UNIT 4 DEFINITION OF CONNECTIVES AND SYMBOLIZATION OF EVERYDAY LANGUAGE

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4.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- explain the meaning of Connective used in symbolic logic.
- explain the need of Connective.
- discuss various Connectives with examples.
- explain the need of symbolization for logical purposes
- discuss different procedures for symbolizing simple and compound statements.

4.2 INTRODUCTION

Logician and mathematician Bertrand Russell states that symbolic logic consists of three parts: the calculus of propositions, the calculus of classes and the calculus of relations. The calculus of propositions or propositional logic, also known as sentential logic or statement logic is that branch of logic that studies ways of combining or altering statements or propositions to form more complicated statements or propositions. In propositional logic, the simplest statements are considered as indivisible units. The most thoroughly researched branch of propositional logic is classical truth-functional propositional logic. Such type of logic studies about logical connectives that are used to produce complex statements whose truth value depends entirely on the truth values of the simpler statements which makes the complex statements, and in which it is assumed that every statement is either true or false but not both. Symbolic logic uses two kinds of symbols to form compound statements. One is logical connectives and the other is propositional variable.

4.3 MEANING OF CONNECTIVE

Connectives in propositional logic are special words or terms which are used to generate statements when used. They connect or join statements to produce other statements. Hence they are called connectives. These connectives are also called truth functional connectives because they function truth functionally. This means that propositional logic deals only with truth functional connectives. The following are the five truth functional connectives are: Not, And, Or, If-then, If and only if.



CHECK YOUR PROGRESS

Q. 1 State whether the following statements are True or False:

a) According to Bertrand Russell, symbolic logic consists of three parts.(True/ False)

b) The number connectives used in propositional logic is four.(True/ False)

Q 2. How many connectives are used in propositional logic and what are they?

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4.4 DEFINITION OF CONNECTIVES

The following points will give you the idea about the nature of the logical connectives

4.4.1 Not/ It is not the case that (~) [Negation]

- Not is a monadic or unary connective as it can connect only one statement at a time.
- The function of Not is referred to as negation.
- The connective Not is represented by the symbol '~'.
- The symbol '~' is called 'curl' or 'tilde'.
- In propositional logic 'Tilde' is always placed in front of the proposition it negates. All operators other than tilde are placed between two propositions. Tilde is used to translate any negated simple proposition. Let us take some examples:

Statement	Symbolized form
a. Philosophers do not commit mistake	~P
b. It is not the case that philosophers commit mistakes	~P
c. It is false that philosophers commit mistakes	~P

- Tilde cannot be used to connect two propositions. Thus P ~M is not a proper expression.
- Tilde is used to negate a simple proposition, but it can also be used to negate a compound proposition, e.g. ~ (P o M). In this case tilde negates the entire expression inside the parantheses.
- Chhanda Chakraborti in her book Logic Informal, Symbolic and Inductive says, "It is important to remember that in propositional logic, negations, i.e., statements with explicit 'not' or its equivalent in them, are compound statements." (p. 76) Let us take an example to understand this point. The statement 'Philosophers do not commit mistake' can be symbolically represented as ~M. Since M is prefixed by '~', ~M is said to be a compound statement.
- The negation of any true statement is false and the negation of any false statement is true. This point can be stated through a truth table.

p	~ p
T	F
F	T

4.4.2 And (•) [Conjunction]

- A compound statement composed by the connective 'and' is called a conjunction.
- 'And' is the dyadic or binary connective as it can connect two statements at a time.
- The two simple statements that are connected through the logical connective 'and' are known as 'conjuncts.'
- The logical connective 'and' is represented by the symbol 'o'.
- The symbol 'o'is called 'dot'.
- In propositional logic 'dot' is placed between two propositions. Dot is used to translate such conjunctions as 'and', 'also', 'but', 'however', 'yet', 'still', 'moreover', 'although' and 'nevertheless'. Let us take the following example:

Sentences	Symbolization
Example 1: Frege is a logician, and Amartya Sen is an economist	F • E
Example 2: Frege is a logician, but Amartya Sen is an economist	F • E
Example 3: Frege is a logician; however Amartya Sen is an economist	F • E
Example 4: Frege and Amartya Sen are thinkers	F • E

Note that example 4 is equivalent to the meaning to 'Frege is a thinker and Amartya Sen is a Thinker'. In order to translate such a statement as a conjunction of two simple statements, the original statement must be equivalent to a compound statement in English. The statement 'Ananya and Adwita are friends' cannot be said to be equivalent in meaning to 'Ananya is a friend and Dvita is a friend'. Hence, the statement 'Ananya is a friend and Advita is a friend' cannot be translated as 'A o D'.

- A conjunction is true only when both the conjuncts are true; it is false in every other case. If p and q are any true conjuncts, then the truth condition for 'p o q' will be:

p	Q	p • q
T	T	T
T	F	F
F	T	F
F	F	F

4.4.3 Either-Or (v) [Disjunction]

- A compound statement formed by connecting two statements with the help of the connective 'or' is called a disjunction.
- 'Or' is the dyadic or binary connective as it can connect two statements at the same time. The individual statements are known as 'disjuncts'.
- The logical connective 'or' is represented by the symbol 'v'.
- The symbol 'v' is called 'wedge'. It is also called 'vee'.

- There are two senses of the word 'or' in English language: one is the inclusive sense and the other is the exclusive sense. The following is the difference between these two senses of the word 'or':

Either-or: (Inclusive sense)

- The inclusive sense of 'or' suggests that it is possible for both the disjuncts to be true at the same time.
- Let us take an example:

  Frege is either a logician or a musician. This means that Frege can both be a logician and a musician at the same time.

Either-or: (Exclusive sense)

- The exclusive sense of 'or' suggests that it is not possible for both the disjuncts to be true at the same time.
- Let us take an example:

  Frege is either literate or an illiterate person. This means that Frege cannot both be a literate and illiterate person at the same time.
- Disjunction in propositional logic represented by 'v' is not an exclusive 'or'. It is an inclusive 'or'.
- In propositional logic 'wedge' is placed between two propositions. Wedge is a symbolic representation of the word 'or' taken in an inclusive sense. Let us take the following example:

Sentences	**Symbolization**
Example 1: Either I shall call him up or I shall write to him.	C V W
Example 2: I shall not call him up unless some emergency pops up.	~ CVE

Note in example 2 that if two propositions are connected by 'unless' then also disjunctive can be used.

- disjunction is true when either of the disjuncts is true and also when both the disjuncts are true; and is false when both the disjuncts are false. If p and q are any true disjuncts, then the truth condition for (p v q) will be:

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

4.4.4 If-then (Material Implication)

- A compound statement formed by connecting two statements with the help of the connective 'or' is called a **material conditional.**
- 'If-then' is the dyadic or binary connective as it can connect two statements at the same time. The component statement which contains 'if' is known as antecedent. Again, the component statement which contains 'then' is known as consequent.
- The logical connective 'if-then' is represented by the symbol '⊃'.
- The symbol '⊃' is called 'horse-shoe' or 'hook'. Customarily, the horse-shoe symbol is also used to translate 'implies'.
- The symbol '⊃' is used to translate such conditionals as 'given that', 'provided that', 'if', 'only if' etc.

Let us take the following examples:

Sentences **Symbolization**

Example 1: If it rains, then we shall not go for shopping R ~S

Example 2: Given that Ananya works, Adwita will also work A D

- An implication is false when antecedent is true and consequent is false. If p is antecedent and q is consequent, then the truth condition for p⊃q will be

4.4.5 If and only if (Equivalence or Biconditional)

- A compound statement formed by the connective 'if and only if' (also used iff for short) is called equivalence or biconditional statement.
- The biconditional, as its name implies, is a two-way conditional.
- The function of 'if and only if' is referred to as bicondition.
- The logical connective 'if and only if' is represented by the symbol '≡'.
- The symbol '⊃' is called 'triple bar' or 'equivalent' sign.
- Let us take the following examples:

Sentences **Symbolization**

Example 1: The shopkeeper will allow us to buy the stuff if and only if we pay him. $S \equiv P$

Example 2: Ashok will attend the ceremony if and only if he will be invited. $A \equiv I$

- A biconditional is true if when both the propositions have the same truth value. A biconditional is false when both the propositions have different truth values. The truth condition for 'p ≡ q' will be:

p	q	p ≡ q
T	T	T
T	F	F
F	T	F
F	F	T



CHECK YOUR PROGRESS

Q 3. State whether the following statements are true or false

a) A conjunction is true if at least one of the conjuncts is true.(True/ False)

b) A disjunction is true if at least one of the disjuncts is true. (True/False)

c) A material conditional is true if the antecedent is true and the consequent is false.(True/ False)

d) A biconditional is true if both the propositions have the same truth value.(True/ False)



ACTIVITY: 4.1

- Why do you think that connectives of propositional logic are truth functional?

Ans .....

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4.5 SYMBOLIZATION OF EVERYDAY LANGUAGE

There are two-fold tasks in the elementary symbolic logic for the beginners. They are the following:

1. Symbolization of propositions and arguments stated in ordinary language. It is very necessary to learn how to symbolize English sentences in logic because symbolization will be incorrect unless punctuation marks are put correctly. If we wrongly symbolize an English sentence, the logical structure of the sentence will become wrong.

2. Testing the validity of various types of arguments. As it is known to us that the validity of an argument depends on the correct form of the argument, it is therefore necessary to symbolize the propositions constituting the argument correctly.

It is to be mentioned here that punctuation is important in mathematics. In the absence of a special convention no number is uniquely denoted by 4 x4 + 5, although when it is made clear how its constituents are grouped, it denotes either 21 or 36: the first when punctuated will be (4x4)+5=21, while the second when punctuated will become 4 x (4+5)=36. To avoid ambiguity and to make meaning clear punctuation marks in mathematics appear in the form of parentheses ( ), which are used to group individual symbols; brackets [ ], which are used to group expressions that include parentheses etc.

In the language of symbolic logic also same punctuation marks as used in mathematics are used. In logic, for example, 'p • q v r' will be considered ambiguous as it is devoid of punctuation mark. It may mean either the conjunction of p with q v r or else the conjunction of p • q with r. The different meaning of two expressions can be explicated by punctuating the given formula as p • (q v r), or else as (p • q) v r. In case p and q both are false and r is true, the first punctuated expression is false (since its first conjunct is false), but the second punctuated expression is true (since its second disjunct is true). It is because punctuation formulas differ in both the cases, the truth or falsity also varies in both.

Parentheses, square brackets are the devices to demarcate clearly the scope of each connective. The scope of a connective is the range of its operation or the extent of its operation within the statement. For example, the scope of negation is always over that which is negated, that is, in '~R' the component 'R' is under the scope of '~'. It is to be remembered here that when there are several connectives in use, the main connective is the truth functional connective in the statement which has the maximum scope. The scope of the main connective covers the entire statement, and all the

component statements. The following is the example showing the maximum scope of a connective:

~[(A • B) v (C ≡ D)]

In the above example it can be observed that '~' ranges over the whole statement and all the components; the scope of 'o' is only over (A • B), again the scope of '≡' is only over (C≡D), and the connective 'v' ranges over both (A • B) and (C≡D). Therefore '~' is said to have the maximum scope.

Chhanda Chakraborti in her book Logic: Informal, Symbolic and Inductive lays down the following statements as the key to the symbolization in propositional logic to be followed closely:

1. Pick out the atomic or simple statements in the given statements.
2. If a list of capital letters to be used as the translation key is not provided, then select an appropriate and unique capital letter for each distinct simple statement.
3. Read the statement carefully to understand the grouping of the statements.
4. Look out for punctuation marks such as comma to identify the main and subordinate connectives.
5. Replace each simple statement by the chosen capital letter.
6. Replace each English connective by the appropriate connective from propositional logic.

4.5. 1 TWO KINDS OF STATEMENTS

This section will concentrate on the basic structure or grammar of one branch of symbolic logic known as sentential or propositional logic. It is that section of symbolic logic which considers simple statements as its atomic or the most basic or elementary units, such as 'A. N. Whitehead is a logician' or 'A. J. Ayer is a logical positivist'. Simple statements form the foundation of propositional logic. In addition to the simple statements we come across compound statements in logic. Thus statements can be divided into two types:

a. Simple statements

b. Compound statements

4.5.1.1 SYMBOLIZATION OF SIMPLE STATEMENTS

Propositional logic deals with propositions. Hence, it is known as the logic of propositions. Propositional logic considers simple statements to be the most basic units. By holding that simple statements are the most basic units, propositional logic exhibits one important point, i.e. simple statements are not further analyzable.

By a simple statement we mean structurally simple statements. A simple statement is one that does not contain any other statement as a component part. For example, 'Colombus discovered America' is a simple statement since it has only one unanalyzable content to be stated.

Examples of simple statements:

1. All Assamese are Indians.
2. I. M. Copi is a famous logician.
3. Krisha Kanta Handique State University is an open university.

Any convenient upper case letter from English alphabet can be selected to represent the statements. Example 1 can be symbolically expressed as 'I', example 2 can be symbolically expressed as 'L', and example 3 can be symbolically expressed as 'O'.

4.5.1.2 SYMBOLIZATION OF COMPOUND STATEMENTS

A compound statement is one which contains more than one simple proposition as its constituent part.

A) Negation:

In section 4.4.1 it has been pointed out that in propositional logic a negated simple proposition containing the connective 'not' is considered a

compound statement. Let us learn about the symbolization of sentences containing negation:

(i) Abhipsha is not a good student.

~ A

(ii) The students will not go to enjoy movie tomorrow.

~ S

(iii) It is not the case that Dispur is the capital of Delhi.

~ D

(iv) It is false that grasses are not green.

~(~ G)

(v) It is not true that truth does not hold.

~(~ T)

(vi) It is untrue that bribery is good.

~(B)

B) Conjunction:

(i) Abhipsha likes rice and Bipasha likes chapatti.

A • B

(ii) Rajesh is poor but honest.

R • H

(iii) Ananya loves Prakash, however he barely tolerates her.

A •T

(iv) It is raining; nevertheless we shall go for shopping.

R • S

(v) Chinmoyee is honest, although not very intelligent.

C • ~ I

(vi) It is cold yet tolerable.

C • T

(vii) 'But' is a truth functional connective, while 'because' is not.

T • ~ B

(viii) Akash will go to the market despite the fact that he is ill.

A • I

(ix) Neelabh is honest, moreover he is intelligent.

$N \bullet I$

C) Disjunction:

(i) Either Apurva is intelligent or he is honest

$A \vee H$

(ii) Either Apurva is not honest or he is not intelligent.

$\sim A \vee \sim I$

D) Implication:

(i) If it rains, then we shall not go for the picnic.

$R \supset \sim P$

(ii) If it does not rain, then we shall go for shopping.

$\sim R \supset S$

(iii) You will come out with flying colours only if you work hard.

$F \supset W$

(iv) You will come out with flying colours if you work hard.

$W \supset F$

E) Biconditional:

(i) You will pass in the examination if and only if you work hard.

$P \equiv W$

4.5.2 COMPLEX SYMBOLIZATIONS

A) Not Both/Both Not

i) Not Both:

Example:

Chapatti and rice will not both be served here.

This sentence can be paraphrased in the following way:

'It is not the case that both chapatti and rice will be served.' The symbolic representation of this sentence will be:

$\sim (C \bullet R)$

[Here '~' stands for 'it is not the case', 'C' stands for 'Chapatti will be served' and 'R' stands for 'Rice will be served'] . This means that at most one of them will be served.

ii) Both Not:

Example:

Chapatti and rice both will not both be served.

This sentence can be paraphrased in the following way:

'Chapatti will not be served and rice will not be served.' The symbolic representation of this sentence will be:

~ C • ~ R

[Here '~' stands for 'it is not the case', 'C' stands for 'Chapatti will be served' and 'R' stands for 'Rice will be served']. The meaning of the given sentence therefore is that none of these two things will be served.

B) Neither-nor:

Example:

Neither Chapatti nor rice will be served.

This sentence can be paraphrased in the following way:

'Chapatti will not be served and rice will not be served.' The symbolic representation of this sentence will be:

~ C • ~ R

[Here '~' stands for 'it is not the case', 'C' stands for 'Chapatti will be served' and 'R' stands for 'Rice will be served']. The meaning of the given sentence therefore is that none of these two things will be served.

C) Unless:

Amrita will go to the market unless she has other commitments.

This sentence can be paraphrased in the following way:

'If Amrita does not have other commitments, she will go to the market.'

The symbolic representation of this sentence will be:

~ C ⊃ M

[Here '~ C' stands for 'Amrita does not have other commitments' and 'M' stands for 'Amrita will go to the market'].



CHECK YOUR PROGRESS

Q 4. What are the two kinds of statements which are taken into consideration for symbolization in propositional logic?

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4.6 LET US SUM UP

Symbolic logic uses two kinds of symbols to form compound statements. One is logical connectives and the other is propositional variable.

- Connectives in propositional logic are special words or terms which are used to generate statements when used. They connect or join statements to produce other statements. Hence they are called connectives.
- These connectives are also called truth functional connectives because they function truth functionally. This means that propositional logic deals only with truth functional connectives. The following are the five truth functional connectives are: Not, And, Or, If-then, If and only if
- There are two-fold tasks in the elementary symbolic logic for the beginners. They are the following: 1. Symbolization of propositions and arguments stated in ordinary language. 2. Testing the validity of various types of arguments. As it is known to us that the validity of an argument depends on the correct form of the argument, it is therefore necessary to symbolize the propositions constituting the argument correctly.
- Parentheses, square brackets are the devices to demarcate clearly the scope of each connective. The scope of a connective is the range of its operation or the extent of its operation within the statement.

- Statements in propositional logic can be divided into two types: a) Simple statements, b) Compound statements. Hence a brief account regarding the process of symbolization of both simple as well as compound statements are described.
- Lastly this unit has briefly taken up complex symbolization of few statements containing words like 'not both/both not', 'neither-nor', 'unless'.

4.7 FURTHER READINGS

1) Chhanda C. (2007), Logic-Informal, Symbolic and Inductive, Prentice-Hall of India Pvt. Ltd.

2) Copi, I.M.(1998), Symbolic Logic (Fifth Edition), Prentice-Hall of India Pvt. Ltd.

3) Hurley, P. J. (2007), Introduction to Logic, Cengage Learning India Private Limited.

4) Klenk,V. (2008), Understanding Symbolic Logic, Dorling Kindersley (India) Pvt. Ltd.

5) Sharma, B. & Deka, J. (2014), A Text Book of Logic, Aank-Baak, Panbazar, Guwahati, Assam.

4.8 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q. No. 1:

a) True b) False

Ans. to Q. No.2: There are five connectives used in propositional logic. They are the following: Not, And, Or, If-then, If and only if.

Ans. to Q. No. 3:

a) False

b) True

c) False

d) True

Ans. to Q. No. 4: The following are the two kinds of statements taken into consideration for symbolization in propositional logic: simple statement and compound statement.

4.9 MODEL QUESTIONS

A) Very short question

Q1: Define connective.

Q2: What are the types of symbols used in propositional logic?

Q3: There are two-fold tasks in the elementary symbolic logic for the beginners?

Q4: Symbolize the following statements:

a) Ananya performed her duties and earned promotion.

b) I went for shopping but shops were closed.

c) He failed though he tried.

d) Unless we work, we shall not progress.

e) It is not true that either Jesmin will not win or that Jaymoti will not win.

f) It is not true that neither Jesmin nor Jaymoti won.

g) If Kahuwa gets highest marks in English, then not both Karabee and Jaya will be happy.

h) Neither it is the case that if a mother is tough then she is not kindhearted, nor is it true that if the mother is kindheared then she is tough.

i) It is not true that Rohit went to the market but did not buy anything.

j) Jaya and Susmita both were not selected for the job.

B) Short question : (Answer in work 100-150 words)

Q1: Why is not (~) is regarded to be a monadic connective?

Q2: State the difference between monadic connective and dyadic connective.

Q3: Distinguish between two senses of or, namely, exclusive or and inclusive or.

C) Long questions (Answer in work 300-500 words)

Q1: Write short note on connective used in propositional logic.

Q2: Illustrate various connectives with suitable examples.

Q3: Give a brief account of symbolizing everyday language.

Q4: What are the statement keys put forward by Chhanda Chakraborti for the symbolization of statements in propositional logic to be followed closely?

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