PG PH S1-04 (DSE)

TWO YEAR MASTER'S DEGREE COURSE

(CBCS in Accordance with NEP 2020)

SEMESTER-I

LOGIC

PHILOSOPHY

Surya Kumar Bhuyan School of Social Sciences

KRISHNA KANTA HANDIQUI STATE OPEN UNIVERSITY

Subject Experts

1. **Professor Sauravpran Goswami**, Dept. of Philosophy, G.U.
2. **Dr. Sabitri Devi**, Associate Professor, Dept. of Philosophy, Cotton College.
3. **Dr. Naba Prasad Nath**, Associate Professor, Dept. of Philosophy, Nowgong College, Autonomous.

Course Coordinator: Dr. Bhaskar Bhattacharyya, Assistant Professor, Dept of Philosophy, KKHSOU

SLM Preparation Team

UNITS	CONTRIBUTORS
1	**Dr. Bhaskar Bhattacharyya**
2	**Meenakshi Gogoi,** Icon Academy
3 & 4	**Dr. Jahnabi Deka,** Gauhati University
5 & 7	**Dr. Minakshi Das,** North Kamrup College
6 .	**Ms. Pallabi Sarmah,** Krishnaguru Adhyatmik Viswavidyalaya
8.	**Dr. Hasen Ali Ahmed,** Indira Gandhi College
9 & 10	**Ms. Sima Baruah,** Cotton University
11 & 12	**Dr. Manashi Bora,** Nowgong Girls' College
13 & 14	**Dr. Banjit Sarma,** BongaigaonCollege
15	**Dr. Maina Sarma,** B.B.Borooah College

Editorial Team

Content	:	**Dr. Bhaskar Bhattacharyya, Dr. Tejasha Kalita,** KKHSOU
Language	:	**Dr. Bhaskar Bhattacharyya,** KKHSOU
Structure, Format & Graphics	:	**Dr. Bhaskar Bhattacharyya,** KKHSOU

November, 2024

This Self Learning Material (SLM) of the Krishna Kanta Handiqui State Open Universityis made available under a Creative Commons Attribution-Non Commercial -Share Alike 4.0 License (international): http://creativecommons.org/licenses/by-nc-sa/4.0/

Printed and published by Registrar on behalf of the Krishna Kanta Handiqui State Open University.

Headquarters: Patgaon, Rani Gate, Guwahati-781017

City Office: Resham Nagar, Khanapara, Guwahati 781022; Web: www.kkhsou.in

The University acknowledges with thanks the financial support provided by the **Distance Education Bureau**, **UGC** for the preparation of this study material.

CONTENTS

		Pages
UNIT 1:	**Fundamental Ideas of Logic** Logic: what it is, Nature of argument, Kinds of argument: Inductive and Deductive, Logic and Deductive argument, Argument and argument form Truth and Validity	**9-23**
UNIT 2:	**Symbolic Logic and Traditional Logic** Nature of Symbolic Logic, Classical Logic and Symbolic Logic, Development of Symbolic Logic Characteristics of Symbolic Logic, functions and the Importance of the Use of Symbols in Symbolic Logic	**24-43**
UNIT 3:	**Truth Functions, Inter-Definitions of Logical Constants** Definition of Truth Function, Basic Truth Functions, Negative or Contradictory function, Conjunctive function, Disjunctive function, Implicative or Conditional function, Equivalence or Biconditional function, Alternative function, Stroke Function, Inter- definitions of logical constants	**44-68**
UNIT: 4	**Definitions of Connectives and Symbolization of Everyday Language** Meaning of connective, Definition of connectives, Not/ It is not the case that (Negation), And (Conjunction), Either-or (Disjunction), If-then (Material Implication), If and only if (Equivalence of Biconditional), Symbolization of everyday language, Two kinds of statements, Symbolization of Simple statements, Symbolization of Simple statements, Complex Symbolizations	**69-87**
UNIT 5:	**Construction of Truth Table** Construction of Truth Tables, Truth Table for Statement, Characterization of Statement forms as Tautologies, Contradictory and Contingent, Truth Table for Arguments, To Decide Logical Equivalences,	**88-102**
UNIT 6:	**Determination Of Validity And Invalidity Of Arguments** Validity of arguments, Invalidity of arguments, Determination of validity and invalidity of arguments-direct truth table method, Indirect truth table method, Formal proof of validity, Conditional proof, Indirect proof	**103-115**
UNIT 7:	**Formal Proof of Validity and Conditional Proof** The Method of Formal Proof of Validity, The Rules of Inference, The Rules of Replacement, The Rule of Conditional Proof	**116-127**

UNIT 8: Indirect Proof and Strengthened Rule Of Conditional Proof **128-144**

Indirect Proof, What is a logical method? ,The use of Indirect Proof in proposition, The use of indirect proof in validity of argument, The Strengthened Rule of Conditional Proof, Types of Conditional Proof, Ordinary Conditional Proof (OCP), Strengthened Conditional Proof (SCP), The concept of the scope of the assumption

UNIT 9: Singular Proposition and General Proposition **145-160**

Singular proposition, General proposition, Traditional subject-predicate propositions

UNIT 10: Quantification and Rules of Quantification **161-178**

Universal and existential quantifiers, Modern square of opposition, Symbolization of four categorical statements with quantifiers, Symbolization of universal and existential propositions Symbolization of propositions by the process of instantiation, Rules of quantification Proving validity

UNIT 11: Set Theory **179-202**

Definition of set, Set membership, Set inclusion, Empty set, Operations on sets Union of sets, Intersection of sets, Difference of sets, Complement of set, Laws of operations on set, Domain of individuals, Using set notation for logical propositions (categorical propositions)

UNIT 12: Membership, Inclusion, the Empty Set **203-213**

The concept of membership, the concept of inclusion, the concept of empty set

UNIT 13: Operation on Sets, Domains of Individuals **214-225**

Operation on sets, Union of sets, Intersection of sets, Difference of sets, Domain of individuals

UNIT-14: Translating Everyday Language **226-239**

Symbolization of ordinary language, Symbolizing Propositions of Predicate Logic, Symbolization of Categorical Propositions by Using Set Notation:

UNIT: 15 Venn Diagram **240-261**

Categorical proposition and standard form categorical proposition, Classes and relation. The four standard form categorical propositions and their class relations, Distribution of terms Venn diagram, Use of Venn diagram, History, Aristotelian and Boolean standpoint Symbolism and diagrams for standard form categorical propositions

COURSE INTRODUCTION

This course deals with Logic. Logic is an important branch of philosophy. The word 'logic' is derived from the Greek word 'logos'. It means thought or reason or law. Logic is defined as the science of reasoning. This definition is not accepted as the real definition of logic because reasoning implies a special kind of inferring or inference. Again, we can define logic as the study of the methods and principles used in distinguishing correct from incorrect arguments. This helps us to make a distinction between correct and incorrect argument. That is why; it is clear that logic is a tool or strategy by which we can find out the distinction between good reasoning and bad reasoning. The study of logic, therefore, enhances our reasoning power to testify the arguments, whether they are valid or invalid. It also helps us in our domestic discourse to argue systematically with others.

This course consists of fifteen units. The first unit starts with **"Fundamental Ideas of Logic".** The issues covered in this unit are- Logic: what it is, Nature of argument, Kinds of argument: Inductive and Deductive, Logic and Deductive argument, Argument and argument form, and Truth and Validity.

The second unit is **'Symbolic Logic and Traditional Logic'.** The concepts included in this unit are- Nature of Symbolic Logic, Classical Logic and Symbolic Logic, Development of Symbolic Logic, Characteristics of Symbolic Logic, and functions and the Importance of the Use of Symbols in Symbolic Logic.

The third unit is **'Truth Functions, Inter-Definitions of Logical Constants'.** The issues covered in this are- Definition of Truth Function, Basic Truth Functions, Negative or Contradictory function, Conjunctive function, Disjunctive function, Implicative or Conditional function, Equivalence or Biconditional function, Alternative function, Stroke Function, and Inter- definitions of logical constants.

The fourth unit is **'Definitions of Connectives and Symbolization of Everyday Language'.** It discusses the concepts, which are- Meaning of connective, Definition of connectives, Not/ It is not the case that (Negation), And (Conjunction), Either-or (Disjunction), If-then (Material Implication), If and only if (Equivalence of Biconditional), Symbolization of everyday language, Two kinds of statements, Symbolization of Simple statements, Symbolization of Simple statements, and Complex Symbolizations.

The fifth unit is **'Construction of Truth Table'.** The concepts covered in this unit are: Construction of Truth Tables, Truth Table for Statement, Characterization of Statement forms as Tautologies, Contradictory and Contingent, Truth Table for Arguments, and To Decide Logical Equivalences.

The Sixth unit is **'Determination of Validity and Invalidity of Arguments'.** The issues included in this unit are: Validity of arguments, Invalidity of arguments, Determination of validity and invalidity of arguments-direct truth table method, Indirect truth table method, Formal proof of validity, Conditional proof, and Indirect proof.

The seventh unit is **"Formal Proof of Validity and Conditional Proof".** After going through this unit, you will be able to get knowledge of the Method of Formal Proof of Validity, The Rules of Inference, The Rules of Replacement, and The Rule of Conditional Proof.

The name of the eighth unit is **'Indirect Proof and Strengthened Rule of Conditional Proof'.** The issues included in this unit are: Indirect Proof, What is a logical method?, The use of Indirect Proof in proposition, The use of indirect proof in validity of argument, The Strengthened Rule of Conditional Proof, Types of Conditional Proof, Ordinary Conditional Proof (OCP), Strengthened Conditional Proof (SCP), and The concept of the scope of the assumption.

The name of the ninth unit is **'Singular Proposition and General Proposition'.** After going through this unit you will be able to get knowledge of Singular proposition, General proposition, and the concept of Traditional subject-predicate propositions.

The tenth unit is **"Quantification and Rules of Quantification".** The concepts included in this unit are: Universal and existential quantifiers, Modern square of opposition, Symbolization of four categorical statements with quantifiers, Symbolization of universal and existential propositions, Symbolization of propositions by the process of instantiation, Rules of quantification, and Proving validity.

The elevenths unit is **"Set Theory".** The issues included in this unit are- Definition of set, Set membership, Set inclusion, Empty set, Operations on sets, Union of sets, Intersection of sets, Difference of sets, Complement of set, Laws of operations on set, Domain of individuals, and Using set notation for logical propositions (categorical propositions).

The twelfth unit is **"Membership, Inclusion, the Empty Set".** The concepts covered in this unit are- The concept of membership, the concept of inclusion, and the concept of empty set.

The thirteenth unit is **"Operation on Sets, Domains of Individuals".** This unit provides us the knowledge of Operation on sets, Union of sets, Intersection of sets, Difference of sets, and Domain of individuals.

The fourteenth unit is **"Translating everyday language".** The concepts covered in this unit are- Symbolization of ordinary language, Symbolizing Propositions of Predicate Logic, and Symbolization of Categorical Propositions by Using Set Notation.

The fifteenth unit is **"Venn Diagram".** The concepts included in this unit are: Categorical proposition and standard form categorical proposition, Classes and relation, The four standard form categorical propositions and their class relations, Distribution of terms, Venn diagram, Use of Venn diagram, History, Aristotelian and Boolean standpoint, and Symbolism and diagrams for standard form categorical propositions.

While going through this course you will come across some boxes which are put on the left side or right side of the text. These boxes will give us the meanings of some words and concepts within the text. Apart from this, there will be some broad and short questions included under Activity and Check Your Progress in every unit. Activities will increase our thinking capacity because questions put in Activity are not directly derived from the text. But answers to the short questions are put in the section Answers to Check Your Progress. Besides, there are some text-related questions which are put in Model Questions. These questions will help you in selecting and mastering probable topics for the examination so that you can prepare for the examination with confidence.

UNIT: 1 FUNDAMENTAL IDEAS OF LOGIC

UNIT STRUCTURE

1.1 Learning Objectives

1.2 Introduction

1.3 Logic: what it is

1.4 Nature of Argument

- 1.4.1 Kinds of argument: Inductive and Deductive
- 1.4.2 Logic and Deductive Argument
- 1.4.3 Argument and Argument Form

1.5 Truth and Validity

1.6 Let Us Sum Up

1.7 Further Reading

1.8 Answers to Check Your Progress

1.9 Model Questions

1.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- define logic
- define argument
- discuss the nature of argument
- explain the distinction between truth and validity in logic

1.2 INTRODUCTION

Argument and argument forms, truth and validity are the fundamental notions of logic. A proposition is the statement which is either true or false. As for instance, Manmohan Sing is the present Prime Minister of India, Sachin Tendulkar is one of the best cricketers of the world and Men are four-footed animals etc. are the examples of propositions. The first two propositions are true, while the third one is false, because we know that men are not four-footed animals.

On the other hand an argument is either valid or invalid. An argument implies a logical relation of premise and conclusion. When a conclusion is necessarily derived from the premises, the argument is valid. On the other hand when a conclusion is not necessarily derived from the premises, the argument will be invalid. On the other hand an argument form is a symbolic representation of propositions. As for instance:

All historians are rationalists
Bipan Chandra is a historian
∴ Bipan Chandra is a rationalist

This is valid argument, because the conclusion is necessarily derived from the premises.

If he comes, I will go
He does not come
∴ I shall not go

This is invalid argument, because the conclusion is not necessarily derived from the premises. From the above examples it is clear that validity and invalidity are concerned with argument.

To be brief, truth and falsity are dealt with propositions and validity and invalidity are concerned with arguments.

1.3 LOGIC: WHAT IT IS

The word 'logic' is derived from the Greek word 'logos'. It means thought or reason or law. Logic is defined as the science of reasoning. This definition is not accepted as the real definition of logic because reasoning implies a special kind of inferring or inference. In inference we proceed from known to unknown. Just as we can draw the conclusion that 'there is fire in the hill' which is derived from the premise that 'there is smoke in the hill'. But the important issue is that logic does not include the act of inferring or inference within the subject matter of logic.

Again we can define logic as the study of the methods and principles used in distinguishing correct from incorrect arguments. This helps us to make a distinction between correct and incorrect argument. That is why, it

is clear that logic is a tool or strategy by which we can find out the distinction between good reasoning and bad reasoning. The study of logic, therefore, enhances our reasoning power to testify the arguments, whether they are valid or invalid. It also helps us in our domestic discourse to argue systematically with others.

Traditional logic is defined as the study of laws of thought. There are three of laws of thought which are indispensable for correct reasoning. These laws are: i) the law of identity, ii) the law of excluded middle iii) law of non-contradiction. Apart from these laws, there are other laws of inference and deduction, which are also equally indispensable for correct reasoning. These three laws of thought refer to mental process, but every mental process is not considered reasoning. As for instance, remembering, imagining, memory, day dreaming etc. are not considered as reasoning. They are only considered as mental states. As logic deals with correctness or incorrectness, so, logic is defined as the science of valid reasoning. Logicians deal with validity or correctness and invalidity or incorrectness of arguments, while psychologists concern with mental process. That is why the traditional definition of logic as the study of laws of thought is not satisfactory from the logicians' points of view.



CHECK YOUR PROGRESS

Q 1: State whether the following statements are true or false:

a. Logic is derived from the Greek word 'logos'.

b. Validity and invalidity are concerned with propositions.

c. Truth and falsity concern with arguments.

d. A proposition is either true or false.

e. There are five laws of thought.

f. Logic is the study of the methods and principles used in distinguishing correct from incorrect arguments.

g. Logic deals with the product of reasoning that is argument

Q 2: Answer short Questions.

a. What is argument?

b. Give an example of argument.

c. What is logic?

d. What are the three laws of thought in traditional logic?

e. What conclusion?

f. What is premise?

1.4 THE NATURE OF ARGUMENT

As it is said that logic deals with the correctness or incorrectness of argument, so the question naturally comes to enquire, what is argument? And what are its kinds?

Arguments consist of a group of proposition, of which one is the conclusion and others are premises which are regarded as providing support or grounds for the truth of the conclusion. Premises are those propositions from which conclusions are derived. On the other hand conclusions are those propositions which are drawn from the premises. There is a necessary relationship between premise and conclusion. It will be clear when we go through examples.

All men are mortal

Socrates is a man

∴ Socrates is mortal

The first two propositions are premises because the last one is drawn from the first two propositions. The last proposition is known as conclusion because it is drawn from the first two propositions. So, in an argument there is a necessary relationship between premise and conclusion. If we do not notice a necessary relationship between premise and conclusion, the argument will be invalid. As for instance-

A. All men are rational

Shibaji is a man

∴ Shibaji is rational

B. If rain comes, we will not go out
Rain comes
∴ We will not go out

C. Most actors are celebraties
Amir khan is an actor
∴ Amir khan is a celebraties

The first two examples show that premises imply the conclusion and which is necessarily derived from the premises. And in the third one premises support the conclusion.

Logic is concerned with the product of reasoning that is argument. It is dealt with the enquiry whether an argument is correct (sound) or incorrect (unsound). The task of the logician is to find out methods and techniques which are essential for making this distinction.

1.4.1 Kinds of Argument: Inductive and Deductive

According to traditional logicians, there are two kinds of argument-deductive argument and inductive argument. And accordingly logicians have classified logic into inductive logic and deductive logic.

In deductive argument the conclusion is necessarily derived from the premises. Generally, we proceed from general premise to particular premise in case of deductive argument. So, the conclusion is less general than the premises.

As for instance:

All babies cry
Munmun is a baby
∴ Munmun cries

Here, the first proposition 'All babies cry' is a major premise. And the second proposition 'Munmun is a baby' is a minor premise. The last proposition 'Munmun cries' is the conclusion which is derived from the above two premises. So, this is called deductive reasoning. According to this reasoning, if the premises are true the conclusion must be true. It claims that premises provide absolutely conclusive grounds for the truth of the

conclusion.The technical terms 'valid' and 'invalid' are used in place of correct and incorrect in characterising deductive argument. A deductive argument is valid when its premises and the conclusion are true and it is absolutely impossible for the premises to be true and the conclusion to be false.

On the other hand, In case of inductive argument the conclusion is derived only with probability from its particular premises. The premises serve as evidence for the conclusion. Inductive arguments involve the claim that the premises provide some grounds for the truth of the conclusion. Here, we proceed from particular premise to general premise. So, in inductive argument the conclusion is more general than the premises. Inductive arguments deal with inductive logic.

As for instance:

Pinky is a baby and it cries
John is a baby and it cries
Paul is a baby and it cries
∴ All babies cry

In this inductive argument the conclusion is derived from particular premises. The premises are collected from experience. Therefore, they serve as evidence or ground for the truth of the conclusion. The conclusion- 'All babies cry' is a general premise which is derived from the particular premises like 'Pinky is a baby and it cries', 'John is a baby and it cries', 'Paul is a baby and it cries' etc. So, in inductive argument we derive a conclusion from particular premises. The argument is true because the premises provide some grounds or evidence for the truth of the conclusion. On the other hand inductive argument will be false when its premises do not provide some support or evidence for the truth of the conclusion.

1.4.2 Logic and Deductive Argument

Logic is specifically dealt with the enquiry into the deductive reasoning in which the conclusion is necessarily derived from the premises. We often use the terms reasoning or argument for logic. Deductive logic also known as exact logic or formal logic.

Logic deals with the enquiry whether an argument is correct (sound) or incorrect (unsound). The task of the logician is to find out methods and techniques to distinguish correct reasoning from incorrect reasoning. The correctness and incorrectness of argument is not dealt with the subject matter of reasoning.

For example,

(1) All cats are dog
Raja is a cat
∴ Raja is a dog

(2) All Abbas are Babbas
Raja is a Abba
∴ Raja is Abba

The above argument is valid or correct. The correctness or validity of the above arguments is not concerned with the subject matter. They are valid because of their form not because of their subject matter. The form of an argument implies the very structure of the argument. When we replace all the terms of an argument by letters of English Alphabet we get the form of the argument. For example, if we use the letters 'S' and 'P' in place of the terms of the above argument and the letter 'a' for the proper name then we get the argument form-

All S are P
a is S
∴ a is P

Here, this argument form is valid. Therefore, any argument with any subject matter will be valid only if conforms with the form of the argument as mentioned above.

1.4.3 Argument and Argument Form

In case of the discussion of the nature of argument we find a distinction between argument and argument form. We already know that argument contains propositions from which conclusion is necessarily derived

from the premises. On the other hand argument form is a symbolic formulation of propositions which contain variables. We can go through examples:

A. All men are rational
Ram is a man
∴ Ram is rational

B. If P then Q
P
∴ Q

The first example is an example of argument, while the other is the example of argument form. It can be mentioned that logic is primarily concerned with the argument form. Argument form is nothing but the logical or symbolic structure of proposition.

Logic is concerned with the product of reasoning that is argument. It deals with the enquiry whether an argument is correct (sound) or incorrect (unsound). The task of the logician is to find out methods and techniques to distinguish correct reasoning from incorrect reasoning. The correctness and incorrectness of argument is not dealt with the subject matter of reasoning.

1.5 TRUTH AND VALIDITY

Truth and falsity, validity and invalidity are the fundamental notions of logic. A proposition is true when it corresponds to reality. As for instance, 'Grass is green' is a true proposition because we see that grass is green in reality. We nowhere see that grass is white. On the other hand the proposition 'All animals are immortal' is a false proposition because we never perceive immortal animals.

On the other hand in case of argument we use the terms 'valid' or 'invalid'. A deductive argument is valid when the conclusion is necessarily derived from the premises. So, the term validity is not applied in case of

propositions. In a valid argument, the propositions which are considered as premises may be true or false. Therefore, a valid argument can have true premises or false premises. If the conclusion in an argument is false, still the argument can be valid, because the premises may imply the conclusion. Again in a valid argument there can be false premises, yet the argument can be valid provided the conclusion follows from the premises. Truth and falsity are the attributes of propositions and validity and invalidity are the attributes of arguments.

The relations between true or false propositions and validity or invalidity of arguments are complex. We generally notice this relationship in deductive logic. An argument may be valid in spite of the fact that premises are false. Again an argument may be valid even when its conclusion and one or its premises are false. The validity or invalidity of argument depends only upon the relationship of the premises and the conclusion.

We can take some examples in order to show that a valid argument can have false premises and an invalid argument can have true premises.

A. All dogs are cats

All cats are horses

∴ All dogs are horses

This argument shows the premises are false but the argument is valid.

B. All dogs are animals

All cats are animals

∴ All cats are dogs

This argument is invalid, although premises are true.

C. All human beings are rich

All businessman are rich

∴ All businessman are human beings.

This is an invalid argument because the premises in the argument are false.

From the combination of false premises and true conclusion we can decide whether the argument is valid or invalid, depending on the arrangement of premises and conclusion. For example:

A. All men are rational
No cats are rational
∴ No cats are men

This is a valid argument. This argument shows that the premises and the conclusion are true.

The above discussion holds that truth and validity are the basic concepts of logic. Validity and invalidity of argument become independent of the truth and falsity of the premises. From the above mentioned examples it is clear that a valid argument can have false premises, similarly an invalid argument can have true premises. Again validity and invalidity of arguments are not concerned with whether the premises are empirically true or not. Irving M. Copy in this context clears that "To determine the truth or falsehood of premises is the task of scientific enquiry in general, since premises may deal with any subject matter at all. But determining the validity or invalidity of arguments is the subject of deductive logic." (Symbolic Logic p-5). Logicians are concerned with the question of validity and invalidity of arguments depending on the logical relation of the premises and conclusion.



CHECK YOUR PROGRESS

Q 3: Answer short questions:

a) What are the two kinds of argument?
b) What is deductive argument?
c) What is inductive argument?
d) Give an example of deductive argument?
e) What do you mean by argument form?
f) Give an example of argument form

Q 4: State whether the following statements are true or false

a) Deductive logic is also known as exact logic or formal logic.

b) In inductive argument we proceed from general premise to particular Premise.

c) The technical terms 'valid' and 'invalid' are used in inductive logic.

d) The correctness and incorrectness of argument is associated with the subject matter of reasoning.



ACTIVITY: 1.1

- Do you find any relationship between truth and falsity of propositions and validity and invalidity of arguments? Discuss.

Ans ……

……

……

……

……



1.6 LET US SUM UP

From the above discussion it becomes clear that the concepts of Argument and Argument forms, truth and validity are the fundamental concepts of logic. These concepts of logic are interdependent. The concepts of argument and argument form without the concepts of truth and validity will be answerless. And again, the concepts of truth and Validity without the concepts of argument and argument form will be contentless discussion. That is why, the subject matter of logic starts with the discussion of argument and argument form and truth and validity. So, these two inter-related concepts are considered as fundamental concepts in logic.

The most important points discussed in this unit are:

- Logic is derived from the Greek word 'Logos'. It means thought or reason or law.
- Logic is defined as the study of the methods and principles used in distinguishing correct from incorrect arguments.
- There are three laws of thought. They are respectively: The law of identity, the law of excluded middle and the law of non-contradiction.
- Argument consists of propositions from which conclusion is necessarily derived from the premises. Argument implies a relation between premise and conclusion.
- An argument form is a symbolic representation of propositions.
- Arguments are generally divided into kinds: Deductive and inductive argument
- Truth and falsity are dealt with propositions and validity and invalidity are concerned with arguments.



1.7 FURTHER READING

1. Copi, Irving M. (2006). Symbolic Logic (fifth edition). New Delhi: Prentice Hall of India Private Ltd.
2. Jain, Krishna.(1998). A textbook of logic. (fourth Revised Edition)
3. Singh, S. Shyam Kishore. Modern Logic (Vol.1) Lamyana Press
4. Copi, Irving M., Cohen, Carl., Jetli, Priyedarshi., & Prabhakar, Monica.(2006). Introduction to Logic (Twelfth edition).Delhi: Dorling Kindersley (India) Pvt. Ltd.



1.8 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1:

a) Ans: True

b) Ans: False

c) Ans: False

d) Ans: True

e) Ans: False

f) Ans: True

g) Ans: True

Ans to Q No 2:

a) Ans : An argument implies a logical relation between premise and conclusion.

b) Ans : All Philosophers are rationalists
Radhakrishnan is a philosopher
∴ Radhakrishnan is a rationalist

c) Ans : Logic is the study of the methods and principles used in distinguishing correct from incorrect arguments.

d) Ans : Conclusion is that proposition which is drawn from the premises.

e) Ans : Premises are those propositions from which conclusions are derived.

Ans to Q No 3:

a) Ans: Inductive and deductive argument

b) Ans: Inductive argument is that argument in which the conclusion is drawn with probability from particular premises

c) Ans: Deductive argument is that argument in which the conclusion is necessarily derived from premises.

d) Ans: All international cricketers are rich man
Sachin Tendulkar is a international cricketer
∴ Sachin Tendulkar is a rich man

e) Ans: An argument form is a symbolic formulation of propositions which contain variables.

f) Ans: If P then Q
P
∴ Q

Ans to Q No 4:

a) Ans: True

b) Ans: False

c) Ans: False

d) Ans: False

1.9 MODEL QUESTIONS

A. Very short Questions

Q 1: What is logic?

Q 2: Give an example of valid argument

Q 3: What are the three laws of thought?

Q 4: What are the kinds of arguments?

Q 5: Define inductive argument

Q 6: Give an example of deductive argument.

Q 7: What is the other name of deductive logic?

Q 8: What do you mean by argument form?

Q 9: What are three fundamental laws of thought?

B. Short Questions (Answer each question in about 150 words)

Q 1: What is the nature of valid argument? Briefly explain

Q 2: Write short notes on Argument

Q 3: Write short notes on Argument form.

Q 4: Write short notes on Laws of thought

Q 3: Distinguish between truth and validity

Q 4: Distinguish between Deductive and inductive argument

C. Long Questions (Answer each question in about 300-500 words)

Q 1: What is logic? Discuss the different definitions of logic and state which one you do accept more.

Q 2: What is argument? Explain the nature of argument with suitable examples.

Q 3: Do you find any relationship between truth and falsity and validity and invalidity of arguments? Illustrate with suitable examples.

Q 4: Are truth and validity the fundamental concepts in logic? What do you think? Discuss with suitable examples.

Q 5: What do you mean by the nature of argument? Explain the distinction between argument and argument form with suitable examples.

Q 6: Is there any relation between logic and deductive argument? Discuss.

*** ***** ***

UNIT: 2 SYMBOLIC LOGIC AND TRADITIONAL LOGIC

UNIT STRUCTURE

2.1 Learning Objectives

2.2 Introduction

2.3 Nature of Symbolic Logic

2.4 Classical Logic and Symbolic Logic

2.5 Development of Symbolic Logic

2.6 Characteristics of Symbolic Logic

2.7 Functions and the Importance of the Use of Symbols in Symbolic Logic

2.8 Let Us Sum Up

2.9 Further Reading

2.10 Answers to Check Your Progress

2.11 Model Questions

2.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- define symbolic logic and classical logic
- discuss a modern comprehensive symbolism
- explain the characteristics of symbolic logic
- identify variables used in symbolic logic
- identify ideograms and phonograms
- describe deductive method
- explain the importance of the use of symbols in logic
- explain the development of symbolic logic.

2.2 INTRODUCTION

This unit will familiarize you with different concepts of symbolic logic, and also will introduce you with different specialized technical symbols

(variables), which help to fulfill the aim of logic. (The aim of Logic is to express the form of the argument and to determine their validity and invalidity).The different concepts are:

- Concept of classical logic in philosophy
- Nature and characteristics of symbolic logic
- Different stages of development in symbolic logic
- The relation between symbolic logic and classical logic
- Functions and the importance of the use of symbols in symbolic logic

2.3 NATURE OF SYMBOLIC LOGIC

Before we discuss it in detail we must know something of its origin. That is why, it will be necessary to begin with classical logic to discuss the nature of symbolic logic.

- **Concept of Classical Logic in Philosophy:** Classical logic is the name given to that part and the treatment of formal logic which have come down substantially unchanged from medieval times. Classical logic is also known as 'Traditional Logic' or 'Aristotelian Logic'. The Greek philosopher Aristotle and his followers propounded it and had made such an elaborate treatment of this branch of study of about 2000 years ago. As Aristotle propounded it so this logic is also known as 'Aristotelian Logic' by his name. Aristotle formulated this logic so brilliantly and thoroughly that it covered the thinking of the Western world till the middle of the 19th century. And most of the Aristotle's successors thought it to be a finished science on which no improvement could be made. Aristotelian logic is also known as 'Classical Logic' or 'Traditional Logic'.

  Symbolic logic is a development of classical deductive logic and has emerged as a result of the use of the large amount of symbols in accordance with modern mathematical methods and principles. Modern logic represents a later phase in the development of logic, which is comparatively recent origin. Symbolic logic is the name

commonly given to the modern treatment of deductive logic. In the middle of the 19th century, some eminent mathematicians started to use the symbols of algebra for explaining the principles of classical or traditional logic. The extensive use symbols brought about significant change in the 'Traditional Logic' and the developed form of this is known as 'Modern Logic'. Modern logic stands for a group of studies and is called by different names as 'Symbolic Logic', 'Mathematical Logic', 'Algebra of Logic', 'Logistics' etc. Modern logic is basically known as Symbolic logic, is evident from its name involves the extensive use of symbols. Symbolic logic is a modern development of logic, used synonymously with Modern formal logic. Aristotle had made an elaborate treatment of this branch of study.

Symbolic logic is that form of logic which is developed in recent past, in which specialized technical symbols are used extensively to express the arguments and to determine their value. Symbolic logic is the developed, refined and mature branch of study, and based on the concepts and techniques used in Aristotelian logic. Like Mathematics the method of Symbolic logic is mathematical, deductive and formal in nature. Symbolic logic as a formal logic is concerned with only formal truth, not with the material truth of propositions.



LET US KNOW

Formal truth: A proposition is formally true if the proposition is true because of the very form or structure of the proposition.

Materially true: A proposition is materially true if the proposition is true as a matter of fact. It is the fact that determines whether the proposition is true or false.

2.4 CLASSICAL LOGIC AND SYMBOLIC LOGIC

Though there is a very close relation between classical logic and symbolic logic but there are some important points of differences involved in the process and the development of symbolic logic. They are:

- DISTINCTION BETWEEN SYMBOLIC LOGIC AND CLASSICAL LOGIC
  - **Classical Logic has a long history where as Symbolic Logic has a very short history:** Classical logic has a very long history behind it which has been formulated by Aristotle about 2000 years ago, where as symbolic logic is very recent origin and is developed in the later part of the nineteenth century before 150 years ago. Thus, symbolic logic is comparatively recent origin and has a very short history. (This modern developmental process began in the 19th century). The success of reasoning in mathematics has inspired some thinkers to extend such apriori, deductive thinking in logic also. As a result of this symbolic logic emerged in its present form. So, it can be said that the difference between classical logic and symbolic logic lies in the difference of different stages of development.
  - **In Classical Logic symbols were used to a limited extent where as in Symbolic Logic symbols are used very extensively:** Aristotle used variables in classical logic also, but in a very restricted way only to represent the terms used in arguments by the letters of the English Alphabet in order to bring out the logical structure of arguments. But the use of variables in symbolic logic is much wider than that of classical logic.
  - **Symbolic Logic has a very comprehensive use of symbols:** Symbolic logic is an improvement upon traditional logic in the sense that it has adopted symbolism in a comprehensive way and through the use of its technical language it has become a powerful tool for analysis and deduction. The exclusive use of deductive method has made it more accurate than the classical logic. But the use of symbols in classical logic is very limited.



Aristotle* (384-322 B.C):-** A Greek Philosopher had born in Stagira, Macedonia in Greece. He is best known as the author of ***Ethics.

- **Regarding the classification of propositions, the modern classification is an improvement upon the classical classification of propositions:** In classical logic a proposition is classified into four types-A, E, I and O only. But in modern logic a proposition is classified into three types - Simple, Compound and General.

A simple proposition is again of four types:

(a) Subjectless,

(b) Subject predicate,

(c) Relational and

(d) Class membership proposition.

A compound proposition is again of four types-

(a) Conjunctive proposition,

(b) Disjunctive proposition,

(c) Implicative proposition

(d) Alternative proposition.

A general proposition is of three types-

(a) Existential proposition,

(b) One predicate universal proposition,

(c) General propositions asserting relation between two classes.

- **Classical treatment of relation is extremely unsatisfactory because classical logicians only recognized one kind of relation between the subject and the predicate (the relation of proposition):** According to classical logicians the copula is only capable to express all types of relations. But modern logicians think that the copula of classical logic is not only capable to express all types of relations. Modern Logic recognizes only a limited function of the copula.

From the above considerations, it may safely be maintained that the difference between classical logic and symbolic logic is one of degree only and not of kind. There can not be any fundamental difference between symbolic logic and classical logic. Though the word symbolic has been applied to modern logic but it was not the case that symbols were used for

the first time by only modern logicians. Aristotle for the first time made use of the symbols namely 'variables' but the use of variables in classical logic was limited to syllogistic inferences only, which helped to express the form of the arguments only. Modern logic grows out of traditional logic itself, and does not contradict traditional logic. The implicit principles and techniques in the classical logic are made more explicit in symbolic logic in the light of modern mathematical development. Thus, it can be said that modern symbolic logic is a developed form of classical logic. So, Basson and O'Connor remarks- 'Classical Logic is related to Symbolic Logic as embryo to adult organism' ***(Introduction to Symbolic Logic, p.3)*** **Thus, the difference between classical logic and modern logic is only that of different stages of development.**



LET US KNOW

Simple Proposition: A simple proposition is that which contains only a singe statement.

Compound Proposition: A compound proposition is that which contains two or more statements.General Proposition: A general proposition is that which asserts that one class is wholly or partially included or excluded from another class.



ACTIVITY: 2.1

- Do you find any similarity between classical logic and symbolic Logic? Discuss

Ans.....

.....

.....

.....

.....



CHECK YOUR PROGRESS

Q 1: State whether the following statements are true or false:

a) Classical logic is also known as 'Traditional logic' or 'Aristotelian Logic'. (True/False)

b) Modern logic is a development of the classical logic.(True/False)

c) Symbolic logic is concerned with formal truth.(True/False)

d) Symbolic logic is a recent origin.

e) The difference between classical logic and modern logic is one of degree only and not of kind. (True/False)

f) According Basson and O, Connor, classical logic is related to symbolic logic and is narrower in scope. (True/False)

Q 2: Who is the founder figure of traditional logic?

.....

Q 3: Mention only two distinctions between classical logic and symbolic logic.

.....

.....

2.5 DEVELOPMENT OF SYMBOLIC LOGIC

Modern logic has adopted a comprehensive symbolism. It has become a powerful tool for analysis and deduction through the use of its technical language. In order to study the historical background of symbolic logic; we have to focus our attention to the time of Aristotle who first build up logic as a separate branch of study.

The triumph of abstract reasoning in the field of pure mathematics throughout the ages, has tempted people to extend the range of such 'a priori' and 'deductive thinking' to other fields also. Accordingly, various

attempts were made, in the seventeenth century, to construct a kind of generalized art or science of reckoning, so as to extend the field of formal logic. The most important worker in this field in the second half of 19th century was Leibnitz, who has brought about revolutionary developments in the traditional logic by using mathematical techniques and methods in logic. Leibnitz developed logic into a full-fledged mathematical language. In his book, "Dissertio de Arte Combinatoria" he has given the hint to develop a universal scientific language through some fundamental ideographic notation as well as by developing a calculus of reasoning, all problems arising out of the universal language can be solved.

Modern logic is said to be a joint venture between mathematics and philosophy. Because after Leibniz the foundation father of symbolic logic includes George Boole, Augustus-De-Morgan, W.S. Jevons, Giuseppe Peano, Gottlob Frege, C.S.Peirce, Russell etc. In the middle of the nineteenth century the two English Mathematicians George Boole and Augustus De Morgan presented a systematic mathematical way to logic. Augustus de Morgan attempted to extend the sphere of formal logic, specially of the doctrine of the syllogism, by taking into account other kinds of relations between the terms of a proposition than those commonly expressed by the copula 'is' or 'are'. He pointed out that the copula in any case has several different meanings, for it expresses sometimes the identity of objects having different names, or one concept has the same content as another. The actual character of a relationship must be determined in each case by knowledge of the relevant facts. Symbols can only be used to express such knowledge, not as a substitute for it. Thus, the foundation of the logic of relations was laid by Augustus De Morgan. W.S.Jevons elaborately simplified and discussed the Algebra of Classes with him.

The first moderately successful attempt to construct an algebraic or quasi-algebraic logic (Symbolic Logic) was made by George Boole. He published two important contributions to the subject, namely, "The Mathematical Analysis of Logic, Being an Essay Towards a Calculus of Deductive Reasoning (1847), and "An Investigation of the Laws of Thought on which are founded the Mathematical Theories of Logic and Probability



***George Boole* (1815-1864):-** A British mathematician born in Lincoln, England.



Augustus de Morgan (1806-1871):- He was an English mathematician and logician born in India.

(1854). A very remarkable contribution of Boole had proved him as the founder father of symbolic logic. He proved that the syllogistic reasoning is a special case of a kind of logical algebra. He also discussed the logical analysis, application of the deductive method and Calculus. He employed the literal symbols x, y, z, etc., to represent 'things as subjects of our conceptions'; the signs of operation +, -, x, for the combination or resolution of conceptions of things 'so as to form new conceptions involving the same elements'; and the sign º for identity (or the equivalent of "is").

During the later decades of the nineteenth century, E.Schroeder (1841-1902), C.S.Pierce (1839-1914) and several others made further attempts and valuable contributions towards symbolic logic by developing algebra of logic. John Venn carried on the works taken up by Boole and Jevons by his significant contribution of diagram known as Venn's diagram to represent class relations. In his 'Symbolic Logic', John Venn tried to criticize and systematize all the results achieved up to his time. From this the name "Symbolic Logic" was used for the first time. He used the term 'class' in the sense of a 'compartment', which might be empty (which was later called a "null class") and aimed at the elaboration of a "class calculus", in which all the propositions were treated as asserting the "existence" (in some "universe" or other) or "non-existence" of certain classes denoted by their terms (or the occupation or emptiness of the "compartment"). Venn represented the four familiar types of categorical propositions as:



C. I. Lewis (1883-1964):- Clarence Irving Lewis was an American Philosopher.



LET US KNOW

Venn's Diagram: A unique technique of representing categorical propositions by diagrams. John Venn (1834-1923) has devised this technique, so it is named by his name. In Venns diagram only the circles are used.

All x' s are y by x y=0

No x' s are y by x y=0

Some x' s are y by x y = 0

Some x' s are not y by x y = 0

(Where "0" means "different from, or more than, nothing"). But the four traditional types of propositions were not sufficient for a calculus; he regarded it as a legitimate convention for the purpose of his calculus.

The most important contributions towards symbolic logic during the past sixty years have been made by mathematicians. There are a few European Scientists started their research on the foundations of Mathematics. They are Gottlob Frege, Guiseppe Peano, Bertrand Russell and A.N.Whitehead. Russell and Whitehead had explained their method in the form of "Mathematical Logic", "Symbolic Logic" and "Logistics" etc. Symbolic logic and Mathematics are said to be closely related with each other by the use of logical reasoning or arguments in terms of mathematical principles as well as the extensive use of symbols.

The most significant development in symbolic logic was made by the German mathematician Gottlob Frege who formulated predicate calculus and is regarded as the second founder of modern symbolic logic after Boole. The works taken up by Frege and Peano were carried on by the two philosophers A.N.Whitehead and Bertrand Russell. They brought out a work on mathematical logic, entitled in Principia Mathematica. In their works a system of symbolic logic is elaborated and made to serve as the foundation of the whole of mathematics and this whole of mathematics with all its intricacies can be deduced to logic. Russell by his work Principia Mathematica made revolutionary change in logic. Another important philosopher Augustus De Morgan introduced symbols for relations and quantifies predicates. After Principia Mathematica a large number of logical systems of symbolic logic have been developed by different logicians and mathematicians who made a significant contribution to symbolic logic. The first important name in the development of logic from its traditional classical form into its modern symbolic form is G.H.Poh.

Thus, the development of classical logic made modern Logic to develop as a powerful human apparatus. Cohen and Nagel said that classical logic is related to symbolic logic as ''embryo to adult organism''. Thus the contributions of these mathematicians and philosophers have so



Leibnitz* (1646-1716):-** An eminent mathematician and philosopher, born at Leipzig in Germany. He wrote the famous book, 'New Essays Concerning the Human Understanding'.***



A priori: A priori means knowledge that we gain through intellect without experience.

thoroughly incorporated into modern symbolic logic by their more appropriate distinctive ideas.

ACTIVITY: 2.1



- Do you find any remarkable contribution of George Boole which had proved him as the founder father of symbolic logic? Discuss.

Ans.....

.....

.....

.....

- Do you accept that the reformation of logic by the use of different symbols establishes a universal scientific language? Discuss.

Ans.....

.....

.....

.....

2.6 CHARACTERISTICS OF SYMBOLIC LOGIC

As a science Symbolic logic possesses certain characteristics. A distinguished logician C. I. Lewis has mentioned three important general characteristics of Symbolic logic. These are:

- The application of Ideograms
- The application of Deductive Method
- The application of Variables
- **Application of Ideogram instead of Phonogram:** One of the most important characteristics of symbolic logic is the use of symbols. Symbol is the sign artificially designed to stand for something. There are two types of symbols:-ideogram and phonogram. The first basic characteristic of symbolic logic is the extensive use of ideograms instead of phonogram. Ideograms are those types of symbols which stand

directly for the concepts or idea and indirectly for the sound or phone. For example, 'x' (multiplication sign), '-'(subtraction sign), '?'(Question mark) etc. On the other hand phonograms are those symbols which stand directly for the sounds or phones and indirectly for the concept or idea. For example, the written words 'multiplication sign', 'question mark', 'division sign', etc. When these words are uttered, first of all we hear the sound and then we understand the meanings.

- **Application of Deductive Method:** The second basic characteristic of symbolic logic is the use of the deductive method. A deductive method is that in which from a limited number of statements we can produce an indefinite number of other statements and for producing these statements we are to employ a limited number of rules. Deductive method is used in case of demonstrative reasoning as like as mathematics. As symbolic logic employs this method in this respect, it is quite close to mathematics. And therefore, symbolic logic is purely deductive in nature.
- **The application of variables:** The third basic characteristic of symbolic logic is the use of variables. Variable is a type of symbol that can assume any value within a definite range. Aristotle introduced the notion of variable into logic for the first time in a restricted way.

Generally, the meaning of variable is changeable but in symbolic logic variable means a symbol which can stand for anyone of a given range of values. For example, if x y=12

Here, x and y can assume any value within the range of 1, 2, 3, 4, 5, 6…12. And if the value of x is 3, then the value of y is 4 (3x4=12) and if the value of x is 6 then the value of y is 2 (6x2=12) and so on.

Thus, variables are those symbols which do not have any fixed meaning. They may be given any value. There are two types of symbols used in symbolic logic- they are variables (e.g. p, q, r, s, t. etc) and constants (~, .,ν , ⊃ , º). There are some other variables used in symbolic logic which stand for statements. Propositional logic which is a branch of symbolic logic deals with the logical relationship between the propositions. The small

(middle) letters of English Alphabet namely, p, q, r, s, t, etc are the variables which stand for statements. On the other hand constants '~ 'is used for 'not', '. 'is used for 'and', 'ν' is used for 'either or' ⊃ ' is used for 'if-then' and 'º' is used for 'if and only if'. All these constants are used to connect more than one simple statement to make a compound proposition. So, they are also known as connectives. But ' ~ ' is not a connective, it is that constant which is used in front of a single statement to make its negation.



CHECK YOUR PROGRESS

Q 4: What are the different names of modern logic?

.....

Q 5: What are the different characteristics of symbolic logic?

.....

.....

Q 6: Name the author of Principia Mathematicia.

.....

Q 7: Write two important names of logicians who made great contributions in the developmemt of symbolic logic.

.....

Q 8: Who is the author of "Dissertiode Arte Combinatoria"?

.....

Q 9: State whether the following statements are true or false:

a) The use of variables is one of the basic characteristics of symbolic logic. (True/False)

b) Ideograms are those type of symbols which stand directly for sound and indirectly for concepts.(True/False)

c) Phonograms are those type of symbols which stand directly for concepts and indirectly for sounds.(True/False)

d) The words 'multiplication sign' ,' question mark', 'substraction sign' etc. are phonograms. (True/False)

2.7 FUNCTIONS AND THE IMPORTANCE OF THE USE OF SYMBOLS IN LOGIC

- **I Necessity or utility of using symbols in Logic:** Logic as a science of reasoning is concerned with the arguments and that arguments contain propositions or statements as their premises and conclusions. The communication of propositions and arguments requires the use of language. But the arguments formulated in any natural language are often difficult to appraise because of their vague and equivocal nature of the words, the ambiguity of their construction, the misleading idioms they may contain, and their pleasing but deceptive metaphorical style etc. To avoid these peripheral difficulties connected with ordinary language, different thinkers have developed specialized symbols (vocabularies). The use of symbols plays a very important role to perform the function perfectly.

  The following are the importance of using symbols in logic:

  - The use of symbols removes certain difficulties of ordinary language: As one of the important functions of logic is to provide methods of testing the validity and invalidity of arguments. But when the arguments are expressed in any ordinary natural language then it becomes very difficult to determine its validity or invalidity. Because sometimes ordinary languages are full of vagueness.
  - The use of symbols helps to exhibit clearly and concisely the form of propositions and arguments: Logic is generally concerned with the validity of arguments. If the arguments are classified and arranged into particular types, the task of testing the validity of such arguments becomes clear. By the use of symbols, the logical form of an argument becomes clear and distinct as a result of which it becomes easy to determine and test the validity of arguments by applying the general rules.

    For example:

    If rains are enough, then the crops will be good.

Rains are enough.

Therefore, the crops are good.

The symbolic form of the argument is-

If p then q

p

Therefore, q

- The use of symbols helps to express the generality of logical rules for testing the validity of arguments: There are some specific rules that can be applied to all the arguments of a particular form. If a particular argument of a particular form is valid, then any other argument belonging to that form may be regarded as valid. Thus, the use of symbols expresses the fact that a particular logical rule is applicable to all the arguments of a particular form. This is the generality of logical rule.
- The use of symbols helps us to save Time, Energy, Labour and Thought: The use of symbols in logic is concise and minimizes the exercise of mental labour. The same is true in case of a logical argument. The use of variables and constants removes the ambiguity, indefiniteness, obscurity and other complexities of arguments. It saves time and energy.
- A convenient symbolism helps us to concentrate upon the essential context: The use of symbols makes concepts and techniques clear and distinct. Symbols can make conciseness, clarity and economy of expression in arguments which would be practically unintelligible when they are expressed in ordinary language. Reasoning depends on the abstract relation.
- Symbols enable us to distinguish and keep distinct different meanings: The use of symbols expresses conveniently the concepts and technical terms which are peculiar to logic itself. Symbols like variables and constants help in representing different logical operations. These are some advantages of the use of symbols that led the logicians to invent some artificial symbolism which are free

from the defects of ordinary language. Modern logic fulfills their investigations by the introduction of many more special symbols and symbolism which has become a powerful device (tool) for analysis and deductions through the development of its own technical language. Special symbols are much better adapted to the actual drawing of inferences than ordinary languages. The use of inferences and the evaluation of arguments are greatly facilitated by the adoption of a special logical notation. Thus, Alfred North Whitehead says: "…..***by the aid of symbolism, we can make transitions in reasoning almost mechanically by the eye, which otherwise would call into play the higher faculties of the brain."***(A.N. Whitehead, ***An Introduction To Mathematics,*** Oxford University Press, Oxford, England, 1911).



ACTIVITY: 2.3

- Do you accept that the use of symbols can remove certain difficulties of ordinary language? Discuss

Ans.....

……..

……..

……..



CHECK YOUR PROGRESS

Q 10: What is the main function of logic?

……..

……..

Q 11: What are the two types of symbols used in symbolic logic?

……..

Q 12: What is symbolic logic? (Answer in about 60 words)

.....

.....

.....

.....

.....

.....

Q 13: State Whether the following statements are true or false:

a) The use of symbols reduces our mental labour.(True/False)

b) Classical logic has a long history as symbolic logic has a very short history. (True/False)

c) According to Cohen and Nagel, "Classical logic is related to symbolic logic as embryo to adult organism." (True/False)

d) Classic logic is said to be a joint venture between mathematics and Philosophy. (True/False)



2.8 LET US SUM UP

The main points discussed in this unit are:

- Classical logic is also known as traditional logic or Aristotelian logic.
- Aristotelian propounded traditional logic. So, it is known as Aristotelian logic.
- Modern logic is also known as symbolic logic, Mathematical logic, Algebra of logic, Logistics.
- The use of symbols brought about significant change in the traditional logic and the developed form of traditional logic is known as symbolic logic.
- The three important general characteristics of Symbolic logic are: The application of Ideograms, the application of deductive method, the application of variables.
- Leibnitz wrote the book- Dissertio de Arte Combinatoria.
- A.N. Whitehead and Bertrand Russell wrote the Book *Principia Mathematicia.*

2.9 FURTHER READING

1) Copy, Irving M.(1997). Symbolic Logic(Fifth Edition). New Delhi: Prentice-Hall of India Private Limited.

2) Basson, A.H.,& O'Connor,D.J.(1992) Introduction To Symbolic Logic. Calcutta: Oxford University Press, Walton Street, Oxford.

3) Wolf, A.(1976). Text Book Of Logic. Delhi:Surjeet Publications, 7-K, Kolhapur Road, Kamala Nagar.

4) Copy, Irving M., & Cohen, Carl.(1990). Introduction To Logic(Eighth Edition). New York. Macmillan Publishing Company.

5) Whitehead, A.N.(1911). An Introduction To Mathematics. Oxford, England: Oxford University Press.

2.10 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1:

(a) True

(b) True

(c) True

(d) True

(e) True

(f) True

Ans to Q No 2: Aristotle

Ans to Q No 3:

a) Classical logic has a long history whereas symbolic logic has a very short history.

b) In classical logic symbols were used to a limited extent where as in symbolic logic symbols are used extensively.

Ans to Q No 4: Classical logic or Aristotelian logic or traditional logic

Ans to Q No 5:

a) The application of ideograms

b) The application of deductive method

c) The application of variables

Ans to Q No 6: Bertrand Russell and R.N. Whitehead

Ans to Q No 7: George Boole, G. Peano

Ans to Q No 8: Leibnitz

Ans to Q No 9:

(a) True

(b) False

(c) False

(d) True

Ans to Q No 10: The main function of logic is to provide methods of testing the validity and invalidity of arguments.

Ans to Q No 11: Variables and Constants

Ans to Q No 12: Symbolic logic is a developed, refined and mature branch of study which is based on the concepts and techniques used in Aristotelian logic. The method of symbolic logic is mathematical, deductive and formal in nature. Symbolic logic as a formal logic deals with formal truth not with the material truth of propositions. It is known as 'Mathematical logic', Algebra of logic'. 'Logistics' etc.

Ans to Q No 13:

a) True

b) False

c) True

d) False



2.11 MODEL QUESTIONS

A. Very short Questions

Q 1: Define modern logic? Explain its nature and characteristics?

Q 2: Who is the founder of classical logic?

B. Short Questions (Answer each question in about 150 words)

Q 1: What are the differences between symbolic logic and classical logic? Explain.

Q 2: What are the utilities of using symbols in logic? Discuss.

Q 3: Explain the different stages of developments in symbolic logic.

C. Write short notes on (Answer each question in about 150 words)

a. Deductive method

b. Variable

c. The history of modern logic

D. Distinguish between (Answer each question in about 300-500words)

a. Ideogram and phonogram

b. Classical logic and modern logic

E. Long Questions (Answer each question in about 300-500 words)

Q 1: Explain the relationship of classical logic and symbolic logic.

Q 2: "Classical logic is related to symbolic as 'embryo to adult organism' Discuss this statement.

*** ***** ***

UNIT- 3: TRUTH FUNCTIONS, INTERDEFINITIONS OF LOGICAL CONSTANTS

UNIT STRUCTURE

3.1 Learning Objectives

3.2 Introduction

3.3 Definition of Truth Function

3.4 Propositional Variable

3.5 Basic Truth Functions

- 3.5.1 Negative or Contradictory Function
- 3.5.2 Conjunctive Function
- 3.5.3 Disjunctive Function
- 3.5.4 Implicative or Conditional Function
- 3.5.5 Equivalence or Biconditional Function

3.6 Alternative Function

3.7 Stroke Function

3.8 Inter- definitions of Logical Constants

3.9 Let Us Sum Up

3.10 Further Reading

3.11 Answers to Check Your Progress

3.12 Model Questions

3.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- explain the meaning of Truth Function
- explain the need of Truth Functions in symbolic logic
- discuss various Truth Functions through truth table
- describe the Inter-definitions of various Logical Constants

3.2 INTRODUCTION

This unit introduces to you the concept of truth functionality. Truth function is said to be the basis of propositional logic. The concept of truth

function plays a pivotal role in propositional logic, because truth function helps determining the truth values of compound statements. The basic logical constants, namely, '~', '•', 'v', '⊃', '≡' are called truth functional operators. These constants are also known as truth functional connectives. These connectives yield truth functional compounds. To put it in other words, by means of the application of these connectives various truth functional compound expressions are generated. A detailed account of the meaning of these connectives is dealt with in the next chapter.

Let us try to know the background of the concept of truth function. Symbolic logic deals with proposition. A proposition is said to have two truth values: Truth (T) and falsity (F). For example, proposition 'p' may be either true or false. A simple proposition is true if it corresponds to a fact. Again, a simple proposition is false if it does not correspond to a fact. Let us take an example, 'it is raining' is a simple proposition and the truth of this proposition will depend on whether it is actually raining or not. Now one question may crop up here: is a compound proposition true if it corresponds to a fact and false if it does not correspond to a fact? The answer will be that the truth value of a compound proposition depends on the truth value of the component parts of the compound proposition. Here, component parts mean the simple propositions that constitute the compound proposition. What is the truth value of the proposition 'if it rains then Nistha will get wet' if 'it rains' is true (T) and 'Nistha will get wet' is false (F). If we look at the proposition carefully we shall see that here we are not concerned with the reasons why Nistha will not get wet, with whether she carried an umbrella or travelled on a car or whatever. We even do not bother about whether the parts of the compound proposition are really true or false. Our sole aim here is to see how the truth value of the whole proposition is dependent on the truth value of its components. In order to determine the truth value of the proposition, i.e., 'if it rains, then Nistha will get wet' we must know the rules of computation for each of five logical constants [detailed meaning of logical constant is discussed in UNIT 4]. The symbolic representation of the proposition 'if it rains then Nistha will get wet' will be: 'p ⊃ q'. Here, 'p' stands for 'it rains', 'q'

stands for 'Nistha will get wet' and ' ' stands for 'if-then'. By following the computation table for implication we can compute the truth value of the proposition: 'p ⊃ q', i.e., for an implicative function if 'p' is 'T' and 'q' is 'F', then the truth value of the compound proposition will be 'F'.

3.3 DEFINITION OF TRUTH FUNCTION

The concept of truth function in propositional logic is mathematical in origin. The notion of function is found in elementary mathematics. An expression is said to be a function of a given variable or variables, if the value of the expression is uniquely determined when the variable or variables take a determinate value. Let us take the following example:

Y = 5x + 4

In the above example y is said to be a function of x, because the value of y is dependent on the value of x. Thus if x takes the value 6, the value of y will be 34. Again, if x takes the value 7, then the value of y will be 39. Since the value of the variable y is dependent on the value of the variable x, therefore y is regarded to be the function of x.

The mathematical idea of function has been extended into logic and the idea of function has come to be known as truth function in propositional logic. Logicians prefer truth function to function, because they are concerned with the truth values of expressions. Symbolic logic, instead of dealing with numerical value, deals with truth values, i.e., 'T' and 'F'. Hence, it is called a bi-valued system.

According to the idea of truth function, the truth or falsity of a compound proposition is uniquely determined by the truth or falsity of the constituent simple propositions. The truth value of the compound proposition can be calculated by knowing the truth value of the constituent propositions and also by keeping in mind the definitions of the logical constants or logical connectives. The five basic logical connectives are: '~', 'v', '•', '⊃', '≡'. These logical connectives yield truth functional compounds. **A truth function can be defined as any compound expression whose truth value is completely determined by the truth value of its constituent simple**

propositions. The logical constants and their symbolic representations are given below:

Symbol	Name Logical	function	Meaning
~	Tilde	Negation	Not, it is not the case that
•	Dot	Conjunction	and, also, moreover
v	Wedge	Disjunction	or, unless
⊃	Horseshoe	Implication	if-then, only-if
≡	Triple bar	Equivalence	if and only if

The following is an example of truth function:

Saul Kripke is a logician and Bertrand Russell is a mathematician. The above compound expression can be symbolically represented in the following way: p • q.

'p • q' is said to be a truth function, because the truth value of the expression is determined by the value of the propositions 'p' and 'q' taken separately. Generally small letters of English alphabet such as p, q, r, s, t are used to form compound expression. These letters are known as propositional variables.



CHECK YOUR PROGRESS

Q 1: State whether the following statements are True or False:

a) Truth function is a compound expression. (True/ False)

b) 'And' ('o') is a propositional variable. (True/ False)

c) There are five basic truth functional operators. (True/ False)

Q 2. What is truth function?

.....

.....

.....

Q 3: What do you mean by truth or falsity of a proposition?

.....

.....

3.4 PROPOSITIONAL VARIABLE

Variable, in algebra, is a symbol (usually a letter) standing in for an unknown numerical value in an equation. Variable is a kind of symbol that can stand for any one of a given range of values. For example, ab=24. Here, 'a' and 'b' can stand for any value within a definite range of 1, 2, 3, 4, 5.….24. If the value of 'a' is '6' and the value of 'b' is '4', then the value of ab = 24.

The lower case letters of the English alphabet 'p', 'q', 'r', 's', 't' etc. in a compound proposition stand for simple propositions. These letters are known as propositional variables, because a variable can stand for any proposition whatsoever.

The meaning of a variable varies from place to place, from time to time. It does not have a definite meaning. Therefore a variable may be given for any range of value. For example:

If p then q

p

Therefore q

Here, p and q are variables. These variables can be substituted by any proposition. Let us take the following example to clarify the meaning of a variable.

Example 1

If you read hard (p), then you will pass in the examination (q).

You read hard (p)

Therefore you will pass in the examination (q).

The above argument can be symbolically represented by the variables, namely, 'p' and 'q' in the following way:

$p \supset q$

p

$\therefore q$

Let us cite another argument

Example 2

If you take the medicine (p), then you will recover speedily (q).

You take the medicine (p)

Therefore you will recover speedily (q).

The above argument can also be symbolically represented by the variables, 'p and 'q' in the following way:

$p \supset q$

p

$\therefore q$

We can see that in example 1 'p' stands for 'you read hard' and in example 2 'p' stands for 'you take the medicine'. Again, in example no. 1 'q' stands for 'you will pass in the examination' and in example no. 2 'q' stands for 'you will recover speedily'. This shows that the variables remain same although the propositions standing for the variables vary.

3.5 BASIC TRUTH FUNCTIONS

The following are the basic truth functions with their symbolic representations

	Truth function	**Symbolic representation**
a)	Negative or Contradictory function	$\sim p$
b)	Conjunctive function	$p \bullet q$
c)	Disjunctive function	$p \vee q$
d)	Implicative function	$p \supset q$
e)	Equivalence or biconditional function	$p \equiv q$

Apart from these five basic truth functions there is another truth function, namely, Alternative
truth function.

3.5.1 Negative or Contradictory function

The function of 'not' is referred to as negation. The negation of a proposition is another proposition whose truth value depends on the original proposition. For example, if the statement 'he is playing' is true, then the

negation of the statement 'he is not playing' would be false. Symbolically, it can be put in this way: "if 'p' is true, then 'not p' is false". Again, if the statement 'he is not playing' is true, then its contradictory statement 'he is playing' would be false. Symbolically the statement can be put in this way: "if 'p' is false, then 'not p' is true". The following truth table for negation will show how any statement having the form of a negation (~ p) is determined by the truth value of the statement that is negated (p). The following is the truth table for negation:

P	**~P**
T	F
F	T

The value of ~ p depends on the value of the variable p, hence ~ p is the truth function of p

3.5.2 Conjunctive function

A truth functionally compound expression composed of two simple propositions by the connective 'and' (o) is called conjunctive function. For example, 'Plato founded the Academy and Aristotle founded the Lyceum'-is a compound proposition. The logical connective 'and' (o) joins two simple propositions, namely, 'Plato founded the Academy' and 'Aristotle founded the Lyceum' and thereby it forms a compound proposition. The symbolic formulation for this conjunctive function is 'p o q'. (Here, 'p' stands for 'Plato founded the Academy' and 'q' stands for 'Aristotle founded the Lyceum').

The constituents of a conjunctive proposition 'p' and 'q' are called conjuncts. Variable 'p' is called 'left conjunct' and variable 'q' is called 'right conjunct'. The truth value of a conjunctive function is calculated according to the following rule covering all possible cases:

1. If the left conjunct is true and the right conjunct is true, then the conjunctive truth function as a whole is true.

2. If the left conjunct is true and the right conjunct is false, then the conjunction as a whole is false.
3. If the left conjunct is false and the right conjunct is true, then the conjunction as a whole is false.
4. If the left conjunct is false and the right conjunct is false, then the conjunction as a whole is false.

From the above conditions the following general rule may be extracted:

A conjunctive function is true when all its conjuncts are true. A conjunctive function is false if one of the conjuncts is false.

By following the definitions of truth operators or logical connectives we can construct a truth table for conjunctive truth function:

p	q	p • q
T	T	T
T	F	F
F	T	F
F	F	F

The truth values under the logical connective 'o' exhibit all the possible truth values of the compound structure 'p o q'.

Conjunctive function has got another important characteristic which is expressible in terms of some 'mathematical jargon'. Conjunctive function can be commutative, associative and idempotent. These three properties can be symbolically represented in the following way:

$(p \bullet q) \equiv (q \bullet p)$ Commutative property

$p \bullet (q \bullet r) \equiv (p \bullet q) \bullet r$ Associative property

$(p \bullet p) \equiv p$ Idempotent property

3.5.3 Disjunctive function

A truth functionally compound expression composed of two simple propositions by the connective 'or' (v) [in the sense of inclusive disjunction]

both the disjuncts can be true, while exclusive disjunction rules out this possibility and declares that only one or the other, not both of the disjuncts is true. For example, 'Bertrand Russell is either a logician or he is a mathematician' is a disjunctive proposition. The symbolic representation of this proposition will be: 'p v q' (Here, 'p' stands for 'Bertrand Russell is a logician' and 'q' stands for 'Bertrand Russell is a mathematician').

The constituents of a disjunctive proposition 'p' and 'q' are called disjuncts. Variable 'p' is called 'left conjunct' and variable 'q' is called 'right conjunct'. The truth value of a conjunctive function is calculated according to the following rule covering all possible cases:

1. If the left disjunct is true and the right disjunct is true, then the disjunction as a whole is true.
2. If the left disjunct is true and the right disjunct is false, then the disjunction as a whole is true.
3. If the left disjunct is false and the right disjunct is true, then the disjunction as a whole is true.
4. If both left disjunct and right disjunct are false, then the disjunction as a whole is false.

From the above conditions the following general rule may be extracted:

A disjunctive function is true when at least one of the disjuncts is true. When all the disjuncts are false, the disjunctive function is false.

By following the definitions of truth operators or logical connectives we can construct a truth table for disjunctive truth function:

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

The truth values under the logical connective 'v' contain all the possible truth values of the compound structure 'p v q'.

Just as conjunctive function, disjunctive function also has certain mathematical properties. Disjunctive function has commutative, associative and idempotent properties.

These properties can be symbolically represented in the following way:

$(p \vee q) \equiv (q \vee p)$	Commutative property
$p \vee (q \vee r) \equiv (p \vee q) \vee r$	Associative property
$(p \vee p) \equiv p$	Idempotent property

3.5.4 Implicative or Conditional function

A truth functionally compound expression composed of two simple propositions by the connective 'if-then' ($\supset$) is called Implicative or conditional function. For example, 'if you work hard, then you will be able to reach your goal.' The logical connective 'if-then' ($\supset$) joins two simple propositions, namely, 'you work hard' and 'you will be able to reach your goal' and thereby it forms a compound proposition. The symbolic formulation for this implicative function is '$p \supset q$'. (Here, 'p' stands for 'you work hard' and 'q' stands for 'you will be able to reach your goal.')

In this type of compound the component between the 'if' and the 'then' is called the antecedent. Here, 'you work hard' is an antecedent. The component proposition that follows the 'then' is called consequent. The truth value of a conjunctive function is calculated according to the following rule covering all possible cases:

1. If the antecedent is true and the consequent is true, then the conditional function as a whole is true.
2. If the antecedent is true and the consequent is false, then the conditional function as a whole is false.
3. If the antecedent is false and the consequent is true, then the conditional function as a whole is true.

4. If the antecedent is false and the consequent is false, then the conditional function as a whole is true.

From the above conditions the following general rule may be extracted:

1. A material conditional will be true whenever its consequent is true, irrespective of what value its antecedent has -true or false.
2. A material conditional will be true whenever its antecedent is false, irrespective of what value its consequent has -true or false.

By following the definitions of truth operator or logical connective '⊃' we can construct a truth table for conditional truth function:

P	q	p ⊃ q
T	T	T
T	F	F
F	T	T
F	F	T

3.5.5 Equivalence or Biconditional function

A truth functionally compound expression composed of two simple propositions by the connective 'if and only if' (≡) is called equivalence or biconditional function. For example, 'One can apply for the post of Assistant Professor if and only if one qualifies NET '-is a compound proposition. The logical connective 'if and only if' (≡) joins two simple propositions, namely, 'One can apply for the post of Assistant Professor' and 'one qualifies NET'; and thereby it forms a compound proposition. The symbolic formulation for this biconditional function is 'p ≡ q'. (Here, 'p' stands for 'One can apply for the post of Assistant Professor' and 'q' stands for 'one qualifies NET')

The truth value of a biconditional function is calculated according to the following rules covering all possible cases:

1. If both the constituents are true, then the biconditional function as a whole is true.

2. If the left constituent is true and the right constituent is false, then the biconditional function as a whole is false.
3. If the left constituent is false and the right constituent is true, then the biconditional function as a whole is false.
4. If both the constituents are false, then the biconditional function as a whole is true.

From the above conditions the following general rule may be extracted:

A biconditional function is true when both the constituent propositions have the same truth value. A biconditional function is false when one constituent proposition is true and the other constituent proposition is false.

By following the definitions of truth operators or logical connectives we can construct a truth table for biconditional truth function:

p	q	p = q
T	T	T
T	F	F
F	T	F
F	F	T



CHECK YOUR PROGRESS

Q 3: State whether the following statement is True or False.

a) There are five basic truth functions. (True/ False)

b) Propositional variables can stand for any proposition whatsoever. (True/ False)

c) The lower case letters of English alphabet are taken as propositional variables. (True/ False)

Q 4: When is a conjunctive function false?

.....

.....

Q 5: When is a disjunctive function false?

.....

.....

Q 6: When is a conditional function false?

.....

.....

Q 7: When is a biconditional function true?

.....

.....



ACTIVITY: 3.2

- Prepare a chart for the basic truth functions with suitable examples.

Ans .....

.....

.....

.....

.....

3.6 ALTERNATIVE FUNCTION

A truth functionally compound expression composed of two simple propositions by the connective 'or' (v) [in exclusive sense] is called alternative truth function. For example, 'Dodere is either literate or illiterate'-is a compound proposition. The logical connective 'either-or' (^) joins two simple propositions, namely, 'Dodere is literate' and 'Dodere is illiterate'; and thereby

it forms a compound proposition. The symbolic formulation for this alternative function is 'p ^ q'. (Here, 'p' stands for 'Dodere is literate' and 'q' stands for 'Dodere is illiterate')

The truth value of an alternative function is calculated according to the following rules covering all possible cases:

1. If both the constituents are true, then the alternative function as a whole is false.
2. If the left constituent is true and the right constituent is false, then the alternative function as a whole is true.
3. If the left constituent is false and the right constituent is true, then the alternative function as a whole is true.
4. If both the constituents are false, then the alternative function as a whole is false.

From the above conditions the following general rule may be extracted:

An alternative function is true when one constituent proposition is true and the other constituent proposition is false. An alternative function is false when either both the constituent propositions are true or both are false.

By following the definitions of truth operators or logical connectives we can construct a truth table for alternative truth function:

p	q	p ^ q
T	T	F
T	F	T
F	T	T
F	F	F

3.7 STROKE FUNCTION

Stroke function was introduced by a logician named Henry M. Sheffer in 1913. In Boolean functions and propositional calculus, the Sheffer stroke is written as "I" (vertical bar) or ""↑" (an upward arrow). It denotes a logical operation that is equivalent to the negation of the conjunction operation. The logical operation, in ordinary language, is known as "not both." The stroke function is also called the alternative denial, since it says that at least one of its operands is false. In Boolean algebra and digital electronics it is also known as the NAND operation ("not and").

It is pointed out by the logicians that various truth functions are inter-definable. We have seen that 'v', '•', '⊃', are definable in terms of each other with the help of the primitive constant '~'. The fact that logical constants are inter-definable in this way makes one ask, "how many of these constants are really indispensable for building up our calculus of propositions?" All functions but two can be dispensed with, since '⊃' and '•' can be defined in terms of '~' and 'v'; and 'v' can be defined in terms of '~' and '•' or alternatively in terms of '~' and '⊃'. Can the number of indispensable constants be reduced still further? The answer is that all logical constants save one can be dispensed with. Stroke function is defined as follows: 'p/q is to be read "at least one of p and q is false". The following truth table represents the definition of stroke function:

p	q	p • q
T	T	T
T	F	F
F	T	F
F	F	F

The following functions can be defined in terms of stroke function:

1. ~ p is defined as p/p

   ~ p = df p/p

2. p • q is defined as (p/q)/(p/q)

   p • q = df (p/q)/(p/q)

3. p v q is defined as (p/p)/ (q/q)

   p v q = df (p/p)/ (q/q)

4. p ⊃ q is defined as p/(q/q)

   p ⊃ q = df p/(q/q)



CHECK YOUR PROGRESS

Q 8: State the condition under which an alternative function is true.

.....

.....

Q 9: How is 'p/q'read?

.....

.....

3.8 INTER-DEFINITIONS OF LOGICAL CONSTANTS

Various logical constants in propositional logic are inter-definable. Each logical constant has its own special characteristics yet they can be reduced and defined into each other. For example, the logical constant 'and' can be defined in terms of other logical constants such as '~' and '⊃'; and '~' and 'V'. Defining and reducing the logical constants into one another is called 'inter-definitions of logical constants' or 'inter-definability of truth functions'. Logical constants such as '~', '•', '⊃', 'v', '≡' are thus inter-definable. What is important here is that while defining one logical constant in terms of other logical constants, the meaning of the original expression

must remain unchanged. The following inter-definitions express inter-definitions of various logical constants:

1. Logician Augustus de Morgan points out that the logical constant 'and'/ '(•)' [conjunction] may be defined in terms of the logical constants 'not' / '(~)' [negation] and 'or/ '(v)' [disjunction]. Again, the logical constant 'and'/ '(•)' [conjunction] can be defined in terms of the logical constants 'not' / '(~)'[negation] and 'if-then'/ '⊃'[implication or material conditional].

   The following are the inter-definitions:

   (i) '~' and 'v' as

   $p \bullet q =_{df} \sim(\sim p \vee \sim q)$

   (ii) '~' and '⊃' as

   $p \bullet q =_{df} \sim(p \supset \sim q)$

   [Here 'df' is abbreviated for 'definition']

   The above inter-definitions can be shown to be logically equivalent by means of truth table. The following are the truth tables for these inter-definitions:

(I)

p	•	q	≡	~	(	~	p	v	~	q)
T	T	T	T	T		F	T	F	F	T
T	F	F	T	F		F	T	T	T	F
F	F	T	T	F		T	F	T	F	T
F	F	F	T	F		T	F	T	T	F

(ii)

p	•	q	≡	~	(	p	⊃	~	q	)
T	T	T	T	T	T	F	F	T		
T	F	F	T	F	T	T	T	F		
F	F	T	T	F	F	T	F	T		
F	F	F	T	F	F	T	T	F		

2. The logical constant 'or'/'v' [disjunction] can be defined in terms of the logical constants 'not'/ '~' [contradiction] and 'and'/ '•' [conjunction]. Again, the logical constant 'or'/'v' [disjunction] can be defined in terms of the logical constants 'not'/ '~' [contradiction] and 'if-then'/'⊃'[implication or material conditional].

The inter-definitions can be presented in the following way:

(i) '∼' and '•' as

p v q = df ∼(∼p • ∼q)

(ii) '∼' and '⊃' as

p v q = df ∼p ⊃ q

With the help of truth table, the above inter-definitions can be shown to be logically equivalent. The following are the truth tables for these inter-definitions:

(i)

p	v	q	≡	∼	(∼	p	•	∼	q)
T	T	T	T	T	F	T	F	F	T
T	T	F	T	T	F	T	F	T	F
F	T	T	T	F	F	F	T		
F	F	F	T	F	T	F	T	T	F

(ii)

p	v	q	≡	∼	p	⊃	q
T	T	T	T	F	T	T	T
T	T	F	T	F	T	T	F
F	T	T	T	T	F	T	T
F	F	F	T	T	F	F	F

3. The logical constant 'if-then'/'⊃'[implication or material conditional] can be defined in terms of the logical constants 'not'/ '∼' [contradiction] and 'or'/'v' [disjunction]. Again, the logical constant 'if-then'/' '[implication or material conditional] can be defined in terms of the logical constants 'not'/ '∼' [contradiction] and 'and'/ '•' [conjunction].

The following are the inter-definitions:

(i) '∼' and 'v' as

p ⊃ q = df ∼p v q

(ii) '∼' and '•' as

p ⊃ q = df ∼(p • q)

With the help of truth table, the above inter-definitions can be shown to be logically equivalent. The following are the truth tables for these inter-definitions:

(i)

p	⊃	q	≡	~	p	v	q
T	T	T	T	F	T	T	T
T	F	F	T	F	T	F	F
F	T	T	T	T	F	T	T
F	T	T	T	T	F	T	F

(ii)

p	⊃	q	≡	~	(	p	•	~	q	)
T	T	T	T	T	T	F	F	F	T	T
T	F	F	T	F	T	T	T	T	F	F
F	T	T	T	T	F	F	F	F	T	T
F	T	T	T	T	F	F	T	T	F	F

4. The logical constant 'if and only if'/ '≡' [bi-conditional] can be defined in terms of the logical constants 'if-then'/'⊃'[implication or material conditional] and 'and'/ '•' [conjunction]. The logical constant 'if and only if'/ '≡' [bi-conditional] can be defined in terms of the logical constants 'and'/ '•' [conjunction], 'not'/ '~' [contradiction] and 'or'/'v' [disjunction].

   The following are the inter-definitions:

   (i) $(p \equiv q) =_{df} (p \supset q) \bullet (q \supset p)$

   (iii) $(p \equiv q) =_{df} (p \bullet q) \vee (\sim p \bullet \sim q)$

The above inter-definitions can be shown to be logically equivalent by means of truth table. The following are the truth tables for these inter-definitions:

(i) (p ≡ q) ≡ (p ⊃ q) • (q ⊃ p)

T	T	T	T	T	T	T	T	T	T	T
T	F	F	T	T	F	F	F	F	T	T
F	F	T	T	F	T	T	F	T	F	F
F	T	T	T	F	T	F	T	F	T	F

(ii) (p ≡ q) ≡ (p • q) v (~ p • ~ q)

T	T	T	T	T	T	T	T	F	T	F	F	T	
T	F	F	T	T	F	F	F	F	T	F	T	F	
F	F	T	T	F	F	T	F	T	F	F	F	T	
F	T	F	T	F	F	F	T	T	F	T	T	F	

[The expression '= df' which is used in all above inter-definitions mean 'is equivalent to by definition'. Let us take the following example to illustrate this point. The proposition 'Russell is both mathematician and philosopher' is equivalent to the proposition-'It is false that either Russell is not intelligent or Russell is not honest'. This means that these two propositions are logically equivalent (≡). Therefore the symbolic expression of this equivalent expression is either '(p o q) ≡ df ~ (~ p v ~ q)' or '(p o q) ≡ ~ (~ p v ~q)'.]



CHECK YOUR PROGRESS

Q 10: Mention various inter-definitions

.....

.....

.....

.....



3.9 LET US SUM UP

- According to the idea of truth function, the truth or falsity of a compound proposition is uniquely determined by the truth or falsity of the constituent simple propositions. The truth value of the compound proposition can be calculated by knowing the truth value of the constituent propositions and also by keeping in mind the definitions of the logical constants or logical connectives. The five basic logical connectives are: '~', 'v', '•', '⊃', '≡'. These logical connectives yield truth functional compounds. **A truth function can be defined as any compound expression whose truth value is completely determined by the truth value of its constituent simple propositions.**
- The lower case letters of the English alphabet 'p', 'q', 'r', 's', 't' etc. in a compound proposition stand for simple propositions. These letters are known as propositional variables, because a variable can stand for any proposition whatsoever.
- There are five basic truth functions. They are:

Truth function	Symbolic representation
a. Negative or Contradictory function	~ p
b. Conjunctive function	p • q
c. Disjunctive function	p v q
d. Implicative function	p ⊃ q
e. Equivalence or biconditional function	p ≡ q

- The function of 'not' is referred to as negation. The negation of a proposition is another proposition whose truth value depends on the original proposition.
- A truth functionally compound expression composed of two simple propositions by the connective 'and' (o) is called conjunctive function.

- A truth functionally compound expression composed of two simple propositions by the connective 'or' (v) [in the sense of inclusive disjunction] is called Implicative or conditional function.
- A truth functionally compound expression composed of two simple propositions by the connective 'if and only if' (≡) is called equivalence or biconditional function.
- A truth functionally compound expression composed of two simple propositions by the connective 'or' (v) [in exclusive sense] is called alternative truth function.
- Stroke function was introduced by a logician named Henry M. Sheffer in 1913. In Boolean functions and propositional calculus, the Sheffer stroke is written as "|" (vertical bar) or ""↑" (an upward arrow). It denotes a logical operation that is equivalent to the negation of the conjunction operation. The logical operation, in ordinary language, is known as "not both." The stroke function is also called the alternative denial, since it says that at least one of its operands is false.
- Various logical constants in propositional logic are inter-definable. Each logical constant has its own special characteristics yet they can be reduced and defined into each other. For example, the logical constant 'and' can be defined in terms of other logical constants such as '□' and '⊃'; and '□' and 'V'. Defining and reducing the logical constants into one another is called 'inter-definitions of logical constants' or 'inter-definability of truth functions'.

3.10 FURTHER READING

1) Chhanda C. (2007), Logic-Informal, Symbolic and Inductive, Prentice-Hall of India Pvt. Ltd.

2) Copi, I.M.(1998), Symbolic Logic (Fifth Edition), Prentice-Hall of India Pvt. Ltd.

3) Hurley, P. J. (2007), Introduction to Logic, Cengage Learning India Private Limited.

4) Klenk,V. (2008), Understanding Symbolic Logic, Dorling Kindersley (India) Pvt. Ltd.

5) Sharma, B. & Deka, J. (2014), A Text Book of Logic, Aank-Baak, Panbazar, Guwahati, Assam.

3.11 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q. No. 1:

a) True

b) False

c) True

Ans. to Q. No.2: A truth function can be defined as any compound expression whose truth value is completely determined by the truth value of its constituent simple propositions. For example: Saul Kripke is a logician and Bertrand Russell is a mathematician. The compound expression can be symbolically represented in the following way: p•q.

Ans. to Q. 3:

a) True

b) True

c) True

Ans. to Q. No 4: A conjunctive function is false if one of the conjuncts is false.

Ans. to Q. No 5: If both left disjunct and right disjunct are false, then the disjunction as a whole is false.

Ans. to Q. No 6: If the antecedent is true and the consequent is false, then the conditional function as a whole is false.

Ans. to Q. No 7: If both the constituents are of same value, then the bi-conditional function is true. This means that if both the constituents are true, the biconditional function is true. Again, if both the constituents are false, the biconditional function is true.

Ans. to Q. No 8: An alternative function is true when one constituent proposition is true and the other constituent proposition is false.

Ans. to Q. No 9: 'p/q is to be read "at least one of p and q is false".

Ans. to Q. No 10:

1. $p \bullet q = df \sim(\sim p \vee \sim q)$
2. $p \bullet q = df \sim(p \supset \sim q)$
3. $p \vee q = df \sim(\sim p \bullet \sim q)$
4. $p \vee q = df \sim p \supset q$
5. $p \supset q = df \sim p \vee q$
6. $p \supset q = df \sim(p \bullet \sim q)$
7. $(p \equiv q) = df (p \supset q) \bullet (q \bullet p)$
8. $(p \equiv q) = df (p \bullet q) \vee (\sim p \bullet \sim q)$

3.12 MODEL QUESTIONS

A) Very Short Questions:

Q 1: What is function?

Q 2: Give an example of truth function.

Q 3: Give an example of propositional variable.

Q 4: What is variable?

Q 5: "Truth function is a compound expression"-give one reason.

B) Short questions : (Answer each question in about 150 words)

Q 1: Write a short note on propositional variable.

Q 2: Distinguish between Logical constant and propositional variable.

Q 3: What is alternative function? Explain with suitable examples.

Q 4: Write a short note on Stroke Function.

C) Long Questions: (Answer each question in about 300-500 words)

Q 1: Discuss various truth functions with suitable examples.

Q 2: Explain various truth functions with the help of truth table.

Q 3: What do you mean by interdefinability of truth function? Explain how various basic truth functions or logical constants are interdefinable.

*** ***** ***

UNIT- 4: DEFINITION OF CONNECTIVES AND SYMBOLIZATION OF EVERYDAY LANGUAGE

UNIT STRUCTURE

4.1 Learning Objectives

4.2 Introduction

4.3 Meaning of Connective

4.4 Definition of Connectives

- 4.4.1 Not/ It is not the case that (Negation)
- 4.4.2 And (Conjunction)
- 4.4.3 Either-or (Disjunction)
- 4.4.4 If-then (Material Implication)
- 4.4.5 If and only if (Equivalence of Biconditional)

4.5 Symbolization of Everyday Language

- 4.5.1 Two kinds of Statements
  - 4.5.1.1 Symbolization of Simple Statements
  - 4.5.1.2 Symbolization of Simple Statements
- 4.5.2 Complex Symbolizations

4.6 Let Us Sum Up

4.7 Further Reading

4.8 Answers to Check Your Progress

4.9 Model Questions

4.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- explain the meaning of Connective used in symbolic logic.
- explain the need of Connective.
- discuss various Connectives with examples.
- explain the need of symbolization for logical purposes
- discuss different procedures for symbolizing simple and compound statements.

4.2 INTRODUCTION

Logician and mathematician Bertrand Russell states that symbolic logic consists of three parts: the calculus of propositions, the calculus of classes and the calculus of relations. The calculus of propositions or propositional logic, also known as sentential logic or statement logic is that branch of logic that studies ways of combining or altering statements or propositions to form more complicated statements or propositions. In propositional logic, the simplest statements are considered as indivisible units. The most thoroughly researched branch of propositional logic is classical truth-functional propositional logic. Such type of logic studies about logical connectives that are used to produce complex statements whose truth value depends entirely on the truth values of the simpler statements which makes the complex statements, and in which it is assumed that every statement is either true or false but not both. Symbolic logic uses two kinds of symbols to form compound statements. One is logical connectives and the other is propositional variable.

4.3 MEANING OF CONNECTIVE

Connectives in propositional logic are special words or terms which are used to generate statements when used. They connect or join statements to produce other statements. Hence they are called connectives. These connectives are also called truth functional connectives because they function truth functionally. This means that propositional logic deals only with truth functional connectives. The following are the five truth functional connectives are: Not, And, Or, If-then, If and only if.



CHECK YOUR PROGRESS

Q.1: State whether the following statements are True or False:

a) According to Bertrand Russell, symbolic logic consists of three parts.(True/ False)

b) The number connectives used in propositional logic is four.(True/ False)

Q.2: How many connectives are used in propositional logic and what are they?

.....

.....

4.4 DEFINITION OF CONNECTIVES

The following points will give you the idea about the nature of the logical connectives

4.4.1 Not/ It is not the case that (~) [Negation]

- Not is a monadic or unary connective as it can connect only one statement at a time.
- The function of Not is referred to as negation.
- The connective Not is represented by the symbol '~'.
- The symbol '~' is called 'curl' or 'tilde'.
- In propositional logic 'Tilde' is always placed in front of the proposition it negates. All operators other than tilde are placed between two propositions. Tilde is used to translate any negated simple proposition. Let us take some examples:

	Statement	**Symbolized form**
a.	Philosophers do not commit mistake	~P
b.	It is not the case that philosophers commit mistakes	~P
c.	It is false that philosophers commit mistakes	~P

- Tilde cannot be used to connect two propositions. Thus P ~M is not a proper expression.
- Tilde is used to negate a simple proposition, but it can also be used to negate a compound proposition, e.g. ~ (P o M). In this case tilde negates the entire expression inside the parantheses.
- Chhanda Chakraborti in her book Logic Informal, Symbolic and Inductive says, "It is important to remember that in propositional logic, negations, i.e., statements with explicit 'not' or its equivalent in them, are compound statements." (p. 76) Let us take an example to understand this point. The statement 'Philosophers do not commit mistake' can be symbolically represented as ~M. Since M is prefixed by '~', ~M is said to be a compound statement.
- The negation of any true statement is false and the negation of any false statement is true. This point can be stated through a truth table.

p	~ p
T	F
F	T

4.4.2 And (•) [Conjunction]

- A compound statement composed by the connective 'and' is called a conjunction.
- 'And' is the dyadic or binary connective as it can connect two statements at a time.
- The two simple statements that are connected through the logical connective 'and' are known as 'conjuncts.'
- The logical connective 'and' is represented by the symbol 'o'.
- The symbol 'o'is called 'dot'.
- In propositional logic 'dot' is placed between two propositions. Dot is used to translate such conjunctions as 'and', 'also', 'but',

'however', 'yet', 'still', 'moreover', 'although' and 'nevertheless'.

Let us take the following example:

Sentences **Symbolization**

Example 1: Frege is a logician, and Amartya Sen is an economist F • E

Example 2: Frege is a logician, but Amartya Sen is an economist F • E

Example 3: Frege is a logician; however Amartya Sen is an economist F • E

Example 4: Frege and Amartya Sen are thinkers F • E

Note that example 4 is equivalent to the meaning to 'Frege is a thinker and Amartya Sen is a Thinker'. In order to translate such a statement as a conjunction of two simple statements, the original statement must be equivalent to a compound statement in English. The statement 'Ananya and Adwita are friends' cannot be said to be equivalent in meaning to 'Ananya is a friend and Dvita is a friend'. Hence, the statement 'Ananya is a friend and Advita is a friend' cannot be translated as 'A o D'.

- A conjunction is true only when both the conjuncts are true; it is false in every other case. If p and q are any true conjuncts, then the truth condition for 'p o q' will be:

p	Q	p • q
T	T	T
T	F	F
F	T	F
F	F	F

4.4.3 Either-Or (v) [Disjunction]

- A compound statement formed by connecting two statements with the help of the connective 'or' is called a disjunction.
- 'Or' is the dyadic or binary connective as it can connect two statements at the same time. The individual statements are known as 'disjuncts'.

- The logical connective 'or' is represented by the symbol 'v'.
- The symbol 'v' is called 'wedge'. It is also called 'vee'.
- There are two senses of the word 'or' in English language: one is the inclusive sense and the other is the exclusive sense. The following is the difference between these two senses of the word 'or':

Either-or: (Inclusive sense)

- The inclusive sense of 'or' suggests that it is possible for both the disjuncts to be true at the same time.
- Let us take an example:

  Frege is either a logician or a musician. This means that Frege can both be a logician and a musician at the same time.

Either-or : (Exclusive sense)

- The exclusive sense of 'or' suggests that it is not possible for both the disjuncts to be true at the same time.
- Let us take an example:

  Frege is either literate or an illiterate person. This means that Frege cannot both be a literate and illiterate person at the same time.
- Disjunction in propositional logic represented by 'v' is not an exclusive 'or'. It is an inclusive 'or'.
- In propositional logic 'wedge' is placed between two propositions. Wedge is a symbolic representation of the word 'or' taken in an inclusive sense. Let us take the following example:

Sentences	**Symbolization**
Example 1: Either I shall call him up or I shall write to him.	C V W
Example 2: I shall not call him up unless some emergency pops up.	~ CVE

Note in example 2 that if two propositions are connected by 'unless' then also disjunctive can be used.

- Disjunction is true when either of the disjuncts is true and also when both the disjuncts are true; and is false when both the disjuncts are false. If p and q are any true disjuncts, then the truth condition for (p v q) will be:

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

4.4.4 If-then (Material Implication)

- A compound statement formed by connecting two statements with the help of the connective 'or' is called a **material conditional.**
- 'If-then' is the dyadic or binary connective as it can connect two statements at the same time. The component statement which contains 'if' is known as antecedent. Again, the component statement which contains 'then' is known as consequent.
- The logical connective 'if-then' is represented by the symbol '⊃'.
- The symbol '⊃' is called 'horse-shoe' or 'hook'. Customarily, the horse-shoe symbol is also used to translate 'implies'.
- The symbol '⊃' is used to translate such conditionals as 'given that', 'provided that', 'if', 'only if' etc.

Let us take the following examples:

Sentences	**Symbolization**
Example 1: If it rains, then we shall not go for shopping	R ~S
Example 2: Given that Ananya works, Adwita will also work	A D

- An implication is false when antecedent is true and consequent is false. If p is antecedent and q is consequent, then the truth condition for p⊃q will be :

4.4.5 If and only if (Equivalence or Biconditional)

- A compound statement formed by the connective 'if and only if' (also used iff for short) is called equivalence or biconditional statement.
- The biconditional, as its name implies, is a two-way conditional.
- The function of 'if and only if' is referred to as bicondition.
- The logical connective 'if and only if' is represented by the symbol '≡'.
- The symbol '⊃' is called 'triple bar' or 'equivalent' sign.
- Let us take the following examples:

Sentences **Symbolization**

Example 1: The shopkeeper will allow us to buy the stuff if and only if we pay him. $S \equiv P$

Example 2: Ashok will attend the ceremony if and only if he will be invited. $A \equiv I$

- A biconditional is true if when both the propositions have the same truth value. A biconditional is false when both the propositions have different truth values. The truth condition for 'p ≡q' will be:

P	q	p = q
T	T	T
T	F	F
F	T	F
F	F	T



CHECK YOUR PROGRESS

Q.3: State whether the following statements are true or false :

a) A conjunction is true if at least one of the conjuncts is true.(True/ False)

b) A disjunction is true if at least one of the disjuncts is true. (True/False)

c) A material conditional is true if the antecedent is true and the consequent is false.(True/ False)

d) A biconditional is true if both the propositions have the same truth value.(True/ False)



ACTIVITY: 4.1

- Why do you think that connectives of propositional logic are truth functional?

Ans.....

.....

.....

4.5 SYMBOLIZATION OF EVERYDAY LANGUAGE

There are two-fold tasks in the elementary symbolic logic for the beginners. They are the following:

1. Symbolization of propositions and arguments stated in ordinary language. It is very necessary to learn how to symbolize English sentences in logic because symbolization will be incorrect unless punctuation marks are put correctly. If we wrongly symbolize an English sentence, the logical structure of the sentence will become wrong.

2. Testing the validity of various types of arguments. As it is known to us that the validity of an argument depends on the correct form of the argument, it is therefore necessary to symbolize the propositions constituting the argument correctly.

It is to be mentioned here that punctuation is important in mathematics. In the absence of a special convention no number is uniquely denoted by 4 x4 + 5, although when it is made clear how its constituents are grouped, it denotes either 21 or 36: the first when punctuated will be (4x4)+5=21, while the second when punctuated will become 4 x (4+5)=36. To avoid ambiguity and to make meaning clear punctuation marks in mathematics appear in the form of parentheses ( ), which are used to group individual symbols; brackets [ ], which are used to group expressions that include parentheses etc.

In the language of symbolic logic also same punctuation marks as used in mathematics are used. In logic, for example, 'p • q v r' will be considered ambiguous as it is devoid of punctuation mark. It may mean either the conjunction of p with q v r or else the conjunction of p • q with r. The different meaning of two expressions can be explicated by punctuating the given formula as p • (q v r), or else as (p • q) v r. In case p and q both are false and r is true, the first punctuated expression is false (since its first conjunct is false), but the second punctuated expression is true (since its second disjunct is true). It is because punctuation formulas differ in both the cases, the truth or falsity also varies in both.

Parentheses, square brackets are the devices to demarcate clearly the scope of each connective. The scope of a connective is the range of its operation or the extent of its operation within the statement. For example, the scope of negation is always over that which is negated, that is, in '~R' the component 'R' is under the scope of '~'. It is to be remembered here that when there are several connectives in use, the main connective is the truth functional connective in the statement which has the maximum scope. The scope of the main connective covers the entire statement, and all the component statements. The following is the example showing the maximum scope of a connective:

~[(A • B) v (C ≡ D)]

In the above example it can be observed that '~' ranges over the whole statement and all the components; the scope of 'o' is only over (A • B), again the scope of '≡' is only over (C≡D), and the connective 'v' ranges over both (A • B) and (C≡D). Therefore '~' is said to have the maximum scope.

Chhanda Chakraborti in her book Logic: Informal, Symbolic and Inductive lays down the following statements as the key to the symbolization in propositional logic to be followed closely:

1. Pick out the atomic or simple statements in the given statements.
2. If a list of capital letters to be used as the translation key is not provided, then select an appropriate and unique capital letter for each distinct simple statement.
3. Read the statement carefully to understand the grouping of the statements.
4. Look out for punctuation marks such as comma to identify the main and subordinate connectives.
5. Replace each simple statement by the chosen capital letter.
6. Replace each English connective by the appropriate connective from propositional logic.

4.5.1 Two Kinds of Statements

This section will concentrate on the basic structure or grammar of one branch of symbolic logic known as sentential or propositional logic. It is that section of symbolic logic which considers simple statements as its atomic or the most basic or elementary units, such as 'A. N. Whitehead is a logician' or 'A. J. Ayer is a logical positivist'. Simple statements form the foundation of propositional logic. In addition to the simple statements we come across compound statements in logic. Thus statements can be divided into two types:

a. Simple statements
b. Compound statements

4.5.1.1 Symbolization of Simple Statements

Propositional logic deals with propositions. Hence, it is known as the logic of propositions. Propositional logic considers simple statements to be the most basic units. By holding that simple statements are the most basic units, propositional logic exhibits one important point, i.e. simple statements are not further analyzable.

By a simple statement we mean structurally simple statements. A simple statement is one that does not contain any other statement as a component part. For example, 'Colombus discovered America' is a simple statement since it has only one unanalyzable content to be stated.

Examples of simple statements:

1. All Assamese are Indians.
2. I. M. Copi is a famous logician.
3. Krisha Kanta Handique State University is an open university.

Any convenient upper case letter from English alphabet can be selected to represent the statements. Example 1 can be symbolically expressed as 'I', example 2 can be symbolically expressed as 'L', and example 3 can be symbolically expressed as 'O'.

4.5.1.2 Symbolization of Compound Statements

A compound statement is one which contains more than one simple proposition as its constituent part.

A) Negation:

In section 4.4.1 it has been pointed out that in propositional logic a negated simple proposition containing the connective 'not' is considered a compound statement. Let us learn about the symbolization of sentences containing negation:

(i) Abhipsha is not a good student.

~ A

(ii) The students will not go to enjoy movie tomorrow.

~ S

(iii) It is not the case that Dispur is the capital of Delhi.

~ D

(iv) It is false that grasses are not green.

~(~ G)

(v) It is not true that truth does not hold.

~(~ T)

(vi) It is untrue that bribery is good.

~(B)

B) Conjunction:

(i) Abhipsha likes rice and Bipasha likes chapatti.

A • B

(ii) Rajesh is poor but honest.

R • H

(iii) Ananya loves Prakash, however he barely tolerates her.

A •T

(iv) It is raining; nevertheless we shall go for shopping.

R • S

(v) Chinmoyee is honest, although not very intelligent.

C • ~ I

(vi) It is cold yet tolerable.

C • T

(vii) 'But' is a truth functional connective, while 'because' is not.

T • ~ B

(viii) Akash will go to the market despite the fact that he is ill.

A • I

(ix) Neelabh is honest, moreover he is intelligent.

N • I

C) Disjunction:

(i) Either Apurva is intelligent or he is honest

A v H

(ii) Either Apurva is not honest or he is not intelligent.

~A v ~ I

D) Implication:

(i) If it rains, then we shall not go for the picnic.

$R \supset \sim P$

(ii) If it does not rain, then we shall go for shopping.

$\sim R \supset S$

(iii) You will come out with flying colours only if you work hard.

$F \supset W$

(iv) You will come out with flying colours if you work hard.

$W \supset F$

E) Biconditional:

(i) You will pass in the examination if and only if you work hard.

$P \equiv W$

4.5.2 Complex Symbolizations

A) Not Both/Both Not

i) Not Both:

Example:

Chapatti and rice will not both be served here.

This sentence can be paraphrased in the following way:

'It is not the case that both chapatti and rice will be served.' The symbolic representation of this sentence will be:

$\sim (C \bullet R)$

[Here '~' stands for 'it is not the case', 'C' stands for 'Chapatti will be served' and 'R' stands for 'Rice will be served'] . This means that at most one of them will be served.

ii) Both Not:

Example:

Chapatti and rice both will not both be served.

This sentence can be paraphrased in the following way:

'Chapatti will not be served and rice will not be served.' The symbolic representation of this sentence will be:

$\sim C \bullet \sim R$

[Here '~' stands for 'it is not the case', 'C' stands for 'Chapatti will be served' and 'R' stands for 'Rice will be served']. The meaning of the given sentence therefore is that none of these two things will be served.

B) Neither-nor:

Example:

Neither Chapatti nor rice will be served.

This sentence can be paraphrased in the following way:

'Chapatti will not be served and rice will not be served.' The symbolic representation of this sentence will be:

~ C • ~ R

[Here '~' stands for 'it is not the case', 'C' stands for 'Chapatti will be served' and 'R' stands for 'Rice will be served']. The meaning of the given sentence therefore is that none of these two things will be served.

C) Unless:

Amrita will go to the market unless she has other commitments.

This sentence can be paraphrased in the following way:

'If Amrita does not have other commitments, she will go to the market.'

The symbolic representation of this sentence will be:

~ C ⊃ M

[Here '~ C' stands for 'Amrita does not have other commitments' and 'M' stands for 'Amrita will go to the market'].



CHECK YOUR PROGRESS

Q.4: What are the two kinds of statements which are taken into consideration for symbolization in propositional logic?

.....

.....



4.6 LET US SUM UP

Symbolic logic uses two kinds of symbols to form compound statements. One is logical connectives and the other is propositional variable.

- Connectives in propositional logic are special words or terms which are used to generate statements when used. They connect or join statements to produce other statements. Hence they are called connectives.
- These connectives are also called truth functional connectives because they function truth functionally. This means that propositional logic deals only with truth functional connectives. The following are the five truth functional connectives are: Not, And, Or, If-then, If and only if
- There are two-fold tasks in the elementary symbolic logic for the beginners. They are the following: 1. Symbolization of propositions and arguments stated in ordinary language. 2. Testing the validity of various types of arguments. As it is known to us that the validity of an argument depends on the correct form of the argument, it is therefore necessary to symbolize the propositions constituting the argument correctly.
- Parentheses, square brackets are the devices to demarcate clearly the scope of each connective. The scope of a connective is the range of its operation or the extent of its operation within the statement.
- Statements in propositional logic can be divided into two types: a) Simple statements, b) Compound statements. Hence a brief account regarding the process of symbolization of both simple as well as compound statements are described.
- Lastly this unit has briefly taken up complex symbolization of few statements containing words like 'not both/both not', 'neither-nor', 'unless'.

4.7 FURTHER READING

1) Chhanda C. (2007), Logic-Informal, Symbolic and Inductive, Prentice-Hall of India Pvt. Ltd.

2) Copi, I.M.(1998), Symbolic Logic (Fifth Edition), Prentice-Hall of India Pvt. Ltd.

3) Hurley, P. J. (2007), Introduction to Logic, Cengage Learning India Private Limited.

4) Klenk,V. (2008), Understanding Symbolic Logic, Dorling Kindersley (India) Pvt. Ltd.

5) Sharma, B. & Deka, J. (2014), A Text Book of Logic, Aank-Baak, Panbazar, Guwahati, Assam.

4.8 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q. No. 1:

a) True

b) False

Ans. to Q. No.2: There are five connectives used in propositional logic. They are the following: Not, And, Or, If-then, If and only if.

Ans. to Q. No. 3:

a) False

b) True

c) False

d) True

Ans. to Q. No. 4: The following are the two kinds of statements taken into consideration for symbolization in propositional logic: simple statement and compound statement.

4.9 MODEL QUESTIONS

A) Very short question

Q 1: Define connective.

Q 2: What are the types of symbols used in propositional logic?

Q 3: There are two-fold tasks in the elementary symbolic logic for the beginners?

Q 4: Symbolize the following statements:

a) Ananya performed her duties and earned promotion.

b) I went for shopping but shops were closed.

c) He failed though he tried.

d) Unless we work, we shall not progress.

e) It is not true that either Jesmin will not win or that Jaymoti will not win.

f) It is not true that neither Jesmin nor Jaymoti won.

g) If Kahuwa gets highest marks in English, then not both Karabee and Jaya will be happy.

h) Neither it is the case that if a mother is tough then she is not kindhearted, nor is it true that if the mother is kindheared then she is tough.

i) It is not true that Rohit went to the market but did not buy anything.

j) Jaya and Susmita both were not selected for the job.

B) Short question : (Answer in work 100-150 words)

Q 1: Why is not (~) is regarded to be a monadic connective?

Q 2: State the difference between monadic connective and dyadic connective.

Q 3: Distinguish between two senses of or, namely, exclusive or and inclusive or.

C) Long questions (Answer in work 300-500 words)

Q 1: Write short note on connective used in propositional logic.

Q 2: Illustrate various connectives with suitable examples.

Q 3: Give a brief account of symbolizing everyday language.

Q 4: What are the statement keys put forward by Chhanda Chakraborti for the symbolization of statements in propositional logic to be followed closely?

*** ***** ***

UNIT- 5: CONSTRUCTION OF TRUTH TABLE

UNIT STRUCTURE

5.1 Learning Objectives

5.2 Introduction

5.3 Construction of Truth Tables

5.4 Truth Table for Statement

5.5 Characterization of Statement forms as Tautologies, Contradictory and Contingent.

5.6 Truth Table for Arguments

5.7 To Decide Logical Equivalences

5.8 Let Us Sum Up

5.9 Further Reading

5.10 Answers to Check Your Progress

5.11 Model Questions

5.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- explain the basic truth table methods
- discuss how to construct truth tables
- explain how to construct truth tables for statements
- discuss whether an expression is tautology, contradictory or contingent
- explain how to construct truth tables for arguments
- explain how to determine the validity or invalidity of an argument by following truth table method
- explain how to determine logical equivalences between statements

5.2 INTRODUCTION

This unit introduces you to the truth table method and how to construct it. In symbolic logic the truth table method is used to determine

whether a compound statement is true or false. Normally, all statements can be divided into simple and compound statement. While a simple statement does not contain any other statement as its component part, a compound statement does contain another statement as its component part. For example, 'roses are red' is a simple statement and 'roses are red and violets are blue is a compound statement'.

Every statement is either true or false. Therefore, every statement contains truth value. The truth value of a true statement is true and the truth value of a false statement is false. In truth table method every statement is typically represented by small letter alphabet or variable, like p, q, r etc. Again, the truth values of statements are decided by the truth value of their connectives. In logic these connectives are regarded as logical constants. In symbolic logic there are five logical constants they are, '.', 'v', '?', '≡' and '~'. These logical constants are used in order to combine the truth functionally compound statements. If there are two statements p and q then there will be four possible sets of truth values and the number of rows will be 2^2= 2x2= 4. And if there are three statements then there will be eight possible sets of truth values likewise the number of rows will be 2^3 = 2x2x2 = 8. Thus, the number of rows increases in the geometrical progression as the number of variable increase. Now, let us discuss how to construct truth table by following the truth table method in logic.

5.3 CONSTRUCTION OF TRUTH TABLES

In propositional logic by using the symbol '.', which is a truth-functional connective we can construct the truth table for the conjunctive function 'p. q'. Representing the truth values true and false by the capital letters 'T' and 'F', respectively, the way in which the truth value of a conjunction is determined by the truth values of its conjuncts can be displayed more briefly by means of a truth table as follows:

p	*q*	*p . q*
T	T	T
T	F	F

F	T	F
F	F	F

In the above truth table as there are two variable, 'p' and 'q' so, we have four possible combinations of truth values i.e. 'TT', 'TF', 'FT', 'FF'. From the above table it is clear that a conjunctive function is true only under 'TT' combination. Again, under the combinations 'TF', 'FT', and 'FF' a conjunctive function is false. Hence, it may be concluded that a conjunctive function is true if and only if both the conjuncts are true, otherwise it is false.

Again, when two statements are combined disjunctively by using the word 'or' between them then it is called a disjunctive statement. Symbolically disjunctive statement is expressed as 'p v q' i.e. "either p or q". In Latin, the word 'vel' expresses the inclusive sense of the word 'or'. The symbol 'v' called as 'wedge' is a truth-functional connective and can be expressed by the following truth table:

p	*q*	*p v q*
T	T	T
T	F	T
F	T	T
F	F	F

From the above table it is clear that a disjunctive function is true under three combinations. These are 'TT', 'TF', and 'FT'. Again, it is false only under the combination 'FF'. Hence, we may conclude that a disjunctive function is true if and only if one of its disjuncts is true. It is false if both the disjuncts are false.

In propositional logic if two statements are connected conditionally then it is called as an implicative or conditional statement. Symbolically an implicative statement is expressed as 'p ⊃ q', i.e. "if p then q". Here the symbol '?' is defined as 'horseshoe'. The truth table of an implicative statement can be defined as follows: ⊃

p	q	p ⊃ q
T	T	T
T	F	F
F	T	T
F	F	T

The above table shows that the function 'p ⊃ q' is true under the combinations 'TT', 'FT' and 'FF'. It is false only under the combination 'TF'. Thus, it can be concluded that a conditional function will be false if and only if its antecedent is true and consequent is false. Apart from this one condition in all the cases it is true.

Followed by the conditional statement we have another statement which is called the bio-conditional statement. Two statements are said to be materially equivalent when they have the same truth value. Thus, a bioconditional statement is symbolized as 'p ≡ q'. Hence the three-bar symbol may be read as 'if and only if'. The truth table of a bioconditional statement is as follows:

p	q	p ≡ q
T	T	T
T	F	F
F	T	F
F	F	T

By following the above table it can be said that a bio-conditional function is true under the combinations of 'TT' and 'FF'. Again, it is false under the combinations of TF' and 'FT'. From this consideration it can be concluded that a bio-conditional function will be true if its components have same truth values and false if its components contain different truth values. As per the rule of symbolization we use the symbol '~' called a curl is used to symbolize a negative function. Symbolically a negative function is expressed as '~ p'. Let 'p' be a true statement, then its negation '~ p' will be false. Again, if 'p' is false, then '~p' is true. This can be shown in the truth table as follows:

~p	
p	~p
T	F
F	T

In the above table 'p' and '~p' are contradictory. So, they have different truth values. In negative function a sentence of the form 'p' is true if its contradiction is false and false if its contradiction is true.



CHECK YOUR PROGRESS

Q 1: State whether the following statements are true or false:

(a) A bio-conditional function is true if its components do not have same truth values. (True/False)

(b) If two statements are connected conditionally then that is called a conjunctive statement. (True/False)

(c) The symbol called 'wedge' is used to symbolize disjunctive statements. (True/False)

(d) 'Dot' is a truth functional connective. (True/False)

Q 2: Fill in the blanks:

(a) … ……is used to symbolize a negative function.

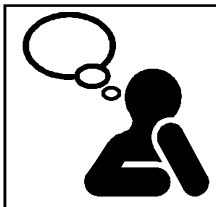
(b) A conjunctive function is true if and only if both the conjuncts are .....

5.4 TRUTH TABLE FOR STATEMENT

The truth values of a compound statement are determined by the truth values of the simple statements it contains and the basic truth tables of the five connectives. Thus, the statement "Either India wins their conference championship and Australia wins their conference championship or New Zealand wins the superbowl" can be symbolically expressed as "(p . q) v r ", whose exact significance is indicated by the following truth table:

p	q	r`	(p	.	q)	v	r
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	F
T	F	T	T	F	F	F	T
T	F	F	T	F	F	F	F
F	T	T	F	F	T	F	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	F	T
F	F	F	F	F	F	F	F

In the above truth table the combinations of the truth values under each of the column for the variables are not similar. Half of the numbers of truth values in the first column under the variable 'p' shall be 'T' (true) and other half shall be 'F' (false). Likewise, in the second column, under the variable ' q ' one fourth of the number of values shall be 'T' which should be written first to be followed by equal numbers of 'F's, which again should be followed by equal number of 'T's and equal number of 'F's. In the third column under the variable ' r ' one eight of the number of values shall be 'T' to be followed by 'F' then 'T' then 'F', whose numbers are the same as that of the first group of 'T's. In the above table the column under the disjunction i. e. 'v' is the answering column. So, to figure out the truth value of the disjunction, first we need to determine the truth values of (p . q) by applying the conjunctive truth table method. After determining the truth values of (p . q) in the final step we need to compare the truth values with the truth values of the column under the variable ' r ' which is connected through the disjunct 'v'. Thus, finally by applying the disjunctive truth table method we can find out the truth values of a statement in propositional logic.



ACTIVITY: 5.1

- Construct truth tables for the following statement :

1. (p v q) . r

Ans:

2. ~ (p . q) | | r

Ans:

5.5 CHARACTERIZATION OF STATEMENT FORMS AS TAUTOLOGIES, CONTRADICTORY AND CONTINGENT

In Propositional Logic, we can find out whether a statement is tautologous, contradictory or contingent on the basis of its possible truth values.

- A statement can be said as tautologous if that is logically true. In a tautologous truth table all the possible truth values of the resulting column are Ts. Let us take an example:

[(p	v	q)	.	~ p]	⊃	q
T	T	T	F	F	T	T
T	T	F	F	F	T	F
F	T	T	T	T	T	T
F	F	F	F	T	T	F

In the above table if we notice it would be clear that in its final column all the truth values are Ts. So, from this it could be concluded that the above statement is a tautological statement.

- A statement is called contradictor if it is logically false. If a statement is logically impossible to be true then that is called a contradictory statement. Let us take another example.

p	.	~	(q	⊃	p)
T	F	F	T	T	T
T	F	F	F	T	T
F	F	T	T	F	F
F	F	F	F	T	F

From the above analysis of this table we may conclude that the above statement is a contradictory statement. In the above table it has been seen that in the final column truth values listed under the connective dot are all Fs. Hence the statement [p . ~ (q □ p)] is a contradictory statement.

- A contingent statement is that which is neither tautologous nor contradictory. In other words, it is logically possible for the statement

to be true and it is also logically possible for it to be false. Hence, a contingent statement is a mixture of both kinds of truth values. Let us discuss it with an example:

p	≡	(~ q		⊃	p)
T	T	F	T	T	T
T	T	T	F	T	T
F	F	F	T	T	F
F	T	T	F	F	F

In the above table it has been seen that in its final column all the truth values are the mixture of Ts and F. The final column has three Ts, representing the logical possibility of being true, and one F, representing the logical possibility of being false. Hence it could be concluded that the above statement is a contingent statement.



ACTIVITY- 5.2

- Use the truth table to decide whether the following symbolized statement is tautologous, contradictory or contingent.

1. d ⊃ (b v ~ b)

Ans:

2. (s ⊃ h) ≡ (~h • s)

Ans:



CHECK YOUR PROGRESS

Q.3: What do you mean by a contingent statement? Explain briefly. (Answer in about 20 words)

Q.4: What is the difference between a tautological statement and a contradictory statement?

Q.5: Fill in the blanks:

(a) Contradictory statements are logically ……

(b) If a statement is logically impossible to be false then that is called a……

5.6 TRUTH TABLE FOR ARGUMENT

A deductive argument is valid if its conclusion necessarily follows from its premises. In a deductive argument if the premises are true, then the conclusion must be true. This means that in a deductive argument if the premises are logically true but the conclusion is false, then the argument is an invalid argument. Since a truth table can provide all the logical possibilities so, it can be used to determine whether a deductive argument is valid or invalid. The whole process has three steps:

- Symbolization of the deductive argument;
- Construction of the truth table for the given argument;
- Determining the validity or invalidity of the argument. In this regard it is necessary to mention here that in a deductive argument the conclusion should necessarily follow from the premises. But, if in the truth table there is found at least one row in which the premises are true and the conclusion is false then that would mean that the argument is invalid. If such a row is not found, this would mean that it is not logically possible to have true premises with a false conclusion. Therefore, the argument would be valid. Let us take an argument:

If young people don't have good economic opportunities, there would be more gang violence. Since there is more gang violence, young people don't have good economic opportunities.

The argument can be symbolized as follows:

$$\begin{array}{l} \sim p \supset q \\ \underline{\quad q \quad} \\ \sim p \end{array}$$

The truth table of the argument is as follows:

(1st premise)			(2nd premise)	(Conclusion)
(~p	⊃	q)	q	~p
F	T	T	T	F
F	T	F	F	F
T	T	T	T	T
T	F	F	F	T

In the above truth table we have seen that in the first row the premises are true but the conclusion is false. This means that the argument is invalid. Let us consider another example:

If I study then I pass

I study

Therefore, I pass

- Symbolically the argument can be expressed as follows

$$\begin{array}{l} p \supset q \\ p \\ \hline q \end{array}$$

- The truth table for the argument will be as follows:

(1st premise)	(2nd premise)	(Conclusion)
p ⊃ q	p	q
T	T	T
F	T	F
T	F	T
T	F	F

After completing the truth table, we check each row of the four final columns, looking for rows with true premises but a false conclusion. But, we do not find any. So the above argument is valid argument.



ACTIVITY- 5.3

Determine the validity or invalidity of the following argument by applying the truth table method:

1. If you invest in the stock market, then you get rich.
   You didn't invest in the stock market.
   Therefore, you didn't get rich.

Ans:



CHECK YOUR PROGRESS

Q 6: Fill in the blanks:

(a) A deductive argument is…… ……if its conclusion necessarily follows from its premises.

(b) In an invalid deductive argument the conclusion …….. follow from the premises.

5.7 TO DECIDE LOGICAL EQUIVALENCES

Two statements are said as logically equivalent if they necessarily have the same truth values. This means that their possible truth values listed in the two final columns are the same in each row. Let us take an example of comprising two statements:

1. The stock market will fall if interest rates are raised.
2. The stock market won't fall only if interest rates are not raised.

Symbolically both the statements can be expressed as follows respectively:

1. $p \supset q$ and 2. $\sim q \supset \sim p$

To see whether both of the statements are logically equivalent or not first we need to construct truth tables for each statement.

p ⊃| q	~q ⊃ ~p
T T T	F T F
T F F	T F F
F T T	F T T
F T F	T T T

After completing both the truth tables, it has been seen that the two main (or final) columns are identical to each other. This shows that the two statements are logically equivalent.



ACTIVITY: 5.4

Use truth tables to determine whether each pair of statements are logically equivalent:

1. p • ~ q
   q V ~p

Ans:

2. ~(p ⊃ q)
   ~q • ~ p

Ans:



5.8 LET US SUM UP

- In symbolic logic the truth table method is used to find out truth or falsity of a statement.
- In truth table method the number of rows increases in the geometrical succession as the number of variable increases.
- There are five truth table methods they are: conjunctive disjunctive, conditional, bio-conditional and negative. By following these five truth table methods we can find out the truth values of statements and arguments.

- On the basis of truth table method we can find out whether a compound expression is tautology, contradictory or contingent.
- We can also use the truth table method in order to test the validity or invalidity of an argument.
- The truth table method also can be used to find out whether two statements are logically equivalent or not.



5.9 FURTHER READING

1) Copy, I. M.; Cohen, C; Jetli, P. Prabhakar, M. (2006), Symbolic Logic. Delhi. Pearson.

2) Hurley, Patrick J.(2008). Introduction to logic. Wordsworth. Cengage Learning.

3) Klenk, Virginia (2009), Understanding Symbolic Logic. Delhi. Pearson Education, Inc and Dorling Kindersley publishing Inc.

4) Baronett, S., & Sen, M. (2009) Logic. Delhi. Pearson.



5.10 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1:

(a) False

(b) False

(c) True

(d) True

Ans to Q No 2:

(a) curl

(b) true

Ans to Q No 3: In a truth table of a contingent statement if in its final column all the truth values are found as the mixture of both truth and false then it is called as contingent statement.

Ans to Q No 4: In a truth table of a tautological statement in its final column all its truth values are found as true on the other hand in a

contradictory statement in its final column all the truth values are found as false.

Ans to Q No 5:

(a) False

(b) Tautological statement

Ans to Q No 6:

(a) valid

(b) does not

5.11 MODEL QUESTIONS

A. Very Short question

Q.1: What is the difference between a simple and compound statement in logic?

Q.2: what is logical constant?

Q.3: What do you mean by variable?

Q.4: What is a conjunctive function?

Q.5: Which symbol represents the disjunctive function?

B. Short question (Answer in about 150 words)

Q.1: What do you mean by Logical Equivalence?

Q.2: What are the conditions of an argument to be valid or invalid? Explain by giving an example.

Q.3: Explain the conjunctive truth table method by giving its truth table.

Q.4: Explain the conditional truth table method by giving its truth table.

C. Long Questions (Answer in about 300- 500 words)

Q.1: Explain the truth table method by giving the truth table.

Q.2: What do you mean by Tautology, contradictory and contingent statement? Explain by giving example.

Q.3: Explain the truth table method for testing the validity of argument.

Q.4: Use the truth table method to determine the validity or invalidity of the following argument forms.

(a) p . q

∴ p

(b) p v p

P

∴ p . q

(c) (p ⊃ q) . (r ⊃ s)

P v r

∴ q v s

(d) (p v q) ⊃ (p . q)

~ (p v q)

∴ ~ (p . q)

(e) p ⊃ (q ⊃ r)

p ⊃ q

∴ p ⊃ r

Q.5: Use the truth table method to characterize the following statement forms as tautologous, contradictory or contingent.

(a) p ⊃ ~ p

(b) (p ⊃ ~p) . (~p ⊃ p)

(c) p ⊃ (p ⊃ p)

(d) (~p . q) . (q ⊃ p)

(e) (p . q) ⊃ p

(f) [(p ⊃ q) ⊃ p] ⊃ p

*** ***** ***

UNIT 6: DETERMINATION OF VALIDITY AND INVALIDITY OF ARGUMENTS

UNIT STRUCTURE

6.1 Learning Objectives

6.2 Introduction

6.3 Validity of Arguments

6.4 Invalidity of Arguments

6.5 Determination of Validity and Invalidity of Arguments-direct truth Table Method

6.6 Indirect Truth Table Method

6.7 Formal Proof of Validity

6.8 Conditional Proof

6.9 Indirect Proof

6.10 Let Us Sum Up

6.11 Further Reading

6.12 Model Questions

6.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- explain validity and invalidity of arguments.
- describe difference between validity and invalidity of arguments.
- explain the methods of determination of validity of arguments.
- describe various methods of validity

6.2 INTRODUCTION

Validity is one of the important concepts of elementary logic. Validity is the attribute of an argument. A deductive argument is evaluated as valid or invalid. The validity of an argument refers to the relation between its propositions between the set of propositions that serves as the premises and the one proposition that serves as the conclusion of that argument. If

the conclusion follows with logical necessity from the premises, one says that the argument is valid. Consequently validity can never apply to any single proposition by itself, as the needed relation can not possibly be found within any one proposition.

Validity of an argument depends on the form of an argument. An argument is valid if and only if its form is valid. But form has nothing to do with subject matter. It follows that what an argument says, its contents, is irrelevant to its validity. So validity and invalidity are formal notions that are applied to formal reasoning and formal logic. However a valid argument can never have true premises and a false conclusion. The basic feature of a good deductive argument is known as validity.

According to W.E. Johnson, valid argument must fulfill two conditions viz. the constitutive condition and the epistemic condition. The constitutive conditions are a) the premises must be true and b) the premises must imply the conclusion. The epistemic conditions are a) the premises should be known to be true and b) the premises should be known to imply the conclusion. For e.g.

1. All army jawans are Indians
   Bahadur is an army jawan
   Therefore Bahadur is an Indian.
2. Either he is a student or a teacher
   He is not a student
   Therefore he is a teacher.

So the validity of an argument is determined independently of the truth of the premises of the argument.

6.3 VALIDITY OF ARGUMENTS

Logic is primarily concerned with valid inference. According to A. Wolf, 'valid' means correct, accurate or sound. An inference may be valid without being true or vice-versa. An inference is valid when it is justified by the evidence adduced in support of it. It is true if it is in accord with the relevant facts i.e. if it describes the facts concerned approximately as they

are. So a valid argument is not the same thing as a true inference. It is possible for an inference to be valid.

The goal of logic is to study the conditions of valid argument. It is not concerned with the conditions of true inference. The study of truth of an inference involves a multinational problem which may relate to mathematics, physics, chemistry, economics, morality etc. Logic is concerned with only general conditions of valid inference. It means an abstraction from the truth of the premises. It means logic does not include it within its scope. Logic is to confine its scope within the search for general conditions of validity of argument.

Generally validity refers that any conclusion drawn from premise or premises can not but are valid. Valid argument implies that valid argument will lead to valid conclusion. In case of valid argument, it is impossible for conclusion to be false if the same is drawn from true premises. This argument also implies relation of entailment which means that the conclusion necessarily follows from premise or premises. There are some characteristics of valid arguments in logic. These are -

- Validity of argument means only formal validity, not material validity.
- It applies only to deduction, not to induction.
- Validity of an argument implies that the conclusion necessarily follows from the premise or premises. So there is a relation of entailment between premises and conclusion.
- In case of valid argument, true premises lead to true conclusion.
- Validity of an argument does not guarantee true conclusion. It implies only valid conclusion.
- All instances having the same form of argument will be valid and conversely all instances having the same form of invalid argument will be invalid.
- Validity of argument depends on some general rules or formulae supplies by logic for reaching valid conclusion.

Few examples are cited in order to explain the above points:

1. a) All men are mortal

Ram is mortal

∴ Ram is mortal.

b) All men are imperfect.

Ram is a man.

∴ Ram is imperfect.

In the above two arguments, the conclusion has been drawn from premises following the rules of deduction. So the argument is valid. Any argument like this argument is also valid. The argument is of deductive nature having the relation of entailment between the premises and the conclusion. For e.g.

2. a) All students read books

Ram is a student.

∴ Ram reads book.

b) All women are wicked.

Anima is a woman.

∴ Anima is wicked.

In the above two arguments are of the same from and the both are valid. But the former is materially true but the later is materially false. The valid argument does not guarantee material truth of premises and conclusion. So the validity of an argument refers valid process for drawing valid conclusion.

6.4 INVALIDITY OF ARGUMENTS

The form of argument plays the pivotal role in determining validity or invalidity of argument. The validity or invalidity of argument has nothing to do with the content of argument. It may so happen that true conclusion has been drawn from true premises but still the argument may be invalid if the argument is not based on proper reasoning. For e.g.

Some men are not educated (true)

All artiest are men (true)

∴ Some artists are not educated (true)

In the above cited example all the propositions including the conclusion are true. But the argument is invalid as it has violated the rule of distribution of terms and has committed the fallacy of undistributed middle. Invalidity refers invalidity of argument or invalidity of argument-form. If any conclusion is drawn from premises breaking the rules of reasoning, the argument is invalid. An argument may be announced invalid if it is found that a false conclusion has been drawn from true premise or premises. With the help of few examples we can explain the invalidity of arguments.

Example-1.

All men are mortal (true)

All poets are men (true)

∴ All poets are mortal (true)

This is the case of valid argument. Such a form of argument is valid as in such a case there is no possibility of violation of any rule of reasoning. But there may be some other forms in which argument may be invalid.

Example-2.

All males are mortal (true);

(P) (M)

All females are mortal (true);

(S) (M)

All females are male (false)

(S) (P)

Here the argument form is

All P are M

All S are M

∴ All S are P.

In this example, we find that the term M which was subject in the example 1, has changed its position in example 1, and has turned to be undistributed in example 2. The argument has fallen in the trap of undistributed middle and the argument in the example 2 has been invalid. So in conclusion it can be said that the form of argument determines the validity or invalidity of argument. Furthermore individuality and falsity are

not identical as invalidity refers to form and falsity refers to content. Logicians are concerned with validity or invalidity of arguments. A. Wolf points out that logic studies the conditions of valid inference and this involves the relations between inference and premises.

6.5 DETERMINATION OF VALIDITY AND INVALIDITY OF ARGUMENTS

The basic function of symbolic logic is to determine the validity or invalidity of arguments. Elementary logic has given various methods to test the validity of arguments and argument forms. Through these methods we are also examine whether statements or statement forms are tautologies or not. These methods are called in logic as Decision procedures. It is the mechanical procedure which will always tried to give an answer for any validity problem. Some of the important decision procedures are

1. Truth table method or direct truth table method.
2. Shorter truth table method or indirect truth table method.
3. Formal proof of validity.
4. CNF (Conjunctive Normal Form)
5. DNF (Disjunctive Normal Form)

Let us discuss here the principle methods of decision procedures-

1. Truth table method is one of the significant decision procedures. It is also known as matrix method. The truth table method or the matrix method was popularized by two philosophers and logicians- E.L. Post and Ludwig Wittgenstein. But prior to them Gottlob Frege also employed truth tables The truth table method is a device for exhibiting the conditions under which a truth functional compound proposition is true or false. By means of this decision procedure we can determine whether arguments are valid or invalid. In this method truth values are given in tabular representations. Such tabular representations are called truth tables. According to J.J. Kelly, truth tables give us operational definitions of the important logical connectives, separate from and independent of their use in ordinary language. They also provide a mechanism whereby the truth values of complicated expressions may be

worked out. In other words truth tables are those which represent a systemic exhaustive enumeration of the possible truth values of combinations of simple propositions. A truth table can be defined as a tabular representation in which all possible truth values of a compound statement is considered through the truth values of their simple constituent proposition. The truth table method or the matrix method may be employed for the following basic purposes-firstly, it may be employed for giving operational definitions of logical constant. Secondly, it may be employed to work out the truth values of the compound propositions on the basis of the knowledge of truth values of the constituent simple propositions. Thirdly, truth tables or matrix method may be employed for testing the validity of arguments.

In order to determine the validity of arguments we are to know how to construct a truth table. For constructing a truth table we require certain steps to follow. First, to construct a truth table is to symbolize the statement given. We are to symbolize a number of constituent propositions with the help of variables and logical constants. Secondly, we are to find out the number of variables by means of which the entire compound proposition is constituted. The number of columns will depend on the number of variables or repeated variables and on the number of logical constants. Thirdly, we have to put truth values 'T' and 'F' under the respective columns. Fourthly, we are to determine the number of rows which is considered to be the important step in constructing a truth table. If the number of variable is only one then we are to put truth values 'T' and 'F'. The formula which is adopted for counting the number of rows is '2n' Where 2 is taken for two truth values of 'T' and 'F' and 'n' represents the number of variable or variables. As soon as variable increase the number of rows increases twice. Fifthly, a truth function is by the truth values of its components. If all the truth values of final logical constants are T then the compound proposition is a tautological proposition. Again if truth value F occurs under the final logical constant then the compound proposition is a contradictory and if truth values 'T' and 'F' occurs under the final constant is a contingent proposition. For constructing a truth table five basic truth functions are necessary with tabular

representation. These are conjunctive function, implicative function, disjunctive function, negative function and equivalent function. Furthermore use of brackets is also important for constructing truth table. Let us take the following example:

(P u Q)	=	(~ P	v	Q)
T T T	T	F T	T	T
T F F	T	F T	F	F
F T T	T	T F	T	T
F T F	T	T F	T	T

In this example in the first set of bracket the given (PuQ) we have assigned the truth value TTFF for 'p' and TFTF for 'q'. Under the logical constant, 'implication' is TFTT here. The same process is done with the second set of bracket and under the logical constant 'vel' is TFTT. The main constant is computed by reconciling the two sets of results and the result under the logical constant '=' is TTTT. Therefore it is a tautological truth function.

6.6 INDIRECT TRUTH TABLE METHOD

The second method is the shorter truth table method. This method is one of the decision procedures in propositional logic to establish directly the invalidity or indirectly the validity of an argument. In this method, the direct technique of application to prove the invalidity of an argument is used in such a way that at least one set of possible truth conditions is demonstrated by assuming that the premises are all true but the conclusion is false. It is also known as the method of Reductio ad Absurdum. Here the invalidity of the argument is to be assumed, for which one is to put every premise as true and the conclusion as false. For this, truth value assignment must be put under the main connectives of the premises and the conclusion. In this process guess work has no function. The truth value assignments are put under the invalidating row being forced by the already put truth values. For instance:

~ P v ~ Q P ~Q

F T T F T T F T

In the above example in the column 1 we can put truth value assignment. Truth value 'T' is forced to write in this column to support its connectives (~) for the truth value 'F'. Then column 2 is putting the same truth value for the same variable with column 1. Thereafter column 3 is newly put truth value, forced to write for justifying the negation of Q. Column 4 is putting the same truth value for the same variable in the second premise. Then column 5 is newly put truth value assignment forced to write for justifying the negation of P. So in order to complete the process, one is either to detect a contradiction among the truth value assignments or assign a truth value against the entire logical constant and propositional variable put there for the concerned statements.

6.7 FORMAL PROOF OF VALIDITY

Besides these two methods the formal proof of validity is also noteworthy. According to I.M. Copi, a formal proof of validity for a given argument is defined to be a sequence of statements, each of which is either a premise of that argument or follows from preceding statements by an elementary valid argument. By means of formal proof of validity we can establish that the conclusion in fact is deducible from the premises. It is a method for establishing the validity of arguments by a sequence of shorter elementary arguments that are already known to be true. These shorter arguments are established to be valid on the basis of certain elementary tautologies. These elementary valid arguments are sometimes called Rules of Inference and Rules of Replacement.

Formal proof of validity is a simple and more clarifying method than the method of truth table. It can not prove the invalidity of an argument as it can only prove a valid argument as valid. It is also the demonstration of the derivability of the consequence from a set of premises is the main concern. A formal proof of validity expresses that the statements constituting a formal proof of validity are logically interconnected. It is purely a logical process of deduction. For e.g.

1. E u F
2. Rv~O
3. ~R / . . ~E
4. ~O (2,3 By D.S)

6.8 CONDITIONAL PROOF

In case of formal proof of validity, we have found another procedure in order to prove the conditionals. It is called conditional proof. This proof does not have premises which we generally use to have in normal proof of validity. In order to infer something when we have no premise from which to infer it, we need some additional proof that depends on assumptions and then we see what follows from the assumptions. In a conditional proof, one is to assume the antecedent of the conditional which is to be proved and to see whether the consequent follows from that or not. If it follows then we may infer the conditional itself. The rule of conditional proof is useful in constructing proofs of conditionals, especially where premises and conclusion are very complex.

6.9 INDIRECT PROOF

The validity of an argument can be established by applying both formal proof of validity and conditional proof. Conditional proof need not necessarily be brief and easier in comparison to formal proof of validity. But the procedure of conditional proof has some limitations. When an assumption is made, does not matter in which stage of the proof process that must eventually be discharged one must have utmost importance on making the proper assumption. We never make an assumption unless we know what exactly it is for. So logicians have suggested another method for removing the limitations of conditional proof. It is Indirect Proof method. In constructing a formal proof of validity, it becomes difficult to prove the conclusion directly by applying the rules of inference. An indirect proof consists of assuming the negation of the statement to be arrived. This assumption seeks to derive

a contradiction and then that the original assumption is false. And proving the falsity of the assumption establishes the truth of the statement to be obtained.

The other two methods are less useful for determining the validity of arguments.



6.10 LET US SUM UP

We have discussed the determination of validity of arguments. The primary points of this unit are:

- The validity of an argument refers to the relation between its propositions between the set of propositions that serves as the premises and the one proposition that serves as the conclusion of that argument.
- In valid arguments, all the propositions including conclusion may be true.
- In invalid argument, all the propositions including the conclusion may be true
- For the determinations of validity of arguments we have found various decision procedures.
- Truth table method can be used to determine the logical status of a single, individual statement form or statement simply by checking its truth value.
- Shorter truth table method establishes directly the invalidity or indirectly the validity of an argument.
- Formal proof of validity is used for proving a valid argument as valid but can not prove the invalidity of an argument.
- The rule of conditional proof is applicable in constructing proofs of conditionals, especially where premises and conclusion are very complex.

- Indirect proof is useful when the formal proof of validity becomes very difficult to prove the conclusion directly just by applying the rules inference.



6.11 FURTHER READING

1) Basson and O'connor 1953 Introduction to Symbolic logic, Oxford University Press, Delhi.
2) Copi, I.M. 1979. Symbolic Logic, Prentice Hall of India, pvt ltd. New Delhi.
3) Copi, I.M. & Kohen Carl. 1984, Introduction to Symbolic Logic, Prentice Hall of India, pvt ltd. New Delhi.
4) Quine, W.V. 1970. Philosophy of Logic, Prentice Hall of India, pvt ltd. New Delhi.
5) Roy, B.N. 1984. Deductive Logic, S.C. Sarkar & Sons pvt ltd, Calctta.
6) Strawson, P.F.1976. Introduction to Logical Theory, Methuen & co. Ltd. London.



6.12 MODEL QUESTIONS

A. Very short Questions

Q. 1: What is the attribute of argument?

Q. 2: What are the conditions that a valid argument can fulfill?

Q. 3: Give an example of valid argument.

Q. 4: What are the various decision procedures?

B. Short questions (Answer in about 100-150 words)

Q. 1: Write short notes on :

a) Indirect proof method

b) Conditional Proof

Q. 2: Write the characteristics of valid argument.

Q. 3: What is elementary valid argument form?

Q. 4: What are the purposes of a truth table method?

C. Long Questions (Answer in about 300-500)

Q. 1: Explain the strategy for constructing a truth table method.

Q. 2: Discuss the procedure of formal proof of validity.

Q. 3: Give an elaboration of the indirect truth table method with an example.

*** ***** ***

UNIT-7: FORMAL PROOF OF VALIDITY AND CONDITIONAL PROOF

UNIT STRUCTURE

7.1 Learning Objectives

7.2 Introduction

7.3 The Method of Formal Proof of Validity

- 7.3.1 The Rules of Inference
- 7.3.2 The Rules of Replacement

7.4 The Rule of Conditional Proof

7.5 Let Us Sum Up

7.6 Further Reading

7.7 Answers to Check Your Progress

7.8 Model Questions

7.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- explain the meaning of formal proof of validity
- discuss the rules of inference and the rules of replacement
- explain how to construct a formal proof of validity by applying the rules
- illustrate the method of conditional proof
- explain how to determine validity by applying the rule of conditional poof.

7.2 INTRODUCTION

This unit introduces you to the concept of formal proof of validity and conditional proof. A formal proof of validity for a given argument is defined to be a sequence of statements, each of which is either a premise of that argument or follows from preceding statement by an elementary valid argument. Thus, the last statement in the sequence is the conclusion of

the argument whose validity is being proved. On the other hand the rule of conditional proof is applied to those arguments whose conclusions are conditional statements. But, in order to describe the method of conditional proof first we need to know the method of formal proof of validity and how to proof the validity of an argument by following it.

7.3 THE METHOD OF FORMAL PROOF OF VALIDITY

In order to proof the validity of the conclusion of an argument by means of formal proof of validity or method of deduction we have to maintain a strategy for it. For testing the validity of an argument by means of formal proof of validity formulated in ordinary language first we have to symbolize the statement to bring out the logical form of the argument. Here we may use capital alphabets for elementary statements. Then, after having the logical form of the argument we may construct proofs or derivations using the elementary valid argument forms. The given premises and the subsequent steps in the deduction shall be numbered. The construction of the proof in a formal proof of validity is generally started with the deduction of a consequence from the premises. Unless the derived consequence is the conclusion it shall be treated as one of the premises for the deduction of further consequences. Each and every subsequent step in the deduction may likewise be treated as an additional premise. Thus, we have to repeat the process of deduction until the conclusion is obtained. On the right side of each step we have to give explanation by writing the number of steps from which the derivation is made and the name of the rule applied in brief. A more and more concise way of writing out this proof of validity is to list the premises and the statements deduced from them in one column, with justifications for the latter written beside them. In each case the justification for a statement specifies the preceding statement and the rule of inference and replacement by which the statement in question is deduced.

Thus, it is necessary to mention here that in order to construct formal proof of validity we need to take the help of certain rules. These rules are called the rules of inference and the rules of replacement. Let us discuss it.

There are all total nineteen rules by following which we can construct the formal proof validity.



CHECK YOUR PROGRESS

Q.1: Answer the following questions:

(a) The method of formal proof of validity is also known as _____.

(b) _____ alphabets are used in formal proof of validity.

Q.2: What are the names of the rules of formal proof of validity?

7.3.1 The Rules of Inference

The first nine rules of the method of deduction is called the rule of inference. These are the nine elementary valid argument forms whose validity can be easily established by following the truth table method. These rules are used to construct formal proof of validity for a wide range of more complicated arguments. They are as follows:

1. Modus Ponens (M. P.)

   p ⊃ q

   p

   ∴ q

2. Modus Tollens (M. T.)

   p ⊃ q

   ~ q

   ∴ ~ p

3. Hypothetical Syllogism (H. S.)

   p ⊃ q

   q ⊃ r

   ∴ p ⊃ r

4. Disjunctive Syllogism (D.S.)

   p v q

   ~ p

   ∴ q

5. Constructive Dilemma (C. D.)

   (p ⊃ q) . (r ⊃ s)

   p v r

   ∴ q v s

6. Destructive Dilemma (D.D.)

   (p ⊃ q) . (r ⊃ s)

   ~ q v ~s

   ∴ ~ p v ~ r

7. Simplification (Simp.)

   p. q

   ∴ p

8. Conjunction (Conj.)

   p

   q

   ∴ p . q

9. Addition (Add.)

   p

   ∴ p v q

Thus, by following the rules of inference we can construct the formal proof of validity of a given argument like follows:

1. (A . B) ↄ [A ↄ (D . E)]
2. (A . B) . C / ∴ D v E
3. A . B 2, Simp.
4. A ↄ (D . E) 1, 3, M.P.
5. A 3, Simp.
6. D . E 4, 5, M. P.
7. D 6, Simp.
8. D v E 7. Add.

As explained above by applying the rules of inference and by means of a chain of deduction we have arrived at the conclusion in step no 8. This proves that the given argument is valid.



ACTIVITY- 7.1

- Construct a formal proof for the following arguments:

A ↄ B

C ↄ D

(~ B v ~D) . (~ A v ~ B)

∴ ~ A v ~ C

Ans:

7.3.2 The Rules of Replacement

Apart from the rules of inference there exist ten more rules which are used to prove the validity of a truth functionally compound statement. There are many truth-functional arguments that cannot be proved valid using only the nine rules of inference. For example:

A . B

∴ B

The above argument is a valid argument. But, we cannot use the rules of inference to prove the validity of the above argument. Hence any part of a compound statement is replaced by an expression that is logically equivalent to the part replaced, the truth value of the resulting statement is the same as that of the original statement. This can be regarded as the Rules of Replacement. It can be said as the additional principle of inference. It permits us to infer from any statement the result of replacing all or part of that statement by any other statement logically equivalent to the part replaced. Thus, by using the rules of replacement we can replace logically equivalent expressions with each other wherever they occur. Following are the rules of replacement by following which we can construct formal proof of validity for a given argument. These rules are numbered consequentively after the first nine rules already stated. They are as followed:

10.	De Morgan's Theorems (De M.):	~ (p . q) ≡ (~ p v ~ q).
		~ (p v q) ≡ (~p . ~q).
11.	Commutation (Com.):	(p v q) ≡ (q v p).
		(p. q) ≡ (q. p).
12.	Association (Assoc.):	[p v (q v r)] ≡ [(p v q) v r].
		[p . (q . r)] ≡ [(p . q) . r].
13.	Distribution (Dist.):	[p . (q v r)] ≡ [(p . q) v (p . r)].
		[p v (q . r)] ≡ [(p v q) . (p v r)].
14.	Double Negation (D. N.):	p ≡ ~ ~ p.
15.	Transposition (Trans.):	(p ⊃ q) ≡ (~ q ⊃ ~ p).
16.	Material Implication (Impl.):	(p ⊃ q) ≡ (~ p v q).

17. Material Equivalence (Equiv.): $(p \equiv q) \equiv [(p \supset q) . (q \supset p)]$.
$(p \equiv q) \equiv [(p . q) \vee (\sim p . \sim q)]$.

18. Exportation (Exp.): $[(p . q) \supset r] \equiv [p \supset (q \supset r)]$.

19. Tautology (Taut.): $p \equiv (p \vee p)$.
$P \equiv (p . p)$.

Thus, the formal proof of validity for the above argument A . B/ ∴ B can now be written as

1. A . B / ∴ B
2. B . A 1, Com.
3. B 2, Simp.

Let us take an example:

1. (A v B) ⊃ (C . D)
2. ~ C / ∴ ~ B
3. ~ C v ~ D 2, Add.
4. ~ (C . D) 3, De. M.
5. ~ (A v B) 1, 4, M.T.
6. ~A . ~ B 5, De. M.
7. ~B . ~A 6, Com.
8. ~B 7, Simp.

Thus, in the above example along with the rules of inference we have also used the rules of replacement and arrived at the conclusion in the line number 8 and finally found that the argument is valid. From the above examples it has been seen that there is an important difference between the first nine rules of inference and the last ten rules of replacement. The first nine rules can be applied only to the whole lines of a proof serving as premisses. On the other hand the last ten rules of replacement can be applied either to whole lines or to parts of lines.



CHECK YOUR PROGRESS

Q.3: State the rule of Exportation.

Q.4: State the rule of material Implication.

Q.5: Write the main point of difference between the rule of inference and the rule of replacement.

Q.6: What is the rule of De Morgan?



ACTIVITY-7.2

• Construct a formal proof of validity of the given argument.

F v (G v H)

(G ⊃ I) . (H ⊃ J)

(I v J) ⊃ (F v H)

~ F / ∴ H

7.4 THE RULE OF CONDITIONAL PROOF

As already explained the rule of conditional proof is applied to those arguments whose conclusions are conditional statements. To every argument there corresponds a conditional statement whose antecedent is the conjunction of the argument's premises and whose consequent is the argument's conclusion. Given any argument whose conclusion is a conditional statement, a proof of its validity, using the rule of conditional proof that is, a conditional proof of its validity is constituted by assuming the antecedent of its conclusion as an additional premise, and then deducing the consequent of its conclusion by a sequence of elementary valid arguments. Thus, the conditional proof of validity of the following argument

1. A ⊃ B /∴ A ⊃ (A . B) will be as follows:

1. A ⊃ B /∴ A ⊃ (A . B)

2.	A	/∴ A . B (C . P.)
3.	B	1, 2, M. P.
4.	A . B	2, 3, Conj.

In the above example in the second slant line and three dot 'therefore' symbol and the parenthesized 'C. P.', indicates that the rule of conditional proof is being used. The rule of conditional proof not only permits us to construct shorter proofs of validity for arguments that could be proved valid by appealing to the original list of nineteen rules of inference and replacements alone. But, it also permits us to establish the validity of valid arguments whose validity could not be proved by reference to the original list alone. Let us take another example:

1.	(A v B) ⊃ (C . D)	
2.	(D v E) ⊃ F	/∴ A ⊃ F
3.	A	/ ∴ F (C. P.)
4.	A v B	3, Add.
5.	C . D	1, 4, M. P.
6.	D . C	5, Com.
7.	D	6. Simp.
8.	D v E	7, Add.
9.	F	2, 8, M. P.

As seen in the above the rule of conditional proof is applied in the conclusion and finally, we got the conclusion 'F' which has been proved in the 9th line. Thus, it proved that the argument is valid. So, the rule of conditional proof can be applied in dealing with any valid argument having a conditional statement as conclusion and it can also be applied more than once in the course of the same deduction.



CHECK YOUR PROGRESS

Q.7: Fill in the blanks

(a) The rule of conditional proof is applied to the statements whose ----------are conditional statements.

(b) ---------- abbreviation is used for the rule of conditional proof.

Q.8: What will be the conditional proof of the following argument?

1. A ⊃ (B ⊃ C)
2. B ⊃ (C ⊃ D) / ∴ A ⊃ (B ⊃ D)



7.5 LET US SUM UP

- A formal proof of validity is defined to be a sequence of statements.
- Each statement in a formal proof of validity is either a premise of that argument or follows from preceding statement by an elementary valid argument.
- The last statement in the sequence of a formal proof of validity is regarded as the conclusion of the argument.
- The derived consequence in a formal proof of validity is treated as one of the premises unless we get the conclusion.
- The process of deduction is repeated in the method of deduction until the conclusion is obtained.
- On the right side of each line in the process of deduction we have to give explanation by writing the number of steps from which the derivation is made and also the name of the rule applied in brief.
- The first nine rules are called the rules of inference and the later ten rules are called the rules of replacement.

- The main point of difference between these two types of rules is that the rules of inference can be applied to the whole line of a proof, but the rules of replacement can also be applied to a part of a proof.
- The rule of conditional proof is applied to the arguments containing conditional statement as its conclusion.
- The abbreviation 'C.P.' is used to indicate the application of the rule.

7.6 FURTHER READING

1) Baronett, S., & Sen, M. (2009) Logic. Delhi. Pearson.
2) Copy, I. M.; Cohen, C; Jetli, P. Prabhakar, M. (2006), Symbolic Logic (Twelfth edition ed.). Delhi. Pearson.
3) Copy, I. M. (2006) Symbolic Logic (Fifth edition ed.). Delhi. Pearson Education, Inc and Dorling Kindersley publishing Inc.
4) Hurley, Patrick J.(2008). Introduction to logic. Wordsworth. Cengage Learning.
5) Klenk, Virginia (2009), Understanding Symbolic Logic. Delhi. Pearson Education, Inc and Dorling Kindersley publishing Inc.

7.7 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1:

(a) method of deduction

(b) Capital

Ans to Q No 2: The names of the rules of formal proof of validity are the rules of inference and the rules of replacement.

Ans to Q No 3: Exportation (Exp.): $[(p \cdot q) \supset r] \equiv [p \supset (q \supset r)]$.

Ans to Q No 4: Material Implication (Impl.): $(p \supset q) \equiv (\sim p \vee q)$.

Ans to Q No 5: The rules of inference can be applied to the whole line of a proof and the rules of replacements can be applied to the whole as well as to the parts of a proof.

Ans to Q No 6: De Morgan's Theorems (De M.): $\sim (p \cdot q) \equiv (\sim p \vee \sim q)$.

$\sim (p \vee q) \equiv (\sim p \cdot \sim q)$.

Ans to Q No 7:

(a) Conclusions

(b) C. P.

Ans to Q No. 8:

1. A Ɔ (B Ɔ C)
2. B Ɔ (C Ɔ D) / ∴ A Ɔ (B Ɔ D)
3. A / ∴ B Ɔ D (C.P.)
4. B / ∴ D (C. P.

7.8 MODEL QUESTIONS

A. Very short questions

Q 1: Define formal proof of validity.

Q.2: What is the condition under which the rule of conditional proof is applied?

Q.3: What is the use of rules of inference?

Q.4: What is the use of rules of replacements?

Q.5: What is the difference between the rules of inference and the rules of replacement?

B. Short question (Answer in about 100- 150 words)

Q.1: Explain briefly the process of deduction.

Q.2: Write a note on the rule of conditional proof.

C. Long Questions (Answer in about 300- 500 words)

Q.1: Write down the rules of inference and rules of replacement by giving examples in support of both.

Q.2: Construct a formal proof of validity of each of the following argument.

(a) 1. (E . F) . G

2. (F ≡ G) Ɔ (H v I) / ∴ I v H

(b) 1. (J . K) Ɔ L

2. (J Ɔ L) Ɔ M

3. ~ K v N /∴ K Ɔ (M . N)

(c) 1. S ↄ (T ↄ U)

2. U ↄ ~ U

3. (V ↄ S) . (W ↄ T) / ∴ (V ↄ ~W)

(d) 1. X ↄ (Y ↄ Z)

2. X ↄ (A ↄ B)

3. X . (Y v A)

4. ~Z / ∴ B

(e) 1. K ↄ L / ∴ K ↄ (L v M)

(f) 1. (~U v V) . (U v W)

2. ~X ↄ ~W / ∴ V v X

(g) 1. A ↄ ~ (B ↄ C)

2. (D . B) ↄ C

3. D / ∴ ~A

(h) 1. O ↄ (P ↄ Q)

2. P ↄ (Q ↄ R) / ∴ O ↄ (P ↄ R)

*** ***** ***

UNIT- 8: INDIRECT PROOF AND STRENGTHENED RULE OF CONDITIONAL PROOF

UNIT STRUCTURE

8.1 learning Objectives

8.2 Introduction

8.3 Indirect Proof

8.4 What is a Logical Method?

8.5 The use of Indirect Proof in Proposition

8.6 The use of Indirect Proof in Validity of Argument

8.7 The Strengthened Rule of Conditional Proof

8.8 Types of Conditional Proof

- 8.8.1 Ordinary Conditional Proof (OCP)
- 8.8.2 Strengthened Conditional Proof (SCP)

8.9 The Concept of the Scope of the Assumption

8.10 Let Sum Up

8.11 Further Reading

8.12 Answers to Check Your Progress

8.13 Model Questions

8.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- explain Indirect Proof (IP), Conditional, and Strengthened Rule of Conditional Proof;
- describe indirect proof as a concept or method;
- explain the use of Indirect Proof in proposition;
- discuss the use of indirect proof in validity of argument;
- explain the types of Conditional Proof
- discuss the concept of the scope of the assumption and so on.

8.2 INTRODUCTION

The concept of proof is the main contention of formal and symbolic logic. Formal logic engages with different types of proofs, such as, indirect proof, direct proof, conditional proof etc. One should be kept in mind that there is a subtle distinction between the concept of proof and the concept of test. Sometimes, it is said that 'test the following' and sometimes it is said that 'proof the following'. To say 'test the following' means the given are valid and invalid and one has to find out or so to speak test which one is valid and which one is not valid. Contrary to that, to say 'proof the following', it means that the given are valid and one has to prove it. More specifically, to say 'test the following argument' is to say that the given argument may be valid or invalid and one has to test it logically and to say that 'proof the following argument' is to say that the given argument is valid and one has to prove it. Thus, the concept of proof 'test' and 'proof' play important role in symbolic or formal logic.

This unit is primarily concerned with Indirect Proof (IP) and Strengthened Rule of Conditional Proof (SCP). Let us explain each of these in turn

8.3 INDIRECT PROOF

What then is Indirect Proof? Is indirect proof a concept or method? According to Irving M. Copi, indirect proof is a method. It is called the method of proof by reductio ad absurdum. The method of proof by reduction ad absurdum besides logic is equally effective in elementary geometry. While dealing with theorems, Euclid often begins by assuming the opposite of what he wants to prove. As the given is true or valid in case of proof, its negation always leads to contradiction. Or simplistically, it can be said that negation of the given in case of indirect proof always leads to absurdity.

For example, if the given is 'p v ~ p' which is valid or tautology without exception, then its negation leads to contradiction or absurdity. The negation of 'p v ~ p' is "p · ~ p'. 'p · ~ p' is a contradiction. It is false without exception.

Thus, the method of indirect proof states that if the negation of the given leads into contradiction or so to speak absurdity, then it must be false. So its negation, i.e., the given must be a theorem and must be true as well. Thus the very nature and objective of indirect proof is clear. Indirect proof thus can be defined as a logical method where the validity or truth of the given is proved by way of negating or denying the given. This method is simplistic and effective in nature. It is simplistic because it is no longer complicated. The student can easily grasp and apply this method.



ACTIVITY- 8.1

- Do you find any difference between the concept of proof and the concept of test?

.....

- What do you mean by 'test the following'?

.....



CHECK YOUR PROGRESS

Q.1: The negation of the given in case of indirect proof always leads to absurdity (True/False).

Q.2: The negation of 'p v ~ p' is "p · ~ p' (True / False)

Q.3: Indirect proof is a concept (True / False)

Q.4: Indirect proof is a method (True / False)

Q.5: p · ~ p' is a contradiction (True / False).

Q.6: Define Indirect Proof.

8.4 WHAT IS A LOGICAL METHOD?

One has to know about the concept of method. We have said that indirect proof is a logical method. Philosophy is all about of methods. The Cartesian method, the phenomenological method, the linguistic method

are cases in point. Likewise, there are various methods applied in logic. Logic as a branch of philosophy is full of methods. Indirect proof is one such method. The main objective of any philosophical or scientific or logical method is to unearth the real or truth. Indirect proof being a logical method tries to establish the truth of the given. Here the given is true. Indirect proof takes a methodological account to prove it true. It does indirectly and that is why it has been named as Indirect Proof.

Logic deals with proposition and argument. Every proposition must be either true or false. Every argument must be valid or invalid. Indirect proof, being a logical method, deals only with true proposition and valid argument. This is because it is a method of proof, not a method of test. Naturally, it has two important tasks. It is used to determine the truth of a proposition. It is also used to determine the validity of an argument.

A proposition according to this method would be determined as true if its negation leads into contradiction. This is so obvious. If the given is a tautology, or true, then its negation would be self-contradictory, or in short, contradictory. Just like, if the proposition P is true, then its negation, i.e., ~ P would be false. So while determining the truth of a proposition by the method Indirect proof, we have to start with the negation of the given and then find out whether it leads to a contradiction or absurdity. Consider the following to make this point clear.

8.5 THE USE OF INDIRECT PROOF IN PROPOSITION

Let us now take an example to demonstrate the use of Indirect Proof in proposition.

$(p \supset q) \supset [(q \supset r) \supset (p \supset r)]$ is given.

Now we start with the method of Indirect proof just by way of denying or negating the given:

1.	$\sim \{(p \supset q) \supset [(q \supset r) \supset (p \supset r)]\}$	(~ given)
2.	$\sim \{\sim (p \supset q) \vee [(q \supset r) \supset (p \supset r)]\}$	1, Implication.
3.	$\sim \sim (p \supset q) \cdot \sim [(q \supset r) \supset (p \supset r)]$	2, De. Morgan.
4.	$(p \supset q) \cdot \sim [(q \supset r) \supset (p \supset r)]$	3, D.N.

5.	$p \supset q$	4, Simp.
6.	$\sim [(q \supset r) \supset (p \supset r)] \cdot (p \supset q)$	4, Com.
7.	$\sim [(q \supset r) \supset (p \supset r)]$	6, Simp.
8.	$\sim [\sim (q \supset r) \vee (p \supset r)]$	7, Impl.
9.	$\sim \sim (q \supset r) \cdot \sim (p \supset r)$	8, De. M.
10.	$\sim \sim (q \supset r)$	9, Simp.
11.	$q \supset r$	10, D.N.
12.	$\sim (p \supset r) \cdot \sim \sim (q \supset r)$	9, Com.
13.	$\sim (p \supset r)$	12 Simp.
14.	$\sim (\sim p \vee r)$	13 Impl.
15.	$\sim \sim p \cdot \sim r$	14, De. M.
16.	$p \cdot \sim r$	15 D.N.
17.	p	16, Simp.
18.	q	5, 17, M.P.
19.	r	11, 18, M. P.
20.	$\sim r \cdot p$	16, Com.
21.	$\sim r$	20, Simp.
22.	$r \cdot \sim r$	19, 21, Conj. Contradiction, Proved.

The above derivation clearly reflects the methodological affectivity of the **Method of Indirect Proof.** The proof is started with the negation of the given in step no. 1. It has ended with a contradiction or absurdity in step no.22. This logical method suggests that if the negation of a given leads into a contradiction then it would indirectly prove that the given is a tautology. This is exactly what is done in the above case. Let us now pass on to apply this method in the case of validity of an argument.

8.6 THE USE OF INDIRECT PROOF IN VALIDITY OF ARGUMENT

Unlike the proposition, the indirect proof while applying in the case of validity of an argument is stated with an additional premise. The additional premise is the negation of the conclusion of the given argument. This is

one of the notable aspects of the method of indirect proof. Let us consider the following argument:

1.	$A \supset (B \cdot C)$	
2.	$(B \vee D) \supset E$	
3.	$D \vee A \ / \therefore E$	
4.	$\sim E$	I. P. (Indirect Proof)
5.	$\sim (B \vee D)$	2, 4, M. T.
6.	$\sim B \cdot \sim D$	5, De. M.
7.	$\sim D \cdot \sim B$	6, Com.
8.	$\sim D$	7, Simp.
9.	A	3, 8, D.S.
10.	$B \cdot C$	1, 9, M. P.
11.	B	10, Simp.
12.	$\sim B$	6, Simp.
13.	$B \cdot \sim B$	11, 12, Conj.

Contradiction, Proved.

In the above argument we have step no. 4 as an additional step. The proof is started with the denial of the conclusion and it takes it as an additional premise and ended with a plain contradiction. The contradiction is absurd. So the demonstration is complete. Thus, the argument is proved valid by indirect proof. The method of indirect proof is effective in strengthening our deductive system. In fact, the method of Indirect Proof along with other logical methods actually makes complete the proof system of deductive logic.

In Indian logic, the same method can be applied in the name of Tarka. In Indian logic there is a popular argument like the following:

Where there is smoke, there is fire.

The Hill is smoky

Therefore, the Hill is fiery.

Indian logicians take the method of Indirect Proof to make the argument firm. Here the conclusion is logically entailed from the premises. More alternatively, it can be said that if one affirms the meaning of the first and

second premises and at the same time denies the meaning of the conclusion, he will involve into a plain contradiction. More simplistically, it can be said that if both the premises are accepted as true and the conclusion is presumed as false, then we involve into a plain contradiction. Thus, there we find the similarity between the East and the West as far as from the methodological point of view. We can say that the concept of Tarka that we observe in Indian logic has the same methodological implication as the concept of Indirect proof that we notice in formal logic. Having said this, whether Indian logic would be regarded as formal logical system or not is altogether a different issue.

8.7 THE STRENGTHENED RULE OF CONDITIONAL PROOF

Before delving into this method, let us make clear about the logical implication of the term 'conditional'. What is conditional? What is the rule of conditionality in formal logic? Conditionality is something that can be comprehended with regard to 'if…then' form where the cause-effect relationship plays the all-important role. To talk of conditionality is to talk of casual conditionality. Human life is all about of conditionality. Everything within the empirical world is determined on the basis of conditionality. Thus, the concept of conditionality is a forceful concept that would determine our form of life. Nothing is unconditional in the empirical world. Everything is hypothetical and conditional.

Thus, when we talk of conditional proof, we ipso-facto mean a kind of proof framed with 'if …then' in the ordinary language. According to Copi, the method of Conditional Proof is applied 'only to arguments whose conclusions were conditional in form'. That is, if the conclusion of an argument is in the form of 'if…then', then it would be treated as Conditional argument and in such a case there remains the relevance of Conditional Proof.

It is important to note here that logic develops out of language. Logical language has its distinctive nature. Logical language must be propositional

language. Logic deals with proposition. Proposition is obtained from propositional language. Propositional language is abstracted out of language we use in our daily life. Therefore, there is no point in saying that language in general would be propositional language. Language expresses by means of sentences. Only those sentences which would express proposition are called logical language. Logical language is expressed by means of proposition. Every proposition must be either true or false. There are various truth-functional operators in logic on the basis of which one can determine the truth-value of composite or compound proposition. They are obtained from ordinary language as well. They are '·' (dot), 'v' (vel or a wedge), '⊃' (horseshoe), '≡' (equivalence), '~' (curl or a tilde). The first one is obtained from 'and', the second one from 'either-or', the third one from 'if …then', the fourth one from 'if and only if' and the fifth one from 'not'. With the help of proposition and the truth-functional operators, the language of first order logic has been developed. When we talk of Conditional Proof, we usually talk argument. Every valid argument in itself is conditional in nature. It states that an argument is valid if its conclusion is logically entailed or implied by its premise or premises. Here the term if registers the conditionality of the argument. Thus, our general perception is that an argument is by its very nature is conditional irrespective of its validity or invalidity. It is equally to be noted here that a statement would equally be treated as conditional as well. All implicative propositions are conditional in nature. Even we can say that every implicative statement or proposition is represented as a conditional argument. Therefore, when we deal with the Conditional Proof, we must take note of both conditional statement as well as conditional argument.

8.8 TYPES OF CONDITIONAL PROOF

There are two types of Conditional Proof, such as, Ordinary Conditional Proof and Strengthened Conditional Proof. Even though the theme of this section is designed in the form of Strengthened Conditional Proof, but it does not bear any sense without the concept of Ordinary Conditional Proof.

8.8.1 Ordinary Conditional Proof (OCP)

Ordinary Conditional Proof is a kind of conditional proof that is primarily concerned with explicit conditional. It is important to note here that not all conditionals are explicit in nature. Conditionals may be expressed both explicitly as well as implicitly. In the case of Ordinary Conditional Proof, the conditional must be explicit in nature. Accordingly, if the conditional is a plain implicative statement, it would then be treated as Ordinary Conditional. In such a case one may take the help of Ordinary Conditional Proof. The statement or proposition $(p \supset q) \supset [(q \supset r) \supset (p \supset r)]$ is a conditional statement where the condition is used or manifested explicitly. Therefore, one may use Ordinary Conditional Proof in such case. Thus, it can be said that Ordinary Conditional Proof is a logical method applied in the case of deduction where the proof of a conditional statement or proposition can be adequately justified provided that the conditional is **explicit in nature.** Let us apply Ordinary Conditional Proof in the case of the stated statement in the following:

$(p \supset q) \supset [(q \supset r) \supset (p \supset r)]$

1.	$p \supset q$	(assumption)
2.	$q \supset r$	(assumption)
3.	p	(assumption) / $\therefore$ r
4.	q	1, 3, M. P.
5.	r	2, 4, M. P. (Proved)

Here the given statement is in the form of explicit conditional and hence Ordinary Conditional Proof is applied here. We take $p \supset q$ as a premise from the antecedent of the given statement. Then again we pass on to the consequent of the given statement. As the consequent of the given statement is itself conditional in nature, we again have taken $q \supset r$ as the second premise. Again we pass on to the consequent of the second, i.e., $p \supset r$, which again is explicit conditional. Again we take p as the third premise from the antecedent of $p \supset r$. Now as residue our conclusion here is r. So we have to deduce r from the supposed premises. This has exactly

been done in the above case. Thus, our all-important observation here is that if the conditional is in explicit form, we can apply Ordinary Conditional Proof. Having said this, in the case of implicit conditional, ordinary Conditional Proof does not function. In such a case we can take the help of Strengthened Conditional Proof. Now let us pass on to Strengthened Conditional Proof.

8.8.2 Strengthened Conditional Proof (SCP)

It has been stated that not all conditional are in the form of explicit conditional. Some conditional are in implicit nature. In such cases we may apply Strengthened Conditional Proof. It is further noted here that we have four binary logical operators with the help of these we can formulate conjunctive, disjunctive, implicative and equivalence statements or proposition. In the case of argument, Strengthened Conditional Proof can be applied just looking at the nature of the conclusion of the given argument. If the conclusion of the given argument is conjunctive, then there is no application of Conditional Proof. If the conclusion would be disjunctive, then one can find the sense of implicit conditional in the disjunctive form of Conditional because one has to know that the disjunctive proposition 'p v q' is identical with the implicative proposition ~ p $\supset$ q. This clearly reflects that p v q $\equiv$ ~p $\supset$ q. Thus, in our sense disjunction is an implicit form of implication or conditional. If the proposition would be implicative, then the implication is expressed in the form of explicit conditional. Therefore, there is no problem as far as Strengthened Conditional Proof is concerned. Finally, if the proposition would be in the form of equivalence, then again there underlies implicit conditional within the equivalence. In logic 'p equivalence q' is defined as '(p $\supset$ q) $\cdot$ (q $\supset$ p)'. It is called biconditional. Thus, biconditional is nothing but of twin conditional. Accordingly, there remains implicit conditionality in biconditional statement.

Thus, it seems that out of four different types of compound logical proposition, there is no use of Conditional Proof whatsoever in the case of argument of which the conclusion is in the form of conjunction. The other remaining three compound logical propositions are: disjunctive, implicative

and equivalence. All these are conditional in nature. In case of disjunctive and equivalence (biconditional) the conditional is used implicitly whereas in the case of implicative proposition, the conditional is explicit in nature. Conditional Proof starts with explicit conditional. In this sense Ordinary Conditional Proof covers only implicative proposition expressed by means of explicit conditional. However, to tackle with implicit conditional as we noted in the case of disjunctive and biconditional propositions, we must take the help of Strengthened Conditional Proof. Thus, when we pass on from Ordinary Conditional Proof to Strengthened Conditional Proof, we can relinquish the relevance or reference of Ordinary Conditional Proof and only stick to Strengthened Conditional Proof.

8.9 THE CONCEPT OF THE SCOPE OF THE ASSUMPTION

The scope of the assumption plays a vital role in case of Strengthened Conditional Proof. According to Copi, 'only a line inferred by the principle of Conditional Proof ends the scope of an assumption and that every use of the rule of Conditional Proof serves to end the scope of an assumption'. Let us make this point clear with the help of an example:

1.	$A \supset B$	
		/ $\therefore A \supset (A \cdot B)$
2.	A	(assumption)
3.	B	1, 2.M.P.
4.	$A \cdot B$	2, 3, Conj.
5.	$A \supset (A \cdot B)$	2-4, C. P.

In the above step no 5 is inferred not from any one or two of the preceding lines, but from the sequence of lines 2, 3, 4, which constitutes a valid deduction of line 4 from lines 1 and 2. By step 5, we infer the validity of the above argument from the demonstrated validity of the argument where an additional premise (in step no.2) is created as an assumption. In the above demonstration step 2 is taken as an assumption of Conditional Proof

and the subsequent steps, such as, 3 and 4 are dependent upon it. Step 5 however is not dependent upon step 2, but only upon step1. Thus, in our case step 5 is outside or beyond the scope of the assumption made as step 2. According to Copi, when an assumption is made in a Conditional Proof of Validity its 'scope' is always limited, never extending all the way to the last line of the demonstration. Thus, assumptions and scope of assumptions are very important in understanding Conditional Proof. It should equally be noted that in the case of Conditional Proof, only a line inferred by the principle of Conditional Proof ends the scope of an assumption, and that every use of the rule of Conditional Proof serves to end the scope of an assumption. When the scope of an assumption has been ended, the assumption is said to have been discharged. In the above case, the assumption has been ended in step 4. Within the assumption everything has been justified with regard to the assumption. However, beyond the scope of the assumption nothing can be justified by reference to it or to any line lying between it and the line inferred by the rule of Conditional Proof that discharges it. According to Copi, it is the case that 'only lines lying between an assumption of limited scope and the line that discharges it, can be justified by reference to that assumptions'. An argument may require more than one assumption. Every assumption has its scope and the scope is limited. One assumption may be lying within the scope of another assumption. Scopes of different assumptions may follow each other. After one assumption of limited scope has been discharged, another such assumption may be made and then discharged. No assumption can embrace the conclusion of the argument. The conclusion of the argument always rests beyond its scope and independent of it.

Let us examine a more complicated argument to apprehend the nature of Conditional Proof and the scope of the assumption what we are talking of.

1. $(A \vee B) \supset [(C \vee D) \supset E]$ / $\therefore A \supset [(C \cdot D) \supset E]$
2. A (assumption 1)
3. $A \vee B$ 2, Add.

4.	(C V D) ⊃ E	1, 3, M. P.
5.	C · D	(assumption 2)
6.	C	5, Simp.
7.	C V D	6, Add.
8.	E	4, 7, M. P.

……

9.	(C · D) ⊃ E	5 - 8, C. P.

……

10.	A ⊃ [(C · D) ⊃ E]	2 - 9, C. P.

In the above argument, there are two assumptions taken for granted as Conditional Proof. The scope of the first assumption contains the steps from 2 - 9. Within the scope of the first assumption, there develops the second assumption containing the steps from 5 - 8. This clearly suggests that within the scope of one assumption other assumption may exist. Here the second assumption lies within the scope of the first assumption. The scope of the first assumption is marked from step 2 to step 9, whereas the scope of the second assumption is marked from step 5 to step 8. It is also important to note that one assumption may exist within the other assumption or may exist independently. When one assumption lies within the scope of another assumption, the assumption that lies within the scope of other assumption must be closed. In our case, the second assumption must be closed before the first assumption. However, if there are assumptions exist independently, then the assumptions shall be closed in hierarchical order.



ACTIVITY - 8.2

Do you think Logic as a branch of philosophy is full of methods?

.....

Explain the methodological affectivity of the Method of Indirect Proof.

.....



CHECK YOUR PROGRESS

Q 7: Indirect proof being a logical method tries to establish the truth of the given (True/ False).

Q 8: Every proposition must be True / False / either true or false.

Q 9: Every argument must be True or False / valid or invalid.

Q 10: Indirect proof deals only with true proposition and valid argument - Is it true?

Q 11: Logic deals with proposition / sentence.

Q 12: $p \supset q \equiv \sim p \vee q$ - Is it true?

Q 13: Give an example of Condi14tional statement.

Q 14: Define Ordinary Conditional Proof.



8.10 LET US SUM UP

- Ordinary Conditional Proof (OCP) is a kind of conditional proof that is primarily concerned with explicit conditional. It is important to note here that not all conditionals are explicit in nature. Conditionals may be expressed both explicitly as well as implicitly. In the case of Ordinary Conditional Proof, the conditional must be explicit in nature.
- Every proposition must be either true or false. Every argument must be valid or invalid. Indirect proof, being a logical method, deals only with true proposition and valid argument. This is because it is a method of proof, not a method of test.
- There are two types of Conditional Proof, such as, Ordinary Conditional Proof and Strengthened Conditional Proof.
- Strengthened Conditional Proof (SCP) can be applied just looking at the nature of the conclusion of the given argument. If the conclusion of the given argument is conjunctive, then there is no application of Conditional Proof. If the conclusion would be disjunctive, then one

can find the sense of implicit conditional in the disjunctive form of Conditional because one has to know that the disjunctive proposition 'p v q' is identical with the implicative proposition ~ p ⊃ q.

- Finally, it can be said even though a distinction between OCP and SCP is made, but after passing from OCP to SCP, one may forgo OCP and stick to SCP. SCP contains OCP. Even Copi remarks that there is nothing wrong to conceive SCP as the Rule of Conditional Proof.



8.11 FURTHER READING

1) Copi, I. M. 2004. Symbolic Logic. New Delhi: Prentice-Hall of India Private Limited.

2) Copi, I. M. 1990. Introduction to Logic. Eight editions, New York: Macmillan Publishing Company.

3) Das, K. L. 2011. Symbolic Logic in Bengali Medium. Sibmandir: Siliguri: Darjeeling: N. L. Publishers.

4) Klenk, V. 2009. Understanding Symbolic Logic. New Delhi: Dorling Kindersley.



8.12 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No 1: True

Ans to Q No 2: True

Ans to Q No 3: False

Ans to Q No 4: True

Ans to Q No 5: Indirect proof can be defined as a logical method where the validity or truth of the given is proved by way of negating or denying the given.

Ans to Q No 6: True.

Ans to Q No 7: Either True or False

Ans to Q No 8: Valid or invalid

Ans to Q No 9: True.

Ans to Q No 10: True.

Ans to Q No 11: Proposition

Ans to Q No 12: True.

Ans to Q No 13: $(p \supset q) \supset [(q \supset r) \supset (p \supset r)]$ is a conditional statement.

Ans to Q No 14: Ordinary Conditional Proof is a logical method applied in the case of deduction where the proof of a conditional statement or proposition can be adequately justified provided that the conditional is explicit in nature

8.13 MODEL QUESTIONS

A) Very short questions

Q 1: What is conditional proof?

Q 2: What is the distinctive mark of SCP?

Q 3: What is the technique of employing Indirect Proof?

Q 4: Is Indirect Proof a logical concept or a logical method?

Q 5: Is Conditional Proof a logical concept or logical method?

Q 6: In what sense Disjunction can be read as Implication?

Q 7: In what sense equivalence can be read as conditional?

Q 8: What is the technique of employing Indirect Proof?

B) Short questions (Answer each question in about 100-150 words)

Q 1: Distinguish between OCP and SCP?

Q 2: Why is CP attributed as SCP? Explain.

Q 3: Distinguish between implicit and explicit conditions.

Q 4: Why does Indirect Proof begin with negation of the given? Briefly explain.

Q 5: What is the relevance of Indirect Proof? Briefly explain.

Q 6: What is the relevance of Conditional Proof? Briefly explain.

Q 7: Is Indirect Proof a logical concept or a logical method? Briefly explain.

Q 8: Is Conditional Proof a logical concept or logical method? Briefly explain.

Q 9: In what sense Disjunction can be read as Implication? Briefly explain.

Q 10: In what sense equivalence can be read as conditional? Briefly explain

Q 11: Why does conjunction not be treated as conditional like disjunction? Briefly explain

Q 12: Use the technique of Indirect Proof to verify that the following are tautologies.

(a) $A \supset [A \,.\, (A \vee B)]$

(B) $(A \supset B) \vee (\sim A \supset C)$

(C) $(A \supset B) \vee (B \supset A)$

Q 13: Use the method of Conditional Proof to verify that the following are tautologies:

(a) $[P \supset (Q \supset R)] \supset [(P \supset Q) \supset (P \supset R)]$

(b) $[P \supset (Q \supset R)] \supset [Q \supset (P \supset R)]$

(c) $[(A \supset B) \supset B] \supset (A \vee B)$

Q 14: Use the techniques of Indirect Proof and Conditional Proof to verify that the following arguments are valid:

(a) $A \supset B \;\; / \therefore A \supset (A \cdot B)$

(b) $A \supset B$

$[A \supset (A \,.\, B)] \;\; C \;\; / \;\; C$

(c) $A \supset B$

$B \supset [(C \supset \sim \sim C) \supset D] \;/\; A \supset D$

Q 15: What is Indirect Proof? Explain the process of the application of indirect proof in proposition with a suitable example.

Q 16: Explain the use of indirect proof in validity of argument with a suitable example.

Q 17: What is conditional? Describe the strengthened rule of conditional proof.

*** ***** ***

UNIT-9: SINGULAR PROPOSITION AND GENERAL PROPOSITION

UNIT STRUCTURE

9.1 Learning Objectives

9.2 Introduction

9.3 Singular Proposition

9.4 General Proposition

9.5 Traditional Subject-predicate Propositions

9.6 Let Us Sum Up

9.7 Further Reading

9.8 Answer to Check Your Progress

9.9 Model Questions

9.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- define what is proposition
- explain the difference between proposition and sentence
- define what is singular proposition
- define what is general proposition
- discuss four-fold classification of proposition

9.2 INTRODUCTION

We live in a world where events, facts, etc. go on occurring. The events or facts are described and judged by the use of language. The judgments concerning these facts or events may either be true or false. Propositions are the expressions of such judgments and hence are either true or false. In other words, propositions are statements which may either be true or false. The truth or the falsity of propositions is determined by the facts of experience or is self-evident. For example, the truth of the proposition "Rose is red" is determined by experience. Further, the truth of the proposition "Two plus two is four" is self-evident.

The propositions must not be confused with sentences. The following points differentiate the sentences from propositions:

"All propositions are sentences, but all sentences are not propositions". Because-

1. A proposition has three parts: subject, copula and predicate. But a sentence has two parts: subject and predicate.
2. A sentence may be of the following types: descriptive, imperative, interrogative, exclamatory etc. But a proposition can only be in a descriptive form.
3. A sentence may be in the past, present or future tense. But a proposition must be in the present tense; it can never be either in the past tense or in the future tense.
4. A sentence may be correct or incorrect, the rules of grammar determine this. But a proposition may be true or false, the facts of experience determine this.
5. In the standard form of proposition, the quality (affirmative or negative) and quantity (universal or particular) are clearly stated, but it is not so in a sentence.

The conception of a 'proposition' found in modern logic is markedly different from the concept found in traditional logic.

According to the traditional logicians a proposition is defined as the statement of a certain relation between two terms. For example- "Books are useful". Here, 'books' and 'useful' are two terms and a certain relation is stated between these two terms. The word 'are' works as a copula which is a sign of relation between two terms. Thus, a proposition consists of three parts, viz., the two terms and the sign of relation between the terms. One term is the subject, the other is the predicate and the sign of relation between the subject and the predicate is the copula.

The subject is that term of a proposition about which something is affirmed or denied. The predicate is that term of a proposition which states something about the subject. The copula is the sign of affirmation or denial. For example, in the proposition "Rose is red", 'rose' is the subject, 'red' is

the predicate and 'is' is the copula. Since the three parts are clearly stated, the proposition "Rose is red" is logical. According to the traditional logicians, the proposition like "Ram likes Laxman" is not logical since the copula is not clearly stated. This can be reduced to the logical form as "Ram is a person who likes Laxman", where 'Ram' is the subject, 'a person who likes Laxman' is the predicate, and 'is' is the copula.

Again according to the modern logicians, a proposition is said to be either true or false. A proposition can be either true or false, but it cannot be both true and false. In a modern proposition, there may have any number of parts or constituents or elements. These elements may be combined variously. Unlike a traditional proposition where in the name of simplification only one logical form was suggested, in a modern proposition there is not only one logical form. It may have diverse logical forms. Here, the logical form of the proposition depends on the way the constituent parts are related to each other.

9.3 SINGULAR PROPOSITION

A proposition in which the subject represents only a single individual, thing or entity is called as singular proposition. In other words, a proposition inwhich the predicate affirms or denies something of only one individual or thing is a singular proposition.A singular proposition is that where no truth functional connective is present. That is to say a singular proposition cannot contain logical connectives like 'and', 'either-or', 'if-then', 'if and only if' etc. Again it is not possible for a singular proposition to contain any quantity indicating words like 'all', 'every', 'some' etc. A singular proposition is that proposition which can be analyzed into terms, but not to propositions, because it is the simplest kind of non-compound proposition.

For example: Socrates is human.

It says that the individual Socrates has the attribute of being human. "Socrates" is the subject term in the present example, and "human" is the predicate term. The subject term denotes a particular individual; the predicate term designates some attribute that individual is said to have.

The same subject term, obviously, can occur in different singular propositions. One may assert that "Socrates is mortal," or "Socrates is fat," or "Socrates is wise," or "Socrates is beautiful." Of these assertions, some are true (the first and the third), and some are false (the second and fourth). Similarly, the very same predicate term can occur in different singular propositions. The term "human"is a predicate that appears in each of the following: "Aristotle is human," "Brazil is human," "Chicago is human," and "O'Keeffe is human"-of which the first and fourth are true, while the second and third are false. An "individual" in this symbolism can refer not only to persons, but to any individual thing, such as a country, a book, a city, or anything of which an attribute(such as "human" or "heavy") can be predicated. Attributes do not have to be adjectives (such as "mortal" or "wise") as in our examples thus far, but can also be nouns (such as "a human"). In grammar the distinction between adjective and noun is important, of course, but in this context it is not significant. We do not need to distinguish between "Socrates is mortal" and "Socrates is a mortal." Predicates can also be verbs, as in "Aristotle writes," which can be expressed alternatively as "Aristotle is a writer."

The Singular Propositions may be either Universal or particular.

A singular proposition is said to be universal if the subject is a definite singular term.

Examples: (a) This book is good.

(b) Shyam is honest.

(c) The sun is hot.

A singular proposition is said to be particular if the subject is an indefinite singular term.

Examples: (a) A boy is good.

(b) A tree is broken.

(c) One dog is not harmful.

Singular propositions can be symbolized. There are two different kinds of symbols for referring to individualsand to attributes. To denote individuals (following a very widely adopted convention) small, or lowercase,

letters, from 'a'to 'w' is used. These symbols are 'individualconstants'. In any particular context in which they may occur, each will designate one particular individual throughout the whole of that context. It is usually convenient to denote an individual by the first letter of its (or his or her) name. We may use the letter 's'to denote Socrates, 'a'to denote Aristotle, 'b'to denote Brazil, 'c'to denote Chicago, and so forth. Capital letters are used to symbolize attributes that individuals may have, and again it is convenient to use the first letter of the attribute referred to: 'H'for human, 'M'for mortal, 'F'for fat, 'W'for wise, and so forth.

By writing an attributesymbol immediately to the left of an individual symbol, we symbolize the singular proposition affirming that the individual named has the attribute specified. Thus the singular proposition, "Socrates is human," will be symbolized simply as 'Hs'. Of course, 'Ha'symbolizes "Aristotle is human," 'Hb'symbolizes "Brazil is human," 'Hc'symbolizes "Chicago is human," and so forth. It is important to note the pattern that is common to these terms. Each begins with the same attribute symbol, H, and is followed by a symbol for some individual, sor aor bor c, and so forth. We could write the pattern as "H_", where the dash to the right of the predicate symbol is a place marker for some individual symbol. This pattern we symbolize as 'Hx'. We use 'Hx'[sometimes written as H(x)] to symbolize the common pattern of all singular propositions that attribute "being human" to some individual. The letter 'x'is called an 'individualvariable'-it is simply a place marker, indicating where the various individual letters 'a'through 'w'(the individual constants) may be written. When one of those constants does appear in place of 'x', we have a singular proposition. The letter 'x'is available to serve as the variable because, by convention, 'a'through 'w'are the only letters we allow to serve as individual constants.

The symbol 'Hx'is called a propositional function. We define a propositional functionas an expression that (1) contains an individual variable and (2) becomes a statement when an individual constant is substituted for the individual variable. So a propositional function is not itself a proposition, although it can become one by substitution. Individual constants may be

thought of as the proper names of individuals. Any singular proposition is a substitution instance of a propositional function; it is the result of substituting some individual constant for the individual variable in that propositional function. A propositional function normally has some true substitution instances and some false substitution instances. If 'H'symbolizes 'human', 's'symbolizes 'Socrates', and 'c'symbolizes 'Chicago', then 'Hs'is true and 'Hc'is false. With the substitution made, what confronts us is a proposition; before the substitution is made, we have only the propositional function. There are an unlimited number of such propositional functions, of course: Hx, Mx, Bx, Fx, and Wx, and so on.

9.4 GENERAL PROPOSITION

A proposition in which the subject represents a group of individuals is called as general propositions. In other words, a proposition in which the predicate affirms or denies something of a part or the entire denotation of the subject term is a general proposition. A general proposition is a proposition about classes.

Thus, general propositions are of two types: Universal propositions andParticular propositions.

A general proposition is said to be universal if its predicate asserts (affirms or denies) something of the entire denotation of its subject.

For example:Everything is mortal.

A general proposition is said to be particular if its predicate asserts something of a partial denotation of its subject.

For example: Something is mortal.

The first general proposition, "Everything ismortal", may be expressed in various ways that are logically equivalent. It can also be expressedas "All things are mortal." It can be expressed by saying:Given any individual thing whatever, it is mortal.

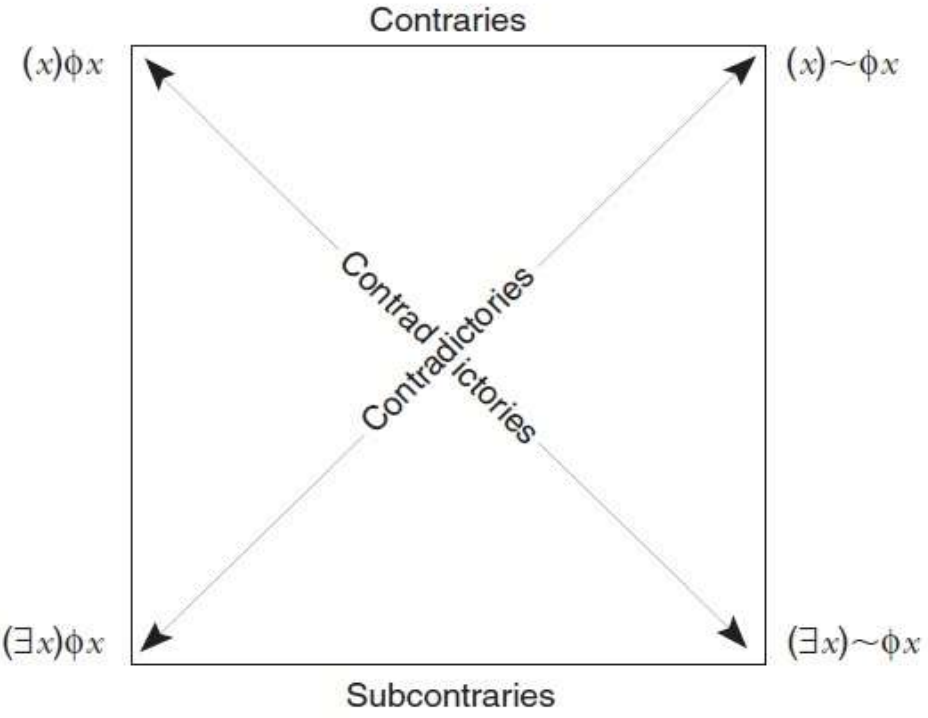
In this latter formulation the word "it" is a pronoun that refers back to the word "thing" that precedes it. We can use the letter 'x', our individual variable, in placeof both the pronoun and its antecedent. So we can rewrite

the first generalproposition as: Given any x, xis mortal.Alternatively, using the notation for predicates we may write: Given any x, Mx.We know that 'Mx'is a propositional function, not a proposition. But here, inthis last formulation, we have an expression that contains 'Mx', and that clearly isa proposition. The phrase "Given any x" is customarily symbolized by "(x)",which is called the universal quantifier. That first general proposition may nowbe completely symbolized as'(x) Mx'which says "Everything is mortal."This analysis shows that we can convert a propositional function into aproposition not only by substitution, but also by generalization, or quantification.

The second general proposition, "Somethingis mortal" may also be expressed as: There is at least one thing that is mortal.In this latter formulation, the word "that" is a relative pronoun referring to theword "thing." Using our individual variable 'x'once again in place of both thepronoun "that" and its antecedent "thing," we may rewrite the second generalproposition as: There is at least one xsuch that xis mortal.Or, using the notation for predicates, we may write: There is at least one xsuch that Mx. Although 'Mx'is a propositional function and not aproposition, we have here an expression that contains 'Mx'that isa proposition.The phrase "there is at least one xsuch that" is customarily symbolized as "$\exists$x";which is called the existential quantifier. Thus the second general propositionmay be completely symbolized as-'($\exists$x)Mx'which says "Something is mortal."

Thus we see that propositions may be formed from propositional functionseither by instantiation that is, by substituting an individual constant for its individualvariable, or by generalization, that is, by placing a universal or existentialquantifier before the propositional function.The universalquantification of a propositional function,(x)Mx, is true if and only if allthe substitution instances of the function are true;that is what universality means here. It is also clear that the existentialquantificationof a propositional function, ($\exists$x)Mx, is true if and only if its propositionalfunction has at least onetrue substitution instance.

A further relationship between universal and existential quantification can be shown by considering two additional general propositions, 'Something is not mortal' and 'Nothing is mortal', which are the respective negations of the first two general propositions just considered.'Something is not mortal' is symbolized as '(∃x)~Mx' and 'Nothing is mortal' is symbolized as '(x)~Mx'. These show that the negation of the universal quantification of a propositional function is logically equivalent to the existential quantification of the new propositional function that results from placing a negation symbol in front of the first propositional function. Where we use the Greek letter '∃'(phi) to represent any attribute symbol whatever, the general connections between universal and existential quantification can be described in terms of the following square array:



Assuming the existence of at least one individual, we can say that-

1. The two top propositions are contraries, that is, they might both be false but cannot both be true.
2. The two bottom propositions are subcontraries, that is, they can both be true but cannot both be false.

3. Propositions that are at opposite ends of the diagonals are contradictories, of which one must be true and the other false.
4. On each side, the truth of the lower proposition is implied by the truth of the proposition directly above it.

9.5 TRADITIONAL SUBJECT-PREDICATE PROPOSITIONS

The standard illustrations of the four types of general propositions are the following:

Universal affirmative(A):(All humans are mortal).

Universal negative(E): (No humans are mortal).

Particular affirmative(I): (Some humans are mortal).

Particular negative(O): (Some humans are not mortal).

Each of these types is commonly referred to by its letter: the two affirmative propositions, Aand I(from the Latin 'affirmo', "I affirm"); and the two negative propositions, Eand O(from the Latin 'nego', "I deny").

In symbolizing these propositions by means of quantifiers, we are led to a further enlargement of our conception of a propositional function. Turning first to the A proposition, "All humans are mortal," we proceed by means of successive paraphrasing, beginning with-

Given any individual thing whatever, if it is human then it is mortal.

The two instances of the pronoun "it" clearly refer to their common antecedent, the word "thing." Because those three words have the same (indefinite) reference, they can be replaced by the letter x and the proposition rewritten as-

Given any x, if x is human then x is mortal.

Now using our previously introduced notation for "if-then," we can rewrite the preceding as

Given any x, x is human $\supset$x is mortal.

Finally, using our now-familiar notation for propositional functions and quantifiers, the original A proposition is expressed as $(x)(Hx \supset Mx)$

In our symbolic translation, the A proposition appears as the universal quantification of a new kind of propositional function. The expression is a propositional function that has as its substitution instances neither affirmative nor negative singular propositions, but conditional statements whose antecedents and consequents are singular propositions that have the same subject term.

Among the substitution instances of the propositional function 'Hx⊃Mx'are the conditional statements'Ha⊃Ma', 'Hb⊃Mb', Hc⊃Mc, and so on. In symbolizing an A proposition, the parentheses serve as punctuation marks. They indicate that the universal quantifier (x) "applies to" or "has within its scope" the entire (complex) propositional function 'Hx⊃Mx'. The notion of the scope of a quantifier is very important, for differences in scope correspond to differences in meaning. The expression (x)(Hx⊃Mx) is a proposition which asserts that all substitution instances of the propositional function 'Hx⊃Mx' are true. On the other hand, the expression '(x)(Hx⊃Mx)' is a propositional function whose substitution instances are '(x)(Ha⊃Ma)', '(x)(Hb⊃Mb)', '(x)(Hc⊃Mc)', and so forth.

The E proposition 'No humans are mortal' may similarly be paraphrased successively as-

Given any individual thing whatever, if it is human, then it is not mortal.

Given any x, if x is human, then x is not mortal.

Given any x, x is human ⊃ x is not mortal.

It is symbolized as- (x)(Hx ⊃~Mx)

Similarly, the I proposition 'Some humans are mortal', may be paraphrased as-

There is at least one thing that is human and mortal.

There is at least one thing such that it is human and it is mortal.

There is at least one x such that x is human and x is mortal.

There is at least one x such that x is human • x is mortal.

It is symbolized as- (∃x)(Hx•Mx)

Finally, the O proposition 'Some humans are not mortal', becomes

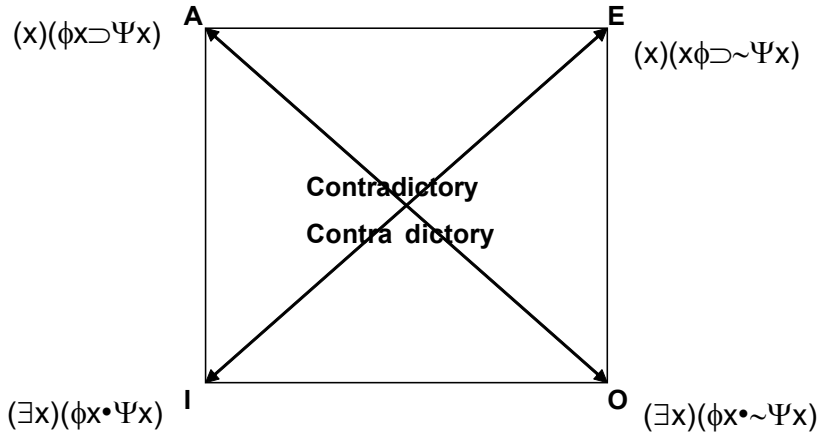
There is at least one thing that is human but not mortal.

There is at least one thing such that it is human and it is not mortal.

There is at least one x such that x is human and x is not mortal.

It is symbolized as- $(\exists x)(Hx \bullet \sim Mx)$

Where the Greek letters 'phi' and 'psi' are used to represent any attribute symbols whatever, the four general subject-predicate propositions of traditional logic may be represented in a square array as-



Of these, the A and the O are contradictories, and the E and the I are contradictories also. But the relation of contraries and sub contraries does not hold for the traditional A,E,I and O propositions, even if we assume that there is at least one individual in the universe. Where 'ϕx' is a propositional function that has no true substitution instances, then regardless of what attribute is symbolized by 'Ψ', the propositional functions '$\phi x \supset \Psi x$' and '$\phi x \supset \sim \Psi x$' have only true substitution instances. This is so because all their substitution instances are conditional statements with false antecedents. In such cases both the A and E propositions that are the universal quantifications of these complex propositional functions are true, so A and E propositions are not contraries. Again where 'ϕx' is a propositional function that has no true substitution instances, then, regardless of what 'Ψx' might be, the propositional functions '$\phi x \bullet \Psi x$' and '$\phi x \bullet \sim \Psi x$' have only false substitution instances, for their substitution instances are conjunctions whose first conjuncts are false. In such cases the I and O propositions that are the existential quantifications of these complex propositional functions

are false. Thus I and O propositions are not sub contraries. In all such cases, then, since the A and E propositions are true and the I and O propositions are false, the truth of a universal does not imply the truth of the corresponding particular; no implication relation holds between them.

If we make the assumption that there is at least one individual, then '(x)(ϕx$\supset\Psi$x)' does imply '($\exists$x)(ϕx•Ψx)'. The latter, however, is not an I proposition. An I proposition of the form 'Some ϕ's are Ψ's is symbolized as '($\exists$x)(ϕx•Ψx)', which asserts that there is at least one thing having both the attribute ϕ and the attribute Ψ. But the proposition '($\exists$x)(ϕx$\supset\Psi$x)' asserts only that there is at least one object that either has the attribute Ψ or does not have the attribute ϕ, which is a very different and much weaker assertion.

The four traditional subject-predicate forms A, E, I and O are not the only forms of general propositions. There are others that involve the quantification of more complicated propositional functions. Thus the general proposition 'All members are either parents or teachers', which does not mean the same as 'All members are parents or all members are teachers', is symbolized as '(x) [Mx$\supset$(Px v Tx)]'. And the general proposition 'Some Senators are either disloyal or misguided', is symbolized as '($\exists$x) [Sx• (Dx v Mx)]'. It should be observed that such a proposition as 'Apples and bananas are nourishing' can be symbolized either as the conjunction of two A propositions, '[(x) (Ax $\supset$ Nx)] • [(x) (Bx $\supset$ Nx)]', or as a single noncompound general proposition, '(x) [(Ax v Bx) $\supset$ Nx]'. It should not, however, be symbolized as '(x) [(Ax • Bx) $\supset$ Nx]', since to say that apples and bananas are nourishing is to say that anything is nourishing which is either an apple or a banana, not that anything is nourishing which is both an apple and a banana. It must be emphasized that there are no mechanical rules for translating statements from English into our logical notation. In every case one must understand the meaning of the English sentence and then re-express that meaning in terms of propositional functions and quantifiers.



CHECK YOUR PROGRESS

Q.1. Objective type questions.

(a) Symbolise the singular proposition 'Aristotle is human'.

(b) Symbolise the general proposition 'All humans are mortal'.

(c) Symbolise the proposition'Some humans are not mortal'

(d) 'The sun is hot' is an example of which proposition?

Q.2. Fill in the blanks.

(a) A proposition in which the predicate affirms or denies something of a part or the entire denotation of the subject term is a _____ proposition

(b) All propositions are sentences, but all sentences are not _____.

(c) Two contrary propositions cannot be both_____ together.

(d) A proposition has three parts: subject, copula and_____.

Q.3. State whether the following statements are true or false.

(a) All sentences are propositions. (True/False)

(b) A proposition in which the subject represents only a single individual, thing or entity is called as singular proposition. (True/False)

(c) The traditional four-fold classification of propositions are termed as A,E,I and O. (True/False)

(d) Two sub-contrary propositions cannot be both false together. (True/False)

Q.4. Answer the following questions.

(a) What is propositional function?

(b) What are the two processes by which we can obtain propositions from propositional function?

(c) What is individual constant?

(d) What is individual variable?



9.6 LET US SUM UP

- All propositions are sentences, but all sentences are not propositions.
- Individual constant is asymbol (byconvention, normally alower case letter, athrough w) which is used inlogical notation todenote an individual.
- Individual variable is a symbol (byconvention, normally thelower case x or y) thatserves as a placeholderfor an individualconstant.
- Propositional functionis an expression thatcontains an individualvariable and becomes astatement when thatvariable is replaced withan individual constant. Apropositional functioncan also become astatement by theprocess ofgeneralization.
- A proposition in which the subject represents only a single individual, thing or entity is called as singular proposition. In other words, a proposition in which the predicate affirms or denies something of only one individual or thing is a singular proposition.
- A proposition in which the subject represents a group of individuals is called as general propositions. In other words, a proposition in which the predicate affirms or denies something of a part or the entire denotation of the subject term is a general proposition.
- Propositions may be formed from propositional functions either by instantiation that is, by substituting an individual constant for its individual variable, or by generalization, that is, by placing a universal or existential quantifier before the propositional function.
- The traditional four-fold classification of propositions are termed as A,E,I and O. A means universal affirmative, E means universal negative, I means particular affirmative and O means particular negative.



9.7 FURTHER READING

1) Copi, I.M (1979), Symbolic Logic, Noida: Pearson India Education Services Pvt. Ltd

2) Angell, R.B (1964), Reasoning and Logic, New York: Meredith, publishing company

3) Rao, K.O.N. (2012), Logic for Degree Students, Ludhiana: Kalyani Publishers

9.8 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q. No 1:

(a) Ha

(b) (x)(Hx⊃Mx)

(c) (∃x)(Hx•~Mx)

(d) Universal singular proposition

Ans to Q. No 2:

(a) General

(b) Propositions.

(c) True

(d) Predicate

Ans to Q. No 3:

(a) False.

(b) True

(c) True

(d) True

Ans to Q. No 4:

(a) Propositional function is an expression that contains an individual variable and becomes a statement when that variable is replaced with an individual constant.

(b) he two processes by which we can obtain propositions from propositional functions are-

1. Substitution or instantiation and
2. Generalisation.

(c) Individual constant is asymbol (by convention, normally a lower case letter, athrough w) which is used in logical notation to denote an individual.

(d) Individual variable is a symbol (by convention, normally the lower case x or y) that serves as a placeholder for an individual constant.

9.9 MODEL QUESTIONS

A. Objective questions

Q 1: What is propositional function?

Q 2: What are the two processes by which we can obtain propositions from propositional function?

Q 3: What is individual constant?

Q 4: What is individual variable?

Q 5: Symbolise the singular proposition 'Aristotle is human'.

Q 6: Symbolise the general proposition 'All humans are mortal'.

Q 7: Symbolise the proposition'Some humans are not mortal'

Q 8: 'The sun is hot' is an example of which proposition?

B. Short questions (Answer in about 100-150 words)

Q 1: What is singular proposition? Explain briefly

Q 2: What is singular proposition? Explain briefly

Q 3: Explain briefly the concept of square of opposition of propositions.

Q 4: Explain briefly traditional subject-predicate propositions.

C) Long questions (Answer in about 300-500 words)

Q.1: Discuss briefly singular proposition.

Q.2: 'All propositions are sentences, but all sentences are not propositions'. Explain.

Q.3: Explain briefly the concept of general proposition.

Q.4: Explain the four-fold classification of propositions.

*** ***** ***

UNIT-10: QUANTIFICATION AND RULES OF QUANTIFICATION

UNIT STRUCTURE

10.1 Learning Objectives

10.2 Introduction

10.3 Universal and Existential Quantifiers

10.4 Modern Square of Opposition

10.5 Symbolization of Four Categorical Statements with Quantifiers

10.6 Symbolization of Universal and Existential Propositions

10.7 Symbolization of Propositions by the Process of Instantiation

10.8 Rules of Quantification

10.9 Proving Validity

10.10 Let Us Sum Up

10.11 Further Reading

10.12 Answers to Check Your Progress

10.13 Model Questions

10.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- define what is predicate logic
- define what is quantification
- explain the types of quantifiers
- discuss the rules of quantification
- discuss the procedures of proving validity of arguments

10.2 INTRODUCTION

Quantification is a twentieth century development which is chiefly credited to GottlobFrege (1848-1945), a great German logician. But the word 'quantification' was introduced by Charles S. Peirce. Quantification falls under the category of predicate logic and not propositional logic.

Predicate logic is defined as that part of symbolic logic where an internal structure of the proposition (both simple and compound) is to be analysed in order to determine the validity of a given argument. Quantification is the process of obtaining a proposition from a propositional function. A propositional function is an expression that contains an individual variable and becomes a statement when an individual constant is substituted for the individual variable. There are two ways of obtaining propositions from propositional functions. One is called Instantiation or Substitution. The other is called Quantification. There are different phrases which are employed to indicate quantity. These phrases are called quantifiers. Words that indicate quantity such as 'all', 'every', 'most' are called quantifiers. Though there are many numbers of quantifiers in English, standard logic uses only two: the maximal 'all' and the minimal 'some'. There are two types of quantifiers: Universal Quantifier and Existential Quantifier. Quantifiers state how many of the individuals or the objects have the property in question. They do not say anything about which one of the individuals have the property.

10.3 UNIVERSAL AND EXISTENTIAL QUANTIFIERS

Universal Quantifiers- A universal quantifier can be defined as an expression that, when prefixed to a propositional function, yields a universal proposition. Examples of universal propositions are:

Everything is dynamic.

Every student is good.

All things are temporary.

Statement which contain words like 'all', 'every', 'each', 'any', 'everything', 'anything' or any word conveying the sense of 'all' are symbolized by the aid of universal quantifier. Let's take the general proposition- 'Everything is temporary'.

It can be expressed as: Given any individual thing whatever, it is temporary.

We can use 'x' (an individual variable) in place of both the pronoun 'it' and its antecedent 'thing'. We can paraphrase the proposition as: Given any x, x is temporary.

Here we have got a property, i.e. 'being temporary'. In order to symbolize this property, as it is already mentioned in the previous section, we need predicate symbol. We can symbolize the property 'being temporary' as 'Tx'. The phrase 'given any x' is called a universal quantifier. 'Given any x' is conventionally symbolized as '(x)'. The general proposition 'everything is temporary' can be symbolized as: (x) Tx

Existential Quantifiers- An existential quantifier can be defined as an expression that, when prefixed to a propositional function, yields an existential proposition. The existential quantifier '(∃x)' is used to assert that some entities (at least one) have a given property. Thus to symbolize the proposition "Something is light" or the proposition "At least one thing is light", start with the proposition from 'Lx' and prefix an existential quantifier to it. The result is the sentence (∃x)Lx, read "For some x, x is light", or "There is at least one x such that x is light", or just "Some x is light". A few more examples are:

Something is expensive. (∃x)Ex.

Something is important. (∃x)Ix.

Let us take a negative proposition:

"Some things are not temporary." It can be written as- There is at least one x, such that x is not temporary.

Here one important point is to be noted that the 'not' appears before the predicate, it does not appear in the quantity part. With the help of existential quantifier it can be translated as: (∃x)~Tx

10.4 MODERN SQUARE OF OPPOSITION

The traditional logicians 'Square of Opposition' are criticized by the modern logicians in relation to the concept of existential import. We know that A and E propositions do not have existential import, but I and O propositions do have the existential import. The result of the acceptance of existential import by the modern logicians led them come to the conclusion that only one relation, i.e. 'Contradictory' among the propositions is valid. Other relations contrary, sub-contrary, subaltern are invalid.

Let us first discuss the contrary relation. The traditional account of the contrary relation states that propositions 'All S is P' and 'No S is P' cannot be true together, though both of them can be false together. We know that A and E propositions do not have existential import. Because of the absence of existential import in A and E propositions, according to the modern logicians, A and E both cannot be true together. Modern logicians therefore hold the view that contrary relation is to be given up.

Again, we know that the traditional account of subcontrary relation between 'Some S is P' and 'Some S is not P' states that they may be true together, but cannot be false together. According to the Boolean interpretation, I and O propositions have existential import and because of the existential import, I and O can be false together. It follows that subcontrary relation is to be given up according to the modern logicians.

According to the Boolean interpretation in the Square of Opposition, only the relation of contradiction is valid. The following equivalences are valid in the modern square of opposition:

A = ~O

E = ~I

I = ~E

O = ~A

These equivalences can be employed to establish the equivalence of the universal and the existential quantifiers in the following way:

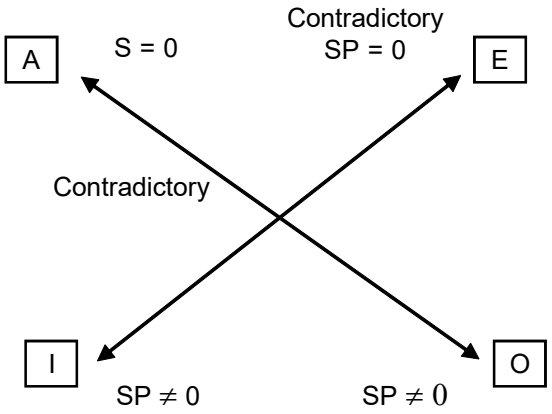
$[(x) (Sx \supset Px)] = \sim [(\exists x) (Sx \bullet \sim Px)]$

$[(x) (Sx \supset \sim Px)] = \sim [(\exists x) (Sx \bullet Px)]$

$[(\exists x) (Sx \bullet Px)] = \sim [(x) (Sx \supset \sim Px)]$

$[(\exists x) (Sx \bullet \sim Px)] = \sim [(x) (Sx \supset Px)]$

The following diagram will represent the modern square of opposition:



These four equivalences are known as Quantifier Negation (Q.N) Equivalences.

10.5 SYMBOLIZATION OF FOUR CATEGORICAL STATEMENTS WITH QUANTIFIERS (A, E, I AND O)

On the basis of quality and quantity, traditional logicians classified propositions into four types: A, E, I and O. Let us take examples for each of these propositions:

Universal affirmative(A):(All humans are mortal).

Universal negative(E): (No humans are mortal).

Particular affirmative(I): (Some humans are honest).

Particular negative(O): (Some humans are not mortal).

Each of these types is commonly referred to by its letter: the two affirmative propositions, Aand I(from the Latin 'affirmo', "I affirm"); and the two negative propositions, Eand O(from the Latin 'nego', "I deny").

Universal affirmative (A)

All humans are mortalcan be written as-

Given any individual thing whatever, if it is human then it is mortal.

Given any x, if x is human then x is mortal.

Given any x, x is human ⊃x is mortal.

It is symbolized as- (x)(Hx⊃Mx)

Universal negative (E)

No humans are mortalcan be written as-

Given any individual thing whatever, if it is human, then it is not mortal.

Given any x, if x is human, then x is not mortal.

Given any x, x is human $\supset$ x is not mortal.

It is symbolized as- $(x)(Hx \supset \sim Mx)$

Particular affirmative (I)

Some humans are honestcan be written as-

There is at least one thing that is human and honest.

There is at least one thing such that it is human and it is honest.

There is at least one x such that x is human and x is honest.

There is at least one x such that x is human • x is honest.

It is symbolized as- $(\exists x)(Hx \bullet Mx)$

(In this proposition both the words 'human' and 'honest' start with the letter 'H'. Therefore in order to distinguish between these two words, we can take 'H' for the word 'human' and 'M' for the word 'honest'.

Particular negative (O)

Some humans are not mortal can be written as-

There is at least one thing that is human but not mortal.

There is at least one thing such that it is human and it is not mortal.

There is at least one x such that x is human and x is not mortal.

It is symbolized as- $(\exists x)(Hx \bullet \sim Mx)$

This is the way of symbolizing A, E, I and O propositions with the help of quantifiers.

10.6 SYMBOLIZATION OF UNIVERSAL AND EXISTENTIAL PROPOSITIONS

Universal and existential propositions are symbolized by the process of generalization by using quantifiers. There are two quantifiers, namely, universal quantifier and existentialquantifier. The universal quantifier is symbolically expressed as '(x)' and theexistentialquantifier as '($\exists$ x)'.

Among the A,E,I and O propositions 'A' and 'E' propositions are symbolized by using the universal quantifier '(x)' and 'I' and 'O' propositions are symbolized by using the existentialquantifier '($\exists$x)'.

Universal proposition-

1.	Everything is movable.	Symbolic form: (x) Mx
2.	Nothing is permanent.	Symbolic form: (x) ~ Px
3.	Only teachers are invited.	Symbolic form: (x) (Ix ⊃Tx)
4.	None but citizens are eligible to vote.	Symbolic form: (x) (Ex ⊃Cx)
5.	All bananas and mangoes are sweet.	Symbolic form: (x) [(Bx v Mx) ⊃Sx]

Existential proposition-

1.	Lions exist.	Symbolic form: (∃x) Lx
2.	Not everything is mortal.	Symbolic form: (∃x) ~ Mx
3.	Not all dogs are mammals.	Symbolic form: (∃x) (Dx• ~Mx)
4.	There are elephants.	Symbolic form: (∃x) Ex
5.	Unicorns do not exist.	Symbolic form: (∃x) ~ Ux

10.7 SYMBOLIZATION OF PROPOSITIONS BY THE PROCESS OF INSTANTIATION

Only singular propositions are symbolized by the process of instantiation. By writing an attributesymbol immediately to the left of an individual symbol, we symbolize the singular proposition affirming that the individual named has the attribute specified. Thus the singular proposition, "Socrates is human," will be symbolized simply as 'Hs'.

Let us take an example of a compound expression consisting of more than one singular proposition. For example, "Neeta is beautiful and intelligent". In this case, the first step is to symbolize the singular propositions separately following the rules of instantiation. In the second step, the logical constant, i.e. 'and' is to be symbolized. In the last step, the symbolized singular propositions are to be conjoined.

Here 'Neeta is beautiful' is a singular proposition. It is to be symbolized as 'Bn'. Again, 'Neeta is intelligent' is also a singular proposition. It is to be symbolized as 'In'. Now these two singular propositions are to be conjoined

with the help of the logical constant '.' (Dot). After symbolization of the compound expression it will take the form: Bn • In

10.8 RULES OF QUANTIFICATION

We now introduce four quantifier rules governing the use of quantifiers.

Universal Instantiation (U.I.)

The rule of Universal Instantiation (U.I.) permits us to drop the universal quantifier. It permits us to move from a universally quantified proposition to a proposition which is not quantified. From a quantified formula '(x)Rx' we allowed by the rule of UI to draw the conclusion 'Ra' or 'Rb' or 'Rc' etc. Let us consider the following example:

All philosophers are honest.

Bertrand Russell is a philosopher.

Therefore, Bertrand Russell is honest.

This argument is clearly a valid argument. It can be symbolized in the following way:

1. (x) (Px⊃Hx)
2. Pb/ ∴ Hb
3. Pb⊃Hb 1 UI (Universal Instantiation)

Then we can assert the conclusion

4. Hb 2, 3 MP

We call the rule of inference that permits the leap from line 1 to line 3 Universal Instantiation, because it yields an instance, (Pb⊃Ga), of the universal generalization (x) (Px⊃Hx).

It is important to note that a universal quantifier must quantify a whole line in a proof to be dropped by UI. Thus, UI cannot be applied to the line

1. (x) Px⊃ (∃x) Hy

To obtain

2. Pa ⊃ (∃x) Hy

Because the (x) quantifier in line 1 does not quantify the whole of that line.

Similarly, UI cannot be applied to the line

1. ~ (x) (Px⊃Hx)

To obtain

2. ~ (Pa ⊃ Ha)

Because the (x) quantifier in line 1 does not quantify the negation sign, and thus does not quantify the entire line.

The general rule for UI can be stated as:

$$\frac{(x)(Mx)}{\therefore My}$$

(where 'y' is an individual symbol)

Universal Generalisation (UG)

The rule of Universal Generalisation (UG)permits us to move from a statement about an individual whether named or taken at random, to a statement about all individuals in the universe. It allows us to move from an unquantified statement to a universally quantified statement. The general rule is presented in the following way:

$$\frac{My}{\therefore (x)(Mx)}$$

(where 'y' denotes "any arbitrarily selected individual. Any arbitrarily selected individual means what is true of an individual is proved to be true of any individual".) We may also state UG as: From the substitution instance of a propositional function with respect to the name of anyarbitrarily selected individual one can validly infer the universal quantification of that propositional function.

Let us consider the following example:

No mortals are perfect.

All humans are mortal.

. ? . No humans are perfect.

1. (x) (Mx⊃ ~ Px)
2. (x) (Hx⊃Mx) / (x) (Hx⊃ ~ Px)
3. Hy⊃ My 2 UI
4. My⊃ ~ Py 1 UI

5. Hy⊃ ~ Py 3,4 H.S
6. (x) (Hx⊃ ~ Px) 5 UG

Existential Generalisation (EG)

The rule of ExistentialGeneralisation (EG) allows us to proceed from an unquantified proposition to a proposition which is existentially quantified. If you have a statement like

Ku • ~ Ru

Then by the application of the rule of EG, it is possible to get

(∃z) (Kz • ~ Rz)

It is important here to note that the statement '(∃z) (Kz • ~ Rz)' can also be obtained from the following:

Kb • ~ Rb

There is an important exception about EG. Though it is a rule of inference, it is not possible in EG to replace some or at least one variable or constant, and necessarily all.

Thegeneral rule for UI can be stated as:

(where 'y' is an individual symbol)

Let us consider the following example:

All dogs are carnivorous.

Some animals are dogs.

∴ Some animals are carnivorous.

1. (x) (Dx⊃Cx)
   (∃x) (Ax • Dx) /∴ (∃x) (Ax • Cx)
2. Aw • Dw 2 EI
3. Dw⊃Cw 1 UI
4. Aw 3 Simp.
5. Dw 3 Simp.
6. Cw 4,6 MP
7. Aw • Cw 5,7 Conj.
8. (∃x) (Ax • Cx) 8 EG

Existential Instantiation (EI)

The rule of Existential Instantiation (EI)permits us to move from an existentially quantified proposition to a proposition which is not quantified. From a quantified formula '(∃x) Rx' we are allowed by the rule of EI to draw the conclusion 'Ry'. Let us consider the following example:

1. (x) (Hx⊃Kx)
2. (∃x) (Hx)/ ∴(?x) (Kx)
3. Hy2 EI (Existential Instantiation)
4. Hy⊃Ky 1 UI
5. Ky 4, 3 M.P
6. (∃x) (Kx) 5 EG

In this proof, line 3 follows from line 2 by the inference rule calledExistential Instantiation (EI).

The general rule for EI can be stated as:

$$\frac{(\exists x)(Mx)}{\therefore My}$$

(where 'y' is an individual constant, that is a substitute for a name. However, there is an important restriction to be placed on this rule, namely, that the individual constant 'y' must not have occurred previously in the proof in which this rule is applied).

10.9 PROVING VALIDITY

The four rules of quantification can be applied to prove the validity of arguments. In order to prove validity of a given argument the following steps are to be adopted.

1. The propositions of the given argument are to be symbolically represented first. In order to represent the propositions one is to adopt the rules of quantification. For example, the proposition "All men are mortal" should not be represented as 'p', because this type of symbolic representation is done in accordance with the rules of propositional logic. We must not forget that we are going to prove the validity of argument in predicate logic. Hence, the proposition is

to be symbolized as "(x) (Mx⊃Rx)" [M-Men, R-Mortal]. In other words, here we have followed the four rules of quantification. Sometimes, a proposition in a given argument may be a singular proposition. We need to follow the rules of symbolizing a singular proposition then.

2. The quantifiers of the propositions of the given argument are to be eliminated by the application of the rules of quantification.
3. The techniques applied for the construction of formal proofs of validity in propositional logic by means of the 19 elementary valid argument forms are to be adopted here and the process will continue until we reach the conclusion.
4. Quantifier is to be added in accordance with the necessity to reach the conclusion.

Example: I

All men are mortal.

All Indians are men.

∴ All Indians are mortal.

The given argument is to be symbolically represented following the rules of quantification.

(Step 1):

(x) (Mx?Rx)

(x) (Ix ?Mx)

∴ (x) (Ix ?Rx)

The validity of the argument can be shown in the following way:

1.	(x) (Mx⊃ Rx)	
2.	(x) (Ix ⊃Mx)	/ ∴(x) (Ix ⊃ Rx)
3.	My⊃ Ry	[1 UI]
4.	Iy⊃My	[2 UI]
5.	Iy⊃Ry	[4, 3 H.S]
6.	(x) (Ix ⊃ Rx)	[5 UG]

Here in line 3 the universal quantifier (x) is found to be dropped applying the rule U.I. to line 1 (step 2). Similarly, in line 4 the universal

quantifier (x) is found to be dropped by applying the rule U.I. to line 2 (step 2). Now we have to consider line 3 and 4 as premises and line 5 is derived via Hypothetical Syllogism (H.S) from these premises (step 3). The universal quantifier (x) is found to be added in line 6 (step 4) via rule U.G. to line 5.

Example: II

No athletes are bookworms.

Carol is a bookworm.

∴ Carol is not an athlete.

The given argument is to be symbolically represented by following the rules of quantification(step 1). The second premise, i.e., 'Carol is a bookworm' is a singular proposition and therefore this proposition is to be symbolized by following the rules of symbolizing a singular proposition. Same process of symbolization will have to be adopted in case of symbolization of the conclusion of the argument.

(x) (Ax ⊃ ~ Bx)

Bc

∴ Ac

The validity of the argument can be shown in the following way:

1. (x) (Ax ⊃ ~ Bx)
2. Bc / ∴ Ac
3. Ac⊃ ~ Bc [1 U I]
4. ~ ~ Bc [2 D. N]
5. ~ Ac [3, 4 M. T]

In this argument, rule U I has been applied in line 1 to eliminate the universal quantifier (x). Line 3 (step 2) is derived via U I. Line 4 is derived from line 2 (step 3) via rule of D N (Double Negation). Line 5 is derived from line 3 and 4 via rule of M T (Modus Tollens). Step 4 is not applicable to this argument, since the conclusion is a singular proposition.



CHECK YOUR PROGRESS

Q.1. Objective type questions.

(a) Name the logician who is credited for the discovery of quantification.

(b) Who introduced the word 'quantification'?

(c) Identify the proposition'Socrates is honest'.

(d) How many phrases are there to indicate quantity in quantification?

Q.2. Symbolize the propositions.

(a) Nothing is permanent in life.

(b) Ram is intelligent.

(c) Shylock is not honest.

(d) Shylock is cleaver and not honest.

(e) Ram is not intelligent but honest.

(f) Hari is either intelligent or a fool.

(g) Something is not desirable.

(h) Something is worth having.

(i) Men are useful creatures.

(j) No person can escape death.

Q.3. State whether the following statements are true or false.

(a) A singular proposition is a substitution instance of a propositional function.

(b) I' and 'O' propositions are symbolized by using the existential quantifier '(∃x)'.

(c) The symbolic form of universal quantifier is '(∃x)'.

(d) Statement which contain words like 'all', 'every', 'each', 'any', 'everything', 'anything' or any word conveying the sense of 'all' are symbolized by the aid of existential quantifier.

Q.4. Answer the following questions.

(a) What are the ways of obtaining propositions from propositional function?

(b) What do you mean by Universal Quantifier?

(c) What is quantification?

(d) What do you mean by Existential Quantifier?



10.10 LET US SUM UP

- Quantification is a twentieth century development which is chiefly credited to GottlobFrege (1848-1945), a great German logician. But the word 'quantification' was introduced by Charles S. Peirce. Quantification falls under the category of predicate logic and not propositional logic.
- There are two ways of obtaining propositions from propositional functions. One is called Instantiation or Substitution. The other is called Quantification.
- There are two types of quantifiers: Universal Quantifier and Existential Quantifier.
- A universal quantifier can be defined as an expression that, when prefixed to a propositional function, yields a universal proposition. Statement which contain words like 'all', 'every', 'each', 'any', 'everything', 'anything' or any word conveying the sense of 'all' are symbolized by the aid of universal quantifier. The symbolic form of universal quantifier is '(x)'.
- An existential quantifier can be defined as an expression that, when prefixed to a propositional function, yields an existential proposition. The existential quantifier '(∃x)' is used to assert that some entities (at least one) have a given property.
- Among the A,E,I and O propositions 'A' and 'E' propositions are symbolized by using the universal quantifier '(x)' and 'I' and 'O' propositions are symbolized by using the existential quantifier '(∃x)'.
- Only singular propositions are symbolized by the process of instantiation. By writing an attributesymbol immediately to the left of an individual symbol, we symbolize the singular proposition affirming that the individual named has the attribute specified.
- There are four rules of quantification. They are-Universal Instantiation (U.I.), Universal Generalisation (UG), Existential Instantiation (EI), ExistentialGeneralisation (EG).

10.11 FURTHER READING

1) Sharma, B., Deka, J. (2014) A text book of Logic, 3rd Ed., Guwahati: AankBaak.

2) Chakraborti, C. (2007) Logic: Informal, symbolic and Inductive, 2nd Ed., New Delhi: Prentice Hall of India Pvt. Ltd.

3) Pollock, J.L. (1969) An Introduction to Symbolic Logic, New York: Holt, Pinehart and Winston, Inc.

4) Gustason, W., Ulrich, D.E. (1973) Elementary Symbolic Logic, New York: Holt, Pinehart and Winston, Inc.

10.12 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q No 1:

(a) GottlobFrege is credited for the discovery of quantification.

(b) The word 'quantification' was introduced by Charles S. Peirce.

(c) Singular proposition.

(d) There are two phrases to indicate quantity in quantification.

Ans. to Q No 2:

(a) (x) ~ Px

(b) Ir

(c) ~ Hs

(d) Cs • ~ Hs

(e) ~ Ir • Hr

(f) Ih v Fh

(g) (∃x) ~ Dx

(h) (∃x) Wx

(i) (x) (Mx⊃Ux)

(j) (x) (Px⊃ ~ Ex)

Ans. to Q No 3:

(a) True

(b) True

(c) False

(d) False

Ans. to Q No 4:

(a) There are two ways of obtaining propositions from propositional function.

1. Instantiation and 2. Quantification

(b) A universal quantifier can be defined as an expression that, when prefixed to a propositional function, yields a universal proposition.Statement which contain words like 'all', 'every', 'each', 'any', 'everything', 'anything' or any word conveying the sense of 'all' are symbolized by the aid of universal quantifier. The symbolic form of universal quantifier is '(x)'.

(c) Quantification is the process of obtaining a proposition from a propositional function.

(d) An existential quantifier can be defined as an expression that, when prefixed to a propositional function, yields an existential proposition. The existential quantifier '(∃x)' is used to assert that some entities (at least one) have a given property.

10.13 MODEL QUESTIONS

A) Objective questions

Q 1: What is quantification?

Q 2: What are the types of quantifiers?

Q 3: What is existential quantifier?

Q 4: What is universal quantifier?

Q 5: State the symbol of universal quantifier.

Q 6: State the symbol of existential quantifier.

B) Short questions (Answer in about 100-150 words)

Q.1: Distinguish between instantiation and quantification.

Q.2: State the distinction between a propositional function and proposition.

Q 3: Explain briefly the concept of quantification.

Q 4: What is universal quantifier? Explain briefly.

Q 5: What is existential quantifier? Explain briefly.

Q.6: Write a short note on: Instantiation.

C) Long questions (Answer in about 300-500 words)

Q.1: Describe how to symbolize a singular proposition.

Q.2: Explain the rules of Quantification.

Q.3: Explain how traditional A,E,I,O propositions are symbolized with the help of quantifiers.

Q.4: Construct formal proof of validity for each of the following arguments.

(a) (x) (Kx⊃ Lx)
(x) [(Kx • Lx) ⊃Mx /∴(x) (Kx⊃Mx)

(b) (x) (Nx⊃ Ox)
(x) (Px⊃ Ox) /∴[(Nx v Px) ⊃ Ox]

(c) (x) (Qx⊃ Rx)
(∃x) (Qx v Rx) /∴(∃x) Rx

(d) (x) (Ax ⊃Bx)
~ Bt/ ∴At

(e) (x) (Cx⊃Dx)
(x) (Ex⊃ ~ Dx) /∴(x) (Ex⊃ ~ Cx)

(f) (x) (Fx⊃ ~ Gx)
(∃x) (Hx • Gx) /∴(∃x) (Hx• ~Fx)

(g) (x) [Sx⊃ (Tx⊃Ux)]
(x) [Ux⊃(Vx • Wx) /∴(x) [Sx⊃ (Tx⊃Vx)]

(h) (x) [(Xx vYx) ⊃ (Zx • Ax)]
(x) [(Zx v Ax) ⊃ (Xx • Yx)] /∴(x) (Xx ≡Zx)

*** ***** ***

UNIT-11: SET THEORY

UNIT STRUCTURE

11.1 Learning Objectives

11.2 Introduction

11.3 Definition of Set

11.4 Set Membership

11.5 Set Inclusion

11.6 Empty Set

11.7 Operations on Sets

- 11.7.1 Union of Sets
- 11.7.2 Intersection of Sets
- 11.7.3 Difference of Sets
- 11.7.4 Complement of Set

11.8 Laws of Operations on Set

11.9 Domain of Individuals

11.10 Using Set Notation for Logical Propositions (Categorical Propositions)

11.11 Let Us Sum Up

11.12 Further Reading

11.13 Answers to Check Your Progress

11.14 Model Questions

11.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- define what is predicate logic
- define what is set
- discuss the concepts of set membership, set inclusion, and empty set
- explain the concepts of union, intersection and difference of sets
- explain a categorical proposition by using some symbolic notations

11.2 INTRODUCTION

The concept of set is one of the most fundamental concepts in mathematics as well as modern logic. Though the application of the concept of set is simpler than arithmetical concept of set, yet it is not familiar to everyone. Many mathematicians and logicians face lots of difficulties in trying to formulate and give an exact definition of the term 'set'. The concept of set was developed at the end of the 19th century by German mathematician George Cantor (1845-1918). In fact, George Cantor is known as the founder of set theory. He defines set as "any collection M into a whole of definite, distinct objects m (which are called the elements of M) of our perception or of our thought." German mathematician Hilbert highly appraised Cantor's set theory by saying that "one of the most fertile and powerful branches of mathematics- a paradise from which no one can drive us out." The study of set theory is highly followed by almost every branchof mathematics as well as its importance is widely recognized in various fields like computer software and application and particularly in modern logic.

English mathematician George Boole recognised the importance of the study of sets in logic for the first time. Boole was mainly known for his Boolean Algebra. He was the primary contributor in the development of logic. He developed symbolic system of mathematical logic that represents relationship between ideas and objects. Boole formulated the basic rules of this system and subsequently redefined by other mathematicians and applied to set theory.



LET US KNOW

George Cantor is known as the founder of set

11.3 DEFINITION OF SET

George Cantor conceived a set to be 'any collection into a whole…of definite and separate objects… The objects are called the elements [members] of the set'. Here, at first, the term 'definite' means that there is some criterion for membership in the set, so that any object can be cognized, at least in theory, to be a member or not to be a member of the set in question. Second, what is meant by 'separate' or [distinct] is that any member can be recognized, at least in theory, as different from any other member, so that no member gets counted more than once; thus the set containing just the objects 1,2, and 2-1, is to be counted as having just two members rather than three. Finally, that a set is 'a collection into a whole' indicates that sets themselves are objects and, therefore, are eligible to be members of other sets.

The words like 'collection', 'aggregate' etc. are used to mean a set. However, these words are in themselves confusing. An 'aggregate' can be said to be 'a class of things' and 'a class' is defined as 'a collection'. Aristotelian logic dealt with the notion of classes. In Aristotelian logic a category, which is a predicate, is one of the fundamental classes of things. There should have a common characteristic or property of the members of the class which can be predicted about all of them. Aristotle also defines a class as a collection of objects or entities. The two terms 'set' and 'class' are used synonymously. Terms like 'collection', 'aggregate' etc. are also used synonymously with the term 'set'.

Some of the examples of set are:

1) The set of all school teachers in Assam.
2) The set of all rational animals.
3) The set of all college teachers of philosophy in Assam.
4) The set of all cricket players of India.
5) The set of all crows in Assam.



LET US KNOW

A set is any collection into a whole…of definite and



CHECK YOUR PROGRESS

Q.1: Define set.

.....

.....

Q.2: How does George Cantor define the concept of set?

.....

.....

11.4 SET MEMBERSHIP

The fundamental relation in set theory is that of membership. Logically, the individuals that are included in a set has a certain property i.e.; the individual is a member of a certain class. But a set may be constituted by only one member also. Again, there may be some sets where there is no member at all. This type of set will be called an empty set. However, the uses of such empty sets in practical fields are very rare, though its theoretical side is only acceptable. For example, the members in the set of continents of Asia include India, Pakistan, China, Bangladesh etc. But, if we talk about the set of female Chief Ministers of Assam then there will be only one member i.e.; SayedAnawaraTaimur. Again, if we discuss about the set of assamese players in Indian cricket team then there will be no member at all and this type of set will be called an empty set as there is no assamese player in Indian team.

Here, while discussing about set there are some importantpoints which are to be noticed like

(A) The words like object, element and member are used synonymously.

(B) Any set is implied by capital letters like A, B, C, X, Y, Z etc.

(C) The members of a set is implied by small letters like a, b, c, x, y, z etc.

The members or elements of set can be anything like numbers, people, letters of the alphabet, animals, or inanimate things, other sets or any distinct or well-defined entity. Each unit or element of the set is called a member of the set.These members that form the set may be objects of any kind except the set itself. Bertrand Russell said that a set is never permitted to be an element of itself. However, a given set may be an element of another set. So, it is logically possible to find a set of sets. If any individual is a member of a certain set andto indicate this membership a Greek letter $\in$ (Epsilon) is used between the individual letter and the class letter.

For example,

Akbar was a king.

Akbar ? king

$a \in K$

Sets may be interrelated. The first interrelationship between two sets is that one set is identical with other. The principle involved here is the principle of 'intentionality of sets'.For example, two sets A and B are identical if and only if, for the value of x, x can be called a member of A, if and only if x is a member of B. The symbolic representation of this point may be given as follows:

$A = B \leftrightarrow (x)\ [(x \in A) \leftrightarrow (x \in B)]$

For example, A = 1,2,3,4

B = 1,2,3,4 therefore, A=B

Similarly, A= Bachelor

B= Unmarried male therefore, A=B

Similarly, A= Human beings

B= Rational animal therefore, A=B

There is another two methods to imply or express a set and its members. They are like: (1) Roster or tabular form (2) Set builder form. The fundamental principle of the theory of set is the 'principle of extensionality'

where the members of the set theory are specified by the way of intentional definition using a rule or semantic description. It is a way of defining a set by the characteristics property of all its elements.

(1) **Roster or tabular form:** According to this method all the members of a set are written separately by using commas (,) within brackets. For example: the set of all the vowels ofEnglish alphabet-{a,e,i,o,u}, the set of all the even numbers less than 10- {2,4,6,8}

(2) **Set builder form:** According to this method it is assumed that there is a common or general characteristicamong the members of a set which is totally absent in those members which are not in the set. For example: the set of vowels of the English alphabet-{a,e,i,o,u}. There is a common characteristic among all these alphabets, i.e.; they are vowels. If this set is expressed by using the letter V, then its symbolic representation will be- V= {x: x the vowels of English alphabet}.



LET US KNOW

Set membership means the individuals that are included in a set has a certain property i.e.; the individual is a member of a certain class.



ACTIVITY: 11.1

- What do you mean by set membership in set theory? Explain.

Ans.....

.....

.....

.....

.....

11.5 SET INCLUSION

Let A and B be two sets. If every element of A is an element of B, then A is called a subset of B. Whenever it is said as 'A is a subset of B', it means that if 'a' is an element of A implies that 'a' is an element of B. If A is a subset of B we say that B contains A or B is a superset of A. This kind of relationship in set theory is known as set inclusion or set containment.

Again, if A is a subset of B, and if there exists at least one element of B which is not an element of A, then A is called a proper subset of B. In other words, if A is a subset of, but not equal to B, then A is called a proper subset of B. For example,

A= {1}

B= {1, 3, 5}

C= {1, 3, 5, 7}

Hence, A is a proper subset of B and C, and B is a proper subset of C. Here, one point that has to be kept in mind that every set is a subset of itself and the empty set is subset of every set. Such types of subsets are known as improper subsets. For example, a subset A of a set B, is called a proper subset of B if, A is not equal to B and it is written as A subset B. In such case it can be said that B is a superset of A.



LET US KNOW

Set inclusion means those kind of relationship in a set theory where A is a subset of B i.e.; B contains A or B is a superset of A.

11.6 EMPTY SET

The concept of empty set is of utmost importance in several mathematical and logical operations. An empty set is also known as a null set, zero set or void set. The concept of empty set can be defined as that kind of set which contains no member or where no kind of member is included. This type of set is expressed by using the Greek symbol ? (phi) or

^ (lambda). For example, let 'A' the set of those stars which revolve round the planet earth. This kind of set is called an empty set which contains no member at all. Or let 'B' be the set which contains those educated women who are the Vice-Chancellors of Gauhati University. But this is also an empty or null set as there is no member in this set. Or let 'C' be the set of people in the world who are older than 250 years. But according to the known statistics 'C' is an empty set.

Here, one important point must be mentioned that logically there can be only one null set. This is so because sets differ from one another only when they have different members. But an empty set has no member or element. Therefore, it is not possible to differentiate one null set from another null set on the basis of the difference of members. For example, the set of even prime number greater than 2 is an empty set. Hence, no null set is different from this set of even prime number greater than 2 from the point of view of the difference of members.

However, the concept of empty is a confusing concept as one may be incorrectly inclined to visualize a set as a heap of things. But, such type of visualization is not mathematically sound though it may be initially and intuitively. If a set is visualized in such a way then the mathematical idea of empty set may appear to be puzzling.

An empty set is symbolically represented in various ways. Generally by means of the Greek letter lambda (^) it is expressed that a particular set has no element of its own. The concept of empty set enables the logicians and mathematicians to perform a number of logical manipulations. For instance, it can be said that if two sets are mutually exclusive then the intersection of them produces an empty set. For example, if the set A contains the members 1, 2, 3 and 4 and the other set B contains the members 5, 6, 7, and 8 then we can say that the intersection of these two sets produces an empty set.

Again, an empty set may be a member of another set logically though nothing belongs to an empty set. Moreover, an empty is a subset of every set. For every x, x is not a member of an empty set. This is the defining property of an empty set.



LET US KNOW

The concept of empty set can be defined as that kind of set which contains no member or where no kind of member is included.



CHECK YOUR PROGRESS

Q.3: What do you mean by set inclusion? Give example

.....

.....

Q.4: What is empty set? Give example.

.....

.....



ACTIVITY: 11.2

- Explain the notion of set inclusion with suitable example in set theory.

Ans.....

.....

.....

.....

.....

11.7 OPERATIONS ON SETS

The various logical problems that occurred practically can be solved by using the concept of set. For this purpose new sets are constructed

from some given sets by applying various methods. Operations on sets are some such ways of constructing new sets from the given sets. For this reason all sets are considered to be the subsets of universe of discourse. Following are some of the operations on sets:

1. Union of sets
2. Intersection of sets
3. Difference of sets

11.7.1 Union Of Sets

The union of two sets, A and B, is the collection of points which are in A or in B or in both. If A and B are two sets, the union of A and B is the set of all those elements which belong either to A or B or both A and B. in other words, the union of set A and B is the set, which consists of all the elements of A and all the elements of B, however the common elements in both these sets are taken only once. For denoting the union of set A and set B the symbol U (U is used to denote the union of two sets) is used. Symbolically it can be shown in the following way

$$(x)\ (x \in AU\ B \leftrightarrow x \in A \text{ or } x \in B)$$

It is read as for every x, x belongs to A union B, if and only if x belongs to B. In other words, we can say that AU B is equal to x, such that, x belongs to A or x belongs to B.

The union of set A and B can also be represented in other way also like

$$AU\ B = \{x: x \in A \text{ or } x \in B\}$$

It is read as A union B is equal to x, such that x belongs to A or x belongs to B.

For example, A= {1, 2, 3, 4, 5, 6}
B= {4, 5, 6, 7, 8, 9}
AU B= {1, 2, 3, 4, 5, 6, 7, 8, 9}

Or, A= {all animals}
B= {all plants}
AU B= {all living organisms}

The example of a set A, which is the set of all animals and a set B, which is the set of all plants, and its union, that is A U B will be the set of all living organisms. If we are to refer to a single element by the variable "x", then, we can say that x is a member of the union, if it is an element present in set A or in set B, or in both. The same elements cannot be used twice in the same set, or, it can be said that sets cannot have duplicate elements. Suppose, we have the set A= {1, 2, 3} and the set B= {2, 3, 4}. Thus, as the operation of A U B we find that A U B= {1, 2, 3, 4}. Multiple occurrences of identical elements have no effect on the cardinality of a set or its contents. A U B is known as the logical sum of A and B. Therefore, A U B can also be replaced by (A + B).

The union of sets can be summarized have certain properties like:

(a) Union of sets has no definite order. It doesn't matter in what order unions are taken, the operation can be used as functional in any order and the parenthesis may be omitted without any ambiguity. The union of sets can be expressed in the following way:
A U (B U C) = (AU B) U C

(b) Union of sets are commutative and as such, the sets can be written in any order i.e.; the operation of union can be represented as (A U B) = (B U A)

(c) The empty set is an identity element for the operation of union i.e.; A U λ = A, for any set A.

(d) One can take the union of several sets at once. For example, the union of A, B, and C contain all elements of A, all elements of B, and all elements of C, and nothing else. So, for an element of x, x is an element of A U B U C, if and only if x is in A, or in B, or in C.

(e) The union of any set with itself is of same value i.e.; (A U A) = A.



LET US KNOW

The union of set A and B is the set, which consists of all the elements of A and all the elements of B, however the common elements in both these sets are taken only once.

11.7.2 Intersection of Sets

The intersection of sets A and B is the set of all elements which are common to both A and B. it's a binary operation. To denote the intersection of sets the symbol ? (read as intersection) is used. Many logicians use . (Dot) or × (multiplication sign) to denote intersection of sets instead of ∩like A ∩ B= A . B= A × B= AB

The intersection of two sets A and B are constituted by all those elements or members that belong to both A and B which can be symbolically represent in the following way:

A ∩ B= (x: x ∈ A and x ∈ B)

This can be read as x belongs to A intersection B if and only if x belongs to A and x belongs to B.

Suppose, A= {1, 2, 3, 4, 5}
B= {2, 4, 5, 6, 7}
A ∩ B= {2, 4, 5}

Or A= {all Indian people}
B= {all Indian Hindu people}
A ∩ B= {all Indian Hindu people}

Or, A= {1, 3, 4, 6}
B= {1, 4, 10, 5}
C= {2, 8, 7, 6}

Then, (A ∩ B) ∩ C= ^, (an empty set) as all the three sets do not contain any common element.

The intersection of sets has certain properties like:

(a) Intersection always has a set value which means that the set of sets is closed under intersection.

(b) Intersection of sets is an associative operation for all sets. That is in intersection the result does not depend on the order of evaluation which symbolically can be represented as
[(A ? B) ? C) = (A ? (B ? C)]

(c) Intersection of sets is commutative in the sense that the result of the intersection of two or more sets do not depend upon the order of the operands. Symbolically it can be represented as $(A \cap B) = (B \cap A)$.

(d) The intersection of any set with itself is the same value i.e.; $(A \cap A) = A$

(e) The operation of intersection distributes over union which can be shown as $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(f) The universal set, U, which includes everythingunder investigation, is a left identity for intersection of sets which symbolically shown as $U \cap A = A$. Again, the universal set, U, is a right identity for intersection of sets which can be represented as $A \cap U = A$



LET US KNOW

The intersection of sets A and B is the set of all elements which are common to both A and B.

11.7.3 Difference of Sets

If A and B are two sets, the difference of the sets A and B in this order, is the set of elements which belong to A but not to B. The difference of sets A and B, is also referred to as the relative complement of B in A. the difference of sets can be symbolically represented as A ~ B (read as A difference B). Hence, it can be said as every x, x ? A ~ B, if and x E A and x E B.

(x) (x E A ~ B $\leftrightarrow$ x E A and x ? B)

Suppose, A= {all people}

B= {all females}

A ~ B= {all males}

Or, A= {1, 2, 3, 4, 5, 6}

B= {2, 4, 6, 8}

A ~ B= {1, 3, 5} because the elements 1, 3, 5 belong to A but not to B. Again, (B ~ A) = {8} because the element 8 belongs to B and not to A.

Like the union of sets and intersection of sets, the difference of sets has also certain properties

(a) The set of sets is closed under set difference. This means that the difference of two sets is a set.

(b) The difference of sets is not associative like the union and intersection of sets. That is, in general, for sets A, B, and C, (A~ B) ~ C $\neq$ A ~ (B ~ C)

(c) Like the union and intersection of sets, the difference of sets is not commutative, i.e.; in general, for sets A and B, A ~ B $\neq$ B ~ A

(d) For difference of sets, the empty set, ?, is a right identity i.e.; in general, for set A, A ~ $\neq$ = A. For set difference, there is no left identity.

(e) The difference of any set with itself is not of the same value i.e.; for set A, A ~ A= $\gamma \neq$ A



LET US KNOW

If A and B are two sets, the difference of the sets A and B in this order, is the set of elements which belong to A but not to B.

11.7.4 Complement of Set

The complement of set is an important issue in the operations of set. Suppose, A is a set of all red things in which all the red things of the world will come together. Here, the complement of this set will be that set which will be constituted by all the non-red things of the world. This will be known as the complement of the former set. Moreover, the Universe of discourse will be constituted by uniting both the set of red things and the complement set of all non-red things. Or in other words, the union of the set of red things and the complement set of non-red things will be called Universe of discourse. Symbolically the complement of set will be represented as A.



CHECK YOUR PROGRESS

Q 5: What is known as union of sets?

.....

.....

Q 6: What do you mean by intersection of sets?

.....

.....

Q 7:Define difference of sets.

.....

.....



ACTIVITY: 11.3

- Explain the operations of sets in set theory with suitable examples.

Ans .....

.....

.....

.....

.....

11.8 LAWS OF OPERATIONS ON SET

For the interrelations between the operations on set, there are certain laws of operations on set. According to the law of distribution, law of double negation and De Morgan's theorems (De M.), there are 31(thirty one) such laws:

1. $A \cup \lambda = A$
2. $A \cup v = v$
3. $A \cup B = B \cup A$

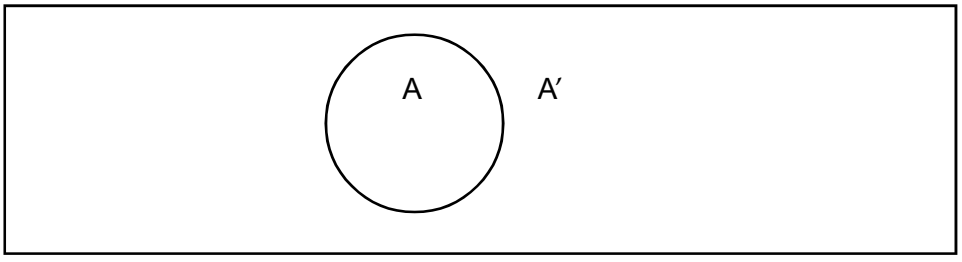
4. $A \cap B = B \cap A$
5. $A \cup (B \cap C) = (A \cup B)$
6. $A \cup (B \cup C) = (A \cap B) \cup (A \cap C)$
7. $A \cup \sim A = v$ (v= universe of discourse)
8. $A \cap \sim A = \lambda$
9. $A \cup A = A$
10. $A \cap A = A$
11. $A \cup V = v$
12. $A \cap \lambda = \lambda$
13. $\lambda \neq v$
14. $\sim \sim A = A$
15. $A = \sim B \rightarrow B = \sim A$
16. $A \cup B \neq \lambda \rightarrow A$? ?

    $v\ B \neq \lambda$
17. $A \cap B \neq \lambda \rightarrow A \neq \lambda$
18. $A \cup (B \cup C) = (A \cup B) \cup C$
19. $A \cap (B \cap C) = (A \cap B) \cap C$
20. $A \cup (A \cap B) = A$
21. $A \cap (A \cup B) = A$
22. $\sim A \neq A$
23. $\sim (A \cup B) = \sim A \cap \sim B$
24. $\sim (A \cap B) = \sim A \cup \sim B$
25. $A \sim A = \lambda$
26. $A \sim (A \cap B) = A \sim B$
27. $A \cap (A \sim B) = A \sim B$
28. $(A \sim B) \sim B = A \sim B$
29. $(A \sim B) \sim A = \lambda$
30. $(A \sim B) \cup B = A \cup B$
31. $(A \cup B) \sim B = A \sim B$

11.9 DOMAIN OF INDIVIDUALS

To understand set theory, the universe of discourse or the domain of individuals is very important. In any application of the set theory all the sets under investigation will likely be subsets of a certain set.This is known as domain of individuals or universal set or domain of discourse. The domain of individuals is defined as the "set of entities over which certain variables of interest in some formal treatment may range." The domain of individuals is a set of things which works as some possible values for all its variables and it bears relations to something else. The term 'domain' is also used in connection with function. So, the domain of function indicates the class of objects to which the function is applicable. Such types of functions are applicable to or defined over a certain class of objects. The class of objects for which a given function is defined is called is known as the domain of that function.

'Domain of individuals' is used synonymously with the terms the 'domain of discourse'. A particular discussion about an individual takes place in connection with this area. According to P. Suppes, "when a fixed set D is taken as given in this way, and one confines himself to the discussion to the subsets of D, we shall call D the domain of individuals, or sometimes the domain of discourse."The following diagrammatic representation will give a precise definition of D.



Here, A + A′ = Domain of discourse

The above diagram shows that any discussion about an individual which belongs to 'A' will be discussed with respect to a particular area which is represented by a rectangle in the above diagram. This diagram represents the domain of discourse. The domain of discourse is constituted of two

classes: class A and class A′. A is the class to which the individual under consideration belongs and A′ is a complementary class which exhaust the domain of discourse.

11.10 USING SET NOTATION FOR LOGICAL PROPOSITIONS (CATEGORICAL PROPOSITIONS)

Following are some of the examples of symbolization of categorical propositions by using set notations:

1. All cows are quadruped: $C \cap \sim Q = \lambda$
2. Every Wise men are humanist: $W \cap \sim H = \lambda$
3. Men are numerous: $M \in N$
4. The Apostles are twelve: $A \in T$
5. No Indian are European: $I \cap E = \lambda$
6. No madmen are reliable: $M \cap R = \lambda$
7. Some singers are not lyrist: $S \cap L \neq \lambda$
8. Some mangoes are not sweet: $M \cap \sim S \neq \lambda$
9. All Indians are religious and patriotic: $I \cap \sim (R \cap P) = \lambda$
10. Some men are intelligent and hardworking: $A \cap (B \cap C) \neq \lambda$
11. Fools and drunk men are truth tellers: $(F \cup D) \cap \sim T = \lambda$
12. Some Americans drink both coffee and milk: $A \cap C \cap M \neq \lambda$
13. Some Americans who drink tea do not drink either coffee or milk: $A \cap T \cap \sim C \cap \sim M \neq \lambda$
14. All S are P: $S \cap \sim P = \lambda$
15. All Americans are philosophers: $A \cap \sim P = \lambda$
16. No S is P: $S \cap P = \lambda$
17. No Americans are philosophers: $A \cap P = \lambda$
18. Some S is P: $S \cap P \neq \lambda$
19. Some Americans are philosophers: $A \cap P \neq \lambda$
20. Some S is not P: $S \cap \sim P \neq \lambda$

21. Some Americans are not philosophers: $A \cap \sim P \neq \lambda$

22. All Americans are clean and strong: $A \cap C \cap S = \lambda$

23. Some Frenchmen drink wine: $F \cap W \neq \lambda$

24. All women are fickle: $W \cap \sim F = \lambda$



CHECK YOUR PROGRESS

Q 8: What is the domain of individuals in set theory?

.....

.....

Q 9: Symbolize the following logical propositions by using set notations.

(a) No Americans are philosophers

(b) Some Frenchmen drink wine

(c) All cows are quadruped

(d) Some mangoes are not sweet



ACTIVITY: 11.4

- Explain the concept of domain of individuals in set theory with suitable diagrammatic representation.

Ans .....

.....

.....

.....

.....



11.11 LET US SUM UP

- The concept of set was developed at the end of the 19th century by German mathematician George Cantor (1845-1918). In fact, George Cantor is known as the founder of set theory.
- George Cantor conceived a set to be 'any collection into a whole…of definite and separate objects… The objects are called the elements [members] of the set'.
- The two terms 'set' and 'class' are used synonymously. Terms like 'collection', 'aggregate' etc. are also used synonymously with the term 'set'.
- The fundamental relation in set theory is that of membership. Logically, the individuals that are included in a set has a certain property i.e.; the individual is a member of a certain class. But a set may be constituted by only one member also.
- The members or elements of set can be anything like numbers, people, letters of the alphabet, animals, or inanimate things, other sets or any distinct or well-defined entity. Each unit or element of the set is called a member of the set.These members that form the set may be objects of any kind except the set itself.
- Let A and B be two sets. If every element of A is an element of B, then A is called a subset of B. Whenever it is said as 'A is a subset of B', it means that if 'a' is an element of A implies that 'a' is an element of B. If A is a subset of B we say that B contains A or B is a superset of A. This kind of relationship in set theory is known as set inclusion or set containment.
- The concept of empty set is of utmost importance in several mathematical and logical operations. An empty set is also known as a null set, zero set or void set. The concept of empty set can be defined as that kind of set which contains no member or where no kind of member is included. This type of set is expressed by using the Greek symbol φ (phi) or ^ (lambda).

- Operations on sets are some such ways of constructing new sets from the given sets. For this reason all sets are considered to be the subsets of universe of discourse.
- The union of two sets, A and B, is the collection of points which are in A or in B or in both. If A and B are two sets, the union of A and B is the set of all those elements which belong either to A or B or both A and B. in other words, the union of set A and B is the set, which consists of all the elements of A and all the elements of B, however the common elements in both these sets are taken only once. For denoting the union of set A and set B the symbol U (U is used to denote the union of two sets) is used.
- The intersection of sets A and B is the set of all elements which are common to both A and B. it's a binary operation. To denote the intersection of sets the symbol ? (read as intersection) is used. Many logicians use . (Dot) or × (multiplication sign) to denote intersection of sets instead of ∩ like A ∩ B= A . B= A × B= AB
- If A and B are two sets, the difference of the sets A and B in this order, is the set of elements which belong to A but not to B. The difference of sets A and B, is also referred to as the relative complement of B in A. the difference of sets can be symbolically represented as A ~ B (read as A difference B).
- The complement of set is an important issue in the operations of set. Suppose, A is a set of all red things in which all the red things of the world will come together. Here, the complement of this set will be that set which will be constituted by all the non-red things of the world. This will be known as the complement of the former set.
- To understand set theory, the universe of discourse or the domain of individuals is very important. In any application of the set theory all the sets under investigation will likely be subsets of a certain set. This is known as domain of individuals or universal set or domain of discourse.

11.12 FURTHER READING

1. Copi, I. M. Symbolic logic, 5th edition, 1996, Prentice hall of India private limited, New Delhi
2. Mates, Benson, Elementary Logic, 1965, New York.
3. Pollck, John L. An introduction to Symbolic logic, 1969, State university of New York, Holt, Rinehart and Winston, INC.
4. Suppes, Patrick., Introduction to logic, 1957, Van Nostrand, New York.

11.13 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No.1: Set is a collection of definite and separate objects into a whole.

Ans to Q No.2: George Cantor conceived a set to be 'any collection into a whole…of definite and separate objects.

Ans to Q No.3: If A is a subset of B we say that B contains A or B is a superset of A. This kind of relationship in set theory is known as set inclusion.

Ans to Q No.4: The concept of empty set can be defined as that kind of set which contains no member or where no kind of member is included.

Ans to Q No.5: The union of set A and B is the set, which consists of all the elements of A and all the elements of B.

Ans to Q No.6: The intersection of sets A and B is the set of all elements which are common to both A and B. it's a binary operation.

Ans to Q No.7: If A and B are two sets, the difference of the sets A and B in this order, is the set of elements which belong to A but not to B.

Ans to Q No.8: The domain of individuals is defined as the "set of entities over which certain variables of interest in some formal treatment may range."

Ans to Q No.9:

(a) $A \cap P = \lambda$

(b) $F \cap W \neq \lambda$

(c) $C \cap \sim Q = \lambda$

(d) $M \cap \sim S \neq \lambda$

11.14 MODEL QUESTIONS

A) Very short questions

Q 1: Who is the founder of set theory?

Q 2: What does the symbol '?' (epsilon) refer to?

Q 3: What does the symbol '?' (lambda) refer to?

Q 4: Can an empty set be the member of another set?

Q 5: Which set is considered as the subset of any other set?

Q 6: "Every set is a subset of the set itself". Is the statement true?

Q 7: Under what operation sets do not have the same value?

Q 8: "Basically there is only one null set." Is the statement true?

Q 9: Write two properties of union of sets?

Q 10: Write two properties of intersection of sets?

Q 11: What do you mean by a member of a set?

Q 12: Is the statement if A=B and B=C, then A=C true for all sets A, B and C?

B) Short questions (answer each question in about 100-150 words)

Q 1: Define difference of sets.

Q 2: Explain Cantor's definition of set.

Q 3: What is set membership?

Q 4: What is meant by domain of individuals?

Q 5: What is known as null set?

C) Short notes (answer each question in about 100-150 words)

Q 1: Domain of individuals

Q 2: Intersection of sets

Q 3: Difference of sets

Q 4: Set inclusion

Q 5: Operations on sets

D) Long questions (answer each question in about 300-500 words)

Q 1: Define set. What is its importance in logical study?

Q 2: What do you mean by set membership? Explain with examples.

Q 3: What do you mean by operation on sets? What are the major set operations? Explain with example.

Q 4: How can we symbolize traditional A E I O propositions by using set notations? Explain with suitable examples.

Q 5: What do you mean by union of sets? Explain with suitable examples.

*** ***** ***

UNIT-12: MEMBERSHIP, INCLUSION, THE EMPTY SET

UNIT STRUCTURE

12.1 Learning Objectives

12.2 Introduction

12.3 The Concept of Membership

12.4 The Concept of Inclusion

12.5 The Concept of Empty Set

12.6 Let Us Sum Up

12.7 Further Reading

12.8 Answers to Check Your Progress

12.9 Model Questions

12.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- define what is predicate logic
- define what is membership
- define what is inclusion
- what is empty set

12.2 INTRODUCTION

The concepts of membership, inclusion and empty set has occupied significant place in set theory. A set is a well-defined collection of objects having same characteristics. These objects are known as members or elements of the set. The concepts of membership, inclusion and empty set which are popularly known as set membership, set inclusion and empty set has arisen depending upon the members of sets or by depending upon those objects or elements which belongs to such sets. Whenever a certain object or element is a member of a certain set, then that object or element is known as the member of that particular set. These members belong to

or are included to such sets. Whenever a certain set includes a certain member then that member is called as a subset of that set or that particular set is known as the superset of that particular member. Hence, the idea of set inclusion has arisen from the idea of subset and superset. However, the concept of empty is quite different which contains no member at all.

12.3 THE CONCEPT OF MEMBERSHIP

The concept of membership is known as set membership. The mathematical use that an individual has a certain property is used in logical sense, that the individual is a member of a certain class. The members or elements of a set can be anything, that is, it can be numbers, people, and letters of the alphabet, other sets or any distinct or well-defined entity. Each of the unit of a set is called a member of the set. The members forming a set may be objects of any kind except the set itself. Following Bertrand Russell, it can be said that a set is never permitted to be an element of itself. But a given set may be an element of another set. Thus it is logically possible to find a set of sets. To say that an individual is a member of a certain set Greek letter '?' (Epsilon) is used between the individual letter and the class letter.

Here, while discussing about set membership there are some important points which are to be noticed like:

(A) The words like object, element and member are used synonymously.

(B) Any set is implied by capital letters like A, B, C, X, Y, Z etc.

(C) The members of a set is implied by small letters like a, b, c, x, y, z etc.

One example of set membership can be given like

Akbar was a king.

Akbar $\in$ king

a $\in$ K

Sets may be interrelated. The first interrelationship between two sets is that one set is identical with other. The principle involved here is the principle of 'intentionality of sets'. For example, two sets A and B are identical

if and only if, for the value of x, x can be called a member of A, if and only if x is a member of B. The symbolic representation of this point may be given as follows:

$A = B \leftrightarrow (x)\ [(x\ A) \leftrightarrow (x \in B)]$

For example, A = 1,2,3,4

B = 1,2,3,4 therefore, A=B

Similarly, A= Bachelor

B= Unmarried male therefore, A=B

Similarly, A= Human beings

B= Rational animal therefore, A=B

There is another two methods to imply or express a set and its members. They are like: (1) Roster or tabular form (2) Set builder form. The fundamental principle of the theory of set is the 'principle of extensionality' where the members of the set theory are specified by the way of intentional definition using a rule or semantic description. It is a way of defining a set by the characteristics property of all its elements.

(1) Roster or tabular form: According to this method all the members of a set are written separately by using commas (,) within brackets. For example: the set of all the vowels of English alphabet-{a,e,i,o,u}, the set of all the even numbers less than 10- {2,4,6,8}

(2) Set builder form: According to this method it is assumed that there is a common or general characteristic among the members of a set which is totally absent in those members which are not in the set. For example: the set of vowels of the English alphabet-{a,e,i,o,u}. There is a common characteristic among all these alphabets, i.e.; they are vowels. If this set is expressed by using the letter V, then its symbolic representation will be- V= {x: x the vowels of English alphabet}

As has been mentioned earlier the fundamental principle of the theory of set is the 'principle of extensionality'. The members of a set can be specified by the way of intentional definition using a rule or semantic description. It is a way of defining a set by the characteristics property of all

its elements. Again, it can be specified by the way of extension or by listing each member of the set. An extensional definition is donated by enclosing the list of members in brackets. Therefore, the members of the sets are called extensional. Extensional specification implies that each of the elements listed is a member of the set. For example:

A= {1, 3, 5, 7}

B={a, e, i, o, u}

Each member of a set is unique. It is so because no two members of a set are identical. If a set contains only a finite number of elements, then it is called a finite set. If an example is taken like if D is the set of the days of the week, then D is finite set. On the other hand, if a non-empty set which is not finite is called an infinite set. For example, if S is the set of the number of stars, then S is an infinite set.



LET US KNOW

Set membership means the individuals that are included in a set has a certain property i.e.; the individual is a member of a certain class.



CHECK YOUR PROGRESS

Q 1: What do you mean by set membership?

.....

.....

Q 2: What are two methods to express a set and its members?

.....

.....

Q 3: What is roster or tabular form?

.....

.....

12.4 THE CONCEPT OF INCLUSION

The concept of inclusion is also known as set inclusion.Let A and B be two sets. If every element of A is an element of B, then A is called a subset of B. This means that "A is contained in B" i.e.; if A is a subset of B whenever $a \in A$, then $a \in B$.It is said as 'A is a subset of B', it means that if 'a' is an element of A implies that 'a' is an element of B. If A is a subset of B we say that B contains A or B is a superset of A. It is often convenient to use the symbol "$\rightarrow$" which means implies. With the help of this symbol we can express a subset in the following way-

A is a subset of B if $a \in A \rightarrow a \in B$

This expression can be read as "A is a subset of B, if a is an element of A implies that a is also an element of B". If A is not a subset of B, then we have to draw a slash through the subset symbol. If A is a subset of B we say that B contains A or B is a superset of A which again means B includes A, or B contains A. This kind of relationship in set theory is known as set inclusion or set containment.

Again, if A is a subset of B, and if there exists at least one element of B which is not an element of A, then A is called a proper subset of B. In other words, if A is a subset of, but not equal to B, then A is called a proper subset of B. For example,

A= {1}

B= {1, 3, 5}

C= {1, 3, 5, 7}

Hence, A is a proper subset of B and C, and B is a proper subset of C.

Here, one point that has to be kept in mind that every set is a subset of itself and the empty set is subset of every set. Such types of subsets are known as improper subsets. For example, a subset A of a set B, is called a proper subset of B if, A is not equal to B and it is written as A subset B. In such case it can be said that B is a superset of A.



LET US KNOW

Set inclusion means those kind of relationship in a set theory where A is a subset of B i.e.; B contains A or B is a superset of A.



CHECK YOUR PROGRESS

Q 4: What is set inclusion?

.....

.....

Q 5: What is known as subset and superset?

.....

.....



ACTIVITY- 12.2

- Explain the concept of set inclusion with suitable examples.

Ans.....

.....

.....

.....

.....

12.5 THE CONCEPT OF EMPTY SET

The concept of empty set is of utmost importance in several mathematical and logical operations. An empty set is also known as a null set, zero set or void set. The concept of empty set can be defined as that kind of set which contains no member or where no kind of member is included. This type of set is expressed by using the Greek symbol ? (phi) or ^ (lambda). For example, let 'A' the set of those stars which revolve round

the planet earth. This kind of set is called an empty set which contains no member at all. Or let 'B' be the set which contains those educated women who are the Vice-Chancellors of Gauhati University. But this is also an empty or null set as there is no member in this set. Or let 'C' be the set of people in the world who are older than 250 years. But according to the known statistics 'C' is an empty set.

Here, one important point must be mentioned that logically there can be only one null set. This is so because sets differ from one another only when they have different members. But an empty set has no member or element. Therefore, it is not possible to differentiate one null set from another null set on the basis of the difference of members. For example, the set of even prime number greater than 2 is an empty set. Hence, no null set is different from this set of even prime number greater than 2 from the point of view of the difference of members.

However, the concept of empty is a confusing concept as one may be incorrectly inclined to visualize a set as a heap of things. But, such type of visualization is not mathematically sound though it may be initially and intuitively. If a set is visualized in such a way then the mathematical idea of empty set may appear to be puzzling.

An empty set is symbolically represented in various ways. Generally by means of the Greek letter lambda (^) it is expressed that a particular set has no element of its own. The concept of empty set enables the logicians and mathematicians to perform a number of logical manipulations. For instance, it can be said that if two sets are mutually exclusive then the intersection of them produces an empty set. For example, if the set A contains the members 1, 2, 3 and 4 and the other set B contains the members 5, 6, 7, and 8 then we can say that the intersection of these two sets produces an empty set.

Again, an empty set may be a member of another set logically though nothing belongs to an empty set. Moreover, an empty is a subset of every set. For every x, x is not a member of an empty set. This is the defining property of an empty set.



LET US KNOW

The concept of empty set can be defined as that kind of set which contains no member or where no kind of member is included.



CHECK YOUR PROGRESS

Q 6: What is empty set? Give example.

.....

.....



ACTIVITY- 12.3

- Explain the notion of empty set with suitable example in set theory.

Ans .....

.....

.....

.....

.....



12.6 LET US SUM UP

- The concepts of membership, inclusion and empty set has occupied significant place in set theory. The concepts of membership, inclusion and empty set which are popularly known as set membership, set inclusion and empty set has arisen depending upon the members of sets or by depending upon those objects or elements which belongs to such sets.
- Whenever a particular person, object or element becomes a member of a particular set, then that relation is called set membership.

- The members or elements of a set can be anything, that is, it can be numbers, people, and letters of the alphabet, other sets or any distinct or well-defined entity. Each of the unit of a set is called a member of the set. The members forming a set may be objects of any kind except the set itself.
- To say that an individual is a member of a certain set Greek letter '?' (Epsilon) is used between the individual letter and the class letter.
- There is another two methods to imply or express a set and its members. They are like: (1) Roster or tabular form (2) Set builder form.
- According to the roster or tabular method all the members of a set are written separately by using commas (,) within brackets.
- According to the set builder method it is assumed that there is a common or general characteristic among the members of a set which is totally absent in those members which are not in the set.
- Each member of a set is unique. It is so because no two members of a set are identical. If a set contains only a finite number of elements, then it is called a finite set.On the other hand, if a non-empty set which is not finite is called an infinite set.
- Set inclusion means those kind of relationship in a set theory where A is a subset of B i.e.; B contains A or B is a superset of A.
- Whenever a certain set includes a certain member then that member is called as a subset of that set or that particular set is known as the superset of that particular member.
- The concept of empty set can be defined as that kind of set which contains no member or where no kind of member is included.
- An empty set is symbolically represented in various ways. Generally by means of the Greek letter lambda (^) it is expressed that a particular set has no element of its own.
- An empty set is also known as a null set, zero set or void set.
- An empty set may be a member of another set logically though nothing belongs to an empty set. Moreover, an empty is a subset of every set.

12.7 FURTHER READING

1. Copi, I. M. Symbolic logic, 5th edition, 1996, Prentice hall of India private limited, New Delhi
2. Mates, Benson, Elementary Logic, 1965, New York.
3. Pollck, John L. An introduction to Symbolic logic, 1969, State university of New York, Holt, Rinehart and Winston, INC.
4. Suppes, Patrick.Introduction to logic, 1957, Van Nostrand, New York.

12.8 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q. No 1: Set membership means that the individual that are included in a certain class has a certain property i.e.; that the individual is a member of the class.

Ans to Q. No 2: The two methods to express a set and its members are roster or tabular form and tabular form.

Ans to Q. No 3: According to the roster or tabular method all the members of a set are written separately by using commas (,) within brackets.

Ans to Q. No 4: Set inclusion means those kind of relationship in a set theory where A is a subset of B i.e.; B contains A or B is a superset of A.

Ans to Q. No 5: Whenever a certain set includes a certain member then that member is called as a subset of that set or that particular set is known as the superset of that particular member.

Ans to Q. No 6: The concept of empty set can be defined as that kind of set which contains no member or where no kind of member is included.

12.9 MODEL QUESTIONS

A) Very short questions:

Q 1: What does the symbol '?' (epsilon) refer to?

Q 2: What does the symbol 'λ' (lambda) refer to?

Q 3: Can an empty set be the member of another set?

Q 4: Which set is considered as the subset of any other set?

Q 5: What is set membership?

Q 6: What is set inclusion?

Q 7: What do you mean by an empty set?

Q 8: What are the two other methods to express a set and its members?

B) Short questions (answer each question in about 100-150 words)

Q 1: What is set membership?

Q 2: What do you understand by set inclusion or set containment?

Q 3: What is null or void set?

Q 4: What do you understand by subset and superset?

C) Short notes (answer each question in about 100-150 words)

Q 1: Set membership

Q 2: Set inclusion

Q 3: Null or zero set

Q 4: Subset and superset

Q 5: Roster or tabular form

D) Long questions (answer each question in about 300-500 words)

Q 1: What do you mean by set membership? Explain with examples.

Q 2: Explain set inclusion with suitable examples?

Q 3: What is null or void or empty set? Is there any logical importance of the empty set in logic? If yes then explain.

*** ***** ***

UNIT-13: OPERATION ON SETS, DOMAINS OF INDIVIDUALS

UNIT STRUCTURE

13.1 Learning Objectives
13.2 Introduction
13.3 Operation on Sets
13.3.1 Union of Sets
13.3.2 Intersection of Sets
13.3.3 Difference of Sets
13.4 Domain of Individuals
13.5 Let Us Sum Up
13.6 Further Reading
13.7 Answers to Check Your Progress
13.8 Model Questions

13.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- defne a set operation
- explain the concept of union of sets
- explain the concept of intersection of sets
- explain difference of sets
- discuss the domain of individuals.

13.2 INTRODUCTION

Logic and set theory are very much intertwined in terms of serving as a foundation for mathematics. Mathematical logic or set theory is more foundational to mathematics. Both are equally important as being foundational, and they are intertwined. With logic, we are concerned with analyzing the justification of a statement. We logically approach the justification of a statement by studying its structure, which is composed of

premises and a conclusion. There are rules to follow that make an argument valid and therefore logical. One such important rule is that there may be several premises, but only one conclusion, and there must be a relation between the premises and the conclusion. Since set theory is the study of relations, it is very convenient to use set theory and logic together. Mathematics studies numbers, geometry, and form. Those require definitions and axioms to characterize the subjects of study. Logic is used to prove properties based on those definitions and axioms. Logic can also be used in the ordinary discourse of nonmathematical topics. Typically, the logic needed for mathematical investigations is great while that for nonmathematical investigations is small.

13.3 OPERATION ON SETS

Sets can be combined in a number of different ways to produce one or more sets. There are logical operations and set operations. If an operator defines an action, an operation is just a part of the expression. The set operator is prescribing to compute the value of an expression and assign it to a specified variable. An expression value can have a logical type; it will use logical operations to compute it. Operations on sets are some ways of constructing new sets from the given sets. Different connectives are used to define the set operations. The connectives allow us to build new sets from given ones. For this purpose, all sets are considered to be the subsets of the universe of discourse. A is a subset of B exactly when every element of A is also an element of B. There are three basic operations on sets.

1. Union of sets
2. Intersection of sets
3. Difference of sets

Inside that set are all the worlds where that variable is "true". With this representation, all set operators correspond exactly to logical operators. "and" corresponds to intersection, "or" to union, "not" to set complement.

13.3.1 Union of Sets

The union of two sets, A and B, is the collection of points which are in A or in B (or in both). If A and B are two sets, the union of A and B is the set of all those elements which belong either to A or B or both A and B. Or the union of set A and B is the set, which consists of all the elements of A and all the elements of B, the common elements being taken only once. In order to denote the union of A and B the symbol "U" is used. Thereby, first set operation is defined using the "U"connective. Thus, the set A U B -read "A union B" or "the union of A and B"-is defined as the set that consists of all elements belonging to either set A or set B (or both). If A is the set of all animals and B is the set of all plants, then A U B is the set of all living organisms. Symbolically it can be shown in the following way:

$(x)(x \in A \cup B \leftrightarrow x \in A \text{ or } x \in B)$

It is read as for every x, x belongs to A union B, if and only if x belongs to A or x belongs to B. In other words we can say that A ? B is equal to x, such that, x belongs to A or x belongs to B. Again the union of set A and B can be represented in the following way:

$A \cup B = \{x: x \in A \text{ or } x \in B\}$

It is read as A union B is equal to x, such that x belongs to A or x belongs to B.

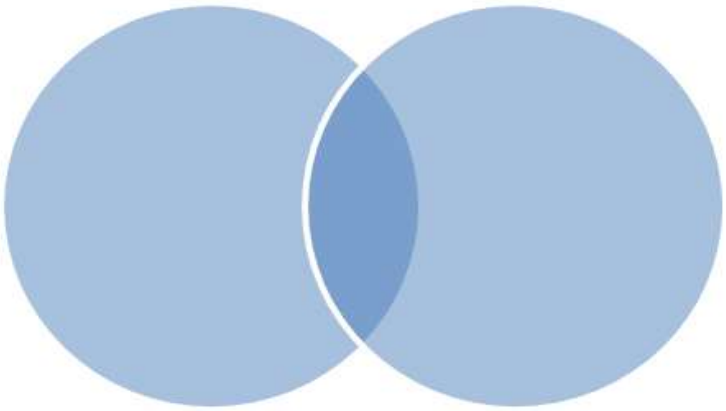
Suppose, $A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

The example of a set A, which is the set of all animals and a set B, which is the set of all plants, and its union, that is AUB will be the set of all living organisms. If we are to refer to a single element by the variable "x", then, we can say that x is a member of the union, if it is an element present in set A or in set B, or in both. The same elements cannot be used twice in the same set, or, it can be said that sets cannot have duplicate elements. Suppose we have the set A = {1, 2, 3} and the set B = {2, 3, 4}. Thus as the operation of A U B, we find that A U B = {1, 2, 3, 4}. Multiple occurrences of identical elements have no effect on the cardinality of a set or its contents.

(A U B) is called the logical sum of A and B. Therefore (A U B) can be replaced by (A + B). The operation of the union can be diagrammatically represented in the following way. A set can be visualized using Venn Diagrams In the diagram we have shaded A U B, the area of A and the area of B.



At the very outset, we can say that binary union is an associative operation. It doesn't matter in what order unions are taken. Here, the operation can be used as functional in any order and the parenthesis may be omitted without any ambiguity. Secondly, the union is commutative and, as such, the sets can be written in any order. Thirdly, one can take the union of several sets at once. Say, for example, the union of A, B, and C contains all elements of A, all elements of B, and all elements of C, and nothing else. Thus, for an element of x, x is an element of A U B U C, if and only if x is in A, or in B, or in C.

13.3. 2 Intersection of Sets

The intersection is a binary operation. The intersection of sets A and B is the set of all elements which are common to both A and B. The symbol "∩" (read as an intersection), is used to denote the intersection. The intersection of two sets A and B is the set of all those elements which belong to both A and B. It can be symbolically represented in the following way:

$$A \cap B = (x: x \in A \text{ and } x \in B)$$

Suppose A = {1, 2, 3, 4, 5}

B = {4, 5, 6, 7}

A ∩ B = {4, 5}

Again, if A = {1, 3, 4, 6}

B = {1, 4, 10, 5}

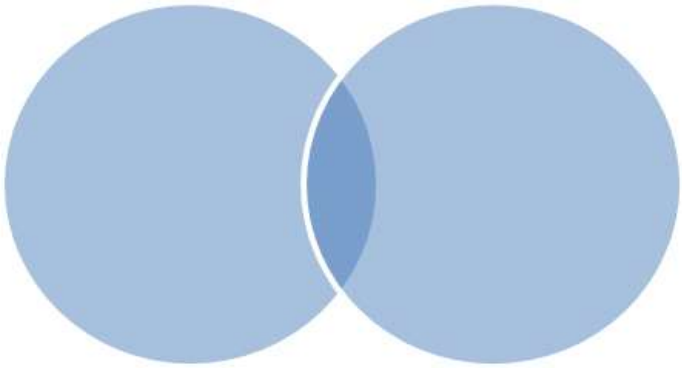
C = {2, 8, 7, 6}

Then, (A ∩ B ∩ C) = ∧ , since, all the three sets do not contain any common element.

Let us see some more examples that will help us to understand the operation on the intersection of sets. Let A be the set of all cricket players and B be the set of all Indian cricket players. Then the intersection of A and B will be the set of all Indian cricket players. The operation of intersection is sometimes used as a function of the product of two sets. In such circumstances, instead of using the symbol intersection, "∩", the symbol dot " ." (A.B) or multiplication sign "×" (A×B) can be used.

From this, it is clear that the operation of intersection is presented as, $x \in A \cap B \leftrightarrow x \in A$ and $x \in B$, and it is read as x belongs to **A** intersection B if and only if x belongs to A and x belongs to B.

The operation of intersection can be diagrammatically represented in the following way. In the diagram, we have shaded A ∩ B, the area that is common to both A and B.



Mutually exclusive sets have nothing in common. If W = { 1, 2, 3 } and S = { a, b, c }, then

W ∩ S = ∧

There some important properties of the operation of the intersection of sets. Firstly, the set of sets is closed under intersection, that is, intersection always has a set value. Secondly, for all sets, intersection is associative. That is, in intersection, the result does not depend on the order of evaluation. Thirdly, for all sets, intersection is commutative. That is, the result does not depend on the order of the operands. Symbolically it can be shown as $(A \cap B) = (B \cap A)$. Lastly, one thing that can be pointed out that, each investigation has its own universal set, which includes everything under investigation, but not everything in the actual known universe. The universal set is also referred to as the set of universe (U). The universal set, U, is a left identity for intersection of sets. Symbolically it can be shown as $U \cap A = A$. Again, the universal set, U, is a right identity for intersection of sets. Symbolically it can be represented as $A \cap U = A$

13.3.3 Difference of Sets

The third operation of the set is difference set. It is a binary operation. It can be understood as if A and B are two sets, the difference of the sets A and B in this order, is the set of elements which belong to A but not to B. The difference of sets A and B, $A \sim B$ or A - B, is also referred to as the relative complement of B in A. This can be symbolically represented as $(A \sim B)$ (read as A difference B). In other words, by $A \sim B$ we mean the set of all things which belong to A but not to B. Thus it can be said that for every x, $x \in A \sim B$, if and only if, $x \in A$ and $x \notin B$. It can be symbolically represented in the following way:

$(x)\ (x \in A \sim B \leftrightarrow x \in A \text{ and } x \notin B)$

It can be read as, x such that x belongs to $A \sim B$ if and only if x belongs to A but not belong to B. Say for example, if A is a set of all human beings, and B is the set of all females, then $A \sim B$ is the set of all human males. By taking some distinct elements as its members, a difference of set can be expressed in the following way:

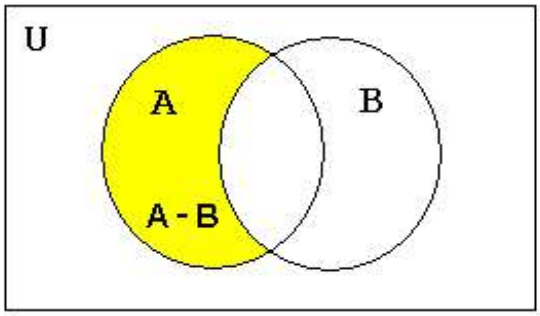
Let A = {a, b, c, d, e, f}

B = {c, d, x, y}

(A ~ B) = {a, b, e, f}, since the elements a, b, e, f belong to A but not to B. Again,

(B ~ A) = {x, y}, since the elements x, y belong to B and not to A

A ~ B is sometimes symbolized as A - B. It can be diagrammatically represented in the following way. In the diagram, we have shaded A - B, the area in A which is not part of B.



It is pertinent to mention that the set of sets is closed under set difference. That is, the difference of two sets is a set. Secondly, difference of sets is not associative, that is, in general, for sets A, B, and C, (A ~ B) ~ C ≠ A ~ (B ~ C). Again, difference of sets is not commutative, that is, in general for sets A and B, A ~ B ≠ B ~ A.



ACTIVITY: 13.1

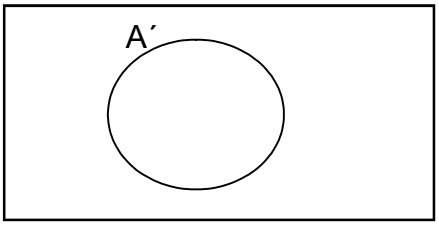
Name of operation	Symbol	Description
Union	U	union = cup
Intersection	∩	intersection = cap
Difference	~	tilde operator = varies with = similar to

13.4 DOMAIN OF INDIVIDUALS

The idea of the domain of individuals or the domain of discourse is the items we want our statements to deal with. In any application of the set

theory all the sets under investigation will likely be subsets of a certain set. It is known as domain of individuals or universal set or domain of discourse. For instance, in arithmetic the domain of discourse is the "world of numbers." In daily life the domain of discourse could be apples, or persons, or even everything. The domain of discourse or universe of discourse for a system for arithmetic is (so to speak) the "world of numbers." Or, more specifically, it might be just, say, all the positive integers. Usually, the domain of discourse is not explicitly specified but assumed to be "everything." Everything, obviously, includes quite a lot. It specifically includes all concrete (as opposed to abstract) things, such as people and trees, and logic books, but it also includes all the numbers, as well as individual times and places. For example, the domain of discourse may be limited to human beings only. In another occasion, the domain of discourse may be restricted to positive whole numbers. In some cases the domain of discourse is infinitely large. When dealing with domains of this size, we cannot replace quantified sentences with their expansions.

The domain of individuals is also known as the 'domain of quantifiers'. It is a set of things that serve as possible values for its variables. The domain of a relation is the set of things that bear the relation to something else. The term 'domain' may also be employed in connection with function. Thus we may speak about the domain of a function which means the class of objects to which the function is applicable. Any function will be applicable to or is only defined over a certain class of objects. The class of objects for which a given function is defined is called the domain of the function. 'Domain of individuals' is used synonymously with the terms the 'domain of discourse'. It is the area in connection with which a particular discussion about an individual takes place. To give a precise definition of D, the following diagrammatic representation will help us widely:



The universal set may also be denoted by U: the set of all objects under the consideration. In the diagram, it is assumed that all elements are members of a fixed universe U. In set theory, the universe is considered to be the set of all possible elements in a given situation. There is the use of circles to represent sets and a rectangle to represent U. The space inside these shapes represents where elements might exist. Shading is used to represent where elements might exist after applying some set operations.

The diagram shows that any discussion about an individual belonging to 'A' will be in or with respect to a particular area. That area is represented by a rectangle in the above diagram. The rectangle, therefore, represents the domain of discourse. This domain of discourse is constituted of two classes. One is the class of A and the other is the class of Á (not A). These two classes together constitute the domain. A is the class to which the individual under consideration belongs. Á is a complimentary class. A class and its complementary class between themselves exhaust the universe or the domain of discourse. It can b expressed as A + Á = U (universe of discourse). In order to introduce a domain of individuals, one can draw a circle, put some dots in it, write some constants on the board (e.g., our "a" and "b", etc.), and then draw a line from each constant to a dot in the circle, making clear that each name goes with one and only one dot and a different name goes to each dot. The name describes nothing and the thing named is not picked out in any obvious sense by a description. It merely marks numerical difference: two names, two individuals.



ACTIVITY: 13.2

Properties of union and intersection of sets:

Associative Laws

$A \cup B \cup C = A \cup (B \cup C)$

$A \cap B \cap C = A \cap (B \cap C)$

Commutative Laws

$A \cup B = B \cup A$

$A \cap B = B \cap A$

Distributive Laws

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup B \cap C = (A \cup B) \cap (A \cup C)$

Idempotent Laws

$A \cup A = A$

$A \cap A = A$.



CHECK YOUR PROGRESS

Q 1: Each of the given propositions is false. Replace the underlined word with another word to make the proposition true.

(a) Intersection is defined using a disjunction.

(b) Set diagrams are used to illustrate set operations.



13.5 LET US SUM UP

- Set theory is nothing but the study of relations. Sets can be combined in a number of different ways to produce one or more sets. There are logical operations and set operations. Operations on sets are some ways of constructing new sets from the given sets. Different connectives are used to define the set operations. The connectives allow us to build new sets from given ones.
- There are three operations of sets: Union of sets, Intersection of sets and Difference of sets.
- In any application of the set theory, all the sets under investigation will likely be subsets of a certain set. It is known as the domain of individuals or universal set or domain of discourse.

- The domain of individuals is also known as the 'domain of quantifiers'. It is a set of things that serve as possible values for its variables.
- The difference of set a binary operation. It can be understood as if A and B are two sets, the difference of the sets A and B in this order, is the set of elements which belong to A but not to B. The difference of sets A and B, A ~ B or A - B, is also referred to as the relative complement of B in A. This can be symbolically represented as (A ~ B) (read as A difference B).
- The intersection is a binary operation. The intersection of sets A and B is the set of all elements which are common to both A and B. The symbol "∩" (read as an intersection), is used to denote the intersection.
- The union of two sets, A and B, is the collection of points which are in A or in B (or in both). If A and B are two sets, the union of A and B is the set of all those elements which belong either to A or B or both A and B. Or the union of set A and B is the set, which consists of all the elements of A and all the elements of B, the common elements being taken only once.



13.6 FURTHER READING

1. O'Leary, Michael L. A First Course in Mathematical Logic and Set Theory. USA: John Wiley & Sons. 2016.
2. Sharma, B. and Jahnabi Deka. A Text Book of Logic. Guwahati: Aak-Baak. 2014
3. Suppes, Patrick. Introduction to Logic. New Delhi: East-West Press. 1957.



13.7 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q No 1:

a) "U"connective

b) Venn diagram

13.8 MODEL QUESTIONS

A) Objective questions

Q. 1: Give two examples of the domain of discourse.

Q. 2: What logical components are used to distinguish the operations of union and intersection?

Q 3: What is union of sets?

Q 4: What is difference of sets?

Q 5: What is intersection of sets?

B) Short questions (Answer in about 100-150 words)

Q 1: Write short notes on

a) Difference of sets.

b) Union of sets

c) Intersection of sets

C) Long questions (Answer in about 300-500 words)

Q. 1: Explain the operation of union of sets with its diagrammatical representation.

*** ***** ***

UNIT-14: TRANSLATING EVERYDAY LANGUAGE

UNIT STRUCTURE

14.1 Learning Objectives

14.2 Introduction

14.3 Symbolization of Ordinary Language

14.4 Symbolizing Propositions of Predicate Logic

14.5 Symbolization of Categorical Propositions by Using Set Notation

14.6 Let Us Sum Up

14.7 Futher Reading

14.8 Answers to Check Your Progress

14.9 Model Questions

14.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- explain the different uses of symbols in everyday language
- discuss different kinds of symbols of predicate logic and propositional logic
- discuss the process of symbolization with the help of quantifiers.
- analyse the process of symbolization with the help of set notations.

14.2 INTRODUCTION

Validity of an argument is a function of its form. But ordinary linguistic usage often obscures the form of an argument. In order to dispel this obscurity, logic introduces various simplifying procedures. For example, a syllogism can be simplified by using of letters to represent its terms. The validity of the argument depends on the arrangement of the terms within the premises and conclusion. By this process of putting letters for terms, form recognition becomes easy. Again, form recognition can be facilitated through the introduction of special symbols called operators, or connective. When we say that the logical operators may be used to translate expressions

in ordinary language, we mean that the operators capture a certain aspect of their correlative linguistic expression. If an argument is expressed in terms of these symbols, determining validity becomes only a matter of visual inspection. Therefore, the process of symbolization of our everyday language plays an important role in formal logic.

In our day to day use of language symbol, sign, mark etc. are used as synonymous. But they are not identical. The national flag or national emblem is used as symbol for the country. When see a national flag of a country we may come to know which country is represented by that flag. That particular flag belongs to a particular country, it is predetermined. Hence regarding symbol it may be stated that symbols are pre determined signs. Prof. Stout mentioned three types of sign. These are- Expressive sign, Suggestive sign and Substitutive sign. Generally the words which are used in our day to day language are expressive sign. Again, different images that are affix on the playing cards such as king, queen etc. are suggestive sign. On the other hand the signs such as: $\supset$, $\therefore$, $\equiv$, $\supseteq$, $\subseteq$, $\in$, $\notin$, p, q, r, s, x, y, z etc. that are used in logic and mathematics are substitutive symbols.

L. S. Stebbing defines symbols as "A sign consciously designated to stand for something will be called a symbol". By following L. S. Stebbing it may be stated that symbols are the artificial signs that are used consciously to represent something. Depending on the nature symbols may be classified into two types' namely verbal symbol and non-verbal symbol. In our day to day language we use different words. These words represent certain things, activities, qualities, command, request etc. All these words represent something by means of words. That is why these symbols are verbal symbols. Verbal symbols are the symbols in which something is deliberately represented by means of words. For example: Book, Pen, Honesty, kindness etc.

Uses of symbols serve different utility in the domain of logic. With the use of symbols we may find out the form of argument adequately and thereby validity and invalidity of argument may be determined easily with the use of general rules. Uses of symbols help us to remove the linguistic difficulties that are arises from the use of ordinary language in the study of

logic. Uses of symbols help us to frame general rules and principles for the study of argument and argument form in logic. The use of symbols ensures clarity and economy of expression. The uses of symbols pave the way for application of mathematical methods into the study of logic. As a result of uses of symbols in logic help us to find out validity or invalidity of argument by means of mathematical operation.

14.3 SYMBOLIZATION OF ORDINARY LANGUAGE

In the Unit IV, there has been a discussion on the basic components and the symbolization of the sentences of propositional logic. With the aid of logical constant and variable we may symbolize the statements of language. Logical constants are the symbols which have definite meaning and whose value cannot be changed from person to person, time to time and place to place. In symbolic logic there are five logical constants which are most commonly used. These are- •, v, ⊃ , ≡ , ~ respectively. In symbolic logic except '~' this all other logical constants are also used as connectives. Generally for symbolization of statement of everyday language as well as to form truth function logical constants are used. Variables are the symbols which are used to substitute something consciously. In the context of logic variable may be defined as follows: Variables are the symbols that are used to substitute something deliberately. For example- p, q, r, s etc.

For the symbolization of the statements of language first of all we have to observe the statement carefully so that we may have the idea about the components of the statement. For the symbolization of the statement according to the need of the statement we have to use variable and logical constants. If the given statement is a simple proposition, such proposition may be symbolized by means of a single variable. If the given proposition or statement is compound, for the symbolization of such statement we have to identify different parts of it and for each par we have to use separate variable. Again, to correlate different parts we have to use logical constants. On the other hand to ensure the scope the logical constant we have to use brackets as per need of the proposition.

Basically two sorts of logic are distinguished depending on the term based and sentence base components. On the one hand there is the propositional or sentential logic, and on the other hand, there is the predicate or quantifier logic. Pointing out the difference between propositional logic or sentential logic and quantifier logic or predicate logic Virginia Klenk in her book *Understanding Symbolic Logic writes*, " in sentential logic we take the sententially simple sentence as an unanalyzed whole, whereas in predicate logic we analyze these simple sentences into their component parts" (p. 220). In order to translate or symbolize into logical forms, the diverse language of everyday life have their own set of rules depending on being a form of propositional or predicate logic.

14.4 SYMBOLIZING PROPOSITIONS OF PREDICATE LOGIC

Predicate logic is concerned with the interior structure of both atomic and compound sentences. The fundamental component in predicate logic is the predicate. The letters which are used to symbolize these predicates are called predicate letters. Predicates are the words or expressions that we use to refer to properties or attributes that things have. For example: "Arne Naess is an environmentalist"-in this proposition the term "environmentalist" is a predicate letter, because "being an environmentalist" is predicated to Arne Naess. The property of "being an environmentalist" may be symbolized by the **predicate letter** 'E', i.e. a predicate is symbolized by uppercase letters (A, B, C,….X, Y, Z). Again, to denote individuals the lowercase letters from a through w (the first twenty three letters of the alphabet) are used. These symbols are known as individual constants. Their function is to denote only one, unique individual or object from the domain of discourse. Since their reference remains fixed or constant within a given context, they are called individual constants. The first letter of an individual or an entity is used to denote the individual. We may use letter r to denote Ramachandra Guha, v to denote Vandana Shiva and so on.

An atomic sentence or singular proposition is that where no truth functional connective is present. That is to say a singular proposition cannot contain logical connectives like 'and', 'either-or', 'if-then', 'if and only if' etc. It is not possible for a singular proposition to contain any quantity indicating words like 'all', 'every' 'some' etc. A singular proposition is that proposition which can be analyzed into terms, but not to propositions, because it is the simplest kind of non-compound proposition. In order to know how to symbolize singular propositions the concepts of "Predicate Letters" or "Predicate Symbols" and "Individual Constants" are important. Consider the sentence "Amal is happy." This sentence asserts that some particular object or entity, Amal, has a certain property-namely, the property of being happy. If we let the capital letter H denote the property of being happy, and the lowercase letter a name the individual, Amal, we can symbolize this sentence as Ha. Similarly, the sentence "Bimal is happy" can be symbolized as Hb, the sentence "Amal is friendly" as Fa, and the sentence "Bimal is friendly" as Fb. The sentences Fa, Fb, Ha, and Hb are alike in that they have the same general structure. In each of these sentences a property is ascribed to some individual entity. This is one of the basic patterns of atomic sentences or singular propositions.

Let us see the process more elaborately. In order to symbolize a singular proposition, we write the symbol for its predicate term to the left of the symbol for its subject term. To put it in other words, the predicate constant in a singular proposition is written first. Then the predicate constant is followed by the individual constant. For example Sankaradeva is a philosopher will be symbolized as 'Ps' ('P' stands for 'philosopher' and 's' stands for 'Sankaradeva'). Again,the proposition "Madhabdeva is a Vaishnava" is to be symbolized as "Vm," where capital letter "V" stands for the predicate "Vaishnava" and lowercase letter "m" stands as individual variable for the individual "Madhabdeva." Individual variables are those which indicate where an individual constant can be written to produce a singular proposition. Moreover, a negative proposition like "Banta is not intelligent" is symbolized as '~ Ib'.

Another basic pattern is illustrated by the sentence "Champak is taller than Dhruba." This sentence asserts that there is a particular property (being taller than) that holds between the two individual objects Champak and Dhruba. If we let c denote Champak, d Dhruba, and T the property of one thing being taller than another we can symbolize this sentence as Tcd. Similarly, we can symbolize the sentence "Dhruba is taller than Champak" as Tdc. And if we let F denote the property of one thing being a friend of another (is a friend of), then we can symbolize the sentence "Champak is a friend of Dhruba" as Fcd, and so on. Properties such as is taller than and is a friend of, which hold between two or more entities, are called relational properties. The particular properties in question are two place relational properties, since they hold between two entities.

In predicate logic two symbols, called quantifiers, are used to state how many. Words that indicate quantity such as 'all', 'every', 'most' are called quantifiers. The first is the universal quantifier, used to assert that all entities have some property or properties. A universal quantifier can be defined as an expression that, when prefixed to a propositional function, yields a universal proposition. Statements which contain words like 'all', 'every', 'each', 'any', 'everything', 'anything' or any word conveying the sense of 'all' are symbolized by the aid of universal quantifier. The symbols (x), (y), and so on-that is, individual variables placed between parentheses-are used for this purpose. Thus, to symbolize the sentence "Everything moves" or "All things move," start with the sentence form Mx and prefix the universal quantifier (x) to obtain the sentence (x)(Mx), which is read as "For all x, Mx," or "Given any x, Mx." Let us try to explain the process of its symbolization. Consider the proposition: 'Everything is temporary'. It can be expressed as:

Given any individual thing whatever, it is temporary.

We can use x (an individual variable) in place of both the pronoun 'it' and its and its antecedent 'thing'. We can paraphrase the proposition as:

Given any x, x is temporary.

Here, we have got a property, i.e. 'being temporary'. In order to symbolize this property, as it is already mentioned in the previous section, we need predicate symbol. We can symbolize the property 'being temporary' as 'Tx'. The phrase 'given any x' is called a universal quantifier. 'Given any x' is conventionally symbolized as '(x).' The general proposition 'everything is temporary' can be symbolized as, (x) Tx.

The existential quantifier is used to assert that some individual or individuals have one or more properties. An existential quantifier can be defined as an expression that, when prefixed to a propositional function, yields an existential proposition. Here we use a symbol that looks like a backward letter E along with an individual variable, placing both symbols within parentheses, as in (∃x), (∃y), and so on. The existential quantifier '(∃x)' is used to assert that some entities (at least one) have a given property. Thus, to symbolize "Some things move," we again start with the sentence form Mx but this time we prefix the existential quantifier (∃x) to obtain the sentence (∃x) (Mx), which is read as "There is an x, Mx" or "For some x, Mx."Thus, to symbolize the proposition "Something is light" or the proposition "At least one thing is light," start with the proposition form Lx and prefix an existential quantifier to it. The result is the sentence ((∃x) Lx, read "For some x, x is light," or "There is at least one x such that x is light," or just "Some x is light."

Let us take a negative proposition: Something is not temporary. It can be written as, There is at least one x, such that x is not temporary. Here one important point is to be noted that the 'not' appears before the predicate, it does not appear in the quantity part. With the help of Existential quantifier it can be translated as:

(∃x) ~Tx

When we use the universal quantifier, (x), we are saying something about all the individuals represented by the variable x in the quantifier. Just how many x's constitute all depends on how many things we want our language to be able to deal with. For instance, in some systems for arithmetic, we want the individual constants to denote numbers, so in such

a system the number of x's will be infinite. Let us see the symbolization of four categorical propositions by means of using quantifiers:

Let us take the sentence "All humans are mortal." It is correctly symbolized as (x)(Hx ⊃ Mx) (where Hx ="x is human" and Mx = "x is mortal") and is read "For all x, if x is human, then x is mortal" or "Given any x, if x is human, then x is mortal," which is roughly what the sentence "All humans are mortal" asserts. Similarly, "No philosopher is dishonest"---can be written as: Given any individual thing whatever, if it is philosopher, then it is not dishonest. Given any x, if x is a philosopher, then x is not dishonest. Given any x, x is philosopher ⊃ x is not dishonest. Thus, it is symbolized as, (x) (Px⊃ ~ Dx).

Let us take a particular affirmative proposition: Some philosophers are philanthropists. This sentence can be written as: There is at least one thing that is philosopher and philanthropist. This again can be restructured as "There is at least one thing such that it is philosopher and it is philanthropist". To be more precise, it is formulated as: There is at least one x such that x is philosopher and x is philanthropist. To quantify it by quantifiers, it is to be shaped as "There is at least one x such that x is philosopher ox is philanthropist". Thus, it can be symbolized as (x) (Px• Hx). Similarly, "Some philosophers are not philanthropists" --- can be written as: There is at least one thing that is philosopher but not philanthropist. It implies, "There is at least one thing such that it is philosopher and it is not philanthropist". To quantify it, "There is at least one x such that x is philosopher and x is not philanthropist". Thereby, it is symbolized as, (x) (Px • ~Hx).

More complicated symbolizations are obtained in other ways. Thus, to symbolize the sentence "Something is both heavy and expensive," conjoin Hx (for "x is heavy") with Ex (for "x is expensive") to get (Hx • Ex), and then add an existential quantifier to get (∃x)(Hx • Ex). Similarly, to symbolize "Something is both sweet and fattening," conjoin Sx (for "x is sweet") with Fx (for "x is fattening") to get (Sx • Fx), and then add an existential quantifier to get (∃x)(Sx • Fx).



ACTIVITY -14.1

Symbolize the following by using quantifiers:

1. Everything is movable.
2. Not everything is movable.
3. Nothing is movable.
4. Everything is immovable.
5. Some people are honest.
6. Some people are not honest.
7. Some honest people are mistreated.
8. Some people are liars and thieves.
9. It's not true that some people are honest.
10. Some people are neither honest nor truthful.
11. Some things are neither expensive nor worthwhile.

14.5 SYMBOLIZATION OF CATEGORICAL PROPOSITIONS BY USING SET NOTATION

A categorical proposition refers to two classes. In order to symbolize a categorical proposition, we use some symbolic notation. The four categorical propositions are symbolized by using set notations. If two classes, s and p, have some members in common, then this common membership is called as the product or intersection of the two classes.

E proposition is the universal negative proposition. It says that no s's are p's, which means one class is totally excluded from another class. Or it says that no member of the class s is members of class p, that is, there are no things that belong to both classes. Thus there is no overlap between the class of p and the class of s. In other words the two classes have no members in common, that they are disjoint; that the region that is both s and p is empty. The intersection of the two classes is symbolized as sp or $s \cap p$. In order to show that a particular class has no member we use

a zero (0) or lambda (λ). When we find that the intersection of two classes is empty then symbolize it as sp = 0 or as $s \cap p = \lambda$. For example:

1. No s is p.

   $S \cap p = \lambda$

2. No americans are philosophers.

   $a \cap p = \lambda$

I proposition is the contradictory of an e proposition. This proposition says that one class is partially included in another class. It is expressed in the way that there are some s's that are p's. It implies that the class of s is partly included in the class of p. From it we understand that there is some overlap between the class of s and the class of p. From the point of view of members of the class there exists at least one member which is both s and p. In other words the product of the classes s and p is not empty. Therefore, an i proposition can be symbolized as $sp \neq 0$ or as $s \cap p \neq \lambda$. For example:

1. Some s is p.

   $S \cap p \neq \lambda$

2. Some americans are philosophers.

   $a \cap p \neq \lambda$

3. Some frenchmen drink wine.

   $f \cap w \neq \lambda$

in order to symbolize a and o proposition the concept of class complement is used. The class complement is the class of all things not belonging to the original class. The complement of class s is symbolized as s (s bar). Proposition 'a' is the universal affirmative proposition. It says that all s's are p's, that whatever is in the class of s must also fall into the class of p. This means there can be no s that is not also a p. In other words, all members of the class s are also the members of class p, or by obversion it can be said as "no s is non-p." Thus this proposition says that the product of the classes designated by its subject and predicate terms is empty. A proposition states that the word "all" attaches itself directly to the s and not to the p. As such, an a proposition asserts something definite about s, but

it leaves the open question of the extent of the p dominion. Thus we symbolize an a proposition as sp = 0 or s $\cap$ ~p = λ. Let us take see the following propositions and their symbolizations.

1. All s are p.

   The set of s is $\subseteq$ p.

   S $\cap$ ~p = λ

2. All americans are philosophers.

   the set of americans is the subset of the set of philosophers.

   a $\cap$ ~p = λ

3. All women are fickle.

   W $\cap$ ~f = λ

O proposition is the particular negative proposition. It says that one class is partially excluded from another class. It is expressed in the way that there is something that is s but not p, that is, the class of s falls partly outside the class of p. In other words the categorical form of o proposition "some s is not p" can be logically obverted equivalently to i proposition. As such it can be symbolized as sp $\neq$? or sp $\neq$ 0. Since, o proposition is contradictory to a proposition, we symbolize it as sp $\neq$ 0 or s $\cap$ ~p $\neq$ λ. For example:

1. Some s is not p.

   S $\cap$ ~p $\neq$ λ

2. Some americans are not philosophers.

   a $\cap$ ~p $\neq$ λ

The following are some more examples that will help us to symbolize relatively complex propositions.

4. All americans are clean and strong.

   A $\subseteq$ c $\cap$ s

5. Fools are drunk men are truth tellers.

   (f u d) $\subseteq$ t

6. Some americans drink both coffee and milk.

   a $\cap$ c $\cap$ m $\neq$ λ

7. Some americans who drink tea do not drink either coffee or milk.

   (a $\cap$ t) $\cap$ ~ (c u m) $\neq$ λ



CHECK YOUR PROGRESS

Q 1: Answer the following questions :

a) '•' What is the name of this symbol?

b) ∃x What is the name of this symbol?

c) '⊃' What is the name of this symbol?

d) ∩ What is the name of this symbol?

e) ' ~ ' is this symbol a logical constant?



ACTIVITY: 14.1

Symbolize the following propositions by means of set notations:

1. No fish are mammals.
2. All tigers are wild.
3. No man is happy and self-controlled.
4. No poem is uninteresting.
5. Some writers are mad.
6. Honest and drunk are truth tellers.
7. All trucks are vehicles.
8. No roses are daisies.
9. Some spaniels are not good hunters.
10. Some carbon compounds are not diamonds.



14.6 LET US SUM UP

- Regarding symbol, it may be stated that symbols are pre determined signs. Prof. Stout mentioned three types of sign. These are- Expressive sign, Suggestive sign and Substitutive sign. Generally the words which are used in our day to day language are expressive sign. Again, different images that are affix on the playing cards such as king, queen etc.

are suggestive sign. On the other hand the signs such as: $\supset$, $\therefore$, $\equiv$, $\supseteq$, $\subseteq$, $\in$, $\notin$, p, q, r, s, x, y, z etc. that are used in logic and mathematics are substitutive symbols.

- L. S. Stebbing defines symbols as "A sign consciously designated to stand for something will be called a symbol".
- Uses of symbols serve different utility in the domain of logic. With the use of symbols we may find out the form of argument adequately and thereby validity and invalidity of argument may be determined easily with the use of general rules.
- Uses of symbols help us to remove the linguistic difficulties that are arises from the use of ordinary language in the study of logic.
- Uses of symbols help us to frame general rules and principles for the study of argument and argument form in logic.
- The use of symbols ensures clarity and economy of expression. The uses of symbols pave the way for application of mathematical methods into the study of logic. Virginia Klenk in her book Understanding Symbolic Logic writes, " in sentential logic we take the sententially simple sentence as an unanalyzed whole, whereas in predicate logic we analyze these simple sentences into their component parts" (p. 220).
- Virginia Klenk in her book Understanding Symbolic Logic writes, " in sentential logic we take the sententially simple sentence as an unanalyzed whole, whereas in predicate logic we analyze these simple sentences into their component parts"

14.7 FURTHER READING

1) Chakraborty, Chhanda (2009), Logic: Informal, Symbolic and Inductive, New Delhi: PHI Private Limited.

2) Hurley, Patrick J (2007), Introduction to Logic, New Delhi: Cengage Learning Private Limited.

3) Sharma, Banajit and Jahnabi Deka (2011), A Textbook of Logic, Guwahati: Aak Baak.

14.8 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q No 1:

a) dot

b) existential quantifier

c) Implication

d) Intersection

e) Negation

14.9 MODEL QUESTIONS

A) Objective questions

Q 1: Symbolize the following propositions by means of set notations:

a) No fish are mammals.

b) All tigers are wild.

c) No man is happy and self-controlled.

d) No poem is uninteresting.

e) Some writers are mad.

f) Honest and drunk are truth tellers.

g) All trucks are vehicles. No roses are daisies.

h) Some spaniels are not good hunters.

Q 2: Answer the following questions :

a) 'o' What is the name of this symbol?

b) x What is the name of this symbol?

c) ' ' What is the name of this symbol?

d) ? What is the name of this symbol?

B) Long questions (Answer in about 300-500 words)

Q 1: How do we symbolize categorical propositions by using set notation? Explain

Q 2: How do we symbolize the propositions of predicate logic? Explain

Q 3: How do we symbolize ordinary language? Explain

*** ***** ***

UNIT: 15 VENN DIAGRAM

UNIT STRUCTURE

15.1 Learning Objectives

15.2 Introduction

15.3 Categorical Proposition and Standard form Categorical Proposition

15.4 Classes and Relation

15.5 The Four Standard form Categorical Propositions and their Class Relations

15.6 Distribution of Terms

15.7 Venn Diagram

15.8 Use of Venn Diagram

15.9 History

15.10 Aristotelian and Boolean Standpoint

15.11 Symbolism and Diagrams for Standard form Categorical Propositions

15.12 Let Us Sum Up

15.13 Further Reading

15.14 Answer to Check Your Progress

15.15 Model Questions

15.1 LEARNING OBJECTIVES

After going through this unit you will be able to-

- define a categorical proposition.
- explain the four standard form categorical propositions.
- describe different types of class relation.
- describe distribution of terms.
- illustrate Venn diagram and its uses.
- discuss the four standard form categorical propositions by means of Venn diagram

15.2 INTRODUCTION

Logic is defined as the science of reasoning. An inference or reasoning when expressed in language is called an argument. An argument is any group of propositions of which one is claimed to follow from others. An argument consists of some propositions. But, it is not a mere sum total of some propositions. Every argument has a definite structure. An argument involves the 'premise' and the 'conclusion'. The proposition which is justified or affirmed on the basis of other proposition or propositions of the argument is called the conclusion. The other proposition or the propositions which provide grounds for accepting the conclusion are called the premise or premises. Logic is chiefly concerned with the argument. Hence, proposition is considered as the basic unit of logical thinking. We need proposition to express our logical thinking. There are different ways of classifying a proposition. The standard form propositions are the typical forms of propositions recognised in Aristotelian logic. The standard form categorical propositions can be represented by using Venn diagram, named after the English mathematician and logician John Venn.

15.3 CATEGORICAL PROPOSITION AND STANDARD FORM CATEGORICAL PROPOSITION

A proposition that relates two classes is called a Categorical proposition. That is, a categorical proposition asserts something about the classes. Classes and their relationship form the content of the categorical propositions. The classes are denoted by the subject term and the predicate term. A categorical proposition asserts that either all or part of the class denoted by the subject term is included in or excluded from the class denoted by the predicate term. Thus, a proposition which asserts something about the relationships between the classes referred to by their subject and predicate terms is known as categorical proposition. Aristotelian logic is exclusively concerned with this kind of statements and with arguments that are formed out of these propositions. Some of the examples of categorical propositions are:

1. All ducks swim
2. A fish is not a mammal
3. A few soldiers are heroes
4. All prisoners are not violent

In other words a categorical proposition is one in which the relation between the subject and the predicate is without any condition. In all the above examples it is seen that the relation between the subject and the predicate is not subject to any condition.

A categorical proposition that expresses these relations with complete clarity is called a standard-form categorical proposition. For example;

All men are animal

No man is perfect

Some students are intelligent

Some men are not wise.

Here the subject and the predicate term refer to two classes and connected by the verb 'to be'. A categorical proposition is in standard form if and only if it is a substitution instance of one of the following four forms:

All S are P

No S are P

Some S are P

Some S are not P.

Many categorical propositions, of course, are not in standard form because; they do not begin with the words 'all' 'no', 'or', 'some'. Categorical propositions have to be translated into standard form before it can be considered in logic.

The words 'all', 'no', and 'some' are called quantifiers because they specify how much of the subject class is included in or excluded from the predicate class. The letters S and P stand for the subject and the predicate terms respectively, and the words 'are' and 'are not' are called the copula because they link the subject and the predicate term. Some of the important points about the standard form categorical proposition are:

- The 'subject term' and the 'predicate term' do not mean the same thing in logic that 'subject' and 'predicate' mean in grammar. The subject of the example statements include the quantifier words such as 'all', 'some' but the subject term does not. Similarly the predicate includes the copula 'are' but the predicate term does not.
- There are exactly three forms of quantifiers (all, no, some) and two forms of copulas (is/are, is not/are not). But in ordinary statements various forms of the verb 'to be' are considered as copula.

Standard- form categorical proposition thus represent an ideal of clarity in language.



LET US KNOW

The theory of categorical propositions originated by Aristotle has constituted one of the core topics in logic for over 2,000 years. It remains important even today.

15.4 CLASSES AND RELATION

A class or category is a group of objects having some recognizable common properties or characteristics which can be predicated about all of them. Classes can also be referred to as categories or sets. Aristotelian logic deals with the logic of classes. Aristotle defined a class as a collection of objects or entities. Thus, a category is simply a group, a set of things. For example, class of players, class of college students etc. But it is not a random collection of members. The members of a class are supposed to have a common property that can be predicated about all of them. Terms like 'class', ' sets', 'collection', 'aggregate' etc. are used synonymously.

There are various ways in which classes may be related to each other. If every member of one class is also a member of another class then the first class is said to be included in the second class. For example, suppose we take two classes viz. 'Indians' and 'Asians'. Here, we know that all the members of the class 'Indians' are also the members of the

class 'Asians'. The class 'Indians' is included in the class 'Asians. This kind of inclusion is called total class inclusion relation as the whole class 'Indians' is included in the class 'Asians'. If some, not all members of one class are also members of another, then the first class may be said to be partially included in the second class. For example, some men are poet. If some of the members of the first class are excluded from being the members of the second class, then the first class is said to be partially excluded from the second class. For example, some men are not poet. Of course, there are two classes having no members in common. For example, No triangles are circles. Such kind of relation is said to be total class exclusion relation.

15.5 THE FOUR STANDARD FORM CATEGORICAL PROPOSITIONS AND THEIR CLASS RELATIONS

Quality and quantity are attributes of categorical proposition. According to quality, propositions are divided into two-affirmative and Negative. Again according to quantity, propositions are divided into two-Universal and Particular. Both universal and particular propositions may be affirmative and negative. Hence, combing the quality and quantity principles there are four kinds of categorical propositions in Aristotelian logic. They are universal affirmative, Universal negative, particular affirmative and particular negative. The letters A, E, I and O are used as names for these four kinds of categorical propositions respectively.

The Universal affirmative proposition (A) asserts that the subject class is wholly contained in the predicate class. That is, every member of the subject class is also a member of the predicate class. Because it refers to 'every member', its quantity is universal. Since it asserts a relationship, its quality is affirmative. In other words, in a universal affirmative proposition predicate is affirmed of the whole subject. Suppose S and P are any two classes, its schematic form is 'All S is P'. It may be illustrated by an actual example: 'All swans are white'. It expresses total class inclusion relation.

The universal negative proposition (E) asserts that there is no relation between the two classes. That is, no member of one class is a member of the other class. Its quality is negative and quantity is universal. In other words, in a universal negative proposition predicate is denied of the whole subject. Its schematic form is - 'No S is P'. It may be illustrated by an actual example: 'No children are cruel'. Here, the first class is wholly excluded from the second class. Thus a universal negative proposition expresses the total class exclusion relation.

Particular affirmative proposition (I) affirms that only some, not all members of one class are also members of another class. Its quality is affirmative and quantity is particular. In other words, in a particular negative proposition predicate is affirmed of the part of the subject. For example: Some flowers are yellow. Its schematic form is 'Some S is P'. This kind of proposition expresses partial class inclusion relation.

Particular negative proposition (O) asserts that some members of a class are not members of another class. Its quality is particular and quantity is negative. Hence, in a particular negative proposition predicate is denied of the part of the subject. For example: 'Some flowers are not yellow'. Its schematic form is 'Some S is not P'. Here, quality is negative and quantity is particular.

Thus, the four categorical propositions, named A, E, I and O stand for universal affirmative, universal negative, particular affirmative, particular negative, respectively are known as standard-form categorical propositions. Standard-form categorical propositions refer to two classes.

15.6 DISTRIBUTION OF TERMS

Like quality and quantity, which are the attributes of propositions, distribution is an attribute of the terms of propositions. A proposition may refer to all members of a class or some of the members of a class designated by its subject and predicate term.

The term 'distribution' refers to the ways in which the terms can occur in the categorical propositions. In other words, by the 'distribution' of

a term, it is meant that 'taking it universally or referring to all or parts of it'. A term is said to be distributed if the proposition makes an assertion about every member of the class denoted by the term; otherwise it is undistributed. That means, in a proposition a term is distributed if it refers to all the members of a class designated by the term. Any 'A' proposition of the form 'All S is P' refers to all members of the class designated by its subject term 'S' but does not refer to all the members of the class designated by its predicate term 'P'. Hence, 'A' proposition distributes its subject term but not the predicate term.

The form of universal negative proposition is 'No S is P'. The universal negative proposition i.e. 'E' proposition refers to all the members of the class designated by its subject term as well as by its predicate term. Thus 'E' proposition distributes both the subject and the predicate term.

The form of particular affirmative proposition is 'Some S is P'. The particular affirmative proposition i.e. 'I' proposition refers to some members of the class designated by its subject term and some members of the class designated by its predicate term. Thus 'I' proposition distributes neither the subject nor the predicate.

The form of particular negative proposition is 'Some S is not P'. The particular negative proposition i.e. 'O' proposition distributes the predicate term but not its subject term.

Thus, 'A' proposition distributes its subject term but not the predicate term.

'E' proposition distributes both the subject and the predicate term.

'I' proposition distributes neither the subject nor the predicate.

'O' proposition distributes the predicate term but not the subject term.

The various relations between classes such as class inclusion or class exclusion either affirmed or denied by the four standard form categorical propositions can be represented by using the Venn diagram.



LET US KNOW

Category: A predicate or a fundamental class of things

Class: A collection of objects or entities with a common characteristic.

Some: In logic, 'some' means at least one.



CHECK YOUR PROGRESS

Q 1: Answer briefly :

a) What is categorical proposition?

.....

b) Give an example of categorical proposition?

.....

c) What do you mean by a class?

.....

d) What are the four different standard form categorical propositions?

.....

Q 2: Define -

a) Universal affirmative proposition

.....

b) Particular negative proposition.

.....

Q 3: State whether the following statements are true (T) or False (F)

a) 'A' proposition distributes subject. (True/False)

b) 'E' proposition distribute predicate.(True/False)

Q 4: Fill in the blanks:

a) Standard-form categorical propositions refer to …… classes.

b) The words 'are' and 'are not' are called the ……..because they link the subject and the predicate term.



ACTIVITY -15.3

- Do you find any distinction between a categorical proposition and a standard form categorical proposition?

Ans.....

.....

.....

.....

15.7 VENN DIAGRAM

A Venn diagram is an illustration of the relationships between sets and among sets, indicated by the arrangement of the circles. Venn diagram uses circles to represent sets, as for example by drawing one circle within another to indicate that the first set is a subset of a second set. Venn diagram is thus a method of representing classes and propositions. Copi has defined Venn as an "Iconic representations of categorical propositions, and of arguments to display their logical forms using overlapping circles". The Venn diagram is named after its inventor, British mathematician and logician John Venn (1834-1923).

15.8 USE OF VENN DIAGRAM

The Venn diagram is used in many fields such as in statistics, computer science, linguistics, logic, and probability. Fundamentally they are used to teach elementary set theory, as well as illustrate simple set relationships in probability, statistics, and computer science. The uses of Venn diagram in formal logic can be brought out in the following way.

- Venn diagram is used to represent the information expressed by the four kinds of categorical propositions. That is, the relationship between the classes designated by the subject and the predicate term of categorical proposition is represented by means of Venn diagram.

- The technique of Venn diagram is used to determine the validity or invalidity of syllogistic argument. Thus it enables us to distinguish valid syllogistic argument from invalid syllogistic argument.
- Venn diagrams are used to explain how the two statements relate to each other.
- The logical relations between sets and operations on sets can be represented by using Venn diagrams.

15.9 HISTORY

The history of the Venn diagram, which is otherwise known as the set diagram goes back to 1880, when British logician and philosopher John Venn introduced it in his research paper "On the Diagrammatic and Mechanical Representation of Propositions and Reasonings", which was published in the 'Philosophical Magazine and Journal of Science'. Venn himself did not use the term "Venn diagram" and referred to his invention as "Eulerian Circles". The first to use the term "Venn diagram" was Clarence Irving Lewis in 1918, in his book "A Survey of Symbolic Logic". At that time, John Venn referred to the Venn diagram as the Eulerian circle, because it resembles Euler diagrams, invented by Leonhard Euler in the 18th century. Venn diagrams differ from the use of Euler circles in a few important ways.

15.10 ARISTOTELIAN AND BOOLEAN STANDPOINT

Before going to discuss about the diagrammatic representation of the standard form categorical propositions through Venn diagram, we are to concentrate at the meaning of universal affirmative (A) and universal negative propositions (E) from Aristotelian and Boolean standpoints.

The Aristotelian standpoint differs from the Boolean standpoint only with regard to universal (A and E) propositions. The two standpoints are identical with regard to particular (I and O) propositions.

Aristotle held that universal propositions have existential import. On the other hand, the nineteenth-century English mathematician and logician, George Boole held that no universal propositions have existential import.

George Boole is one of the founders of modern symbolic logic. Boole is now chiefly remembered for Boolean Algebra, named after him. He applied algebraic techniques to traditional Aristotelian logic. Boole proposed a calculus for syllogism. It involved translating each syllogism in arithmetical notation. This idea led to invent propositional calculus and Boolean algebra. His chief works were: The Mathematical Analysis of Logic and An Investigation of the Laws of Thought. The four standard form categorical propositions have the following meaning from the Boolean standpoint.

A-All S are P= No members of S are outside P

E- No S are P= No members of S are inside P

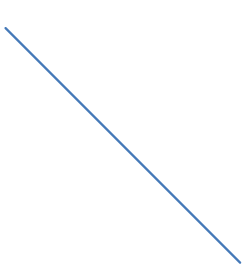
I-Some S are P=A least one S exists, and that S is a P

O-Some S are not P=At least one S exists, and that S is not a P.

John Venn has adopted this interpretation of categorical propositions, while he developed the diagrammatic technique for representing the information expressed by categorical propositions. In Venn diagram, there is an arrangement of overlapping circles in which each circle represents the class denoted by a term in a categorical proposition.

The Boolean square of opposition or the modern square of opposition

Drawing on some ideas of Boole we can build a square of opposition. In Boolean symbolic interpretation, the interrelation among the four standard form categorical propositions appears very clearly. It is obvious that A and O propositions are contradictories. Analogously, E and I propositions contradict each other. This relationship of mutually contradictory pairs of propositions is represented in a diagram called Modern Square of opposition or Boolean square of opposition. The diagram can be represented by the following figure-



* First, we notice that both I and O propositions have existential import, because both assert the existence of at least one entity. On the other hand, A and E propositions both do not have existential import.

* Secondly, if two propositions are related by the contradictory relation, they necessarily have opposite truth value. Thus, if a certain A proposition is given as true, the corresponding O proposition must be false. Similarly, if a certain I proposition is given as false, the corresponding E proposition must be true. But no other inferences are possible. In particular, given the truth value of an A or O proposition, nothing can be determined about the truth value of the corresponding E or I propositions. These propositions are said to have logically undetermined truth value. Similarly, given the truth value of an E or I proposition, nothing can be determined about the truth value of the corresponding A or O propositions. They, too, are said to have logically undetermined truth value.



LET US KNOW

Category: A predicate or a fundamental class of things

Class: A collection of objects or entities with a common characteristic.

Some: In logic, 'some' means at least one.

15.11 SYMBOLISM AND DIAGRAMS FOR STANDARD FORM CATEGORICAL PROPOSITIONS

The symbolisation of the standard form categorical propositions involves set notation. The symbols used for set notation are: ?, =, ∧, ?, ∩ The Boolean interpretation of categorical propositions depends upon the notion of an empty set. The symbols that represent empty class are ? (Lambda) or O (zero). For example, when we say that "the class 'S' is empty", it means that "the class 'S' does not have any member". We symbolise it as S= ∧ or S= O. On the other hand, to say that the class designated by 'S' is not empty is to say the class 'S' have members. We

symbolise the denial by drawing line through the equality sign. For example, S ≠ ∧ or S ≠ O. It means that the class S is not empty.

The concept of complementary class is necessary to explain in this connection. A complementary class is the collection of all things that do not belong to the original class. For example, if S designates the class of 'all students', then its complementary class would be "all things that are not students" which is to be symbolised as ?. It means not S.

If two classes suppose S and P, have some members in common, this common membership will be called the product or intersection of the two classes; it is symbolised as SP or S∩P and it is read as S intersection P. The symbol '?' is known as 'intersection. Thus the statement that claims that the product of the two classes is empty is symbolised as SP= ∧ or SP=O or S∩P = ∧

On the other hand the statement which expresses that the product of two classes is not empty is symbolised as SP≠∧ or SP≠ O or S∩ ? P = ∧.

SYMBOLISATION: The Standard form categorical propositions are symbolised in set notation as:

A- All S is P = SP=O or SP=∩.

The A proposition asserts that the part of the S circle that lies outside the P circle is empty.

E- No S is P= SP =O or SP= ∩.

The E proposition asserts that the area where the two circles overlap is

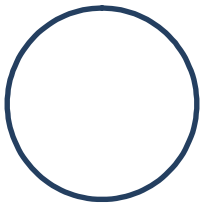
empty.

I-Some S is P= SP ≠ O or SP ≠ ∩.

The I proposition asserts that the area where the two circles overlap is not empty.

O-Some S is not P= SP ≠ O or SP= ∧. The O proposition asserts that the part of the S circle that lies outside the P circle is not empty.

VENN DIAGRAM: After discussing the symbolization procedure, we still need to learn how to diagram a standard form categorical proposition. In Venn diagram, we represent a class by a circle labelled with the term that designates the class. Thus, the class 'S' is diagrammed as shown below:



S **Figure I**

It represents the class 'S' but says nothing about it. Hence to represent the information by the class, two kinds of marks are used: (1) shading an area and (2) Placing an X in an area. Shading an area means that the shaded area is empty and placing an X in an area means that at least one thing exists in that area. Thus, we shade the interior of the circle to indicate that the class 'S' has no members. And we place an X any where in the interior of the circle to represent that there is at least one member of the class 'S'. This is represented by the following diagram:

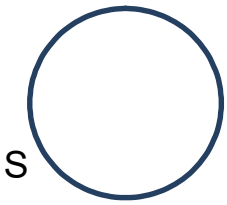


Figure II

S=O

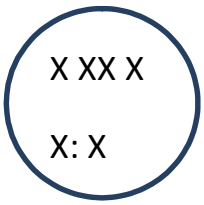
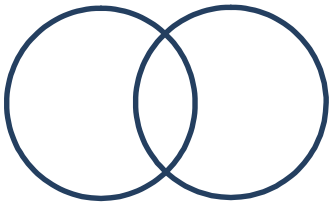


Figure III

S¹O

Standard form categorical propositions have exactly two terms. Therefore, to diagram the standard form categorical propositions two overlapping circles are required. In all the A, E, I and O propositions the subject and the predicate terms that represent classes are abbreviated by S and P. Usually, the left hand circle represents the subject term (S) and the right hand circle represents the predicate (P) term. Such a diagram looks like this:



S **P** **Figure iv**

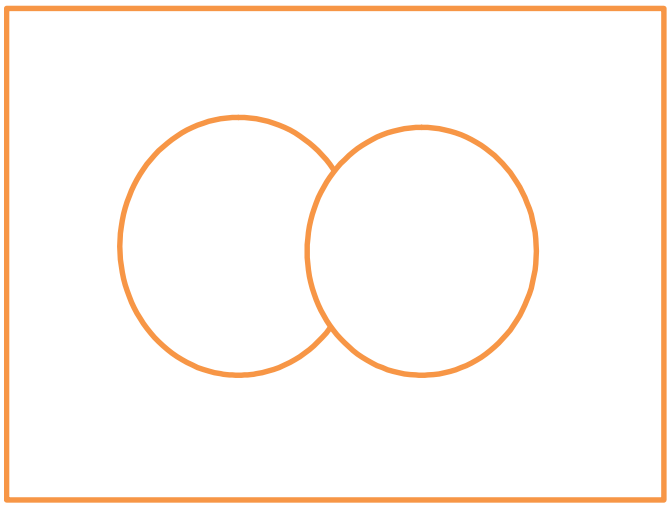
This figure diagrams only the two classes but does not give any information about their class relationship. The above diagram of the two overlapping circles represents different types of class relationship expressed by the four kinds of categorical propositions.

Firstly, the area that designates those members of S that are not members of P is symbolised as SP. The symbol P (P-bar) has been used to indicate the regions which are not P

Secondly, the area that designates those members of P that are not members of S is symbolised as SP. The symbol S (S-bar) has been used to indicate the regions which are not S.

Thirdly the area designating members of S that are at the same time members of P is symbolised as SP. This part represents the intersection of the class S and P or the product of S and P i.e., all things that belong to both S and P.

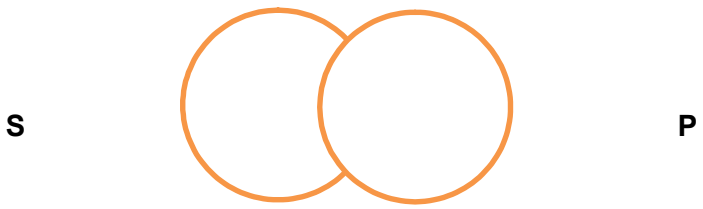
Finally, the area that designates where no members of either S or P can be found is symbolised as SP. Diagrammatically, we may represent this as follows:



Thus, this diagram that represents various classes is known as Venn diagram. Universe of discourse: The rectangle outlining the diagram represents the universe of discourse (U.D) which means the context or everything that is assumed or believed for the discussion. There are many kinds of things in the world, such as people, plants, animals, physical objects etc. The collection of things that we are talking about on a given occasion constitutes the universe of discourse for that occasion.

Diagrams for standard form categorical propositions: Now we are in position to learn how to complete the diagrams for our four standard form categorical propositions. So, let us first diagram the Universal affirmative (A) proposition.

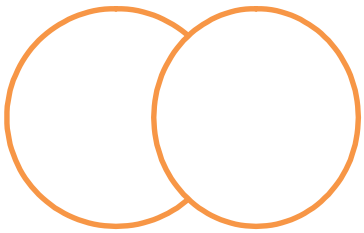
The 'A' proposition asserts that no members of S are outside P. In other words, all members of the class S are also the members of class P. Its schematic form is - "All S is P". It is symbolised as SP=O or SP= ∩. It means the part of the circle S which is not in P is empty. This is represented by shading the part of the S circle that lies outside the P circle. The diagram for the 'A' proposition is given below:



A: All S is P

SP=O

E Proposition: 'E' Proposition is the universal negative proposition. Its schematic form is -"No S is P". It asserts that no members of S are inside P. In other words, the class S is totally excluded from the class P. It is symbolised as SP= ∩ or SP=O. It means the common area of the circle S and P is empty. This is represented by shading the part of the S circle that lies inside the P circle. The diagram for the 'E' proposition is given below:

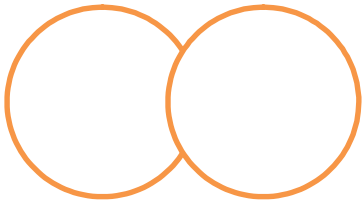


E=No S is P

SP=O

I Proposition: 'I' proposition is the particular affirmative proposition. Its schematic form is - "Some S is P". It asserts that there exists at least one member in the class S and that is also a member of the class P. In other words, the class of S is partly included in the class P. It implies the product of the class S and P is not empty. It is symbolised as SP≠O or SP≠∩. This is represented by placing an X in the area where the S and P circles overlap. The diagram for the 'I' proposition is given below:

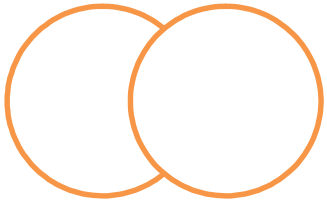
S **P**



I: Some S is P

SP ≠ O

O Proposition: 'O' proposition is the particular negative proposition. Its schematic form is- "Some S is not P". The 'O' proposition asserts that at least one S exists, and that S is not a P. It is symbolised as SP≠O or SP≠∩. . The 'O' proposition is asserts that at least one S exists, and that S is not a P. This is represented by placing an X in the part of the S circle that lies outside the P circle. This X represents an existing thing that is an S but not a P. The diagram for the 'O' proposition is given below:



O: Some S is not P

SP ≠ O



CHECK YOUR PROGRESS

Q 5: Give brief answer :

a) State the symbol which represents the empty class.

.....

b) State the symbol which represents the intersection of the two classes.

.....

c) Symbolise universal negative proposition.

.....

Q 6: What is Venn diagram?

.....

Q 7: Mention two uses of Venn diagram.

.....

Q 8: Draw Venn diagram for universal negative proposition.

.....



15.12 LET US SUM UP

- Proposition is the basic unit of logical thinking. There are different ways of classifying a proposition. A proposition that relates two classes or categories is called a categorical proposition. The classes are denoted respectively by the subject term and the predicate term. For example, 'All students are honest', 'grass is green'.

- The subject and the predicate term express different types of class relations such as total class inclusion, total class exclusion, partial class inclusion and partial class exclusion relations. These relations are expressed by the four standard form categorical propositions.
- The four standard form categorical propositions are: A-Universal affirmative (All S is P), E-universal negative (No S is P), I-particular affirmative (some S is P) and O-particular negative (some S is not P). Thus, these four propositions are referred to as standard form categorical propositions.
- The nineteenth century English logician and mathematician John Venn developed a system of diagrams to represent the categorical propositions. These diagrams have come to be known as Venn Diagrams

15.13 FURTER READING

1) Chakraborti. C., Logic-Informal, Symbolic & Inductive
2) Hurley. J. P., Introduction to Logic
3) Baronett. S. and Sen. M., Logic
4) Copi. I.M. & Cohen. C., Introduction to Logic
5) Sharma. B. and Deka J., A text Book of logic
6) (https://www.lucidchart.com/blog/2013/01/17/a-history-of-the-venn-diagram)

15.14 ANSWER TO CHECK YOUR PROGRESS

Ans to Q No 1:

a) A proposition that relates two classes or categories is called a Categorical proposition. For example, All ducks swim.

b) Some students are honest.

c) A class or category is a group of objects having some recognizable common properties or characteristics which can be predicated about all of them.

d) The four standard form categorical propositions are universal affirmative Universal negative, particular affirmative, particular negative. The letters A, E I and O are used as names for these four kinds of categorical propositions respectively.

Ans to Q No 2:

a) The Universal affirmative proposition is one which asserts that the subject class is wholly contained in the predicate class. In other words, in a universal affirmative proposition predicate is affirmed of the whole subject.

b) Particular negative proposition is one which asserts that some members of a class are not members of another class. Hence, in a particular negative proposition predicate is denied of the part of the subject.

Ans to Q No 3:

a) True

b) False.

Ans to Q No 4:

a) Two

b) quantifiers

Ans to Q No 5:

a) $\wedge$

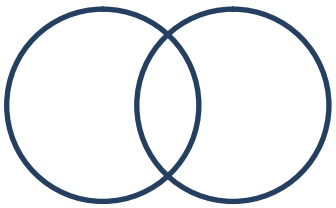
b) $\cap$

c) A-All S is P= SP=O

Ans to Q No 6: A Venn diagram is an illustration of the relationships between sets and among sets, indicated by the arrangement of the circles.

Ans to Q No 7: I) Venn diagram is used to represent the four standard form categorical propositions. II) The technique of Venn diagram is used to determine the validity or invalidity of syllogistic argument.

Ans to Q No 8:



S

E: No S is P

SP=O

15.15 MODEL QUESTIONS

A) Very short questions

Q 1: Define categorical propositions.

Q 2: What is standard form categorical proposition?

Q 3: Give an example of categorical proposition

Q 4: What is venn diagram?

Q 5: What do you mean by total class inclusion relation?

B) Short questions (Answer each question in about 150 words)

Q 1: Write short notes on:

a) Class and relation

b) Uses of Venn diagram

c) Symbolisation of A, E, I and O propositions in Boolean interpretation.

Q 2: Briefly explain the concept of categorical proposition.

Q 3: What is standard form categorical proposition? Briefly explain.

Q 4: What is Boolean square of opposition? Briefly explain.

C) Long questions (Answer each question in about 300-500 words)

Q 1: What is categorical proposition? Explain.

Q 2: Explain the four standard form categorical propositions with examples.

Q 3: What is Venn diagram? Explain how to represent the four standard form categorical propositions by Venn diagram.

*** ***** ***

REFERENCES:

1) Copi, Irving M. (2006). Symbolic Logic (fifth edition). New Delhi: Prentice Hall of India Private Ltd.
2) Jain, Krishna.(1998). A textbook of logic. (fourth Revised Edition)
3) Singh, S. Shyam Kishore. Modern Logic (Vol.1) Lamyana Press
4) Copi, Irving M., Cohen, Carl., Jetli, Priyedarshi., & Prabhakar, Monica.(2006). Introduction to Logic (Twelfth edition).Delhi: Dorling Kindersley (India) Pvt. Ltd.
5) Basson, A.H.,& O'Connor,D.J.(1992) Introduction To Symbolic Logic. Calcutta: Oxford University Press, Walton Street, Oxford
6) Wolf, A.(1976). Text Book Of Logic. Delhi:Surjeet Publications, 7-K, Kolhapur Road, Kamala Nagar
7) Copy, Irving M., & Cohen, Carl.(1990). Introduction To Logic(Eighth Edition). New York. Macmillan Publishing Company
8) Whitehead, A.N.(1911). An Introduction To Mathematics. Oxford, England: Oxford University Press
9) Chhanda C. (2007), Logic-Informal, Symbolic and Inductive, Prentice-Hall of India Pvt. Ltd.
10) Hurley, P. J. (2007), Introduction to Logic, Cengage Learning India Private Limited.
11) Klenk,V. (2008), Understanding Symbolic Logic, Dorling Kindersley (India) Pvt. Ltd.
12) Sharma, B. & Deka, J. (2014), A Text Book of Logic, Aank-Baak, Panbazar, Guwahati, Assam.
13) Copi, I.M. & Kohen Carl. (1984), Introduction to Symbolic Logic, Prentice Hall of India, pvt ltd. New Delhi.
14) Quine, W.V. (1970). Philosophy of Logic, Prentice Hall of India, pvt ltd. New Delhi.
15) Roy, B.N. (1984). Deductive Logic, S.C. Sarkar & Sons pvt ltd, Calctta.
16) Strawson, P.F.(1976). Introduction to Logical Theory, Methuen & co. Ltd. London.

17) Baronett, S., & Sen, M. (2009) Logic. Delhi. Pearson
18) Copy, I. M.; Cohen, C; Jetli, P. Prabhakar, M. (2006), Symbolic Logic (Twelfth edition ed.). Delhi. Pearson
19) Das, K. L. (2011). Symbolic Logic in Bengali Medium. Sibmandir: Siliguri: Darjeeling: N. L. Publishers.
20) Rao, K.O.N. (2012), Logic for Degree Students, Ludhiana: Kalyani Publishers
21) Chakraborti, C. (2007) Logic: Informal, symbolic and Inductive, 2nd Ed., New Delhi: Prentice Hall of India Pvt. Ltd.
22) Pollock, J.L. (1969) An Introduction to Symbolic Logic, New York: Holt, Pinehart and Winston, Inc.
23) Gustason, W., Ulrich, D.E. (1973) Elementary Symbolic Logic, New York: Holt, Pinehart and Winston, Inc.
24) Mates, Benson(1965), Elementary Logic, , New York.
25) Pollck, John L. (1969). An introduction to Symbolic logic, State university of New York, Holt, Rinehart and Winston, INC.
26) Suppes, Patrick. (1957) , Introduction to logic, Van Nostrand, New York.
27) O'Leary, Michael L. (2016) A First Course in Mathematical Logic and Set Theory. USA: John Wiley & Sons..
28) (https://www.lucidchart.com/blog/2013/01/17/a-history-of-the-venn-diagram)