UNIT-7: FORMAL PROOF OF VALIDITY AND CONDITIONAL PROOF

UNIT STRUCTURE

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7.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- explain the meaning of formal proof of validity
- discuss the rules of inference and the rules of replacement
- explain how to construct a formal proof of validity by applying the rules
- illustrate the method of conditional proof
- explain how to determine validity by applying the rule of conditional poof.

7.2 INTRODUCTION

This unit introduces you to the concept of formal proof of validity and conditional proof. A formal proof of validity for a given argument is defined to be a sequence of statements, each of which is either a premise of that argument or follows from preceding statement by an elementary valid argument. Thus, the last statement in the sequence is the conclusion

of the argument whose validity is being proved. On the other hand the rule of conditional proof is applied to those arguments whose conclusions are conditional statements. But, in order to describe the method of conditional proof first we need to know the method of formal proof of validity and how to proof the validity of an argument by following it.

7.3 THE METHOD OF FORMAL PROOF OF VALIDITY

In order to proof the validity of the conclusion of an argument by means of formal proof of validity or method of deduction we have to maintain a strategy for it. For testing the validity of an argument by means of formal proof of validity formulated in ordinary language first we have to symbolize the statement to bring out the logical form of the argument. Here we may use capital alphabets for elementary statements. Then, after having the logical form of the argument we may construct proofs or derivations using the elementary valid argument forms. The given premises and the subsequent steps in the deduction shall be numbered. The construction of the proof in a formal proof of validity is generally started with the deduction of a consequence from the premises. Unless the derived consequence is the conclusion it shall be treated as one of the premises for the deduction of further consequences. Each and every subsequent step in the deduction may likewise be treated as an additional premise. Thus, we have to repeat the process of deduction until the conclusion is obtained. On the right side of each step we have to give explanation by writing the number of steps from which the derivation is made and the name of the rule applied in brief. A more and more concise way of writing out this proof of validity is to list the premises and the statements deduced from them in one column, with justifications for the latter written beside them. In each case the justification for a statement specifies the preceding statement and the rule of inference and replacement by which the statement in question is deduced.

Thus, it is necessary to mention here that in order to construct formal proof of validity we need to take the help of certain rules. These rules are

called the rules of inference and the rules of replacement. Let us discuss it. There are all total nineteen rules by following which we can construct the formal proof validity.



CHECK YOUR PROGRESS

Q 1: Answer the following questions:

(a) The method of formal proof of validity is also known as _____.

(b) _____ alphabets are used in formal proof of validity.

Q. 2: What are the names of the rules of formal proof of validity?

7.3.1 THE RULES OF INFERENCE

The first nine rules of the method of deduction is called the rule of inference. These are the nine elementary valid argument forms whose validity can be easily established by following the truth table method. These rules are used to construct formal proof of validity for a wide range of more complicated arguments. They are as follows:

1. Modus Ponens (M. P.)

   p ⊃ q

   p

   ∴ q

2. Modus Tollens (M. T.)

   p ⊃ q

   ~ q

   ∴ ~ p

3. Hypothetical Syllogism (H. S.)

   p ⊃ q

   q ⊃ r

   ∴ p ⊃ r

4. Disjunctive Syllogism (D.S.)

   p v q

   ~ p

   ∴ q

5. Constructive Dilemma (C. D.)

   (p ⊃ q) . (r ⊃ s)

   p v r

   ∴ q v s

6. Destructive Dilemma (D.D.)

   (p ⊃ q) . (r ⊃ s)

   ~ q v ~s

   ∴ ~ p v ~ r

7. Simplification (Simp.)

   p. q

   ∴ p

8. Conjunction (Conj.)

   p

   q

   ∴ p . q

9. Addition (Add.)

   p

   ∴ p v q

Thus, by following the rules of inference we can construct the formal proof of validity of a given argument like follows:

1. (A . B) ⊃ [A ⊃ (D . E)]
2. (A . B) . C / ∴ D v E
3. A . B 2, Simp.
4. A ⊃ (D . E) 1, 3, M.P.
5. A 3, Simp.
6. D . E 4, 5, M. P.
7. D 6, Simp.
8. D v E 7. Add.

As explained above by applying the rules of inference and by means of a chain of deduction we have arrived at the conclusion in step no 8. This proves that the given argument is valid.



ACTIVITY: 7.1

- Construct a formal proof for the following arguments:

A ⊃ B

C ⊃ D

(~ B v ~D) . (~ A v ~ B)

∴ ~ A v ~ C

Ans:

7.3.2. THE RULES OF REPLACEMENT

Apart from the rules of inference there exist ten more rules which are used to prove the validity of a truth functionally compound statement. There are many truth-functional arguments that cannot be proved valid using only the nine rules of inference. For example:

A . B

∴ B

The above argument is a valid argument. But, we cannot use the rules of inference to prove the validity of the above argument. Hence any part of a compound statement is replaced by an expression that is logically equivalent to the part replaced, the truth value of the resulting statement is the same as that of the original statement. This can be regarded as the Rules of Replacement. It can be said as the additional principle of inference. It permits us to infer from any statement the result of replacing all or part of that statement by any other statement logically equivalent to the part replaced. Thus, by using the rules of replacement we can replace logically equivalent expressions with each other wherever they occur. Following are the rules of replacement by following which we can construct formal proof of validity for a given argument. These rules are numbered consequentively after the first nine rules already stated. They are as followed:

10.	De Morgan's Theorems (De M.):	~ (p . q) ≡ (~ p v ~ q).
		~ (p v q) ≡ (~p . ~q).
11.	Commutation (Com.):	(p v q) ≡ (q v p).
		(p. q) ≡ (q. p).
12.	Association (Assoc.):	[p v (q v r)] ≡ [(p v q) v r].
		[p . (q . r)] ≡ [(p . q) . r].
13.	Distribution (Dist.):	[p . (q v r)] ≡ [(p . q) v (p . r)].
		[p v (q . r)] ≡ [(p v q) . (p v r)].
14.	Double Negation (D. N.):	p ≡ ~ ~ p.
15.	Transposition (Trans.):	(p ⊃ q) ≡ (~ q ⊃ ~ p).
16.	Material Implication (Impl.):	(p ⊃ q) ≡ (~ p v q).

17. Material Equivalence (Equiv.): $(p \equiv q) \equiv [(p \supset q) . (q \supset p)]$.
$(p \equiv q) \equiv [(p . q) \vee (\sim p . \sim q)]$.

18. Exportation (Exp.): $[(p . q) \supset r] \equiv [p \supset (q \supset r)]$.

19. Tautology (Taut.): $p \equiv (p \vee p)$.
$P \equiv (p . p)$.

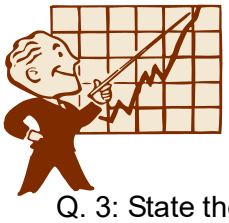
Thus, the formal proof of validity for the above argument A . B/ ∴ B can now be written as

1. A . B / ∴ B
2. B . A 1, Com.
3. B 2, Simp.

Let us take an example:

1. (A v B) ⊃ (C . D)
2. ~ C / ∴ ~ B
3. ~ C v ~ D 2, Add.
4. ~ (C . D) 3, De. M.
5. ~ (A v B) 1, 4, M.T.
6. ~A . ~ B 5, De. M.
7. ~B . ~A 6, Com.
8. ~B 7, Simp.

Thus, in the above example along with the rules of inference we have also used the rules of replacement and arrived at the conclusion in the line number 8 and finally found that the argument is valid. From the above examples it has been seen that there is an important difference between the first nine rules of inference and the last ten rules of replacement. The first nine rules can be applied only to the whole lines of a proof serving as premisses. On the other hand the last ten rules of replacement can be applied either to whole lines or to parts of lines.



CHECK YOUR PROGRESS

Q. 3: State the rule of Exportation.

Q. 4: State the rule of material Implication.

Q. 5: Write the main point of difference between the rule of inference and the rule of replacement.

Q. 6: What is the rule of De Morgan?



ACTIVITY: 7.2

• Construct a formal proof of validity of the given argument.

F v (G v H)

(G ⊃ I) . (H ⊃ J)

(I v J) ⊃ (F v H)

~ F / ∴ H

7.4. THE RULE OF CONDITIONAL PROOF

As already explained the rule of conditional proof is applied to those arguments whose conclusions are conditional statements. To every argument there corresponds a conditional statement whose antecedent is the conjunction of the argument's premises and whose consequent is the argument's conclusion. Given any argument whose conclusion is a conditional statement, a proof of its validity, using the rule of conditional proof that is, a conditional proof of its validity is constituted by assuming the antecedent of its conclusion as an additional premise, and then deducing the consequent of its conclusion by a sequence of elementary valid arguments. Thus, the conditional proof of validity of the following argument

1. A ⊃ B /∴ A ⊃ (A . B) will be as follows:

1. A ⊃ B /∴ A ⊃ (A . B)

2. A /∴ A . B (C . P.)
3. B 1, 2, M. P.
4. A . B 2, 3, Conj.

In the above example in the second slant line and three dot 'therefore' symbol and the parenthesized 'C. P.', indicates that the rule of conditional proof is being used. The rule of conditional proof not only permits us to construct shorter proofs of validity for arguments that could be proved valid by appealing to the original list of nineteen rules of inference and replacements alone. But, it also permits us to establish the validity of valid arguments whose validity could not be proved by reference to the original list alone. Let us take another example:

1. (A v B) ⊃ (C . D)
2. (D v E) ⊃ F/∴ A ⊃ F
3. A / ∴ F (C. P.)
4. A v B 3, Add.
5. C . D 1, 4, M. P.
6. D . C 5, Com.
7. D 6. Simp.
8. D v E 7, Add.
9. F 2, 8, M. P.

As seen in the above the rule of conditional proof is applied in the conclusion and finally, we got the conclusion 'F' which has been proved in the 9th line. Thus, it proved that the argument is valid. So, the rule of conditional proof can be applied in dealing with any valid argument having a conditional statement as conclusion and it can also be applied more than once in the course of the same deduction.



CHECK YOUR PROGRESS

Q. 7: Fill in the blanks

(a) The rule of conditional proof is applied to the statements whose ----------are conditional statements.

(b) ---------- abbreviation is used for the rule of conditional proof.

Q. 8: What will be the conditional proof of the following argument?

1. A ⊃ (B ⊃ C)

2. B ⊃ (C ⊃ D) / ∴ A ⊃ (B ⊃ D)



7.5 LET US SUM UP

- A formal proof of validity is defined to be a sequence of statements.
- Each statement in a formal proof of validity is either a premise of that argument or follows from preceding statement by an elementary valid argument.
- The last statement in the sequence of a formal proof of validity is regarded as the conclusion of the argument.
- The derived consequence in a formal proof of validity is treated as one of the premises unless we get the conclusion.
- The process of deduction is repeated in the method of deduction until the conclusion is obtained.
- On the right side of each line in the process of deduction we have to give explanation by writing the number of steps from which the derivation is made and also the name of the rule applied in brief.
- The first nine rules are called the rules of inference and the later ten rules are called the rules of replacement.

- The main point of difference between these two types of rules is that the rules of inference can be applied to the whole line of a proof, but the rules of replacement can also be applied to a part of a proof.
- The rule of conditional proof is applied to the arguments containing conditional statement as its conclusion.
- The abbreviation 'C.P.' is used to indicate the application of the rule.

7.6 FURTHER READINGS

1) Baronett, S., & Sen, M. (2009) Logic. Delhi. Pearson

2) Copy, I. M.; Cohen, C; Jetli, P. Prabhakar, M. (2006), Symbolic Logic (Twelfth edition ed.). Delhi. Pearson

3) Copy, I. M. (2006) Symbolic Logic (Fifth edition ed.). Delhi. Pearson Education, Inc and Dorling Kindersley publishing Inc.

4) Hurley, Patrick J.(2008). Introduction to logic. Wordsworth. Cengage Learning.

5) Klenk, Virginia (2009), Understanding Symbolic Logic. Delhi. Pearson Education, Inc and Dorling Kindersley publishing Inc.

7.7 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1:

(a) method of deduction

(b) Capital

Ans to Q No 2: The names of the rules of formal proof of validity are the rules of inference and the rules of replacement.

Ans to Q No 3: Exportation (Exp.): $[(p \cdot q) \supset r] \equiv [p \supset (q \supset r)]$.

Ans to Q No 4: Material Implication (Impl.): $(p \supset q) \equiv (\sim p \vee q)$.

Ans to Q No 5: The rules of inference can be applied to the whole line of a proof and the rules of replacements can be applied to the whole as well as to the parts of a proof.

Ans to Q No 6: De Morgan's Theorems (De M.): $\sim (p \cdot q) \equiv (\sim p \vee \sim q)$.

$\sim (p \vee q) \equiv (\sim p \cdot \sim q)$.

Ans to Q No 7:

(a) Conclusions

(b) C. P.

Ans to Q No. 8:

1. A ↄ (B ↄ C)
2. B ↄ (C ↄ D) / ∴ A ↄ (B ↄ D)
3. A /∴ B ↄ D (C.P.)
4. B / ∴ D (C. P.

7.8 MODEL QUESTIONS

A. Very short questions

Q 1: Define formal proof of validity.

Q.2: What is the condition under which the rule of conditional proof is applied?

Q.3: What is the use of rules of inference?

Q.4: What is the use of rules of replacements?

Q.5: What is the difference between the rules of inference and the rules of replacement?

B. Short question (Answer in about 100- 150 words)

Q.6: Explain briefly the process of deduction.

Q.7: Write a note on the rule of conditional proof.

C. Long Questions (Answer in about 300- 500 words)

Q.8: Write down the rules of inference and rules of replacement by giving examples in support of both.

Q. 9: Construct a formal proof of validity of each of the following argument.

(a) 1. (E . F) . G

2. (F ≡ G) ↄ (H v I) / ∴ I v H

(b) 1. (J . K) ↄ L

2. (J ↄ L) ↄ M

3. ~ K v N /∴ K ↄ (M . N)

(c) 1. S ↄ (T ↄ U)

2. U ↄ ~ U

3. (V ⊃ S) . (W ⊃ T) / ∴ (V ⊃ ~W)

(d) 1. X ⊃ (Y ⊃ Z)

2. X ⊃ (A ⊃ B)

3. X . (Y v A)

4. ~Z / ∴ B

(e) 1. K ⊃ L / ∴ K ⊃ (L v M)

(f) 1. (~U v V) . (U v W)

2. ~X ⊃ ~W / ∴ V v X

(g) 1. A ⊃ ~ (B ⊃ C)

2. (D . B) ⊃ C

3. D / ∴ ~A

(h) 1. O ⊃ (P ⊃ Q)

2. P ⊃ (Q ⊃ R) / ∴ O ⊃ (P ⊃ R)

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