PG MA S4-04 (DSE)

TWO YEAR MASTER'S DEGREE COURSE

(CBCS in Accordance with NEP 2020)

SEMESTER - IV

DIFFERENTIAL GEOMETRY

MATHEMATICS

Hiranya Chandra Bhuyan School of Science and Technology

KRISHNA KANTA HANDIQUI STATE OPEN UNIVERSITY

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COURSE INTRODUCTION

"**Differential Geometry**" is the fourth Discipline Specific elective course of MSc (Mathematics) programme. This course provides a systematic introduction to the geometry of curves and surfaces using analytical and tensor-based methods. Beginning with tensor algebra and Riemannian metrics, it develops the tools needed to study curvature, torsion, and the differential properties of space curves. The course then advances to the geometry of surfaces, covering fundamental forms, developable surfaces, geodesics, and curvature theory. Emphasis is placed on building geometric intuition alongside computational skill, enabling learners to understand how curves and surfaces behave, interact, and evolve within three-dimensional space. By the end of the course, learners gain a strong foundation for further study in geometry, mathematical physics, and related fields.

Unit 1: Tensor Algebra

This unit introduces the foundational language of modern geometry—tensors. Learners explore contravariant and covariant vectors, tensor operations, inner products, contraction, and symmetry properties. The unit builds the algebraic framework required for studying geometric objects independent of coordinate choices.

Unit 2: Riemannian Metric

Learners study the Riemannian metric as a tool for measuring lengths, angles, and magnitudes on manifolds. The unit builds intuition for scalar products, projections, and geometric interpretation of coordinate curves and hypersurfaces.

Unit 3: Christoffel Symbols and Covariant Differentiation

In this unit, learners study Christoffel symbols and their transformation laws, followed by covariant differentiation of vectors, tensors, and scalars. The unit explains divergence, curl, and covariant constancy, enabling differentiation on curved spaces.

Unit 4: Curves in Space – I

The unit introduces the differential geometry of space curves, covering tangents, normals, osculating planes, arc length, curvature, torsion, and the Serret–Frenet formulas that describe the motion of the moving trihedron.

Unit 5: Curves in Space – II

Building on Unit 4, this unit studies special curves such as helices and curves defined by the intersection of surfaces. Learners explore intrinsic equations, fundamental theorems of space curves, and the geometry of the osculating circle and osculating sphere.bridge between abstract measure theory and practical applications.

Unit 6: Involute and Evolute

This unit examines involutes and evolutes of space curves, showing how new curves arise from geometric constructions. Learners compute curvature and torsion of evolutes and understand their geometric significance.

Unit 7: Concept of a Surface, Envelopes, and Developables

Learners are introduced to surfaces, regular points, singularities, and parameter transformations. The unit covers tangent planes, normals, and families of surfaces, along with the idea of envelopes and developable surfaces.

Unit 8: Fundamental Forms and Curvature of Surfaces

This unit develops the first and second fundamental forms, which encode metric and curvature information. Learners compute areas, angles, and curvature quantities and study surfaces of revolution and families of curves on surfaces.

Unit 9: Local Non-Intrinsic Properties of a Surface

Focusing on extrinsic geometry, this unit covers normal curvature, principal directions, minimal surfaces, developable surfaces, and lines of curvature. It highlights how surfaces behave relative to the surrounding space.

Unit 10: Conjugate and Asymptotic Lines

This unit introduces conjugate directions and examines their geometric significance on a surface. Learners study the conditions for conjugacy and understand how conjugate direction systems contribute to describing and analysing the local geometry of surfaces.

Unit 11: Asymptotic Lines

In this unit, learners study asymptotic lines, their orthogonality conditions, and their behaviour on ruled surfaces. The unit also includes formulas for curvature and torsion of asymptotic lines.

Unit 12: Fundamental Equations of Surface Theory and Parallel Surfaces

This unit presents the fundamental equations governing surface theory and explores parallel surfaces. Learners compute Gaussian and mean curvature for parallel surfaces and understand how curvature behaves under offsets.

Unit 13: Geodesics

This unit introduces geodesics as the shortest paths on surfaces. Learners study canonical geodesic equations, geodesics on surfaces of revolution, normal properties, torsion of geodesics, and differential equations derived from geometric principles.

Unit 14: Geodesic Curvature

The final unit focuses on geodesic curvature, geodesic parallels, and coordinate systems such as geodesic and geodesic polar coordinates. It also covers angles between curves and geodesics and introduces geodesic mapping.

Each unit includes some along-side boxes to help you know some of the difficult, unseen terms. Some “EXERCISES” have been included to help you apply your own thoughts. You may find some boxes marked with: “LET US KNOW”. These boxes will provide you with some additional interesting and relevant information. Again, you will get “CHECK YOUR PROGRESS” questions. These have been designed to self-check your progress of study. It will be helpful for you if you solve the problems put in these boxes immediately after you go through the sections of the units and then match your answers with “ANSWERS TO CHECK YOUR PROGRESS” given at the end of each unit.