PG MA S4-03(DSC)

TWO YEAR MASTER'S DEGREE COURSE

(CBCS in Accordance with NEP 2020)

SEMESTER - IV

GRAPH THEORY

MATHEMATICS

Hiranya Chandra Bhuyan School of Science and Technology

KRISHNA KANTA HANDIQUI STATE OPEN UNIVERSITY

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COURSE INTRODUCTION

"**Graph Theory**" is a Discipline-Specific Core (DSC) course offered in the fourth semester of the M.Sc. Mathematics programme. The course Graph Theory is an important branch of modern mathematics that deals with the study of graphs, which are mathematical structures used to represent relationships between objects. Graph theory has wide applications in computer science, communication networks, transportation systems, operations research, social networks, electrical circuits, biological sciences, and optimization problems. This course is designed to provide learners with both the theoretical foundations and practical applications of graph theory.

Unit 1: Introduction to Graph Theory

This unit introduces the historical origin and basic concepts of graph theory through the famous Königsberg Bridge Problem. Learners will study the definition of graphs, vertices, edges, degree of a vertex, graph isomorphism, and directed graphs (digraphs). The unit builds the foundational understanding necessary for the study of advanced graph structures.

Unit 2: Graph Operations

This unit focuses on bipartite graphs and their important properties. Learners will study the characterization of bipartite graphs and applications related to matching problems. Cayley's Formula for counting trees is also introduced as an important combinatorial result in graph theory.

Unit 3: Bipartite Graphs

This unit explains the concepts of walks, paths, and cycles in graphs. It further discusses edge connectivity and vertex connectivity, which measure the strength and reliability of connections within a graph or network.

Unit 4: Connectivity

This unit deals with different types of connected graphs, including 2-connected and edge-connected graphs. Learners will understand the importance of connectedness in maintaining network stability and communication systems.

Unit 5: Connected Graphs

This unit deals with different types of connected graphs, including 2-connected and edge-connected graphs. Learners will understand the importance of connectedness in maintaining network stability and communication systems.

Unit 6: Trees

This unit introduces trees, one of the most important structures in graph theory. Topics include definitions and properties of trees, center and centroid of trees, independent cycles, co-cycles, and block cut-point trees. Trees are widely used in data structures, hierarchical systems, and network design.

Unit 7: Cut-Sets and Cut-Vertices

This unit studies cut-sets and cut-vertices, which play a significant role in determining graph vulnerability and network disconnection. Fundamental circuits and cut-sets are also discussed to understand graph decomposition and structural analysis.

Unit 8: Bridges and Algorithms

This unit introduces bridges in graphs and important graph traversal and optimization algorithms such as Breadth-First Search (BFS), Depth-First Search (DFS), Prim's Algorithm, Kruskal's Algorithm, and Dijkstra's Algorithm. These algorithms are widely used in computer science and network optimization.

Unit 9: Traversability

This unit focuses on Eulerian and Hamiltonian graphs along with important optimization problems such as the shortest path problem, Chinese Postman Problem, and Travelling Salesman Problem. These concepts have practical applications in routing, logistics, and transportation planning.

Unit 10: Matching and Coverings

This unit discusses matchings, coverings, dominating sets, domination numbers, factorization, and critical vertices and edges. These concepts are important in combinatorial optimization, scheduling, and resource allocation problems.

Unit 11: Colorability

This unit introduces graph coloring techniques including vertex coloring, chromatic polynomials, and edge coloring. Graph coloring has applications in scheduling, map coloring, register allocation in compilers, and frequency assignment problems.

Unit 12: Planarity

This unit deals with planar graphs and related concepts such as Kuratowski's Theorem, outerplanar graphs, the five-color problem, and dual graphs. The study of planar graphs is important in circuit design and geographical mapping.

Unit 13: Matrices Associated with Graphs

This unit introduces algebraic representations of graphs using matrices such as adjacency matrices, incidence matrices, cycle matrices, path matrices, and cut-set matrices. These matrix methods provide powerful tools for analyzing graph properties mathematically.

Unit 14: Digraphs

This final unit focuses on directed graphs (digraphs), directional duality, acyclic digraphs, and matrices associated with digraphs. Digraphs are important in modeling one-way relationships such as traffic systems, communication networks, and dependency structures.

Each unit includes some along-side boxes to help you know some of the difficult, unseen terms. Some "EXERCISES" have been included to help you apply your own thoughts. You may find some boxes marked with: "LET US KNOW". These boxes will provide you with some additional interesting and relevant information. Again, you will get "CHECK YOUR PROGRESS" questions. These have been designed to self-check your progress of study. It will be helpful for you if you solve the problems put in these boxes immediately after you go through the sections of the units and then match your answers with "ANSWERS TO CHECK YOUR PROGRESS" given at the end of each unit.