UNIT 11: TESTING OF HYPOTHESES

UNIT STRUCTURE

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11.1 LEARNING OBJECTIVES

After going through this unit, you will be able to

- learn the concept of hypothesis
- discuss the basic concepts concerning testing of hypothesis
- discuss the test statistic and critical region
- outline the procedure for hypothesis testing
- describe the different test statistic

11.2 INTRODUCTION

Hypothesis is usually considered as the principal instrument in research. Its main function is to suggest new experiments and observations. In fact, many experiments are carried out with the deliberate object of testing hypotheses. Decision-makers often face situations wherein they are

interested in testing hypotheses on the basis of available information and then take decisions on the basis of such testing.

11.3 CONCEPTS IN TESTING OF HYPOTHESIS

Most people including researchers make rough confidence statements on many aspects of their areas of interest, be it business research, social science or industrial engineering, etc. These statements or hypotheses are often about particular facets of their work areas. These hypotheses need verification for possible acceptance or rejection. To this end, researcher gather data and allow them to support or cast doubt on their hypotheses. This process is termed hypothesis testing and is one of two main areas of statistical inference.

Hypothesis testing is a statistical technique that is used in a variety of situations. Though the technical details differ from situation to situation, all hypothesis tests use the same core set of terms and concepts.

Null Hypothesis: The null hypothesis is a clear statement about the relationship between two (or more) statistical objects. These objects may be measurements, distributions, or categories. Typically, the null hypothesis, as the name implies, generally expresses the idea of no difference and usually given the symbol H_0.

Alternative Hypothesis: The alternative hypothesis, which we denote by H1 , expresses the idea of some difference. Alternative hypotheses may be one-sided or two-sided. Usually the setting of the problem determines the alternative even before the data has been collected.

For example suppose we are interested in testing whether there is any statistical evidence, from a random sample of potential customers, to support the hypothesis that more than 10% of the potential customers will purchase a new product. In this case the null and alternative hypotheses are

H_0 : P = 0.01 (Null Hypothesis)

H_1 : P ‘“ 0.01 (Alternate Hypothesis)

Note: Alternative hypothesis determines the tail of a test.

For example: If we consider the null hypothesis as

$H_0 : \mu = 100$ then the alternative hypothesis may be

H_1 : μ ‘“ 100 (Two tail test) or

H_1 : μ > 100 (Right tail test) or

H_1 : μ < 100 (Left tail test)

Errors in Testing of Hypothesis: All hypothesis testing is liable to errors. There are two basic kinds of error:

- Type I error: Reject H_0 when it is, in fact, true; the probability of committing a type I error is denoted by α.
- Type II error: Accept H_0 when it is, in fact, false; the probability of committing a type II error is denoted by β.

The objective in all hypothesis testing is to set the Type I error level, also known as the significance level, at a low enough value, and then to use a test statistic which minimises the Type II error level for a given sample size. As we fix the Type I error level, it is best to devise the test in such a way that the Type I error is most serious, in terms of cost.

Table: 11.1 possible Solutions in Hypothesis Testing

Actual	Decision	
	Accept H_0	Reject H_0
H_0 is true	Correct Decision	Wrong Decision (Type I Error)
H_0 is false	Wrong Decision (Type II Error)	Correct Decision

Level of Significance: The level of significance is a measure of the statistical strength of the hypothesis test. It is often characterized as the probability of incorrectly concluding that the null hypothesis is false. The significance level is something that you should specify up front. In applications, the significance level is typically one of three values: 10%, 5%, or 1%. A 1% significance level represents the strongest test of the three. For this reason, 1% is a higher significance level than 10%.

Power: Related to significance, the power of a test measures the probability of correctly concluding that the null hypothesis is true.

Test Statistic: After setting up the null hypothesis and alternative hypothesis we compute the test statistic. The test statistic is a statistic based on

appropriate probability distribution. It is used to test whether the null hypothesis set up should be accepted or rejected. It is the main yardstick which helps us to decide whether to accept or reject the null hypothesis.

$$\text{TestStatistic} = \frac{\text{Sample Statistic - Hypothesised Parameter}}{\text{Standard Error of Statistic}}$$

Different probability distribution values are used in appropriate cases while testing the null hypothesis. The following are the most commonly used test statistic.

Z-Distribution: We use Z-Distribution under normal curve for large sample (i.e. if the sample size n > 30). The Z-statistic is defined as

$$Z = \frac{t - E(T)}{S.E.(t)} \sim N(0,1) \text{ as } n \to \infty$$

t being the statistic and T the parameter.

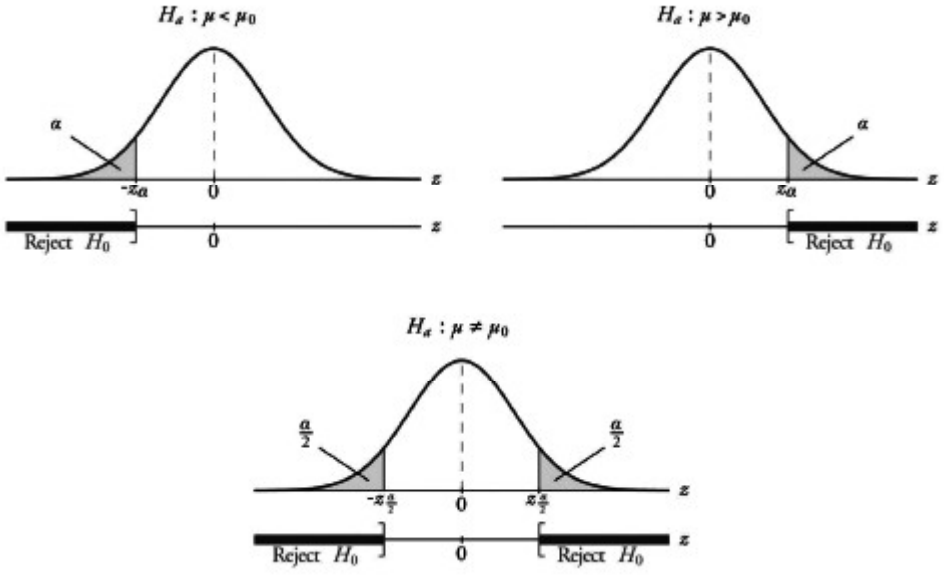
t - Test: t-test is used for small sample i.e., if the sample size n<30.

Critical Region

In testing of hypothesis each test statistic follow certain sampling distribution. As these are also probability distribution so each distribution covers some specified region. Based on the level of significance the region covered by each probability distribution is divided into two mutually exclusive regions, viz. acceptance region and critical region. The critical region of the sampling distribution of a statistic is also known as the α region. It is the area, or areas, of the sampling distribution of a statistic that will lead to the rejection of the hypothesis tested when that hypothesis is true.

Example: When we are testing the null hypothesis $H_0 : \mu = 60$ against the two-sided alternative hypothesis $H_1 : \mu$ '"60, the critical region consists of both tails of the sampling distribution of the test statistic. Such a test is a *two-tailed test.*

On the other hand, if we are testing the null hypothesis $H_0 : \mu = 60$ against one-sided alternative $H_1 : \mu > 60$ *or* $H_1 : \mu < 60$, the critical regions are the *right tail* or *left tail* of the sampling distribution of the test statistic respectively.



11.4 STEPS IN TESTING OF HYPOTHESIS

Steps in Hypothesis Testing: The steps follows in testing of hypothesis are as follows:

1. The first and foremost step in hypothesis testing is tospecify the null hypothesis (H_0) and the alternative hypothesis (H_1).
2. The second step is to specify the level of significance α.
3. The third step is the calculation of test statistic which is given by

$$\text{Test Statistic} = \frac{\text{Sample Statistic} - \text{Hypothesised Parameter}}{\text{Standard Error of Statistic}}$$

4. The next step is to locate a critical region depending on the level of significance.
5. The final step is to describe the result and the statistical conclusion in a concrete way.



CHECK YOUR PROGRESS

Q1: Distinguish between the following:

(i) Null hypothesis and alternative hypothesis

(ii) One-tailed test and two-tailed test

(iii) Type I error and Type II error

Q2: The procedure of testing hypothesis requires a researcher to adopt several steps. Describe in brief all such steps.

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11.5 APPLICATIONS OF HYPOTHESIS TESTING

In this section, some of the standard tests that are most widely used in applications are presented. These are:

1. Test of Single Mean (Large Sample)
2. Test of Single Mean (Small Sample)
3. Test of Difference between Means (Large Sample)
4. Test of Difference between Means (Small Sample)
5. Test of Single Population (Large Sample)
6. Tests for Difference between Two Population Proportions

11.6 TEST OF SINGLE MEAN (LARGE SAMPLE)

Suppose we want to test the null hypothesis $H_0 : \mu = \mu_0$ against one of the alternatives

$H_0 : \mu \neq \mu_0$ or $H_0 : \mu > \mu_0$ or $H_0 : \mu < \mu_0$ on the basis of a large sample n ($n \geq 30$) from the normal population with known variance σ^2. The appropriate test statistic for this test is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Example : Suppose that it is known from experience that the standard deviation of the weight of 8-ounce packages of cookies made by a certain

bakery is 0.16 ounce. To check whether its production is under control on a given day, they select a random sample of 25 packages and find that their mean weight is 8.112 ounce. Is the production process under control? Use 5% level of significance.

Solution:

1. Null hypothesis : $H_0 : \mu = 8$

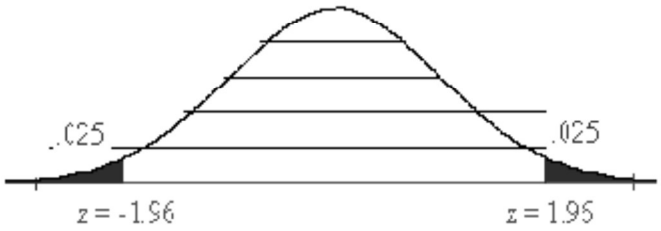
2. Alternative hypothesis : $H_1 : \mu \neq 8$

3. Value of Test Statistic,

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{8.112 - 8}{0.16/\sqrt{25}} = 3.5$$

4. Significance level : 5 %

5. Critical regions :



6. Conclusion : Since calculated *z* = 3.50 exceeds the critical value 1.96, we reject the null hypothesis at the 5% significance level and conclude that the process is not under control.

11.7 TEST OF SINGLE MEAN (SMALL SAMPLE)

Example: The height of 10 soldiers selected at random had a mean height of 158 cm and variance of 39.0625 cms. Assuming a significance level of 5%, test the hypothesis that the soldiers of the population are on the average lesser than 162.5 cms tall.

Solution

1. Null hypothesis : $H_0 : \mu = 162.5$

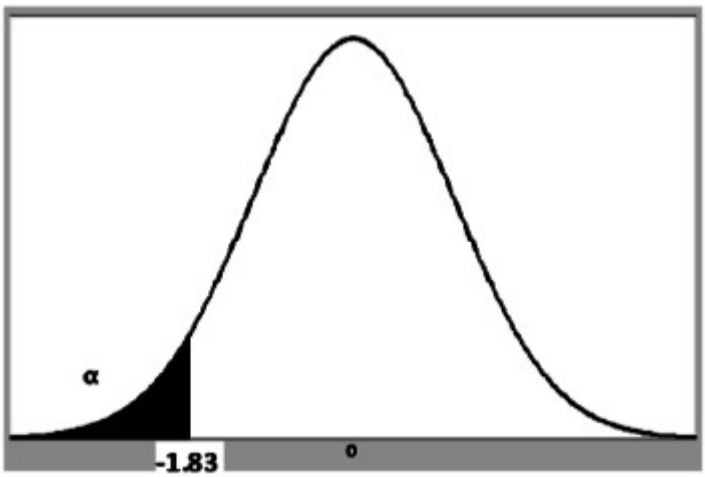
2. Alternative hypothesis : $H_1 : \mu < 162.5$

3. Value of Test Statistic,

$$t = \frac{\bar{x} - \mu}{\sqrt{s^2/(n-1)}} = \frac{158 - 162.5}{\sqrt{39.0625/(10-1)}} = -2.163$$

4. Significance level: 5 %

5. Critical regions: Critical value of t at 5% level of significance with n-1 = 10-1 = 9 degrees of freedom is -1.83.



6. Conclusion: Since calculated t = -2.163 falls in the region of rejection, we reject the null hypothesis at the 5% significance level and conclude that the average height of the soldiers is less than 162.5cm.



CHECK YOUR PROGRESS

Q3: The average score in an aptitude test administered at the national level is 80.To evaluate a state's education system, the average score of 100 of the state's students selected on random basis was 75 with a standard deviation. The state wants to know if there is a significant difference between the local scores and national scores. Conduct the testing of hypothesis for the above situation.

Q.4: The height of ten children selected at random from a given locality had a mean 63.2 cms and variance 6.25 cms. Test at 5% level of significance the hypothesis that the children of given locality are on the average less than 65

11.8 TEST OF DIFFERENCE BETWEEN MEANS (LARGE SAMPLE)

This test is done to detect whether or not the means of two samples drawn from two different sources differ significantly. It is also done to check whether the difference is due to a chance and whether the samples belong to the same population.

The steps in the process of testing the significance of the difference between the two means are as follows:

1. The null hypothesis to be tested is written as H_0: $\mu_1 = \mu_2$ against the alternative hypothesis H_1: $\mu_1 \neq \mu_2$

2. Specify the level of significance α.

3. Computing the test statistic

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left[\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right]}}$$

where $\bar{x}_1$ and $\bar{x}_2$ are sample means, σ_1 and σ_2 are standard deviations of population 1 and population 2 respectively and n_1, n_2 are the sample sizes.

4. Determination of critical value and conclusion.

Example: Given that the standard deviations of each of the two populations is 2.5, and the means of two large samples of sizes 1000 and 2000 are 67.5 and 68 respectively. Test the equality of means.

Solution:

1. Null Hypothesis H_0: $\mu_1 = \mu_2$ (H_0: There is no difference between means)

2. Alternative Hypothesis H_1: $\mu_1 \neq \mu_2$ (H_1: There is difference between means)

3. α = 5% or 0.05

4. Test Statistic, z

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left[\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right]}} = \frac{67.5 - 68}{\sqrt{\left[\frac{6.25}{1000} + \frac{6.25}{2500}\right]}} = -5.16$$

5. From the Standard Normal Distribution Table, the critical value of z, at 5% level of significance is ± 1.96.

6. As the computed value of z is greater than critical value of z, it falls in the region of rejection and hence the null hypothesis is rejected.

It can be concluded that the means are significantly different from each other.

11.9 TEST OF DIFFERENCE BETWEEN MEANS (SMALL SAMPLE)

The steps in the process of testing the significance of the difference between the two means in case of small samples are as follows:

1. The null hypothesis to be tested is written as H_0: $\mu_1 = \mu_2$ against the alternative hypothesis H_1: $\mu_1 \neq \mu_2$

2. Specify the level of significance α.

3. Computing the test statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

$$\text{where } S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

and s_1, s_2 are the sample standard deviations and n_1 and n_2 are the sample sizes.

The test statistic follows Student's t-distribution with $n_1 + n_2 - 2$ d.f.(Degrees of Freedom).

4. Determination of critical value and conclusion.

Example: Samples of two types of electric bulbs were tested for length of life and following data were obtained.

	Type I	Type II
Sample No.	8	7
Sample Mean	1234 hours	1036 hours
Sample SD	36 hours	40 hours

Is the difference in the means sufficient to warrant that Type I is superior to Type II regarding the length of life?

Solution:

1. Null Hypothesis H_0: $\mu_1 = \mu_2$ (H_0: There is no difference between mean life of bulbs)

2. Alternative Hypothesis H_1: $\mu_1 > \mu_2$ (H_1: Type I is superior to Type II)

3. α = 5% or 0.05

4. Test Statistic is given by

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{1}{13}\left[8X(36)^2 + 7X(40)^2\right] = 1659.08$$

$$\therefore t = \frac{1234 - 1036}{\sqrt{1659.08\left[\frac{1}{8} + \frac{1}{7}\right]}} = 9.39$$

5. The critical value of t, at 5% level of significance with 13 d.f. is 1.77.

6. As the computed value of t is greater than critical value of t, it falls in the region of rejection and hence the null hypothesis is rejected.

It can be concluded that the Type I is superior to Type II.

11.10 TEST OF SINGLE PROPORTION (LARGE SAMPLE)

If the observations on various items are categorized into two classes C_1 and C_2, we often want to test the hypothesis, whether the proportion of items in a particular class, say C_1, is P_0 or not. In such cases the null hypothesis becomes

$H_0 : P = P_0$

and $H_1 : P \neq P_0$ or $H_1 : P > P_0$ or $H_1 : P < P_0$

The test statistic for the single proportion is

$$z = \frac{\hat{P} - P_0}{\sqrt{P_0 Q_0 / n}}$$

where $Q_0 = 1 - P_0$ and $\hat{P}$ is the estimated proportion in class C_1.

z follows the standard normal distribution.

Example: To test the conjecture of the management that 60 percent employees favour a new bonus scheme, a sample of 150 employees was drawn and their opinion was taken whether they favour it or not. Only 55 employees out of 150 favoured the bonus scheme. Test the validity of the conjecture of the management at 1% level of significance.

Solution:

1. Null Hypothesis H_0: P = 0.60 (H_0: 60 percent employees favoured the bonus scheme)

2. Alternative Hypothesis H_1: $P \neq 0.60$

3. α = 1% or 0.01

4. Test Statistic, z

$$z = \frac{\hat{P} - P_0}{\sqrt{P_0 Q_0 / n}} = \frac{0.367 - 0.60}{\sqrt{(0.60 X 0.40)/150}} = -5.825, \text{ since } \hat{P} = \frac{55}{150} = 0.367$$

5. From the Standard Normal Distribution Table, the critical value of z, at 1% level of significance is ± 2.58.

6. As the computed value of z is greater than critical value of z, it falls in the region of rejection and hence the null hypothesis is rejected.

It can be concluded that 60 percent employees do not favour the new bonus scheme.

11.11 TESTS FOR DIFFERENCE BETWEEN TWO POPULATION PROPORTIONS

If we have two populations and each item of a population belonged to either of the two classes C_1 and C_2. One may be interested to know whether the proportion of items in C_1 in the both the populations is the same or not. In such cases we apply tests for difference between two population proportions.

The null hypothesis is

$$H_0 : P_1 = P_2$$

and $H_1 : P_1 \neq P_2$ or $H_1 : P_1 > P_2$ or $H_1 : P_1 < P_2$

The test statistic for tesing the difference between two population proportions is

$$z = \frac{P_1 - P_2}{\sqrt{\hat{P}\hat{Q}\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}}$$

where $\hat{Q} = 1 - \hat{P}$ and $\hat{P}$ is the pooled estimate of proportion given by

$$\hat{P} = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$$

z follows the standard normal distribution.

Example: A sample of 400 families in an old city is selected randomly, and a sample of 500 families is randomly selected from several new colonies of the same city. A survey is conducted for the number of houses possessing washing machine. The number of washing machine holders in old city is 48 out of 400 selected families wheras it was 120 in new colonies out of 500. Test whether there is any difference between proportion of washing machine holders in two colonies.

Solution:

1. Null Hypothesis H_0: P_1 =

2. Alternative Hypothesis H_1: $P_1 \neq P_2$

3. α = 5% or 0.05

4. Calculation: $P_1 = 0.12$, $P_2 = 0.24$

$$\hat{P} = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = 0.187 \text{ and } \hat{Q} = 1 - \hat{P} = 0.813$$

$$z = \frac{P_1 - P_2}{\sqrt{\hat{P}\hat{Q}\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}} = \frac{0.12 - 0.24}{\sqrt{(0.187)(0.813)(0.0025 + 0.002)}} = -4.59$$

5. From the Standard Normal Distribution Table, the critical value of z, at 1% level of significance is ± 2.58.

6. As the computed value of z is greater than critical value of z, it falls in the region of rejection and hence the null hypothesis is rejected.

It can be concluded that the proportion of washing machine holders in old city and in new colonies is not the same at 1% level of significance.

11.12 LET US SUM UP



After going through the unit we have come through the following:

- Hypothesis testing is a statistical technique that is used in a variety of situations including decision making.
- The null hypothesis, as the name implies, generally expresses the idea of no difference and usually given the symbol H_0.
- The alternative hypothesis, which we denote by H1, expresses the idea of some difference. Alternative hypothesis determines the tail of a test.
- There are two types of errors in testing of hypothesis: Type I error and Type II error.
- Level of significance plays an important role in testing of hypothesis.

- Test statistic is used to test whether the null hypothesis set up should be accepted or rejected. It is the main yardstick which helps us to decide whether to accept or reject the null hypothesis.
- $$\text{TestStatistic} = \frac{\text{Sample Statistic - Hypothesised Parameter}}{\text{Standard Error of Statistic}}$$
- Some of the standard tests that are most widely used in applications are presented.

  These are:
  - Test of Single Mean (Large Sample)
  - Test of Single Mean (Small Sample)
  - Test of Difference between Means (Large Sample)
  - Test of Difference between Means (Small Sample)
  - Test of Single Population (Large Sample)
  - Tests for Difference between Two Population Proportions

11.13 FURTHER READING



1. Malhotra, N.K. and Dash, S. (2011). Marketing research: An Applied Orientation (6th edition), New Delhi: Pearson Education.

11.14 ANSWERS TO CHECK YOUR PROGRESS



Ans to Q.1. (i) Null Hypothesis: The statistical hypothesis that is set up for testing a hypothesis is known as null hypothesis and it is denoted by H_0. **Alternative Hypothesis:** The negation of null hypothesis is called the alternative hypothesis and it is denoted by H_1.

(ii) The alternative hypothesis determines the tail of a test. For example, let us consider the null hypothesis to be tested be

$H_0 : \mu_1 = \mu_2$ against the alternative hypothesis

$H_1: \mu_1 \neq \mu_2$

Then it is called two tailed test.

If the alternative hypothesis is

$H_1: \mu_1 > \mu_2$ (Right Tail Test)

Or $H_1: \mu_1 < \mu_2$ (Left Tail Test)

Then it is known as one tail test.

(iii) Type I Error: Rejection of H_0 when it is, in fact, true.

Type II Error: Acceptance of H_0 when it is false.

Ans to Q2: Steps in Hypothesis Testing: The steps follows in testing of hypothesis are as follows:

1. The first and foremost step in hypothesis testing is tospecify the null hypothesis (H_0)
   and the alternative hypothesis (H_1).
2. The second step is to specify the level of significance á.
3. The third step is the calculation of test statistic which is given by

$$\text{Test Statistic} = \frac{\text{Sample Statistic - Hypothesised Parameter}}{\text{Standard Error of Statistic}}$$

4. The next step is to locate a critical region depending on the level of significance.

5. The final step is to describe the result and the statistical conclusion in a concrete way.

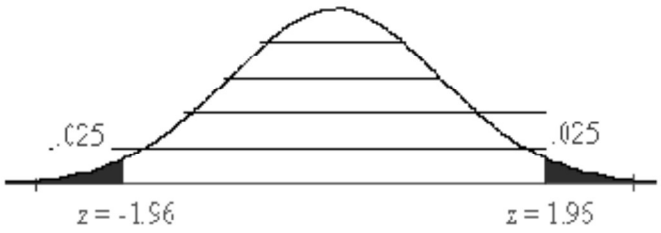
Ans to Q3:

1. Null hypothesis : $H_0 : \mu = 80$
2. Alternative hypothesis : $H_1 : \mu \neq 80$
3. Value of Test Statistic,

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{75 - 80}{14/\sqrt{100}} = -3.57$$

$$\therefore |z| = |-3.57| = 3.57$$, since it is a two tail test

4. Significance level : 5 %
5. Critical region :



6. Conclusion: Since calculated z = 3.57 exceeds the critical value 1.96, we reject the null hypothesis at the 5% significance level and conclude that there is a significant difference between the local scores and the national scores.

Ans to Q4:

1. Null hypothesis : $H_0 : \mu = 65$
2. Alternative hypothesis : $H_1 : \mu < 65$
3. Value of Test Statistic,

$$t = \frac{\bar{x} - \mu}{\sqrt{s^2/(n-1)}} = \frac{63.2 - 65}{\sqrt{6.25/(10-1)}} = -2.158$$

4. Significance level: 5 %

5. Critical regions: Critical value of t at 5% level of significance with (n – 1) = (10 – 1) = 9 degrees of freedom is -1.83.

6. Since $|t| = 2.158 > t_{0.05,9} = 1.83$, therefore the null hypothesis is rejected and can be concluded that the mean height of the children is less than 65 cms.

11.15 MODEL QUESTIONS



Q.1. In a random sample of 400 persons from a large population, 120 is females. Can it be said that males and females are in the ratio 5:3 in the population? Use 10% level of significance.

Q.2. The mean I.Q. of a sample of 1600 children was 99. Is it likely that this was a random sample from a population with mean I.Q. 100 and standard deviation 15?

Q.3. The mean weekly sales of the chocolate bar in candy stores was 140.3 bars per store. After an advertisement campaign, the mean weekly sales in 22 stores for a typical week increased to 15307 and showed a standard deviation of 17.2. Was the advertising campaign successful?

Q.4. On a certain day, 74 trains were arriving on time at Delhi station during the rush hours and 83 were late. At New Delhi there were 65 on time and 107 late. Is there any difference in the proportions arriving on time at the two stations?

Q.5. The mean yield of wheat from a district A was 210 kgs with standard deviation 10 kgs. per acre from a sample of 100 plots. In another district B, the mean yield was 220 kgs with standard deviation 12 kg from a sample of 150 plots. Assuming that the standard deviation of the yield in the entire state was 11 kg, test whether there is any significant difference between the mean yield of crops in the two districts.

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