UNIT 8: INDIRECT PROOF AND STRENGTHENED RULE OF CONDITIONAL PROOF

UNIT STRUCTURE

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8.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- explain Indirect Proof (IP), Conditional, and Strengthened Rule of Conditional Proof;
- describe indirect proof as a concept or method;
- explain the use of Indirect Proof in proposition;
- discuss the use of indirect proof in validity of argument;
- explain the types of Conditional Proof
- discuss the concept of the scope of the assumption and so on.

8.2 INTRODUCTION

The concept of proof is the main contention of formal and symbolic logic. Formal logic engages with different types of proofs, such as, indirect proof, direct proof, conditional proof etc. One should be kept in mind that there is a subtle distinction between the concept of proof and the concept of test. Sometimes, it is said that 'test the following' and sometimes it is said that 'proof the following'. To say 'test the following' means the given are valid and invalid and one has to find out or so to speak test which one is valid and which one is not valid. Contrary to that, to say 'proof the following', it means that the given are valid and one has to prove it. More specifically, to say 'test the following argument' is to say that the given argument may be valid or invalid and one has to test it logically and to say that 'proof the following argument' is to say that the given argument is valid and one has to prove it. Thus, the concept of proof 'test' and 'proof' play important role in symbolic or formal logic.

This unit is primarily concerned with Indirect Proof (IP) and Strengthened Rule of Conditional Proof (SCP). Let us explain each of these in turn

8.3 INDIRECT PROOF

What then is Indirect Proof? Is indirect proof a concept or method? According to Irving M. Copi, indirect proof is a method. It is called the method of proof by reductio ad absurdum. The method of proof by reduction ad absurdum besides logic is equally effective in elementary geometry. While dealing with theorems, Euclid often begins by assuming the opposite of what he wants to prove. As the given is true or valid in case of proof, its negation always leads to contradiction. Or simplistically, it can be said that negation of the given in case of indirect proof always leads to absurdity.
For example, if the given is 'p v ~ p' which is valid or tautology without exception, then its negation leads to contradiction or absurdity. The negation of 'p v ~ p' is "p · ~ p'. 'p · ~ p' is a contradiction. It is false without exception.

Thus, the method of indirect proof states that if the negation of the given leads into contradiction or so to speak absurdity, then it must be false. So its negation, i.e., the given must be a theorem and must be true as well. Thus the very nature and objective of indirect proof is clear. Indirect proof thus can be defined as a logical method where the validity or truth of the given is proved by way of negating or denying the given. This method is simplistic and effective in nature. It is simplistic because it is no longer complicated. The student can easily grasp and apply this method.



ACTIVITY: 8.1

- Do you find any difference between the concept of proof and the concept of test?

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- What do you mean by 'test the following'?

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CHECK YOUR PROGRESS

Q 1: The negation of the given in case of indirect proof always leads to absurdity (True/False).

Q.2 : The negation of 'p v ~ p' is "p · ~ p' (True / False)

Q.3 : Indirect proof is a concept (True / False)

Q.4: Indirect proof is a method (True / False)

Q.5 : p · ~ p' is a contradiction (True / False).

Q.6: Define Indirect Proof.

8.4 WHAT IS A LOGICAL METHOD?

One has to know about the concept of method. We have said that indirect proof is a logical method. Philosophy is all about of methods. The Cartesian method, the phenomenological method, the linguistic method are cases in point. Likewise, there are various methods applied in logic.

Logic as a branch of philosophy is full of methods. Indirect proof is one such method. The main objective of any philosophical or scientific or logical method is to unearth the real or truth. Indirect proof being a logical method tries to establish the truth of the given. Here the given is true. Indirect proof takes a methodological account to prove it true. It does indirectly and that is why it has been named as Indirect Proof.

Logic deals with proposition and argument. Every proposition must be either true or false. Every argument must be valid or invalid. Indirect proof, being a logical method, deals only with true proposition and valid argument. This is because it is a method of proof, not a method of test. Naturally, it has two important tasks. It is used to determine the truth of a proposition. It is also used to determine the validity of an argument.

A proposition according to this method would be determined as true if its negation leads into contradiction. This is so obvious. If the given is a tautology, or true, then its negation would be self-contradictory, or in short, contradictory. Just like, if the proposition P is true, then its negation, i.e., ~ P would be false. So while determining the truth of a proposition by the method Indirect proof, we have to start with the negation of the given and then find out whether it leads to a contradiction or absurdity. Consider the following to make this point clear.

8.5 THE USE OF INDIRECT PROOF IN PROPOSITION

Let us now take an example to demonstrate the use of Indirect Proof in proposition.

$(p \supset q) \supset [(q \supset r) \supset (p \supset r)]$ is given.

Now we start with the method of Indirect proof just by way of denying or negating the given:

1.	$\sim \{(p \supset q) \supset [(q \supset r) \supset (p \supset r)]\}$	(~ given)
2.	$\sim \{\sim (p \supset q) \vee [(q \supset r) \supset (p \supset r)]\}$	1, Implication.
3.	$\sim \sim (p \supset q) \cdot \sim [(q \supset r) \supset (p \supset r)]$	2, De. Morgan.
4.	$(p \supset q) \cdot \sim [(q \supset r) \supset (p \supset r)]$	3, D.N.

5.	$p \supset q$	4, Simp.
6.	$\sim [(q \supset r) \supset (p \supset r)] \cdot (p \supset q)$	4, Com.
7.	$\sim [(q \supset r) \supset (p \supset r)]$	6, Simp.
8.	$\sim [\sim (q \supset r) \vee (p \supset r)]$	7, Impl.
9.	$\sim \sim (q \supset r) \cdot \sim (p \supset r)$	8, De. M.
10.	$\sim \sim (q \supset r)$	9, Simp.
11.	$q \supset r$	10, D.N.
12.	$\sim (p \supset r) \cdot \sim \sim (q \supset r)$	9, Com.
13.	$\sim (p \supset r)$	12 Simp.
14.	$\sim (\sim p \vee r)$	13 Impl.
15.	$\sim \sim p \cdot \sim r$	14, De. M.
16.	$p \cdot \sim r$	15 D.N.
17.	p	16, Simp.
18.	q	5, 17, M.P.
19.	r	11, 18, M. P.
20.	$\sim r \cdot p$	16, Com.
21.	$\sim r$	20, Simp.
22.	$r \cdot \sim r$	19, 21, Conj. Contradiction, Proved.

The above derivation clearly reflects the methodological affectivity of the **Method of Indirect Proof.** The proof is started with the negation of the given in step no. 1. It has ended with a contradiction or absurdity in step no.22. This logical method suggests that if the negation of a given leads into a contradiction then it would indirectly prove that the given is a tautology. This is exactly what is done in the above case. Let us now pass on to apply this method in the case of validity of an argument.

8.6 THE USE OF INDIRECT PROOF IN VALIDITY OF ARGUMENT

Unlike the proposition, the indirect proof while applying in the case of validity of an argument is stated with an additional premise. The additional premise is the negation of the conclusion of the given argument. This is

one of the notable aspects of the method of indirect proof. Let us consider the following argument:

1.	A ⊃ (B · C)	
2.	(B V D) ⊃ E	
3.	D V A / ∴E	
4.	~ E	I. P. (Indirect Proof)
5.	~ (B V D)	2, 4, M. T.
6.	~ B · ~ D	5, De. M.
7.	~ D · ~ B	6, Com.
8.	~ D	7, Simp.
9.	A	3, 8, D.S.
10.	B · C	1, 9, M. P.
11.	B	10, Simp.
12.	~ B	6, Simp.
13.	B · ~ B	11, 12, Conj.

Contradiction, Proved.

In the above argument we have step no. 4 as an additional step. The proof is started with the denial of the conclusion and it takes it as an additional premise and ended with a plain contradiction. The contradiction is absurd. So the demonstration is complete. Thus, the argument is proved valid by indirect proof. The method of indirect proof is effective in strengthening our deductive system. In fact, the method of Indirect Proof along with other logical methods actually makes complete the proof system of deductive logic.

In Indian logic, the same method can be applied in the name of Tarka. In Indian logic there is a popular argument like the following:

Where there is smoke, there is fire.

The Hill is smoky

Therefore, the Hill is fiery.

Indian logicians take the method of Indirect Proof to make the argument firm. Here the conclusion is logically entailed from the premises. More alternatively, it can be said that if one affirms the meaning of the first and

second premises and at the same time denies the meaning of the conclusion, he will involve into a plain contradiction. More simplistically, it can be said that if both the premises are accepted as true and the conclusion is presumed as false, then we involve into a plain contradiction. Thus, there we find the similarity between the East and the West as far as from the methodological point of view. We can say that the concept of Tarka that we observe in Indian logic has the same methodological implication as the concept of Indirect proof that we notice in formal logic. Having said this, whether Indian logic would be regarded as formal logical system or not is altogether a different issue.

8.7 THE STRENGTHENED RULE OF CONDITIONAL PROOF

Before delving into this method, let us make clear about the logical implication of the term 'conditional'. What is conditional? What is the rule of conditionality in formal logic? Conditionality is something that can be comprehended with regard to 'if…then' form where the cause-effect relationship plays the all-important role. To talk of conditionality is to talk of casual conditionality. Human life is all about of conditionality. Everything within the empirical world is determined on the basis of conditionality. Thus, the concept of conditionality is a forceful concept that would determine our form of life. Nothing is unconditional in the empirical world. Everything is hypothetical and conditional.

Thus, when we talk of conditional proof, we ipso-facto mean a kind of proof framed with 'if …then' in the ordinary language. According to Copi, the method of Conditional Proof is applied 'only to arguments whose conclusions were conditional in form'. That is, if the conclusion of an argument is in the form of 'if…then', then it would be treated as Conditional argument and in such a case there remains the relevance of Conditional Proof.

It is important to note here that logic develops out of language. Logical language has its distinctive nature. Logical language must be

propositional language. Logic deals with proposition. Proposition is obtained from propositional language. Propositional language is abstracted out of language we use in our daily life. Therefore, there is no point in saying that language in general would be propositional language. Language expresses by means of sentences. Only those sentences which would express proposition are called logical language. Logical language is expressed by means of proposition. Every proposition must be either true or false. There are various truth-functional operators in logic on the basis of which one can determine the truth-value of composite or compound proposition. They are obtained from ordinary language as well. They are '·' (dot), 'v' (vel or a wedge), '⊃' (horseshoe), '≡' (equivalence), '~' (curl or a tilde). The first one is obtained from 'and', the second one from 'either-or', the third one from 'if …then', the fourth one from 'if and only if' and the fifth one from 'not'. With the help of proposition and the truth-functional operators, the language of first order logic has been developed. When we talk of Conditional Proof, we usually talk argument. Every valid argument in itself is conditional in nature. It states that an argument is valid if its conclusion is logically entailed or implied by its premise or premises. Here the term if registers the conditionality of the argument. Thus, our general perception is that an argument is by its very nature is conditional irrespective of its validity or invalidity. It is equally to be noted here that a statement would equally be treated as conditional as well. All implicative propositions are conditional in nature. Even we can say that every implicative statement or proposition is represented as a conditional argument. Therefore, when we deal with the Conditional Proof, we must take note of both conditional statement as well as conditional argument.

8.8 TYPES OF CONDITIONAL PROOF

There are two types of Conditional Proof, such as, Ordinary Conditional Proof and Strengthened Conditional Proof. Even though the theme of this section is designed in the form of Strengthened Conditional

Proof, but it does not bear any sense without the concept of Ordinary Conditional Proof.

8.8.1 ORDINARY CONDITIONAL PROOF (OCP)

Ordinary Conditional Proof is a kind of conditional proof that is primarily concerned with explicit conditional. It is important to note here that not all conditionals are explicit in nature. Conditionals may be expressed both explicitly as well as implicitly. In the case of Ordinary Conditional Proof, the conditional must be explicit in nature. Accordingly, if the conditional is a plain implicative statement, it would then be treated as Ordinary Conditional. In such a case one may take the help of Ordinary Conditional Proof. The statement or proposition $(p \supset q) \supset [(q \supset r) \supset (p \supset r)]$ is a conditional statement where the condition is used or manifested explicitly. Therefore, one may use Ordinary Conditional Proof in such case. Thus, it can be said that Ordinary Conditional Proof is a logical method applied in the case of deduction where the proof of a conditional statement or proposition can be adequately justified provided that the conditional is **explicit in nature.** Let us apply Ordinary Conditional Proof in the case of the stated statement in the following:

$(p \supset q) \supset [(q \supset r) \supset (p \supset r)]$

1.	$p \supset q$	(assumption)
2.	$q \supset r$	(assumption)
3.	p	(assumption) / $\therefore r$
4.	q	1, 3, M. P.
5.	r	2, 4, M. P. (Proved)

Here the given statement is in the form of explicit conditional and hence Ordinary Conditional Proof is applied here. We take $p \supset q$ as a premise from the antecedent of the given statement. Then again we pass on to the consequent of the given statement. As the consequent of the given statement is itself conditional in nature, we again have taken $q \supset r$ as the second premise. Again we pass on to the consequent of the second, i.e., $p \supset r$, which again is explicit conditional. Again we take p as the third

premise from the antecedent of $p \supset r$. Now as residue our conclusion here is r. So we have to deduce r from the supposed premises. This has exactly been done in the above case. Thus, our all-important observation here is that if the conditional is in explicit form, we can apply Ordinary Conditional Proof. Having said this, in the case of implicit conditional, ordinary Conditional Proof does not function. In such a case we can take the help of Strengthened Conditional Proof. Now let us pass on to Strengthened Conditional Proof.

8.8.2 STRENGTHENED CONDITIONAL PROOF (SCP)

It has been stated that not all conditional are in the form of explicit conditional. Some conditional are in implicit nature. In such cases we may apply Strengthened Conditional Proof. It is further noted here that we have four binary logical operators with the help of these we can formulate conjunctive, disjunctive, implicative and equivalence statements or proposition. In the case of argument, Strengthened Conditional Proof can be applied just looking at the nature of the conclusion of the given argument. If the conclusion of the given argument is conjunctive, then there is no application of Conditional Proof. If the conclusion would be disjunctive, then one can find the sense of implicit conditional in the disjunctive form of Conditional because one has to know that the disjunctive proposition 'p v q' is identical with the implicative proposition $\sim p \supset q$. This clearly reflects that $p \vee q \equiv \sim p \supset q$. Thus, in our sense disjunction is an implicit form of implication or conditional. If the proposition would be implicative, then the implication is expressed in the form of explicit conditional. Therefore, there is no problem as far as Strengthened Conditional Proof is concerned. Finally, if the proposition would be in the form of equivalence, then again there underlies implicit conditional within the equivalence. In logic 'p equivalence q' is defined as '$(p \supset q) \cdot (q \supset p)$'. It is called biconditional. Thus, biconditional is nothing but of twin conditional. Accordingly, there remains implicit conditionality in biconditional statement.

Thus, it seems that out of four different types of compound logical proposition, there is no use of Conditional Proof whatsoever in the case of argument of which the conclusion is in the form of conjunction. The other remaining three compound logical propositions are: disjunctive, implicative and equivalence. All these are conditional in nature. In case of disjunctive and equivalence (biconditional) the conditional is used implicitly whereas in the case of implicative proposition, the conditional is explicit in nature. Conditional Proof starts with explicit conditional. In this sense Ordinary Conditional Proof covers only implicative proposition expressed by means of explicit conditional. However, to tackle with implicit conditional as we noted in the case of disjunctive and biconditional propositions, we must take the help of Strengthened Conditional Proof. Thus, when we pass on from Ordinary Conditional Proof to Strengthened Conditional Proof, we can relinquish the relevance or reference of Ordinary Conditional Proof and only stick to Strengthened Conditional Proof.

8.9 THE CONCEPT OF THE SCOPE OF THE ASSUMPTION

The scope of the assumption plays a vital role in case of Strengthened Conditional Proof. According to Copi, 'only a line inferred by the principle of Conditional Proof ends the scope of an assumption and that every use of the rule of Conditional Proof serves to end the scope of an assumption'. Let us make this point clear with the help of an example:

1.	$A \supset B$	
		/ $\therefore A \supset (A \cdot B)$
2.	A	(assumption)
3.	B	1, 2.M.P.
4.	$A \cdot B$	2, 3, Conj.
5.	$A \supset (A \cdot B)$	2-4, C. P.

In the above step no 5 is inferred not from any one or two of the preceding lines, but from the sequence of lines 2, 3, 4, which constitutes a valid deduction of line 4 from lines 1 and 2. By step 5, we infer the validity

of the above argument from the demonstrated validity of the argument where an additional premise (in step no.2) is created as an assumption. In the above demonstration step 2 is taken as an assumption of Conditional Proof and the subsequent steps, such as, 3 and 4 are dependent upon it. Step 5 however is not dependent upon step 2, but only upon step1. Thus, in our case step 5 is outside or beyond the scope of the assumption made as step 2. According to Copi, when an assumption is made in a Conditional Proof of Validity its 'scope' is always limited, never extending all the way to the last line of the demonstration. Thus, assumptions and scope of assumptions are very important in understanding Conditional Proof. It should equally be noted that in the case of Conditional Proof, only a line inferred by the principle of Conditional Proof ends the scope of an assumption, and that every use of the rule of Conditional Proof serves to end the scope of an assumption. When the scope of an assumption has been ended, the assumption is said to have been discharged. In the above case, the assumption has been ended in step 4. Within the assumption everything has been justified with regard to the assumption. However, beyond the scope of the assumption nothing can be justified by reference to it or to any line lying between it and the line inferred by the rule of Conditional Proof that discharges it. According to Copi, it is the case that 'only lines lying between an assumption of limited scope and the line that discharges it, can be justified by reference to that assumptions'. An argument may require more than one assumption. Every assumption has its scope and the scope is limited. One assumption may be lying within the scope of another assumption. Scopes of different assumptions may follow each other. After one assumption of limited scope has been discharged, another such assumption may be made and then discharged. No assumption can embrace the conclusion of the argument. The conclusion of the argument always rests beyond its scope and independent of it.

Let us examine a more complicated argument to apprehend the nature of Conditional Proof and the scope of the assumption what we are talking of.

1.	$(A \vee B) \supset [(C \vee D) \supset E]$	/ $\therefore A \supset [(C \cdot D) \supset E]$
2.	A	(assumption 1)
3.	$A \vee B$	2, Add.
4.	$(C \vee D) \supset E$	1, 3, M. P.
5.	$C \cdot D$	(assumption 2)
6.	C	5, Simp.
7.	$C \vee D$	6, Add.
8.	E	4, 7, M. P.
……		
9.	$(C \cdot D) \supset E$	5 - 8, C. P.
……		
10.	$A \supset [(C \cdot D) \supset E]$	2 - 9, C. P.

In the above argument, there are two assumptions taken for granted as Conditional Proof. The scope of the first assumption contains the steps from 2 - 9. Within the scope of the first assumption, there develops the second assumption containing the steps from 5 - 8. This clearly suggests that within the scope of one assumption other assumption may exist. Here the second assumption lies within the scope of the first assumption. The scope of the first assumption is marked from step 2 to step 9, whereas the scope of the second assumption is marked from step 5 to step 8. It is also important to note that one assumption may exist within the other assumption or may exist independently. When one assumption lies within the scope of another assumption, the assumption that lies within the scope of other assumption must be closed. In our case, the second assumption must be closed before the first assumption. However, if there are assumptions exist independently, then the assumptions shall be closed in hierarchical order.



ACTIVITY: 8.2

Do you think Logic as a branch of philosophy is full of methods?

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Explain the methodological affectivity of the Method of Indirect Proof.

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CHECK YOUR PROGRESS

Q 7: Indirect proof being a logical method tries to establish the truth of the given (True/ False).

Q 8: Every proposition must be True / False / either true or false.

Q 9: Every argument must be True or False / valid or invalid.

Q 10: Indirect proof deals only with true proposition and valid argument - Is it true?

Q 11: Logic deals with proposition / sentence.

Q 12: $p \supset q \equiv \sim p \vee q$ - Is it true?

Q 13: Give an example of Condi14tional statement.

Q 14: Define Ordinary Conditional Proof.



8.10 LET US SUM UP

- Ordinary Conditional Proof (OCP) is a kind of conditional proof that is primarily concerned with explicit conditional. It is important to note here that not all conditionals are explicit in nature. Conditionals may be expressed both explicitly as well as implicitly. In the case of Ordinary Conditional Proof, the conditional must be explicit in nature.
- Every proposition must be either true or false. Every argument must be valid or invalid. Indirect proof, being a logical method, deals only with true proposition and valid argument. This is because it is a method of proof, not a method of test.

- There are two types of Conditional Proof, such as, Ordinary Conditional Proof and Strengthened Conditional Proof.
- Strengthened Conditional Proof (SCP) can be applied just looking at the nature of the conclusion of the given argument. If the conclusion of the given argument is conjunctive, then there is no application of Conditional Proof. If the conclusion would be disjunctive, then one can find the sense of implicit conditional in the disjunctive form of Conditional because one has to know that the disjunctive proposition 'p v q' is identical with the implicative proposition ~ p ⊃ q.
- Finally, it can be said even though a distinction between OCP and SCP is made, but after passing from OCP to SCP, one may forgo OCP and stick to SCP. SCP contains OCP. Even Copi remarks that there is nothing wrong to conceive SCP as the Rule of Conditional Proof.



8.11 REFERENCES:

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8.12 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: True

Ans to Q No 2: True

Ans to Q No 3: False

Ans to Q No 4: True

Ans to Q No 5: Indirect proof can be defined as a logical method where the validity or truth of the given is proved by way of negating or denying the given.

Ans to Q No 6: True.

Ans to Q No 7: Either True or False

Ans to Q No 8: Valid or invalid

Ans to Q No 9: True.

Ans to Q No 10: True.

Ans to Q No 11: Proposition

Ans to Q No 12: True.

Ans to Q No 13: $(p \supset q) \supset [(q \supset r) \supset (p \supset r)]$ is a conditional statement.

Ans to Q No 14: Ordinary Conditional Proof is a logical method applied in the case of deduction where the proof of a conditional statement or proposition can be adequately justified provided that the conditional is explicit in nature

8.13 MODEL QUESTIONS

A) Very short questions

Q 1: What is conditional proof?

Q 2: What is the distinctive mark of SCP?

Q 3: What is the technique of employing Indirect Proof?

Q 4: Is Indirect Proof a logical concept or a logical method?

Q 5: Is Conditional Proof a logical concept or logical method?

Q 6: In what sense Disjunction can be read as Implication?

Q 7: In what sense equivalence can be read as conditional?

Q 8: What is the technique of employing Indirect Proof?

B) Short questions (Answer each question in about 100-150 words)

Q 1: Distinguish between OCP and SCP?

Q 2: Why is CP attributed as SCP? Explain

Q 3: Distinguish between implicit and explicit conditions

Q 4: Why does Indirect Proof begin with negation of the given? Briefly explain

Q 5: What is the relevance of Indirect Proof? Briefly explain

Q 6: What is the relevance of Conditional Proof? Briefly explain

Q 7: Is Indirect Proof a logical concept or a logical method? Briefly explain

Q 8: Is Conditional Proof a logical concept or logical method? Briefly explain

Q 9: In what sense Disjunction can be read as Implication? Briefly explain

Q 10: In what sense equivalence can be read as conditional? Briefly explain

Q 11: Why does conjunction not be treated as conditional like disjunction? Briefly explain

Q 12: Use the technique of Indirect Proof to verify that the following are tautologies.

(a) $A \supset [A . (A \vee B)]$

(B) $(A \supset B) \vee (\sim A \supset C)$

(C) $(A \supset B) \vee (B \supset A)$

Q 13: Use the method of Conditional Proof to verify that the following are tautologies:

(a) $[P \supset (Q \supset R)] \supset [(P \supset Q) \supset (P \supset R)]$

(b) $[P \supset (Q \supset R)] \supset [Q \supset (P \supset R)]$

(c) $[(A \supset B) \supset B] \supset (A \vee B)$

Q 14: Use the techniques of Indirect Proof and Conditional Proof to verify that the following arguments are valid:

(a) $A \supset B \quad / \therefore A \supset (A \cdot B)$

(b) $A \supset B$

$[A \supset (A . B)] \quad C \quad / \quad C$

(c) $A \supset B$

$B \supset [(C \supset \sim \sim C) \supset D] / \quad A \supset D$

Q 15: What is Indirect Proof? Explain the process of the application of indirect proof in proposition with a suitable example.

Q 16: Explain the use of indirect proof in validity of argument with a suitable example.

Q 17: What is conditional? Describe the strengthened rule of conditional proof.

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