UNIT 12 : COMBINATORICS

UNIT STRUCTURE

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12.1 LEARNING OBJECTIVES

After going through this unit, you will able to know

- the rule of sum
- the rule of product
- permutation
- combination

12.2 INTRODUCTION

Combinatorics is an important branch of discrete mathematics. It deals with counting the number of ways of arranging or selecting objects from a finite collection under specified rules, i.e. it deals with problems of permutation and combinations. Combinatorics is widely applied in computer science.

12.3 THE RULE OF SUM

If there are n_1 ways to do a work A and distinct from them, n_2 ways to do work B,then the number of ways to do A or B is n_1+n_2. Here the two works cannot be done at the same time.

Note : The rule can be extended to any finite number of works. Thus, if r works can be done in n_1, n_2, ...,n_r ways and if they cannot be done simultaneously, then any one of the r works can be performed in $n_1+n_2+n_3$ . . . . .+ n_r ways.

Example 1 : There are 26 boys and 28 girls in a class. Find the number of ways of selecting one student as class representative.

Solution : One boy can be selected in 26 ways (we can select any one of these 26 boys). One girl can be selected in 28 ways.

$\therefore$ One student (a boy or a girl, not both of them) can be selected in 26+28 i.e. 54 ways.

Example 2 : There are 30 mathematics books and 25 science books in a library. In how many ways a mathematics book or a science book can be selected ?

Solution : We are to select one book, a mathematics book or a science book.

One mathematics book can be selected in 30 ways. One science book can be selected in 25 ways.

$\therefore$ a mathematics book or a science book can be selected in 30+25 i.e. 55 ways.

Example 3 : In how many ways a heart or a club can be selected from a pack of cards ?

Solution : A pack of cards contains 52 cards out of which there are 13 hearts, 13 diamonds, 13 clubs, 13 spades.

$\therefore$ One heart can be selected in 13 ways, one club can be selected in 13 ways.

$\therefore$ a heart or a club can be selected in 13+13 i.e, 26 ways.

12.4 THE RULE OF PRODUCT (OR SEQUENTIAL RULE)

If a work can be done in two successive steps, and the first step can be done in n_1 ways, the second step can be done in n_2 ways, then the work can be done in $n_1 \times n_2$ ways.

Note : The rule can be extended to any finite number of works. Thus, if a work can be done in r successive steps, and the first step can be done in n_1

ways, the second step can be done in n_2 ways, and in this way, the r th step can be done in n_r ways, then the work can be done in $n_1 \times n_2 \times ..... \times n_r$ ways.

Example 4 : There are 25 boys and 20 girls in a class. A teacher wants to select one boy and one girl. In how many ways can this be done ?

Solution : One boy can be selected in 25 ways. One girl can be selected in 20 ways. Thus, the first step (i.e. to select one boy) can be done in 25 ways, the second step (i.e. to select one girl) can be done in 20 ways.

$\therefore$ One boy and one girl can be selected in 25×20 i.e. 500 ways.

Example 5 : What is the number of two-digit odd positive integers ?

Solution : Ones place can be filled up in 5 ways (any one of the five digits 1, 3, 5, 7, 9). Tens place can be filled up in 9 ways (any one of the nine digits 1, 2, 3, 4, 5, 6, 7, 8, 9). Thus, the first step can be done in 5 ways, the second step can be done in 9 ways.

$\therefore$ the work can be done in 5×9 i.e. 45 ways.

$\therefore$ the number of two odd-digit positive integers = 45



CHECK YOUR PROGRESS 1

Q 1: One man or a woman is to be selected from 24 men and 12 women. In how many ways can this be done ?

Q 2: There are 24 boys and 12 girls in a class. A teacher wants to select one boy and one girl. In how many ways can this be done ?

Q 3: Find the number of two-digit even positive integers.

Q 4: Find the number of three-digit odd positive integers.

Q 5: Find the number of three-digit even positive integers.

Q 6: Find the number of four-digit odd positive integers.

Q 7: Find the number of four-digit even positive integers.

Q 8: There are 40 mathematics books and 28 science books in a library. In how many ways one mathematics book or one science book can be selected ?

Q 9: There are 40 mathematics books and 28 science books in a library. In how many ways one mathematics book and one science book can be selected ?

Q 10: A big hall has 12 doors. In how many ways can a person enter the hall by one door and come out by a different door ?

Q 11: There are 15 buses plying between Guwahati and Dibrugarh. In how many ways can a man go from Guwahati to Dibrugarh and return by a different bus ?

Q 12: In how many ways can 7 students stand in a queue ?

Q 13: A man has six letters and there are three latter boxes. In how many ways can he post the letters ?

Q 14: In how many ways can three prizes be distributed among six students, if each of the students is eligible for each of the prizes ?

Q 15: Find the number of license plates that can be made where each plate contains two distinct English letters followed by four distinct digits.

Q 16: In how many ways a condidate can answer all the questions of a true-false test consisting of 10 question ?

Q 17: How many functions are there from a set with 5 elements to a set with 7 elements ?

Q 18: How many functions are there from a set with 7 elements to a set with 5 elements ?

Q 19: How many one-one functions are there from a set with 5 elements to a set with 7 elements ?

Q 20: How many one-one functions are there from a set with m elements to a set with n elements ?

Q 21: Find the number of seven-character palindromes that can be formed using the letters of the English alphabet such that no letter appears more than twice in each of them.

12.5 PERMUTATION

Let there be a set containing n distinct elements. An ordered arrangement of r elements of the set is called a permutation (or an r-permutation of n elements), $r \leq n$.

Symbolically it is denoted by ${}^nP_r$, or P(n, r), or $P(n_r)$.

Theorem : (Number of r-permutations without repetition) The number of r-permutations of n distinct objects is

P(n, r) = n(n-1) n(-2) . . . . . (n-r+1)

$$= \frac{n!}{(n-r)!},\ r \leq n$$

Proof : The first object can be closen in n different ways. So, we are left with (n-1) objects. Therefore, the second object can be chosen in (n-1) ways. Similarly, the third object can be chosen in (n-2) ways. In this way, the last object, i.e. the rth object can be chosen in (n-r+1) ways .Consequently, by the product rule, there are n(n-1)(n-2) . . . . (n-r+1) way of ordered arrangment Thus,

P(n, r) = n(n-1)(n-2) . . . . . (n-r+1)

$$= \frac{n(n-1)(n-2)\ .....(n-r+1)(n-r)(n-r-1).....3.2.1}{(n-r)(n-r-1)....\ 3.2.1}$$

$$= \frac{n!}{(n-r)!}$$

Note : $P(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!,\ 0!=1$

Example 6: Find the value of n if P(n, 4) = 360

Solution : P (n, 4) = 360

or, $\frac{n!}{(n-4)!} = 360$

or, $\frac{n(n-1)(n-2)(4-3)(n-4)!}{(n-4)!} = 360$

or, n(n-1)(n-2)(n-3) = 6 x 5 x 4 x 3

$\therefore$ n = 6

Note : Left hand side is the product of 4 consecutive positive integers.

Therefore, the right hand side has also been expressed as the product of 4 consecutive positive integers.

Example 7 : How many four-digit numbers can be formed from the six digits 1, 2, 3, 5, 7, 8, if repetitions of digits are not allowed ?

Solution : The number of 4 digit numbers.

= the number of 4-permutations of 6 objects (digits)

= P (6, 4)

$= \frac{6!}{(6-4)!} = \frac{6!}{2!}$ = 6 x 5 x 4 x 3 = 360

Example 8: In how many ways can 10 different examination answer scripts be arranged so that the best and the worst always come together ?

Solution : The best and the worst come together. So, we can consider the best script and the worst script as a single script. Thus, we can consider 10 objects to be '9' objects. So, these '9' objects can be arranged in ${}^9P_9$ i.e. 9! ways.

Again, the best script and the worst script can be arranged among themselves in 2! ways.

$\therefore$ the reqiored number of ways

= 9! $\times$ 2!

= 725760.



CHECK YOUR PROGRESS 2

Q 1: Three persons enter a railway carriage, where there are 6 vacant seats. In how many ways can they seat themselves ?

Q 2: A gentleman has 6 friends to invite. In how many ways can he send invitation cards to them, if he has three bearers to carry the cards ?

Q 3: There are 20 candidates in an examination for awarding 5 prizes. In how many ways can the prizes be distributed, each candidate being eligible for one prize ?

Q 4: How many numbers lying between 100 and 1000 can be formed with the digits 1, 2, 3, 4, 5 if the digits in the same

number are not repeated ? Fund also the number of numbers lying between 100 and 10,000.

Q 5: How many four-digits even numbers cab be formed from the digits 1, 2, 3, 5, 7, 8 if repetition of digits are not allowed ?

Q 6: How many four-digit odd numbers can be formed from the digits 1, 2, 3, 5, 7, 8 if repetition of digits are not allowed ?

Q 7: How many four-digit numbers divigible by 5 can be formed from the digits 1, 2, 3, 5, 7, 8 if repetition of digits are not allowed ?

Q 8: In how many ways can 10 different examination answer scripts be arranged so that the best and the worst never come together ?

Q 9: Mr X has 4 mathematics books, 3 statistics book and 5 economics books. In how many ways can he arrange them on a shelf of books if books of each subject are placed together, and in the following order : mathematics, statistics, economics ?

Q 10: Find the number of ways in which letters of the word TRIANGLE can be arranged among themselves, if each arrangement begins with T and ends with E.

12.6 COMBINATION

Let there be a set containing n distinct elements. An unordered selection of r elements of the set is called a combination (or an r-combination of n elements), $r \leq n$.

Symblically it is denoted by ${}^{n}C_{r}$, or C(n, r), or $C(n_{r})$

Theorem : (Number of r-combinations without repetition) The number of r-combinations of n distinct objects is

$$C(n, r) = \frac{P(n, r)}{n!} = \frac{n!}{r!(n-r)!}, \quad r \leq n$$

Proof : By r-combination we mean an unerdered selection of r-objects from n distinct objects. Each r-combination contains r objects. These r objects can be ananged among themseleves in ${}^{r}P_{r}$, i.e., r! ways.

Therefore, each r-combination yields r! permutations.

$\therefore$ C(n, r) combinations will yoeld C(n, r) r! permutations.

We know that the number of r-permutations of n distinct objects is given by P(n, r)

$\therefore$ C (n, r) r ! = P (n, r)

or, C(n, r) = $\frac{P(n, r)}{r!}$

$= \frac{n!}{r!(n-r)!}$

Note : $C(n, n) = \frac{n!}{n!(n-n)!} = 1,\ 0.! = 1$

Theorem : C(n, r) = C(n, n-r), $r \le n$

Proof : If r objects are chosen from n objects, then (n-r) objects are left.

Thus, selection of r objects from n objects is the same as leaving (n-r) objects.

Thus, to each r-combination there corresponds an (n-r) combination.

$\therefore$ C(n, r) = C(n, n-r)

Theorem : (Pascal's Identity)

C(n+1, r) = C(n, r) + C(n, r-1), $r \le n$

Proof : Let us counsider a finite set A having (n+1) elements.

Let B be a subset of A, and let the number of elements of A be n.

Let $p \in A$ such that $p \notin B$

Clearly $A = B \cup \{p\}$

Every r-element subset of A is either an r-element subset of B or the union of an (r-1) element subset of B and the singleton set {p}.

In the first case, the number of subsets is C(n, r); whereas in the second case, the number of subsets is C(n, r-1).

$\therefore$ the total number of r-element subsets of A is C(n, r) + C(n, r-1).

But we know that the total number of r-element subsets of A is C(n+1, r).

$\therefore$ C(n+1, r) = C(n, r) + C(n, r-1).

Note : Blaise pascal was a famous french mathematician of the seventeenth century.

Example 9 : In how many ways can a committee of 6 gentlemen and 3 ladies be formed out of 9 gentlemen and 6 ladies?

Solution : 6 gentlemen can be selected from 9 gentlement in ${}^9C_6$ ways. 3 ladies can be selected from 6 ladies can be selected in ${}^6C_3$ ways.

$\therefore$ by the product rule, the number of committees = ${}^9C_6 \times {}^6C_3$

$$= \frac{9!}{6!\ 3!} \times \frac{6!}{3!\ 3!}$$

$$= \frac{9\times 8\times 7}{3\times 2\times 1} \times \frac{6\times 5\times 4}{3\times 2\times 1}$$

$= 3\times 4\times 7\times 5\times 4$

$= 1680$

Example 10 : A question paper contains 3 different sections A, B and C containing 4, 5 and 6 questions respectively. In how many ways a caudidate can make a selection of 7 questions, selecting at least 2 questions from each sections ?

Solution : There are three possibilities.

I 2 questions from A, 2 from B, 3 from C

II 2 from A, 3 from B, 2 from C

III 3 from A, 2 from B, 2 from C

The number of ways in I

$= {}^4C_2 \times {}^5C_2 \times {}^6C_3 = 6\times 10\times 20 = 1200$

The number of ways in II

${}^4C_2 \times {}^5C_3 \times {}^6C_2 = 6\times 10\times 15 = 900$

The number of ways in III

${}^4C_3 \times {}^5C_2 \times {}^6C_2 = 4\times 10\times 20 = 800$

$\therefore$ the required number of ways

$= 1200 + 900 + 800$

$= 2900$



CHECK YOUR PROGRESS 3

Q 1: There are 6 men and 7 women working in an office. In how many ways can a committee of 3 men and 4 women be selected ?

Q 2: There are 6 men and 7 women working in an office. In how many ways can a committee of 4 persons which has at least one woman be selected ?

Q 3: There are 6 men and 7 women working in an office. In how many ways can a committee of 4 persons which has at most one man be selected ?

Q 4: From 8 gentlemen and 4 ladies, a team of 5 is to be formed. In how many ways can this be done so as to include at least one lady?

Q 5: A man has 7 friends, 4 of them are ladies and 3 men. His wife has also 7 friends, 3 of them are ladies and 4 men. In how many ways can they invite a dinner party of 3 ladies and 3 men so that there are 3 of man's friends and 3 of wife's friends ?



12.7 LET US SUM UP

- If there are n_1 ways to do a work A and distinct from them, n_2 ways to do another work B, then the number of ways to do A or B is n_1+n_2. Here the two works cannot be done at the same time. This is known as the rule of sum.
- If a work can be done in two successive steps, and the first step can be done in n_1 ways, the second step can be done in n_2 ways, then the work can be done in $n_1 \times n_2$ ways, This is known as the rule of product.
- Let there be a set containing n distinct elements. An ordered arrangement of r elements of the set is called an r-permutation of n elements, $r \leq n$. An unordered selection of r elements of the set is called an r-combination of n elements, $r \leq n$.
- The number of r-permutations of n distinct objects is

  P(n, r) = n(n-1)(n-2) . . . . (n-r+1)

$$= \frac{n!}{(n-r)!},\ r \le n$$

- The number of r-combinations of n distinct objects is

$$C(n, r) = \frac{P(n, r)}{n!} = \frac{n!}{r!(n-r)!},\ r \le n$$

- C(n, r) = C(n, n-r), $r \le n$
- C(n+1, r) = C(n, r) + C(n, r-1)



12.8 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

Ans to Q No 1: One man can be selected in 24 ways,

One woman can be selected in 12 ways.

$\therefore$ One woman or one woman can be selected in 24+12 i. e. 36 ways.

Ans to Q No 2: One boy can be selected in 24 ways.

One girl can be selected in 12 ways.

$\therefore$ One boy and one girl can be selected in 24×12 i.e. 288 ways.

Ans to Q No 3: Ones place can be filled up in 5 ways (any one of the digits 0, 2, 4, 6, 8).

Tens place can be filled up in 9 ways (any one of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9).

$\therefore$ the required number of numbers

$= 5 \times 9 = 45$

Ans to Q No 4: Ones place can be filled up in 5 ways (any one of the digits 1, 3, 5, 7, 9).

Tens place can be filled up in 10 ways (any one of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

Hundreds place can be filled up in 9 ways (any one of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9).

$\therefore$ the required number of numbers = $5 \times 10 \times 9 = 450$

Ans to Q No 5: Ones place can be filled up in 5 ways (any one of the digits 0, 2, 4, 6, 8).

Tens place can be filled up in 10 ways. (any one of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9).

Hundreds place can be filled up in 9 ways (any one of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9).

$\therefore$ the required number of numbers

$= 5 \times 10 \times 9 = 450$

Ans to Q No 6: Do yourself. Ans 4500

Ans to Q No 7: Do yourself. Ans 4500

Ans to Q No 8: One mathematies book or one science book can be selected in 40+28 i.e 69 ways.

Ans to Q No 9: One mathematies book and one science book can be selected in 40×28 i.e. 1120 ways.

Ans to Q No 10: A person can enter the hall in 12 ways, i.e, the first step can be performed in 12 ways. the person can come out from the hall in 11 ways, i.e., the second step can be performed in 11 ways.

$\therefore$ the required number of ways.

$= 12 \times 11 = 132$

Ans to Q No 11: Do yourself. Ans 210

Ans to Q No 12: Do yourself. Ans 5040

Ans to Q No 13: Do yourself. Ans $3^6 = 729$

Ans to Q No 14: Do yourself. Ans $6^3 = 216$

Ans to Q No 15: First letter can be filled up in 26 ways.

Second letter can be filled up in 25 ways.

First digit can be filled up in 10 ways.

Second digit can be filled up in 9 ways.

Third digit can be filled up in 8 ways.

Fourth digit can be filled up in 7 ways.

$\therefore$ the required number of ways

$= 26 \times 25 \times 10 \times 9 \times 8 \times 7$

Ans to Q No 16: First question can be answered in 2 ways (either true of

false).

Similarly, each of the questions can be answered in 2 ways.

$= 2\times2\times2\times2\times2\times2\times2\times2\times2\times2$

$= 2^{10} = 1024$

Ans to Q No 17: Number of elements in the domain = 5

Number of elements in the codomain = 7

First element can be associated in 7 ways.

Similarly each of the elements of the domain can be associated in 7 ways.

$\therefore$ the required number of functions

$= 7\times7\times7\times7\times7 = 7^5$

Ans to Q No 18: Number of elements in the domain = 7

Number of elements in the codomain = 5

Each of the elements of the domain can be associated in 5 ways.

$\therefore$ the required number of functions

$= 5\times5\times5\times5\times5\times5\times5 = 5^7$

Ans to Q No 19: Number of elements in the domain = 5

Number of elements in the codomain = 7

First element can be associated in 7 ways

Second element can be associated in 6 ways

Third element can be associated in 5 ways

Fourth element can be associated in 4 ways

Fifth element can be associated in 3 ways

$\therefore$ the required number of one-one functions

$= 7\times6\times5\times4\times3 = 2520$

Ans to Q No 20: We note that ; if m > n, there cannot be one-one function from the first set to the second set.

$\therefore m \le n$

Now, 1st element can be associated in n ways

2nd element can be associated in (n-1) ways

3rd element can be associated in (n-2) ways.

In this way, mth element can be associated in (n-m+1) ways.

$\therefore$ the required number of one-one functions.

= n(n-1)(n-2) . . . . (n-m+1)

Ans to Q No 21: [Dictionary meaning of the word palindrome is given below.

'A word, phrase, number or any other sequence of units which has the property of reading the same forwards as it does backwards, character for character, sometimes disregarding punctuation, capitalization and diacritics.']

The required number of palindromes

$= 26 \times 25 \times 24 \times 23$

= 358800

Check Your Progress 2

Ans to Q No 1: Number of ways $= {}^6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times 3!}{3!}$

= 120

Ans to Q No 2: Number of ways = $3 \times 3 \times 3 \times 3 \times 3 \times 3$ = 729

Ans to Q No 3: Number of ways $= {}^{20}P_5 = \frac{20!}{(20-5)!}$

$= \frac{20!}{15!} = \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15!}{15!}$

= 1860480

Ans to Q No 4: Do yourself. Ans 60,180

Ans to Q No 5: Ones place can be filled up in 2 ways (one of the digits 2, 8). The other three places can be filled up in ${}^5P_3$ ways.

$\therefore$ the required number of numbers

$= 2 \times {}^5P_3 = 2 \times \frac{5!}{(5-3)!} = 2 \times \frac{5!}{2!} = 2 \times \frac{5 \times 4 \times 3 \times 2!}{2!}$

= 120

Ans to Q No 6: Ones place can be filled up in 4 ways (one of the digits 1, 3, 5, 7). The other three places can be filled up in ${}^5P_3$ ways.

$\therefore$ the required number of numbers

$= 4 \times {}^5P_3 = 4 \times 5 \times 4 \times 3 = 240$

Ans to Q No 7: Ones place can be filled up in 1 way (5). The other three places can be filled up in ${}^5P_3$ ways.

$\therefore$ the required number of numbers

$= 1 \times {}^5P_3 = 1 \times 5 \times 4 \times 3 = 60$

Ans to Q No 8: If there is no restriction, 10 answer scripts can be arranged among themselves in ${}^{10}P_{10}$ i.e. 10! ways.

Again, we know that if the best and the worst scripts come together, the number of arrangements $= 9! \times 2!$

$= 725760$ (Example 8)

$\therefore$ if the best and the worst do not come together, the number of arrangements

$= 10! - 2 \times 9!$

$= 3628809 - 725760$

$= 2903040$

Ans to Q No 9: Number of ways

$= 4! \times 3! \times 5!$

$= 4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 17280$

Ans to Q No 10: Each arrangement begins with T, and ends with E. So, we are left with six letters R. I. A, N, G, L.

These six letters can be arranged among themselves in ${}^6P_6$ ways, i.e. 6! ways, i.e., $6 \times 5 \times 4 \times 3 \times 2 \times 1$ ways, i.e. 729 ways.

$\therefore$ the number of ways = 720

Check Your Progress 3

Ans to Q No 1: 3 men can be selected from 6 men in ${}^6C_3$ ways.

4 women can be selected from 7 women in ${}^7C_4$.

$\therefore$ the required number of ways

$$= {}^6C_3 \times {}^7C_4 = \frac{6!}{3!\,3!} \times \frac{7!}{4!\,3!} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times \frac{7 \times 6 \times 5}{3 \times 2 \times 1}$$

$= 20 \times 35 = 700$

Ans to Q No 2: At least one woman is to be selected. Therefore number of women may be 1, or 2 or 3 or 4. So we have the following possibilities

Men (6)	Women (7)
3	1
2	2

1	3
0	4

$\therefore$ the required number of ways

$= {}^6C_3 \times {}^7C_1 + {}^6C_2 \times {}^7C_2 + {}^6C_1 \times {}^7C_3 + {}^6C_0 \times {}^7C_4$

$= 20 \times 7 + 15 \times 21 + 6 \times 35 + 1 \times 35$

$= 140 + 315 + 210 + 35$

$= 700$

Ans to Q No 3: At most one man is to be selected. Therefore number of men may be 0, or 1. So, we have the following possibilities

Men (6)	Women (7)
0	4
1	3

$\therefore$ the required number of ways

$= {}^6C_0 \times {}^7C_4 + {}^6C_1 \times {}^7C_3 = 1 \times 35 + 6 \times 35 = 35 + 210 = 245$

Ans to Q No 4: Do yourself. Ans 736

Ans to Q No 5: We have the following possibilities.

Man		Wife	
Men (3)	Ladies (4)	Men (4)	Ladies (3)
3	0	0	3
2	1	1	2
1	2	2	1
0	3	3	0

$\therefore$ the required number of ways

$= ({}^3C_3 \times {}^4C_0 \times {}^4C_0 \times {}^3C_3) + ({}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2) + ({}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1)$

$+ \quad ({}^3C_0 \times {}^4C_3 \times {}^4C_3 \times {}^3C_0)$

$= 485$, after simplification.



12.9 FURTHER READINGS

1) *Discrete Mathematical Structures with Application to Computer Science.* J.P Tremblay & R. Manohar.

2) *Discrete Structures and Graph Theory,* G. S. S. Bhishma Rao.



12.10 MODEL QUESTIONS

Q 1: Write down the sum rule for counting.

Q 2: Write down the product rule for countting

Q 3: Prove using combinational argument that

C(2n, 2) = 2 C (n, 2) + n^2

Q 4: Prove, using combinational argument, that

$C(n+m,r)=\sum_{i=0}^{r} C(m,r-i)C(n,i)$ for integers $n \geq r \geq 0$, and $m \geq r \geq 0$.

Q 5: Two distinguishable dice are rolled. In how many ways can we get a sum of 7 or 9 ?

Q 6: In how many way can we draw a king or a queen from a pack of cards ?

Q 7: A license plate contains two English letters, followed by two digits, followed by two English letters, followed by four digits. How many license plates can be made if repetition of letters and digits are allowed?

Q 8: There are 10 married couple in a group. Find the number of ways of selecting a woman and a man who is not her husband from this group.

Q 9: From 8 boys and 6 girls a team of 8 players is to be forwed. In how many ways can the team be formed so that exactly 3 girls are included excluding a paiticular girl and including a particular boy ?

Q 10: Find the number of one-one functions from a set with n elements to itself.

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