UNIT 6: DETERMINATION OF VALIDITY AND INVALIDITY OF ARGUMENTS

UNIT STRUCTURE

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6.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- explain validity and invalidity of arguments.
- describe difference between validity and invalidity of arguments.
- explain the methods of determination of validity of arguments.
- describe various methods of validity

6.2 INTRODUCTION

Validity is one of the important concepts of elementary logic. Validity is the attribute of an argument. A deductive argument is evaluated as valid or invalid. The validity of an argument refers to the relation between its propositions between the set of propositions that serves as the premises and the one proposition that serves as the conclusion of that argument. If

the conclusion follows with logical necessity from the premises, one says that the argument is valid. Consequently validity can never apply to any single proposition by itself, as the needed relation can not possibly be found within any one proposition.

Validity of an argument depends on the form of an argument. An argument is valid if and only if its form is valid. But form has nothing to do with subject matter. It follows that what an argument says, its contents, is irrelevant to its validity. So validity and invalidity are formal notions that are applied to formal reasoning and formal logic. However a valid argument can never have true premises and a false conclusion. The basic feature of a good deductive argument is known as validity.

According to W.E. Johnson, valid argument must fulfill two conditions viz. the constitutive condition and the epistemic condition. The constitutive conditions are a) the premises must be true and b) the premises must imply the conclusion. The epistemic conditions are a) the premises should be known to be true and b) the premises should be known to imply the conclusion. For e.g.

1. All army jawans are Indians
   Bahadur is an army jawan
   Therefore Bahadur is an Indian.
2. Either he is a student or a teacher
   He is not a student
   Therefore he is a teacher.

So the validity of an argument is determined independently of the truth of the premises of the argument.

6.3 VALIDITY OF ARGUMENTS

Logic is primarily concerned with valid inference. According to A. Wolf, 'valid' means correct, accurate or sound. An inference may be valid without being true or vice-versa. An inference is valid when it is justified by the evidence adduced in support of it. It is true if it is in accord with the relevant facts i.e. if it describes the facts concerned approximately as they

are. So a valid argument is not the same thing as a true inference. It is possible for an inference to be valid.

The goal of logic is to study the conditions of valid argument. It is not concerned with the conditions of true inference. The study of truth of an inference involves a multinational problem which may relate to mathematics, physics, chemistry, economics, morality etc. Logic is concerned with only general conditions of valid inference. It means an abstraction from the truth of the premises. It means logic does not include it within its scope. Logic is to confine its scope within the search for general conditions of validity of argument.

Generally validity refers that any conclusion drawn from premise or premises can not but are valid. Valid argument implies that valid argument will lead to valid conclusion. In case of valid argument, it is impossible for conclusion to be false if the same is drawn from true premises. This argument also implies relation of entailment which means that the conclusion necessarily follows from premise or premises. There are some characteristics of valid arguments in logic. These are

- Validity of argument means only formal validity, not material validity.
- It applies only to deduction, not to induction.
- Validity of an argument implies that the conclusion necessarily follows from the premise or premises. So there is a relation of entailment between premises and conclusion.
- In case of valid argument, true premises lead to true conclusion.
- Validity of an argument does not guarantee true conclusion. It implies only valid conclusion.
- All instances having the same form of argument will be valid and conversely all instances having the same form of invalid argument will be invalid.
- Validity of argument depends on some general rules or formulae supplies by logic for reaching valid conclusion.

Few examples are cited in order to explain the above points:

1. a) All men are mortal

Ram is mortal

∴ Ram is mortal.

b) All men are imperfect.

Ram is a man.

∴ Ram is imperfect.

In the above two arguments, the conclusion has been drawn from premises following the rules of deduction. So the argument is valid. Any argument like this argument is also valid. The argument is of deductive nature having the relation of entailment between the premises and the conclusion. For e.g.

2. a) All students read books

Ram is a student.

∴ Ram reads book.

b) All women are wicked.

Anima is a woman.

∴ Anima is wicked.

In the above two arguments are of the same from and the both are valid. But the former is materially true but the later is materially false. The valid argument does not guarantee material truth of premises and conclusion. So the validity of an argument refers valid process for drawing valid conclusion.

6.4 INVALIDITY OF ARGUMENTS

The form of argument plays the pivotal role in determining validity or invalidity of argument. The validity or invalidity of argument has nothing to do with the content of argument. It may so happen that true conclusion has been drawn from true premises but still the argument may be invalid if the argument is not based on proper reasoning. For e.g.

Some men are not educated (true)

All artiest are men (true)

∴ Some artists are not educated (true)

In the above cited example all the propositions including the conclusion are true. But the argument is invalid as it has violated the rule of distribution of terms and has committed the fallacy of undistributed middle. Invalidity refers invalidity of argument or invalidity of argument-form. If any conclusion is drawn from premises breaking the rules of reasoning, the argument is invalid. An argument may be announced invalid if it is found that a false conclusion has been drawn from true premise or premises. With the help of few examples we can explain the invalidity of arguments.

Example-1.

All men are mortal (true)

All poets are men (true)

∴ All poets are mortal (true)

This is the case of valid argument. Such a form of argument is valid as in such a case there is no possibility of violation of any rule of reasoning. But there may be some other forms in which argument may be invalid.

Example-2.

All males are mortal (true);

(P) (M)

All females are mortal (true);

(S) (M)

All females are male (false)

(S) (P)

Here the argument form is

All P are M

All S are M

∴ All S are P.

In this example, we find that the term M which was subject in the example 1, has changed its position in example 1, and has turned to be undistributed in example 2. The argument has fallen in the trap of undistributed middle and the argument in the example 2 has been invalid. So in conclusion it can be said that the form of argument determines the validity or invalidity of argument. Furthermore individuality and falsity are

not identical as invalidity refers to form and falsity refers to content. Logicians are concerned with validity or invalidity of arguments. A. Wolf points out that logic studies the conditions of valid inference and this involves the relations between inference and premises.

6.5 DETERMINATION OF VALIDITY AND INVALIDITY OF ARGUMENTS

The basic function of symbolic logic is to determine the validity or invalidity of arguments. Elementary logic has given various methods to test the validity of arguments and argument forms. Through these methods we are also examine whether statements or statement forms are tautologies or not. These methods are called in logic as Decision procedures. It is the mechanical procedure which will always tried to give an answer for any validity problem. Some of the important decision procedures are

1. Truth table method or direct truth table method.
2. Shorter truth table method or indirect truth table method.
3. Formal proof of validity.
4. CNF (Conjunctive Normal Form)
5. DNF (Disjunctive Normal Form)

Let us discuss here the principle methods of decision procedures-

1. Truth table method is one of the significant decision procedures. It is also known as matrix method. The truth table method or the matrix method was popularized by two philosophers and logicians- E.L. Post and Ludwig Wittgenstein. But prior to them Gottlob Frege also employed truth tables The truth table method is a device for exhibiting the conditions under which a truth functional compound proposition is true or false. By means of this decision procedure we can determine whether arguments are valid or invalid. In this method truth values are given in tabular representations. Such tabular representations are called truth tables. According to J.J. Kelly, truth tables give us operational definitions of the important logical connectives, separate from and independent of their use in ordinary language. They also provide

a mechanism whereby the truth values of complicated expressions may be worked out. In other words truth tables are those which represent a systemic exhaustive enumeration of the possible truth values of combinations of simple propositions. A truth table can be defined as a tabular representation in which all possible truth values of a compound statement is considered through the truth values of their simple constituent proposition. The truth table method or the matrix method may be employed for the following basic purposes-firstly, it may be employed for giving operational definitions of logical constant. Secondly, it may be employed to work out the truth values of the compound propositions on the basis of the knowledge of truth values of the constituent simple propositions. Thirdly, truth tables or matrix method may be employed for testing the validity of arguments.

In order to determine the validity of arguments we are to know how to construct a truth table. For constructing a truth table we require certain steps to follow. First, to construct a truth table is to symbolize the statement given. We are to symbolize a number of constituent propositions with the help of variables and logical constants. Secondly, we are to find out the number of variables by means of which the entire compound proposition is constituted. The number of columns will depend on the number of variables or repeated variables and on the number of logical constants. Thirdly, we have to put truth values 'T' and 'F' under the respective columns. Fourthly, we are to determine the number of rows which is considered to be the important step in constructing a truth table. If the number of variable is only one then we are to put truth values 'T' and 'F'. The formula which is adopted for counting the number of rows is '2n' Where 2 is taken for two truth values of 'T' and 'F' and 'n' represents the number of variable or variables. As soon as variable increase the number of rows increases twice. Fifthly, a truth function is by the truth values of its components. If all the truth values of final logical constants are T then the compound proposition is a tautological proposition. Again if truth value F occurs under the final logical constant then the compound proposition is a contradictory and if truth values 'T' and 'F' occurs under the final constant is a contingent proposition. For

constructing a truth table five basic truth functions are necessary with tabular representation. These are conjunctive function, implicative function, disjunctive function, negative function and equivalent function. Furthermore use of brackets is also important for constructing truth table. Let us take the following example:

(P u Q)	=	(~ P	v	Q)
T T T	T	F T	T	T
T F F	T	F T	F	F
F T T	T	T F	T	T
F T F	T	T F	T	T

In this example in the first set of bracket the given (PuQ) we have assigned the truth value TTFF for 'p' and TFTF for 'q'. Under the logical constant, 'implication' is TFTT here. The same process is done with the second set of bracket and under the logical constant 'vel' is TFTT. The main constant is computed by reconciling the two sets of results and the result under the logical constant '=' is TTTT. Therefore it is a tautological truth function.

6.6 INDIRECT TRUTH TABLE METHOD

The second method is the shorter truth table method. This method is one of the decision procedures in propositional logic to establish directly the invalidity or indirectly the validity of an argument. In this method, the direct technique of application to prove the invalidity of an argument is used in such a way that at least one set of possible truth conditions is demonstrated by assuming that the premises are all true but the conclusion is false. It is also known as the method of Reductio ad Absurdum. Here the invalidity of the argument is to be assumed, for which one is to put every premise as true and the conclusion as false. For this, truth value assignment must be put under the main connectives of the premises and the conclusion. In this process guess work has no function. The truth value assignments are put under the invalidating row being forced by the already put truth values. For instance:

~ P v ~ Q P ~Q

F T T F T T F T

In the above example in the column 1 we can put truth value assignment. Truth value 'T' is forced to write in this column to support its connectives (~) for the truth value 'F'. Then column 2 is putting the same truth value for the same variable with column 1. Thereafter column 3 is newly put truth value, forced to write for justifying the negation of Q. Column 4 is putting the same truth value for the same variable in the second premise. Then column 5 is newly put truth value assignment forced to write for justifying the negation of P. So in order to complete the process, one is either to detect a contradiction among the truth value assignments or assign a truth value against the entire logical constant and propositional variable put there for the concerned statements.

6.7 FORMAL PROOF OF VALIDITY

Besides these two methods the formal proof of validity is also noteworthy. According to I.M. Copi, a formal proof of validity for a given argument is defined to be a sequence of statements, each of which is either a premise of that argument or follows from preceding statements by an elementary valid argument. By means of formal proof of validity we can establish that the conclusion in fact is deducible from the premises. It is a method for establishing the validity of arguments by a sequence of shorter elementary arguments that are already known to be true. These shorter arguments are established to be valid on the basis of certain elementary tautologies. These elementary valid arguments are sometimes called Rules of Inference and Rules of Replacement.

Formal proof of validity is a simple and more clarifying method than the method of truth table. It can not prove the invalidity of an argument as it can only prove a valid argument as valid. It is also the demonstration of the derivability of the consequence from a set of premises is the main concern. A formal proof of validity expresses that the statements constituting

a formal proof of validity are logically interconnected. It is purely a logical process of deduction. For e.g.

1. E u F
2. Rv~O
3. ~R / . . ~E
4. ~O (2,3 By D.S)

6.8 CONDITIONAL PROOF

In case of formal proof of validity, we have found another procedure in order to prove the conditionals. It is called conditional proof. This proof does not have premises which we generally use to have in normal proof of validity. In order to infer something when we have no premise from which to infer it, we need some additional proof that depends on assumptions and then we see what follows from the assumptions. In a conditional proof, one is to assume the antecedent of the conditional which is to be proved and to see whether the consequent follows from that or not. If it follows then we may infer the conditional itself. The rule of conditional proof is useful in constructing proofs of conditionals, especially where premises and conclusion are very complex.

6.9. INDIRECT PROOF

The validity of an argument can be established by applying both formal proof of validity and conditional proof. Conditional proof need not necessarily be brief and easier in comparison to formal proof of validity. But the procedure of conditional proof has some limitations. When an assumption is made, does not matter in which stage of the proof process that must eventually be discharged one must have utmost importance on making the proper assumption. We never make an assumption unless we know what exactly it is for. So logicians have suggested another method for removing the limitations of conditional proof. It is Indirect Proof method. In constructing a formal proof of validity, it becomes difficult to prove the

conclusion directly by applying the rules of inference. An indirect proof consists of assuming the negation of the statement to be arrived. This assumption seeks to derive a contradiction and then that the original assumption is false. And proving the falsity of the assumption establishes the truth of the statement to be obtained.

The other two methods are less useful for determining the validity of arguments.



6.10 LET US SUM UP

We have discussed the determination of validity of arguments. The primary points of this unit are:

- The validity of an argument refers to the relation between its propositions between the set of propositions that serves as the premises and the one proposition that serves as the conclusion of that argument.
- In valid arguments, all the propositions including conclusion may be true.
- In invalid argument, all the propositions including the conclusion may be true
- For the determinations of validity of arguments we have found various decision procedures.
- Truth table method can be used to determine the logical status of a single, individual statement form or statement simply by checking its truth value.
- Shorter truth table method establishes directly the invalidity or indirectly the validity of an argument.
- Formal proof of validity is used for proving a valid argument as valid but can not prove the invalidity of an argument.
- The rule of conditional proof is applicable in constructing proofs of conditionals, especially where premises and conclusion are very complex.

- Indirect proof is useful when the formal proof of validity becomes very difficult to prove the conclusion directly just by applying the rules inference.



6.11 FURTHER READINGS

1) Basson and O'connor 1953 Introduction to Symbolic logic, Oxford University Press, Delhi.
2) Copi, I.M. 1979. Symbolic Logic, Prentice Hall of India, pvt ltd. New Delhi.
3) Copi, I.M. & Kohen Carl. 1984, Introduction to Symbolic Logic, Prentice Hall of India, pvt ltd. New Delhi.
4) Quine, W.V. 1970. Philosophy of Logic, Prentice Hall of India, pvt ltd. New Delhi.
5) Roy, B.N. 1984. Deductive Logic, S.C. Sarkar & Sons pvt ltd, Calctta.
6) Strawson, P.F.1976. Introduction to Logical Theory, Methuen & co. Ltd. London



6.12 MODEL QUESTIONS

A. Very short Questions

Q.1. What is the attribute of argument?

Q.2. What are the conditions that a valid argument can fulfill?

Q.3. Give an example of valid argument.

Q.4. What are the various decision procedures?

B. Short questions (Answer in about 100-150 words)

Q.1. Write short notes on

a) Indirect proof method

b) Conditional Proof.

Q.2 Write the characteristics of valid argument.

Q.3 What is elementary valid argument form?

Q.4 what are the purposes of a truth table method?

C. Long Questions (Answer in about 300-500)

Q.1. Explain the strategy for constructing a truth table method.

Q.2. Discuss the procedure of formal proof of validity.

Q.3. Give an elaboration of the indirect truth table method with an example

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