UNIT 5: CONSTRUCTION OF TRUTH TABLE

UNIT STRUCTURE

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5.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- explain the basic truth table methods
- discuss how to construct truth tables
- explain how to construct truth tables for statements
- discuss whether an expression is tautology, contradictory or contingent
- explain how to construct truth tables for arguments
- explain how to determine the validity or invalidity of an argument by following truth table method
- explain how to determine logical equivalences between statements

5.2 INTRODUCTION

This unit introduces you to the truth table method and how to construct it. In symbolic logic the truth table method is used to determine

whether a compound statement is true or false. Normally, all statements can be divided into simple and compound statement. While a simple statement does not contain any other statement as its component part, a compound statement does contain another statement as its component part. For example, 'roses are red' is a simple statement and 'roses are red and violets are blue is a compound statement'.

Every statement is either true or false. Therefore, every statement contains truth value. The truth value of a true statement is true and the truth value of a false statement is false. In truth table method every statement is typically represented by small letter alphabet or variable, like p, q, r etc. Again, the truth values of statements are decided by the truth value of their connectives. In logic these connectives are regarded as logical constants. In symbolic logic there are five logical constants they are, '.', 'v', '?', '≡' and '~'. These logical constants are used in order to combine the truth functionally compound statements. If there are two statements p and q then there will be four possible sets of truth values and the number of rows will be 2^2= 2x2= 4. And if there are three statements then there will be eight possible sets of truth values likewise the number of rows will be 2^3 = 2x2x2 = 8. Thus, the number of rows increases in the geometrical progression as the number of variable increase. Now, let us discuss how to construct truth table by following the truth table method in logic.

5.3 CONSTRUCTION OF TRUTH TABLES

In propositional logic by using the symbol '.', which is a truth-functional connective we can construct the truth table for the conjunctive function 'p. q'. Representing the truth values true and false by the capital letters 'T' and 'F', respectively, the way in which the truth value of a conjunction is determined by the truth values of its conjuncts can be displayed more briefly by means of a truth table as follows:

p	*q*	*p . q*
T	T	T
T	F	F

F	T	F
F	F	F

In the above truth table as there are two variable, 'p' and 'q' so, we have four possible combinations of truth values i.e. 'TT', 'TF', 'FT', 'FF'. From the above table it is clear that a conjunctive function is true only under 'TT' combination. Again, under the combinations 'TF', 'FT', and 'FF' a conjunctive function is false. Hence, it may be concluded that a conjunctive function is true if and only if both the conjuncts are true, otherwise it is false.

Again, when two statements are combined disjunctively by using the word 'or' between them then it is called a disjunctive statement. Symbolically disjunctive statement is expressed as 'p v q' i.e. "either p or q". In Latin, the word 'vel' expresses the inclusive sense of the word 'or'. The symbol 'v' called as 'wedge' is a truth-functional connective and can be expressed by the following truth table:

p	*q*	*p v q*
T	T	T
T	F	T
F	T	T
F	F	F

From the above table it is clear that a disjunctive function is true under three combinations. These are 'TT', 'TF', and 'FT'. Again, it is false only under the combination 'FF'. Hence, we may conclude that a disjunctive function is true if and only if one of its disjuncts is true. It is false if both the disjuncts are false.

In propositional logic if two statements are connected conditionally then it is called as an implicative or conditional statement. Symbolically an implicative statement is expressed as 'p ⊃ q', i.e. "if p then q". Here the symbol '?' is defined as 'horseshoe'. The truth table of an implicative statement can be defined as follows: ⊃

p	q	p ⊃ q
T	T	T
T	F	F
F	T	T
F	F	T

The above table shows that the function 'p ⊃ q' is true under the combinations 'TT', 'FT' and 'FF'. It is false only under the combination 'TF'. Thus, it can be concluded that a conditional function will be false if and only if its antecedent is true and consequent is false. Apart from this one condition in all the cases it is true.

Followed by the conditional statement we have another statement which is called the bio-conditional statement. Two statements are said to be materially equivalent when they have the same truth value. Thus, a bioconditional statement is symbolized as 'p ≡ q'. Hence the three-bar symbol may be read as 'if and only if'. The truth table of a bioconditional statement is as follows:

p	q	p ≡ q
T	T	T
T	F	F
F	T	F
F	F	T

By following the above table it can be said that a bio-conditional function is true under the combinations of 'TT' and 'FF'. Again, it is false under the combinations of TF' and 'FT'. From this consideration it can be concluded that a bio-conditional function will be true if its components have same truth values and false if its components contain different truth values. As per the rule of symbolization we use the symbol '~' called a curl is used to symbolize a negative function. Symbolically a negative function is expressed as '~ p'. Let 'p' be a true statement, then its negation '~ p' will be false. Again, if 'p' is false, then '~p' is true. This can be shown in the truth table as follows:

p	~p
T	F
F	T

In the above table 'p' and '~p' are contradictory. So, they have different truth values. In negative function a sentence of the form 'p' is true if its contradiction is false and false if its contradiction is true.



CHECK YOUR PROGRESS

Q 1: State whether the following statements are true or false

(a) A bio-conditional function is true if its components do not have same truth values. (True/False)

(b) If two statements are connected conditionally then that is called a conjunctive statement. (True/False)

(c) The symbol called 'wedge' is used to symbolize disjunctive statements. (True/False)

(d) 'Dot' is a truth functional connective. (True/False)

Q 2: Fill in the blanks:

(a) … ……is used to symbolize a negative function.

(b) A conjunctive function is true if and only if both the conjuncts are

5.4. TRUTH TABLE FOR STATEMENT

The truth values of a compound statement are determined by the truth values of the simple statements it contains and the basic truth tables of the five connectives. Thus, the statement "Either India wins their conference championship and Australia wins their conference championship or New Zealand wins the superbowl" can be symbolically expressed as "(p . q) v r ", whose exact significance is indicated by the following truth table:

p	q	r`	(p	.	q)	v	r
T	T	T	T	T	T	T	T

T	T	F	T	T	T	F	F
T	F	T	T	F	F	F	T
T	F	F	T	F	F	F	F
F	T	T	F	F	T	F	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	F	T
F	F	F	F	F	F	F	F

In the above truth table the combinations of the truth values under each of the column for the variables are not similar. Half of the numbers of truth values in the first column under the variable 'p' shall be 'T' (true) and other half shall be 'F' (false). Likewise, in the second column, under the variable ' q ' one fourth of the number of values shall be 'T' which should be written first to be followed by equal numbers of 'F's, which again should be followed by equal number of 'T's and equal number of 'F's. In the third column under the variable ' r ' one eight of the number of values shall be 'T' to be followed by 'F' then 'T' then 'F', whose numbers are the same as that of the first group of 'T's. In the above table the column under the disjunction i. e. 'v' is the answering column. So, to figure out the truth value of the disjunction, first we need to determine the truth values of (p . q) by applying the conjunctive truth table method. After determining the truth values of (p . q) in the final step we need to compare the truth values with the truth values of the column under the variable ' r ' which is connected through the disjunct 'v'. Thus, finally by applying the disjunctive truth table method we can find out the truth values of a statement in propositional logic.



ACTIVITY: 5.1

- Construct truth tables for the following statement

1. (p v q) . r

Ans:

2. ~ (p . q) | | r

Ans:

5.5. CHARACTERIZATION OF STATEMENT FORMS AS TAUTOLOGIES, CONTRADICTORY AND CONTINGENT

In Propositional Logic, we can find out whether a statement is tautologous, contradictory or contingent on the basis of its possible truth values.

- A statement can be said as tautologous if that is logically true. In a tautologous truth table all the possible truth values of the resulting column are Ts. Let us take an example:

[(p	v	q)	.	~ p]	⊃	q
T	T	T	F	F	T	T
T	T	F	F	F	T	F
F	T	T	T	T	T	T
F	F	F	F	T	T	F

In the above table if we notice it would be clear that in its final column all the truth values are Ts. So, from this it could be concluded that the above statement is a tautological statement.

- A statement is called contradictor if it is logically false. If a statement is logically impossible to be true then that is called a contradictory statement. Let us take another example.

p	.	~	(q	⊃	p)
T	F	F	T	T	T
T	F	F	F	T	T
F	F	T	T	F	F
F	F	F	F	T	F

From the above analysis of this table we may conclude that the above statement is a contradictory statement. In the above table it has been seen that in the final column truth values listed under the connective dot are all Fs. Hence the statement [p . ~ (q □ p)] is a contradictory statement.

- A contingent statement is that which is neither tautologous nor contradictory. In other words, it is logically possible for the statement

to be true and it is also logically possible for it to be false. Hence, a contingent statement is a mixture of both kinds of truth values. Let us discuss it with an example:

p	≡	(~ q	⊃	p)
T	T	F T	T	T
T	T	T F	T	T
F	F	F T	T	F
F	T	T F	F	F

In the above table it has been seen that in its final column all the truth values are the mixture of Ts and F. The final column has three Ts, representing the logical possibility of being true, and one F, representing the logical possibility of being false. Hence it could be concluded that the above statement is a contingent statement.



ACTIVITY: 5.2

- Use the truth table to decide whether the following symbolized statement is tautologous, contradictory or contingent.

1. d ⊃ (b v ~ b)

Ans:

2. (s ⊃ h) ≡ (~h • s)

Ans:



CHECK YOUR PROGRESS

Q 5: What do you mean by a contingent statement? Explain briefly. (Answer in about 20 words)

Q 6: What is the difference between a tautological statement and a contradictory statement?

Q 7: Fill in the blanks:

(a) Contradictory statements are logically ……

(b) If a statement is logically impossible to be false then that is called a……

5.6. TRUTH TABLE FOR ARGUMENT

A deductive argument is valid if its conclusion necessarily follows from its premises. In a deductive argument if the premises are true, then the conclusion must be true. This means that in a deductive argument if the premises are logically true but the conclusion is false, then the argument is an invalid argument. Since a truth table can provide all the logical possibilities so, it can be used to determine whether a deductive argument is valid or invalid. The whole process has three steps:

- Symbolization of the deductive argument;
- Construction of the truth table for the given argument;
- Determining the validity or invalidity of the argument. In this regard it is necessary to mention here that in a deductive argument the conclusion should necessarily follow from the premises. But, if in the truth table there is found at least one row in which the premises are true and the conclusion is false then that would mean that the argument is invalid. If such a row is not found, this would mean that it is not logically possible to have true premises with a false conclusion. Therefore, the argument would be valid. Let us take an argument:

If young people don't have good economic opportunities, there would be more gang violence. Since there is more gang violence, young people don't have good economic opportunities.

The argument can be symbolized as follows:

$$\begin{array}{l} \sim p \supset q \\ \underline{\quad q \quad} \\ \sim p \end{array}$$

The truth table of the argument is as follows:

(1st premise)			(2nd premise)		(Conclusion)		
(~p	⊃	q)	q		~p		
F	T	T	T		F		
F	T	F		F		F	
T	T	T		T		T	
T	F	F	F		T		

In the above truth table we have seen that in the first row the premises are true but the conclusion is false. This means that the argument is invalid. Let us consider another example:

If I study then I pass

I study

Therefore, I pass

- Symbolically the argument can be expressed as follows

$$\begin{array}{l} p \supset q \\ \underline{\quad p \quad} \\ q \end{array}$$

- The truth table for the argument will be as follows:

(1st premise)	(2nd premise)	(Conclusion)
p ⊃ q	p	q
T	T	T
F	T	F
T	F	T
T	F	F

After completing the truth table, we check each row of the four final columns, looking for rows with true premises but a false conclusion. But, we do not find any. So the above argument is valid argument.



ACTIVITY: 5.3

Determine the validity or invalidity of the following argument by applying the truth table method:

1. If you invest in the stock market, then you get rich.
   You didn't invest in the stock market.
   Therefore, you didn't get rich.

Ans:



CHECK YOUR PROGRESS

Q 8: Fill in the blanks:

(a) A deductive argument is..... .....if its conclusion necessarily follows from its premises.

(b) In an invalid deductive argument the conclusion ... .follow from the premises.

5.7. TO DECIDE LOGICAL EQUIVALENCES

Two statements are said as logically equivalent if they necessarily have the same truth values. This means that their possible truth values listed in the two final columns are the same in each row. Let us take an example of comprising two statements:

1. The stock market will fall if interest rates are raised.
2. The stock market won't fall only if interest rates are not raised.

Symbolically both the statements can be expressed as follows respectively:

1. $p \supset q$ and 2. $\sim q \supset \sim p$

To see whether both of the statements are logically equivalent or not first we need to construct truth tables for each statement.

$p \supset q$	$\sim q \supset \sim p$
T T T	F T F
T F F	T F F
F T T	F T T
F T F	T T T

After completing both the truth tables, it has been seen that the two main (or final) columns are identical to each other. This shows that the two statements are logically equivalent.



ACTIVITY: 5.4

Use truth tables to determine whether each pair of statements are logically equivalent:

1. $p \bullet \sim q$
   $q \vee \sim p$

Ans:

2. $\sim(p \supset q)$
   $\sim q \bullet \sim p$

Ans:



5.8 LET US SUM UP

- In symbolic logic the truth table method is used to find out truth or falsity of a statement.
- In truth table method the number of rows increases in the geometrical succession as the number of variable increases.
- There are five truth table methods they are: conjunctive disjunctive, conditional, bio-conditional and negative. By following these five truth

table methods we can find out the truth values of statements and arguments.

- On the basis of truth table method we can find out whether a compound expression is tautology, contradictory or contingent.
- We can also use the truth table method in order to test the validity or invalidity of an argument.
- The truth table method also can be used to find out whether two statements are logically equivalent or not.

5.9 FURTHER READINGS

1) Copy, I. M.; Cohen, C; Jetli, P. Prabhakar, M. (2006), Symbolic Logic. Delhi. Pearson

2) Hurley, Patrick J.(2008). Introduction to logic. Wordsworth. Cengage Learning.

3) Klenk, Virginia (2009), Understanding Symbolic Logic. Delhi. Pearson Education, Inc and Dorling Kindersley publishing Inc.

4) Baronett, S., & Sen, M. (2009) Logic. Delhi. Pearson

5.10 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1:

(a) False

(b) False

(c) True

(d) True

Ans to Q No 2:

(a) curl

(b) true

Ans to Q No 5: In a truth table of a contingent statement if in its final column all the truth values are found as the mixture of both truth and false then it is called as contingent statement.

Ans to Q No 6: In a truth table of a tautological statement in its final column all its truth values are found as true on the other hand in a

contradictory statement in its final column all the truth values are found as false.

Ans to Q No 7:

(a) False

(b) Tautological statement

Ans to Q No 8:

(a) valid

(b) does not

5.11 MODEL QUESTIONS

A. Very Short question

Q.1: What is the difference between a simple and compound statement in logic?

Q. 2: what is logical constant?

Q. 3: What do you mean by variable?

Q. 4: What is a conjunctive function?

Q. 5: Which symbol represents the disjunctive function?

B. Short question (Answer in about 150 words)

Q.6: What do you mean by Logical Equivalence?

Q 7: What are the conditions of an argument to be valid or invalid? Explain by giving an example.

Q 8: Explain the conjunctive truth table method by giving its truth table.

Q 9: Explain the conditional truth table method by giving its truth table.

C. Long Questions (Answer in about 300- 500 words)

Q. 10: Explain the truth table method by giving the truth table.

Q. 11: What do you mean by Tautology, contradictory and contingent statement? Explain by giving example.

Q. 12: Explain the truth table method for testing the validity of argument.

Q. 13: Use the truth table method to determine the validity or invalidity of the following argument forms.

(a) p . q

∴ p

(b) p v p

P

$\therefore$ p . q

(c) (p $\supset$ q) . (r $\supset$ s)

P v r

$\therefore$ q v s

(d) (p v q) $\supset$ (p . q)

~ (p v q)

$\therefore$ ~ (p . q)

(e) p $\supset$ (q $\supset$ r)

p $\supset$ q

$\therefore$ p $\supset$ r

Q. 14: Use the truth table method to characterize the following statement forms as tautologous, contradictory or contingent.

(a) p $\supset$ ~ p

(b) (p $\supset$ ~p) . (~p $\supset$ p)

(c) p $\supset$ (p $\supset$ p)

(d) (~p . q) . (q $\supset$ p)

(e) (p . q) $\supset$ p

(f) [(p $\supset$ q) $\supset$ p] $\supset$ p

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