UNIT 9: 3-D OBJECT REPRESENTATION

UNIT STRUCTURE

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9.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- Learn about 3-D object representation
- Learn about Polygon surfaces
- Learn about Quadric surfaces
- Learn about Basic illumination models
- Learn about Polygon rendering methods

9.2 INTRODUCTION

We have already been acquainted with the idea of different clipping algorithms of line and polygon like Cohen-Sutherland and Cyrus-beck line clipping algorithms, Sutherland -Hodgeman polygon clipping algorithm in the previous unit.

In this unit, we will learn about 3-D Object representation. We will be able to gather knowledge on Polygon surfaces, quadric surfaces, spline representation, basic illumination models, and polygon rendering methods.

In the next unit we will explore the concept of Hermite curve, Bezier curve and B-spline curves, Bezier and B-spline surfaces.

9.3 3-D OBJECT REPRESENTATION

In 3-D representation x, y, and z coordinates are used to represent an object. For 3-D objects, transition, scaling, and rotation are used to represent the object. 3-D object representation is divided into the following two categories:

- **Boundary Representations:** Here, the object representation is possible in the form of polygon surfaces. It is a three-dimensional object model. It is enclosed by surfaces and has its interior and exterior. Example: Polygon surfaces
- **Space-partitioning representations:** It is used to represent the internal features of the polygon which is used to represent a 3D object. In non-overlapping regions, the space is divided.



EXERCISE

1. What are the major division of 3-D object representation?

9.4 POLYGON SURFACES

In this type of representation, the object is represented with a set of polygon surfaces. This is the most commonly used boundary representation of 3D objects. We may not be able to represent all the 3D objects using polygon surfaces. We can represent polygon surfaces using the following three approaches:

- **Polygon Tables:** In this method, the surface is represented using a set of coordinates and attributes. Here the object information is stored by using three tables
  - Vertex table
  - Edge table
  - Polygon-surface table

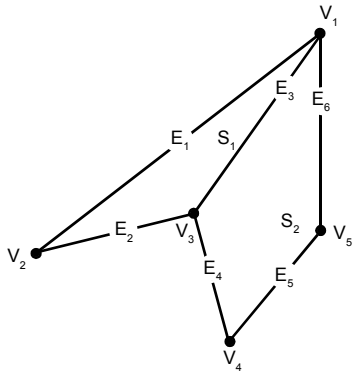


Figure 9.1: Polygon

- **Vertex table:** As shown in figure 9.1, there are five vertices v_1, v_2, v_3, v_4, and v_5. Each vertex stores the information of x, y, and z coordinates. This information can be written using the following vertex table for the five vertices.

Vertex table	
V_1	x_1, y_1, z_1
V_2	X_2, y_2, z_2
V_3	X_3, y_3, z_3
V_4	X_4, y_4, z_4
V_5	X_5, y_5, z_5

Table1: Vertex table of figure 9.1

- **Edge table:** As shown in figure 9.1, there are six edges E_1, E_2, E_3, E_4, E_5, and E_6. The Edge table is used to store the edge information of the polygon. For example, in figure 9.1, the E_1 edge connects the vertices V_1 and V_2. So we can write this information using the edge table for all the edges as given in the following table.

Edge table	
E_1	V_1 , V_2
E_2	V_2 , V_3
E_3	V_1 , V_3
E_4	V_3, V_4
E_5	V_4, V_5
E_6	V_1 ,V_5

Table 2: Edge table of figure 1

- **Polygon-surface table:** As shown in figure 9.1, there are two surfaces S_1 and S_2. It stores the number of surfaces present in the polygon. For example, in figure 9.1 surface S_1 is covered by the edges E_1, E_2, and E_3, and surface S_2 is covered by the edges E_3, E_4, E_5, and E_6. We can write the surface information using the following polygon-surface table.

Polygon-surface table	
S_1	E_1, E_2, E_3
S_2	E_3, E_4, E_5, E_6

Table 3: Polygon-surface table of figure 9.1

Polygon equation is:

$Ax + By + Cz + D = 0$

Where, x, y, and z are points on polygon and A, B, C, and D are the constants.

We can represent this equation like the following also:

$(A/D)x + (B/D)y + (C/D)z = -1$

Let us assume three vertices are (x1, y1, z1), (x2, y2, z2), and (x3, y3, z3).

Now, if we apply Cramer's rule, we can simplify the above in the following way:

$$A = \begin{bmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{bmatrix} \quad B = \begin{bmatrix} x_1 & 1 & z_1 \\ x_2 & 1 & z_2 \\ x_3 & 1 & z_3 \end{bmatrix} \quad C = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} \quad D = -\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

If we simplify this, we will finally get the values as

$A = y_1 (y_2z_3 - y_3z_2) + y_2(y_3z_1 - y_1z_3) + y_3(y_1z_2 - y_2z_1)$

$B = x_1 (x_2z_3 - x_3z_2) + x_2(x_3z_1 - x_1z_3) + x_3(x_1z_2 - x_2z_1)$

$C = x_1 (x_2y_3 - x_3y_2) + x_2(x_3y_1 - x_1y_3) + x_3(x_1y_2 - x_2y_1)$

$D = x_1 (y_2z_3 - y_3z_2) + x_2(y_3z_1 - y_1z_3) + x_3(x_1y_2 - x_2y_1)$

This is how we get the constant values of A, B, C, and D.

For any point x, y, or z with parameters A, B, C, and D, we can say that -

- $A_x + B_y + C_z + D \neq 0$, the point is not on the plane.
- $A_x + B_y + C_z + D < 0$, the point is inside the surface.
- $A_x + B_y + C_z + D > 0$, the point is outside the surface.

- **Polygon Meshes:** The objects which we cannot represent with the help of the polygon surfaces can be represented using polygon meshes. We can have two types of representation: i) Triangle representation and ii) Quadrilateral representation
- **Triangle representation:** Here if the object has 'n' vertices then the object has an (n-1) triangle.
- **Quadrilateral representation:** Here, the surface is divided in the form of small cubes. If the object has n × m array of vertices it will have (n-1)×(m-1) array of Quadrilaterals



CHECK YOUR PROGRESS

Q 1: How can we represent polygon surface?

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9.5 QUADRIC SURFACES

Quadric surfaces are represented in the form of second-degree equations. Using this surface we can represent a complex object. We can represent spheres, ellipsoids, torus, paraboloids, and hyperboloids using quadric surfaces.

- **Sphere:** The equation of the sphere can be represented using the following second-degree equation.

$$x^2+y^2+z^2=r^2$$

where, $x = r\cos\varphi\cos\theta$
$y = r\cos\varphi\sin\theta$
$z = r\sin\varphi$

where φ : Latitude, $-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$
φ : Longitude, $-\pi \le \theta \le \pi$

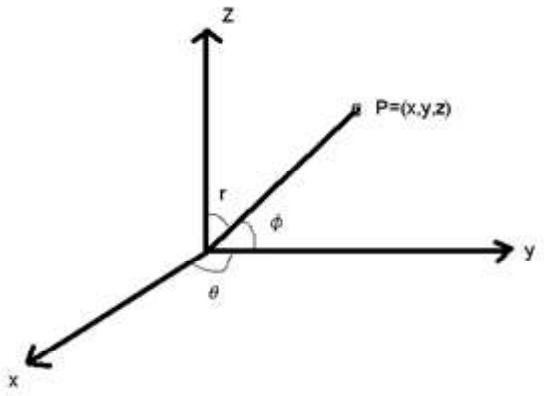


Figure 9.2: Axis angle

- **Ellipsoid:** This is an extension of a spherical surface. This can also be represented in the form of second-degree equations. Here we have three radii, which are perpendicular to each other. The equation for Ellipsoid is given below:

  $(x/r_x)^2+(y/r_y)^2+(z/r_z)^2=1$

  And parametric representation is:

$$x=r_x \cos\phi \cos\theta$$

$$y=r_y \cos\phi \sin\theta$$

$$z=r_z \sin\phi$$

where $-\frac{\pi}{2} \le \phi \le \frac{\pi}{2}$

$-\pi \le \theta \le \pi$ and rx is the radius of the x axis, ry is the radius of the y axis and rz is the radius of z axis.

- **Torus:** A torus is a doughnut-shaped object. It can be generated by rotating a circle, about a specific axis. This can also be represented in the form of second-degree equations.

The equation of Torus is given below:

r-v $(x/r_x)^2+(y/r_y)^2+(z/r_z)^2 =1$

$$x = r_x(r+\cos\phi)\cos\theta$$
$$y = r_y(r+\cos\phi)\sin\theta$$
$$z = r_z \sin\phi$$
$$where -\frac{\pi}{2} \le \phi \le \frac{\pi}{2}$$

$-\pi \leq \theta \leq \pi$ and rx is the radius of x axis, ry is the radius of y axis and rz is the radius of z axis.



CHECK YOUR PROGRESS

Q 2: Define quadric surface.

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9.6 SPLINE REPRESENTATION

A spline is a flexible strip used to produce a smooth curve through a designated set of points. These points are termed control points. The equation of the spline curve is cubical. Spline curves are piecewise polynomial curves. These curves are cubic, polynomial parametric. We use Spline curves to create the model of automobile bodies. It is a free-form curve. We join the given control points to form the spline curve. Control points define the shape of a curve. We can manipulate curves with the help of these control points.

There are two types of spline curves. They are as follows:

- **Interpolation spline:** If a curve is passing through all the control points then the curve is termed as interpolation spline curve. Figure 9.2 shows the interpolation spline curve.
- **Approximation spline:** When the polynomials are fitted to the general control point path without necessarily passing through the control points such type of spline curve is termed as approximation spline. Figure 9.3 shows the Approximation spline curve.



Figure 9.3: Interpolation spline Figure 9.4: Approximation spline

Properties of Spline curves: Spline curve has both local as well as global properties. The property that defines spline curve at a particular place only is termed as local property. Example of local property may be continuity of the curve, curvature of the curve. On the other hand, global

properties define the whole curve. For example global properties will be like the curve is open or close.



CHECK YOUR PROGRESS

Q 3: Write the properties of spline curve.

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9.7 BASIC ILLUMINATION MODELS

The illumination model is used to calculate the intensity of the light that is reflected at a given point on a surface. The lighting effect depends on the light source, surface, and observer. These factors are explained below:

- **Light Source:** It is the light emitting source. There are three types of light sources:
  - **Point Sources:** this is the source that emits rays in all directions. For example a bulb.
  - **Parallel Sources:** The point source which is far from the surface is termed as parallel source. For example: The sun
  - **Distributed Sources:** The source which has rays originated from a finite area is termed as distributed source. For example: A tubelight.
- **Surface:** When light falls on a surface part of it is reflected and part of it is absorbed. The surface structure, the position of the surface and all the nearby surfaces decide the amount of light reflection and absorption.
- **Observer:** The position and sensor sensitivities of the observer also have a huge impact on the lightning effect. The different types of observer's reflection are explained below:
- **Ambient Illumination:** When source of light is indirect, then it is termed as Ambient Illumination. For example, if an observer is standing on a road and facing a building which has glass exterior and the sun rays are falling on that building and rays are reflecting back from the building and the falling on an object which is under observation then it is termed as Ambient Illumination.

The equation of reflected intensity Iamb of any point is:

$I_{amb}=K_a I_a$

Where, I_a : ambient light intensity

K_a: surface ambient reflectivity, the value of K_a varies from 0 to 1

- **Diffuse Reflection:** On rough or grainy surfaces diffuse reflection occurs. The brightness of a point is dependent on the angle made by the light source and the surface.

  The equation of the reflected intensity I_{diff} of a point is:

  $I_{diff} = K_d I_p \cos(?)=K_d I_p (N.L)$

  Where, I_p: the point light intensity

  K_d: the surface diffuse reflectivity, value of K_d varies from 0 to 1

  N: the surface normal

  L: the light direction

- **Specular Reflection:** When light falls on any shiny surface most of it is reflected back and this reflection is termed as specular reflection.

  Phong Model is an empirical model for Specular Reflection which provides the formula for calculation the reflected intensity I_{spec}:

  $I_{spec}=W(\theta)I_l\cos n(\phi)$

  Where $W(\theta)$:K_s

  L: Direction of light source

  N: Normal to the surface

  R: Direction of reflected ray

  V: Direction of observer

  θ: Angle between L and R

  ϕ: Angle between R and V



CHECK YOUR PROGRESS

Q 4: Define Diffuse Reflection

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9.8 POLYGON RENDERING METHODS

IRendering is a process to create realistic image of an object with

the help of computer programs. For rendering techniques, normally two models are used i.e. Illumination Model and Shading Model. We have already discussed about Illumination Model in 9.7. So in this section we will discuss about shading model.

Shading model: The process of altering color of an object or surface in 3-D scene is termed as shading.

There are two types of shading. They are:

- **Constant Intensity Shading:** This shading type is fast. It is also termed as flat shading. It renders an object with polygon surfaces. Here, a single intensity is calculated for each polygon. All points over the surface of the polygon are displayed with the same intensity value. Constant Shading can be useful for quickly displaying the general appearances of the curved surface as shown in the following figure:



Figure 9.5: object A



Figure 9.6: A is rendered with flat shading

- **Gouraud shading:** It is developed by Gouraud. It renders a polygon surface by linear interpolating intensity value across the surface.

  Here each polygon surface is rendered by performing the following calculations:

  - Determining the average unit normal vector at each polygon vertex.
  - Apply an illumination model to each vertex to determine the vertex intensity.
  - Linear interpolate the vertex intensities over the surface of the polygon.

  At each polygon vertex, we obtain a normal vector by averaging the surface normal of all polygons staring that vertex as shown in the following figure:

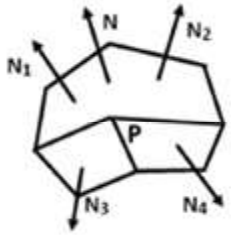


Figure 9.9: the normal vertex at vertex V is calculated as an average of the surface normal for each polygon sharing the vertex

Thus, for any vertex position V, we acquire the unit vertex normal with the calculation

$$Nv = \frac{\sum_{k=1}^{n} Nk}{\left|\sum_{k=1}^{n} Nk\right|}$$

Once we have the vertex normals, we can determine the intensity at the vertices from a lighting model.

- **Phong Shading:** The more accurate method for rendering a polygon surface is to interpolate the normal vector after that apply the illumination model to each surface point. This method is developed by Phong Bui Tuong.

Here a polygon surface is rendered by carrying out the following steps:

- Determine the average unit normal vector at each polygon vertex.
- Linearly interpolate the vertex normals over the surface of the polygon.
- Apply an illumination model along each scan line to calculate projected pixel intensities for the surface points.

Interpolation of the surface normal along a polynomial edge between two vertices as shown in the following figure:

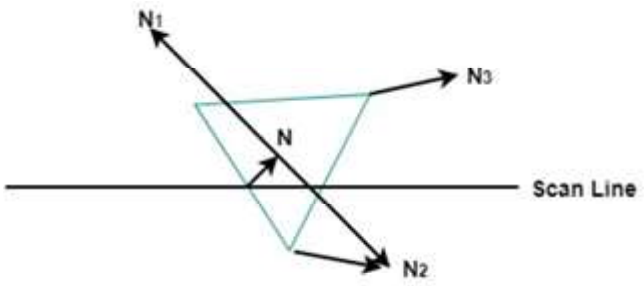


Figure 9.7: interpolation of the surface normal along a polygon edge

$$N = \frac{Y - Y2}{y1 - y2} N_1 + \frac{y1 - y}{y1 - y2} N_2$$

Here, incremental methods are used to evaluate normals between scan lines and along each scan line. At each pixel position along a scan line, the illumination model is applied to determine the surface intensity at that point.



CHECK YOUR PROGRESS

Q 5: Define gouraud shading.

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9.9 LET US SUM

- In 3-D, three coordinates x, y and z are used to represent an object. For 3-D image transition, scaling and rotation are needed.
- Quadric surface are described with second-degree equations. They include sphere, ellipsoids, torus, paraboloids and hyperboloids.
- Spline curves are used to create animation paths. We use Spline curves to create the model of automobile bodies and other drawings and fonts. It is a free-form curve.
- The illumination model is used to calculate the intensity of light that is reflected at a given point on the surface.
- There are two polygon rendering methods namely the Illumination model and the Shading model.



9.10 FURTHER READINGS

1) D. Hearn and P.M. Backer, Computer Graphics, Prentice Hall of India, 1986.
2) W. Newman and R.F. Sproul, Principles of Interactive Computer Graphics, McGraw Hill Publication, 1980.
3) W.K. Giloi, Interactive Computer Graphics, Prentice Hall of India, 1978.

9.11 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q No 1: We can represent polygon surfaces using the following different ways:

- **Polygon Tables:** In this method, the surface is specified by the set of vertex coordinates and associated attributes.
- **Polygon Meshes:** 3D surfaces and solids can be approximated by a set of polygonal and line elements. Such surfaces are called polygonal meshes. In polygon mesh, each edge is shared by at most two polygons. The set of polygons or faces, together form the "skin" of the object.

Ans. to Q No 2: Quadric surface are described with second-degree equations. They include sphere, ellipsoids, torus, paraboloids and hyperboloids.

Ans. to Q No 3: Spline curve has both local as well as global properties. The property that defines spline curve at a particular place only is termed as local property.

Ans. to Q No 4: Diffuse reflection occurs on the surfaces which are rough or grainy. In this reflection the brightness of a point depends upon the angle made by the light source and the surface.

Ans. to Q No 5: This Intensity-Interpolation scheme, developed by Gouraud and usually referred to as Gouraud Shading, renders a polygon surface by linear interpolating intensity value across the surface. Intensity values for each polygon are coordinate with the value of adjacent polygons along the common edges, thus eliminating the intensity discontinuities that can occur in flat shading.

9.12 MODEL QUESTIONS

Q 1: Explain the different approaches of 3D object representation with example.

Q 2: Explain the concept of vertex table with the help of an example.

Q 3: What is the use of polygon surface table?

Q 4: What is sphere? How can we represent the radius of sphere in parametric notation?

Q 5: What is Ellipsoid?

Q 6: How Spline is represented?

Q 7: Write about the illumination models.

Q 8: Discuss polygon rendering methods.

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