a compound proposition. The logical connective 'either-or' (^) joins two simple propositions, namely, 'Dodere is literate' and 'Dodere is illiterate'; and thereby it forms a compound proposition. The symbolic formulation for this alternative function is 'p ^ q'. (Here, 'p' stands for 'Dodere is literate' and 'q' stands for 'Dodere is illiterate')

The truth value of an alternative function is calculated according to the following rules covering all possible cases:

1. If both the constituents are true, then the alternative function as a whole is false.
2. If the left constituent is true and the right constituent is false, then the alternative function as a whole is true.
3. If the left constituent is false and the right constituent is true, then the alternative function as a whole is true.
4. If both the constituents are false, then the alternative function as a whole is false.

From the above conditions the following general rule may be extracted:

An alternative function is true when one constituent proposition is true and the other constituent proposition is false. An alternative function is false when either both the constituent propositions are true or both are false.

By following the definitions of truth operators or logical connectives we can construct a truth table for alternative truth function:

p	q	p ^ q
T	T	F
T	F	T
F	T	T
F	F	F

3.7 STROKE FUNCTION

Stroke function was introduced by a logician named Henry M. Sheffer in 1913. In Boolean functions and propositional calculus, the Sheffer stroke is written as "I" (vertical bar) or ""↑" (an upward arrow). It denotes a logical operation that is equivalent to the negation of the conjunction operation. The logical operation, in ordinary language, is known as "not both." The stroke function is also called the alternative denial, since it says that at least one of its operands is false. In Boolean algebra and digital electronics it is also known as the NAND operation ("not and").

It is pointed out by the logicians that various truth functions are inter-definable. We have seen that 'v', '•', '⊃', are definable in terms of each other with the help of the primitive constant '~'. The fact that logical constants are inter-definable in this way makes one ask, "how many of these constants are really indispensable for building up our calculus of propositions?" All functions but two can be dispensed with, since '⊃' and '•' can be defined in terms of '~' and 'v'; and 'v' can be defined in terms of '~' and '•' or alternatively in terms of '~' and '⊃'. Can the number of indispensable constants be reduced still further? The answer is that all logical constants save one can be dispensed with. Stroke function is defined as follows: 'p/q is to be read "at least one of p and q is false". The following truth table represents the definition of stroke function:

p	q	p • q
T	T	T
T	F	F
F	T	F
F	F	F

The following functions can be defined in terms of stroke function:

1. ~ p is defined as p/p

   ~ p = df p/p

2. p • q is defined as (p/q)/(p/q)

   p • q = df (p/q)/(p/q)

3. p v q is defined as (p/p)/ (q/q)

   p v q = df (p/p)/ (q/q)

4. p ⊃ q is defined as p/(q/q)

   p ⊃ q = df p/(q/q)



CHECK YOUR PROGRESS

Q 8. State the condition under which an alternative function is true.

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Q 9. How is 'p/q'read?

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3.8 INTER-DEFINITIONS OF LOGICAL CONSTANTS

Various logical constants in propositional logic are inter-definable. Each logical constant has its own special characteristics yet they can be reduced and defined into each other. For example, the logical constant 'and' can be defined in terms of other logical constants such as '~' and '⊃'; and '~' and 'V'. Defining and reducing the logical constants into one another is called 'inter-definitions of logical constants' or 'inter-definability of truth functions'. Logical constants such as '~', '•', '⊃', 'v', '≡' are thus inter-definable. What is important here is that while defining one logical constant in terms of other logical constants, the meaning of the original expression must remain unchanged. The following inter-definitions express inter-definitions of various logical constants:

1. Logician Augustus de Morgan points out that the logical constant 'and'/ '(•)' [conjunction] may be defined in terms of the logical constants 'not' / '(~)' [negation] and 'or/ '(v)' [disjunction]. Again, the logical constant 'and'/ '(•)' [conjunction] can be defined in terms of the logical constants 'not' / '(~)'[negation] and 'if-then'/ '⊃'[implication or material conditional].

   The following are the inter-definitions:

   (i) '~' and 'v' as

   p • q = df ~ (~ p v ~ q)

   (ii) '~' and '⊃' as

   p • q = df ~(p ⊃ ~ q)

   [Here 'df' is abbreviated for 'definition']

The above inter-definitions can be shown to be logically equivalent by means of truth table. The following are the truth tables for these inter-definitions:

(I) p • q ≡ ~ (~ p v ~ q)

T	T	T	T	T	F	T	F	F	T
T	F	F	T	F	F	T	T	T	F
F	F	T	T	F	T	F	T	F	T
F	F	F	T	F	T	F	T	T	F

(ii) p • q ≡ ~ (p ⊃ ~ q)

T	T	T	T	T	T	F	F	T
T	F	F	T	F	T	T	T	F
F	F	T	T	F	F	T	F	T
F	F	F	T	F	F	T	T	F

2. The logical constant 'or'/'v' [disjunction] can be defined in terms of the logical constants 'not'/ '~' [contradiction] and 'and'/ '•' [conjunction]. Again, the logical constant 'or'/'v' [disjunction] can be defined in terms of the logical constants 'not'/ '~' [contradiction] and 'if-then'/'⊃'[implication or material conditional].

The inter-definitions can be presented in the following way:

(i) '~' and '•' as

p v q = df ~ (~ p • ~ q)

(ii) '~' and '⊃' as

p v q = df ~ p ⊃ q

With the help of truth table, the above inter-definitions can be shown to be logically equivalent. The following are the truth tables for these inter-definitions:

(i) p v q ≡ ~ (~ p • ~ q)

T	T	T	T	T	F	T	F	F	T
T	T	F	T	T	F	T	F	T	F
F	T	T	T	F	F	F	T		
F	F	F	T	F	T	F	T	T	F

(ii) p v q ≡ ~ p ⊃ q

T	T	T	T	F	T	T	T
T	T	F	T	F	T	T	F
F	T	T	T	T	F	T	T
F	F	F	T	T	F	F	F

3. The logical constant 'if-then'/'⊃'[implication or material conditional] can be defined in terms of the logical constants 'not'/ '~' [contradiction] and 'or'/'v' [disjunction]. Again, the logical constant 'if-then'/' '[implication or material conditional] can be defined in terms of the logical constants 'not'/ '~' [contradiction] and 'and'/ '•' [conjunction].

The following are the inter-definitions:

(i) '~' and 'v' as

p ⊃ q = df ~ p v q

(ii) '~' and '•' as

p ⊃ q = df ~ (p • q)

With the help of truth table, the above inter-definitions can be shown to be logically equivalent. The following are the truth tables for these inter-definitions:

(i) p ⊃ q ≡ ~ p v q

T	T	T	T	F	T	T	T
T	F	F	T	F	T	F	F
F	T	T	T	T	F	T	T
F	T	T	T	T	F	T	F

(ii) p ⊃ q ≡ ~ (p • ~ q)

T	T	T	T	T	T	F	F	T
T	F	F	T	F	T	T	T	F
F	T	T	T	T	F	F	F	T
F	T	T	T	T	F	F	T	F

4. The logical constant 'if and only if'/ '≡' [bi-conditional] can be defined in terms of the logical constants 'if-then'/'⊃'[implication or material conditional] and 'and'/ '•' [conjunction]. The logical constant 'if and only if'/ '≡' [bi-conditional] can be defined in terms of the logical constants 'and'/ '•' [conjunction], 'not'/ '~' [contradiction] and 'or'/'v' [disjunction].

The following are the inter-definitions:

(i) $(p \equiv q) =_{df} (p \supset q) \bullet (q \supset p)$

(iii) $(p \equiv q) =_{df} (p \bullet q) \vee (\sim p \bullet \sim q)$

The above inter-definitions can be shown to be logically equivalent by means of truth table. The following are the truth tables for these inter-definitions:

(i) $(p \equiv q) \equiv (p \supset q) \bullet (q \supset p)$

T	T	T	T	T	T	T	T	T	T	T
T	F	F	T	T	F	F	F	F	T	T
F	F	T	T	F	T	T	F	T	F	F
F	T	T	T	F	T	F	T	F	T	F

(ii) $(p \equiv q) \equiv (p \bullet q) \vee (\sim p \bullet \sim q)$

T	T	T	T	T	T	T	T	F	T	F	F	T	
T	F	F	T	T	F	F	F	F	T	F	T	F	
F	F	T	T	F	F	T	F	T	F	F	F	T	
F	T	F	T	F	F	F	T	T	F	T	T	F	

[The expression '= df' which is used in all above inter-definitions mean 'is equivalent to by definition'. Let us take the following example to illustrate this point. The proposition 'Russell is both mathematician and philosopher' is equivalent to the proposition-'It is false that either Russell is not intelligent or Russell is not honest'. This means that these two propositions are logically equivalent (≡). Therefore the symbolic expression of this equivalent expression is either '$(p \circ q) \equiv_{df} \sim(\sim p \vee \sim q)$' or '$(p \circ q) \equiv \sim(\sim p \vee \sim q)$'.]



CHECK YOUR PROGRESS

Q 10. Mention various inter-definitions

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3.9 LET US SUM UP

- According to the idea of truth function, the truth or falsity of a compound proposition is uniquely determined by the truth or falsity of the constituent simple propositions. The truth value of the compound proposition can be calculated by knowing the truth value of the constituent propositions and also by keeping in mind the definitions of the logical constants or logical connectives. The five basic logical connectives are: '~', 'v', '•', '⊃', '≡'. These logical connectives yield truth functional compounds. **A truth function can be defined as any compound expression whose truth value is completely determined by the truth value of its constituent simple propositions.**
- The lower case letters of the English alphabet 'p', 'q', 'r', 's', 't' etc. in a compound proposition stand for simple propositions. These letters are known as propositional variables, because a variable can stand for any proposition whatsoever.
- There are five basic truth functions. They are:

Truth function	Symbolic representation
a. Negative or Contradictory function	~ p
b. Conjunctive function	p • q
c. Disjunctive function	p v q
d. Implicative function	p ⊃ q
e. Equivalence or biconditional function	p ≡ q

- The function of 'not' is referred to as negation. The negation of a proposition is another proposition whose truth value depends on the original proposition.
- A truth functionally compound expression composed of two simple propositions by the connective 'and' (o) is called conjunctive function.
- A truth functionally compound expression composed of two simple propositions by the connective 'or' (v) [in the sense of inclusive disjunction] is called Implicative or conditional function.
- A truth functionally compound expression composed of two simple propositions by the connective 'if and only if' (≡) is called equivalence or biconditional function.
- A truth functionally compound expression composed of two simple propositions by the connective 'or' (v) [in exclusive sense] is called alternative truth function.
- Stroke function was introduced by a logician named Henry M. Sheffer in 1913. In Boolean functions and propositional calculus, the Sheffer stroke is written as "|" (vertical bar) or ""↑" (an upward arrow). It denotes a logical operation that is equivalent to the negation of the conjunction operation. The logical operation, in ordinary language, is known as "not both." The stroke function is also called the alternative denial, since it says that at least one of its operands is false.
- Various logical constants in propositional logic are inter-definable. Each logical constant has its own special characteristics yet they can be reduced and defined into each other. For example, the logical constant 'and' can be defined in terms of other logical constants such as '□' and '⊃'; and '□' and 'V'. Defining and reducing the logical constants into one another is called 'inter-definitions of logical constants' or 'inter-definability of truth functions'.

3.10 FURTHER READINGS

1) Chhanda C. (2007), Logic-Informal, Symbolic and Inductive, Prentice-Hall of India Pvt. Ltd.

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3) Hurley, P. J. (2007), Introduction to Logic, Cengage Learning India Private Limited.

4) Klenk,V. (2008), Understanding Symbolic Logic, Dorling Kindersley (India) Pvt. Ltd.

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3.11 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q. No. 1:

a) True

b) False

c) True

Ans. to Q. No.2: A truth function can be defined as any compound expression whose truth value is completely determined by the truth value of its constituent simple propositions. For example: Saul Kripke is a logician and Bertrand Russell is a mathematician. The compound expression can be symbolically represented in the following way: p•q.

Ans. to Q. 3:

a) True

b) True

c) True

Ans. to Q. 4: A conjunctive function is false if one of the conjuncts is false.

Ans. to Q. 5: If both left disjunct and right disjunct are false, then the disjunction as a whole is false.

Ans. to Q. 6: If the antecedent is true and the consequent is false, then the conditional function as a whole is false.

Ans. to Q. 7: If both the constituents are of same value, then the bi-conditional function is true. This means that if both the constituents

are true, the biconditional function is true. Again, if both the constituents are false, the biconditional function is true.

Ans. to Q. 8: An alternative function is true when one constituent proposition is true and the other constituent proposition is false.

Ans. to Q. 9: 'p/q is to be read "at least one of p and q is false".

Ans. to Q. 10:

1. $p \bullet q = df \sim(\sim p \vee \sim q)$
2. $p \bullet q = df \sim(p \supset \sim q)$
3. $p \vee q = df \sim(\sim p \bullet \sim q)$
4. $p \vee q = df \sim p \supset q$
5. $p \supset q = df \sim p \vee q$
6. $p \supset q = df \sim(p \bullet \sim q)$
7. $(p \equiv q) = df (p \supset q) \bullet (q \bullet p)$
8. $(p \equiv q) = df (p \bullet q) \vee (\sim p \bullet \sim q)$



3.12 MODEL QUESTIONS

A) Very Short Questions:

Q 1: What is function?

Q 2: Give an example of truth function.

Q 3: Give an example of propositional variable.

Q 4: What is variable?

Q 5: "Truth function is a compound expression"-give one reason.

B) Short questions

Q 1: Write a short note on propositional variable.

Q 2: Distinguish between Logical constant and propositional variable.

Q 3: What is alternative function? Explain with suitable examples.

Q 4: Write a short note on Stroke Function.

C) Long Questions

Q 1: Discuss various truth functions with suitable examples.

Q 2: Explain various truth functions with the help of truth table.

Q 3: What do you mean by interdefinability of truth function? Explain how various basic truth functions or logical constants are interdefinable.

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