UNIT 6: TRANSFORMATIONS

UNIT STRUCTURE

6.1 Learning Objectives

6.2 Introduction

6.3 Basic concept of transformation

6.4 Types of transformation

6.4.1 Geometric transformation

6.4.1.1 Translation

6.4.1.2 Rotation

6.4.1.3 Scaling

6.4.1.4 Shearing

6.4.2 Coordinate transformation

6.5 Homogenous Coordinate

6.7 Let Us Sum Up

6.8 Further Reading

6.9 Answer to Check Your Progress

6.10 Model Questions

6.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- Basic concept of transformation.
- Types of Transformation
- Geometric transformation

6.2 INTRODUCTION

Transformation means moving an object, rotating, resizing an object from one place to another. The need for transformation arises when several object independently defined in its coordinate need to represent in a master coordinate system. It is required to reposition the images on the screen and change their size or orientation.

6.3 BASIC CONCEPT OF TRANSFORMATION

Transformation is any change in the position, shape,or size of an object. There are two types of transformations - geometric transformation, and coordinate transformation. Geometric transformation in which change in the object size, shape, and position takes place,keeping the coordinate system constant. But in coordinate transformation, the object is held constant while the coordinate system is transformed.

6.4 TYPES OF TRANSFORMATION

6.4.1 Geometric Transformation:

Here the objects movesto keep the coordinate system fixed at one place.

TRANSLATION

The translation operation shifts the object from one position to other position. To translate a point from a coordinate position (x,y) to another point (x1,y1),we add algebraically the translation distances Tx and Ty to the original coordinates.

X1=x+Tx

Y1=y+Ty

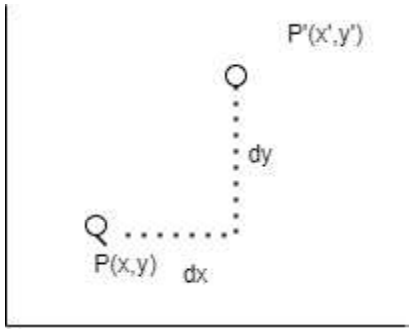
Tx and Ty are called shift vector.

To translate a point from one position to another, matrix representation is:

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix}$$

Where x' and y' are coordinates of new point,dx and dy are translation distance along x and y axis.

It can be written as P'=P+T where P' is the new position of point P, P is the old position and T is the translation distance.



Question 1. Given a triangle A(2,2),B(10,2) and C(5,5). Translate the triangle with translation distance dx=5,dy=6.

Solution: Given A=(2,2), B(10,2) ,C=(5,5) and translation distance dx=5,dy=6.

We know in order to get new coordinates we have P'=P+t

Now the new coordinates for A ,B,C can be written as

A'=A+T

B'=B+T

C'=C+T

Where A',B',C' are new coordinates after translation and A,B,C are original coordinates and

T represents translation distance along the x and y-axis.

$$A' = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

$$B' = \begin{bmatrix} 10 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 15 \\ 8 \end{bmatrix}$$

$$C' = \begin{bmatrix} 5 \\ 5 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \end{bmatrix}$$

Thus the new coordinates after translation is A'(7,8),B'(15,8),C'(10,11)

ROTATION

Rotation is a process of changing the angle of the object. It can be clockwise or anticlockwise.

Let coordinate of P in the polar form

X=rcosφ

Y=rsinφ

Where φ is the angle between x-axis and point P, r is the distance from origin to P.

After rotation the new coordinates of the point P is represented by P'(x',y')

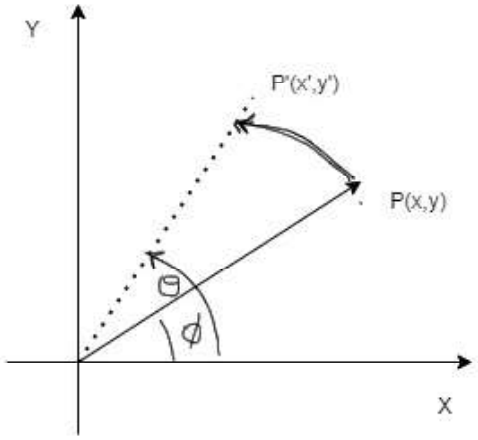
X'=rcos($\varphi + \varphi$)

Y'=rsin($\varphi + \varphi$)

So finally, we can say that

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

i.e P'=R.P



Question 2. Given a triangle whose coordinates are A (2,2),B(8,2) and C(5,5). Rotate the triangle by 90degree.

Solution. We know rotation matrix is given by following.

$$R = \begin{bmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

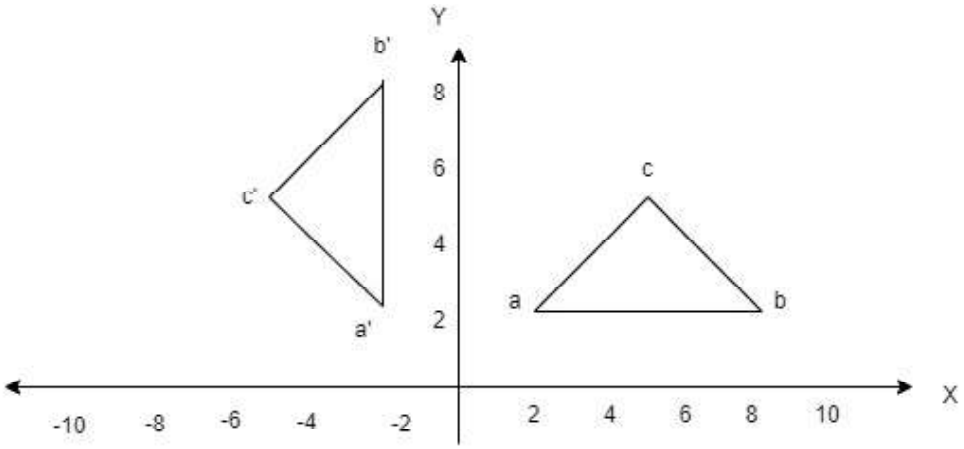
Therefore to get the coordinates of triangle after rotation, we have to multiply rotation matrix with the A,B,C coordinates.

$$A' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$B' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 8 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$$

$$C' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} -5 \\ 5 \end{bmatrix}$$

Thus the new coordinates A',B',C' after rotation is A'(-2,2),B'(-2,8) and C'(-5,5)



The figure showsthe original triangle which is on the right side and the triangle after rotation is on the left side with coordinates A'(-2,2), B(-2,8), and C'(-5,5).



CHECK YOUR PROGRESS

Q 1: Given a circle C with radius 10 and center coordinates (1, 4). Apply the translation with distance 5 towards X axis and 1 towards Y axis. Obtain the new coordinates of C without changing its radius.

Q 2: Given a triangle with corner coordinates (0, 0), (1, 0) and (1, 1). Rotate the triangle by 90 degree anticlockwise direction and find out the new coordinates.

SCALING

Scaling is used to alter or change the size of an object based on scaling factor. There are two types of scaling factor:

Sx- Scaling which is in x-direction.

Sy-Scaling which is in y-direction

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} Sx & 0 \\ 0 & Sy \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$P' = S.P$$

If scaling factor(SF)=1, the object size will not change.

If SF>1, the object gets expands, and if SF<1, the object size gets reduced.

The above matrix can be represented in 3*3 matrix as written below

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

QUESTION 3. Scale the polygon with coordinates A(2,5),B(7,10) and C(10,2) by two units in x-direction and two units in the Y direction.

Solution: Here given A=(2,5), B=(7,10),(10,2) and Sx=Sy=2

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Putting the value of Sx,Sy and A,B,C in the above,we have

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 7 & 10 \\ 5 & 10 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 14 & 20 \\ 10 & 20 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

Scaling matrix

3 points of polygon in matrix form

Since we have three points in the polygon A(x1,y1),B(x2,y2) and C(x3,y3) ,so we have written these three points in the above matrix form with scaling matrix.

SHEARING

Shearing transformation makes the object to slant either in the X-direction or in Y-direction. Two types of shearing transformation are there-X -shear and Y-shear.X-shear maintains the y coordinates and changes take place in x values which causes the vertical line to tilt left or right. The Y-shear preserves the x-coordinate but changes take place in y coordinates.

The new coordinates after applying shearing is:

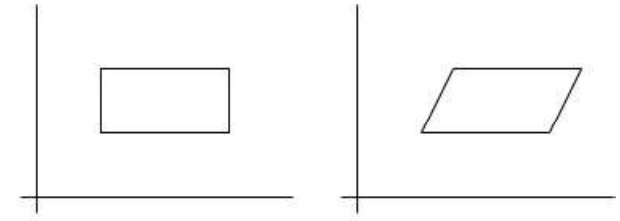
$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ shx & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Shearing matrix in X-axis

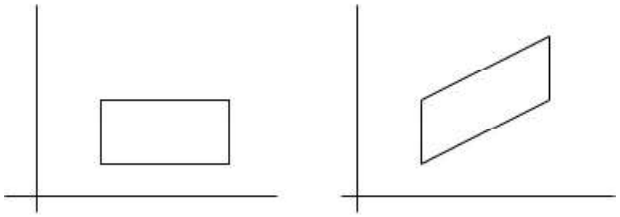
$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Shy & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Shearing matrix in Y-axis

After applying shearing, the effects are shown below:



Before X-shear After X-Shear



Before Y-shear After Y-shear



CHECK YOUR PROGRESS

Q 3: Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.

Q 4: Given a triangle with points (1, 1), (0, 0) and (1, 0). Apply shear parameter 2 on X axis and 2 on Y axis and find out the new coordinates of the object.

6.4.2 Coordinate Transformation

We can apply the effects of translation, scaling and rotation by moving the observer who views the object and by keeping the object stationary. This type of transformation is called coordinate transformation.

6.5 HOMOGENOUS COORDINATE

A 2-dimensional coordinate can be represented in 3 dimensions using homogenous coordinates. The two-dimensional transformation can be represented as 3*3 matrices so that usage of the operations becomes relatively easy. Generally, this concept of homogenous is used in design and construction applications. This conversion is done in translation, scaling, shearing, and rotation to fit the picture into the proper position.

Consider coordinates (x,y) in 2-D. This can be represented in 3 D as (x_h, y_h, h) where

X_h=x. h and Y_h=y. h.

In order to convert from 3-D to 2-D

$X=X_h/h$

$Y=Y_h/h$

Therefore matrix representation of translation in homogenous coordinates is given by:

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 1 & 0 & ty \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Rotation matrix in homogenous is given by:

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Y \end{bmatrix}$$

The scaling matrix in homogenous coordinates is given by:

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Y \end{bmatrix}$$

The Shearing matrix in homogenous coordinates is given by

$$\begin{bmatrix} X' \\ Y' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$



CHECK YOUR PROGRESS

Q 5: Convert coordinates (2,3) into homogenous coordinates if h=2.

Q 6: Convert homogenous coordinates (8,4,2) into 2D.



6.6 LET US SUM UP

- Ability to simulate the manipulation of objects in space is transformation.
- Geometric transformation is in which the object itself is transformed relative to a stationary background.
- Coordinate transformation in which the object is held stationary while coordinate system is transformed relative to the object.
- In translation, an object is displaced a given distance and direction from its original position.

- In rotation, the object is rotated theta degree about the origin.
- Scaling is the process of expanding or compressing the dimensions of an object.
- In a homogenous coordinate system, two-dimension coordinate positions are transformed into three-dimensioncoordinates.



6.7 FURTHER READINGS

1) Computer Graphics y Zhigang Xiang and Roy A Plastock
2) Computer Graphics By Hearn and Baker



6.8 ANSWER TO CHECK YOUR PROGRESS

Ans to Q No 1: New center coordinates of C = (6, 5).

Ans to Q No 1: New coordinates of the triangle after rotation = A (0, 0), B(0, 1), C(-1, 1).

Ans to Q No 1: New coordinates after scaling =A(0,9),B(6,9),C(6,0),D(0,0)

Ans to Q No 1: New coordinates of the triangle after shearing in Y axis = A (1, 3), B(0, 0), C(1, 2).

New coordinates of the triangle after shearing in X axis = A (3, 1), B(0, 0), C(1, 0).

Ans to Q No 1: Homogenous coordinates of (2,3) is (4,6,2)

Ans to Q No 1: x=8/2=4,y=4/2=2 So the original coordinates in 2-D is (4,2).



6.9 MODEL QUESTIONS

Q 1: Find the matrix that represents rotation of an object by 30 degree about the origin.

Q 2: What are the new coordinates of the point P(2,-4) after the rotation?

Q 3: Write the general form of the matrix for rotation about a point P(h,k).

Q 4: Perform a 45 degree rotation of a triangle A(0,0),B(1,1) ,C(5,2) (a) about a origin (b)about P(-1,-1)

Q 5: Write the general form of a scaling matrix with respect to a fixed point P(h,k)

Q 6: Magnify the triangle with vertices A(0,0),B(1,1),C(5,2) to twice its size while keeping C(5,2) fixed.

Q 7: Prove that 2D Scaling transformations are commutative i.e, S1 S2=S2 S1.

Q 8: Prove that 2D rotations about the origin are commutative i.e. R1 R2=R2 R1.

Q 9: Rotate a line CD whose endpoints are (3, 4) and (12, 15) about origin through a 45° anticlockwise direction.

Q 10: Rotate line AB whose endpoints are A (2, 5) and B (6, 12) about origin through a 30° clockwise direction.

*** ***** ***