# UNIT 14 : BOOLEAN ALGEBRA

## UNIT STRUCTURE


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## 14.1 LEARNING OBJECTIVES

After going through this unit, you will able to know

- boolean algebra, its properties, examples
- basic laws of boolean algebra
- boolean expression.

## 14.2 INTRODUCTION

Boolean Algebra is an algebraic structure after the name of British logician George Boole (1813-1864). By an algebraic structure we mean a set equipped with one or more binary operations defined on it satisfying certain axioms. Though it is an abstract algebraic sturucture, it can be used for analysing electrical circuits. Nowadays it is immensely used for the analysis and design of electronic computers.

## 14.3 DEFINITION, PROPERTIES AND EXAMPLES

**Definition :** Let B a non-empty set equipped with two binary operations + and . , a unary operation $'$, and two distinct elements 0 and 1. Then B is called a Boolean Algebra if the following conditions are satisfied.

1. Commutative laws

   (i) a + b = b + a, (ii) a.b = b.a, $\forall a, b \in B$

2. Distributive laws

   (i) a + (b.c) = (a + b).(a + c)

   (ii) a. (b + c) = (a.b) + (a.c), $\forall a, b, c \in B$

3. Identity laws

   (i) a + 0 = a, (ii) a.1 = a

4. Complement laws

   (i) a + $a^{/}$ = 1, (ii) a.$a^{/}$ = 0

**Note :** 1. From the definition it is clear that there cannot a Boolean Algebra with one element.

2. 1 is called unit element, and 0 is called zero element.
3. $a^{/}$ is called complement of a.
4. A Boolean Algebra is generally denoted by (B, + , . , $^{/}$, 0, 1) or (B, +, . ,$^{/}$ ) or simply by B.
5. The symbols 0 and 1 are not necessaily real numbers 0 and 1 respectively.

**Example 1 :** B = {0, 1} is a Boolean Algebra under +, . , $^{/}$ defined by

| + | 1 | 0 |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 0 |

| . | 1 | 0 |
|---|---|---|
| 1 | 1 | 0 |
| 0 | 0 | 0 |

| | / |
|---|---|
| 1 | 0 |
| 0 | 1 |

**Solution :** From the definition of +, . , $^{/}$, it follows that

1. (i) a + b = b + a

   (ii) a.b = b.a, $\forall a, b \in B$

2. (i) a + (b.c) = (a+b).(a + c)

   (ii) a.(b+c) = (a.b) + (a.c), $\forall a, b, c \in B$

3. (i) a + 0 = a, (ii) a.1 = a, $\forall a \in B$

4. (i) a + $a^{/}$ = 1 (1 + $1^{/}$ = 1 + 0 = 1),

   0 + $0^{/}$ = 0 + 1 = 1)

   (ii) a. $a^{/}$ = 0 (1. $1^{/}$ = 1.0 = 0,

   0.$0^{/}$ = 0.1 = 0)

   $\therefore$ (b, + , . , $^{/}$ ) is a Boolean Algebra.

**Properties**

1. In a Boolean Algebra B, zero element is unique.

**Proof :** Let (B, + , . , $^{/}$ ) be a Boolean Algebra.

If possible, let there be two zero elements 0 and $0_1$

$\therefore$ a + 0 = a, $\forall a \in B$

a + $0_1$ = a, $\forall a \in B$

$\therefore$ $0_1$ + 0 = 0 ($\because$ 0 is zero element)

Again $0_1$+0 = 0 + $0_1$ (by commutative law)

= 0 ($\because$ $0_1$ is zero element)

$\therefore$ $0_1$ = 0

i.e. zero element is unique.

**Note :** In view of the above property, we will write as “the” zero element.

2. In a Boolean Algebra B, unit element is unique.

**Proof :** Let (B, +, . , $^{/}$ ) be a Boolean Algebra.

If possible, let there be two unit elements 1, and $1_u$.

$\therefore$ $a.1 = a, \forall a \in B$

$a.1_u = a, \forall a \in B$

$\therefore$ $1_u$.1 = $1_u$ ($\because$ 1 is unit element)

$\therefore$ $1_u$ = 1

i.e unit element is unique.

**Note :** In view of the above theorem, we will write as “the” unit element.

3. In a Boolean Algebra B, complement of each element is unique.

**Proof :** Let (B, + , . , $^{/}$ ) be a Boolean Algebra.

Let 0 be the zero element, and

1 be the unit element.

Let $a \in B$

If possible, let $a^{/}$ and $a_1$ be complements of a

$\therefore$ a + $a^{/}$ = 1, a . $a^{/}$ = 0 ($\because$ $a^{/}$ is complement of a)

a + $a_1$ = 1,a . $a_1$ = 0 ($\because$ $a_1$ is complement of a)

Now $a^{/}$ = $a^{/}$.1

= $a^{/}$. (a + $a_1$)

= ($a^{/}$.a) + ($a^{/}$.$a_1$)

= (a.$a^{/}$) + ($a^{/}$.$a_1$)

$= 0 + (a^{/}.a_1)$

$= (a^{/}.a_1) + 0$

$= a^{/}.a_1$ (1)

Again $a_1 = a_1.1$

$= a_1.(a + a^{/})$

$= (a_1.a) + (a_1.a^{/})$

$= (a.a_1) + (a_1.a^{/})$

$= 0 + (a_1.a^1)$

$= (a^{/}.a_1) + 0$

$= a^{/}.a_1$ (2)

From (1) and (2), we get

$a^{/} = a_1$

**Note :** In view of the above theorem, we write as "$a^{/}$" is the complement of a.

4. In a Boolean Algebra B,

(i) $1^{/} = 0$, (ii) $0^{/} = 1$, where 0 is the zero element, and 1 is the unit element.

**Proof :** (i) Let B be a Boolean Algebra,

Now, by identity laws, we have a + 0 = 1 and a.1 = a, $\forall a \in B$

Replacing a by 1 in a + 0 = a, we get 1 + 0 = 1.

Replacing a by 0 in a.1 = 1, we get 0.1 = 0

i.e. 1.0 = 0 (0.1 = 1.0, by commutative law)

Thus, 1 + 0 = 1 (1)

1.0 = 0 (2)

From (1) and (2), we conclude that $0 = 1^{/}$ i.e. $1^{/} = 0$ (complement laws)

(ii) Let B be a Boolean Algebra.

Now, by identity laws, we have a.1 = 1 and a + 0 = a, $\forall a \in B$

Replacing a by 0 in a.1 = a, we get 0.1 = 0.

Replacing a by 1 in a + 0 = a, we get 1 + 0 = 1.

i.e. 0 + 1 = 1, (1+ 0 = 0 + 1, by commutativeled)

Thus, 0.1 = 0 (1)

0 + 1 = 1 (2)

From (1) and (2), we conclude that $0^{/} = 1$ (complement law)

**Note :** The property (ii), viz $0' = 1$ can be derived from property (i), viz $1' = 0$ by interchanging the binary operations + and . , and by interchanging 0 and 1. Here (ii) is called the dual of (i), and vice versa, i.e. (i) is the dual of (ii) This principle is known as the principle of duality. The principle holds good for every statement and every theorem in a Boolean Algebra.

**Example 2 :** Let X be a non-empty set Let P(x) be the power set of A, i.e P(x) is the set of all subsets of x.

Let $A, B \in P(x)$, i.e. $A \subseteq X,\ B \subseteq X$

We define

$A + B = A \cup B,$

$A.B = A \cap B$

$A'$ = the complement of A.

Clearly, union is a binary operation on P(x), intersection is a binary operation on P(x), complementation is a unary operation on P(x).

Now 1. (i) $A + B = A \cup B = B \cup A = B + A$

(ii) $A.B = A \cap B = B \cap A = B.A,\ \forall\ A, B \in P(x)$

2. (i) $A + (B.C) = A \cup (B \cap C)$

$= (A \cup B) \cap (A \cup C)$

$= (A + B).(A + C),\ \forall\ A, B, C \in P(x)$

(ii) $A.(B + C) = A \cap (B \cup C)$

$= (A \cap B) \cup (A \cap C)$

$= (A.B) + (A.C)$

3. (i) $A + \phi = A \cup \phi = A,\ \ \phi \in P(x)$

(ii) $A.X = A \cap X = A$

4. (i) $A + A' = A \cup A' = X$

(ii) $A.A' = A \cap A' = \phi$

$\therefore\ (P(x), \cup, \cap, ')$ is a Boolean Algebra, where $\phi$ is the zero element and X is the unit element.

**Example 3:** Let B = {1, 2, 5, 10, 11, 22, 55, 110} be the set of all positive divisiors of 110. We define

a + b = l.c.m. of a, b,

a.b = g.c.d. of a, b,

$a' = \frac{110}{a}, \ a, b \in B$

We construct the composition tables for +, . , $'$ as follow?

| + | 1 | 2 | 5 | 10 | 11 | 22 | 55 | 110 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 5 | 10 | 11 | 22 | 55 | 110 |
| 2 | 2 | 2 | 10 | 10 | 22 | 22 | 110 | 110 |
| 5 | 5 | 10 | 5 | 10 | 55 | 110 | 55 | 110 |
| 10 | 10 | 10 | 110 | 10 | 110 | 110 | 110 | 110 |
| 11 | 11 | 22 | 110 | 110 | 11 | 22 | 55 | 110 |
| 22 | 22 | 22 | 110 | 110 | 22 | 22 | 110 | 110 |
| 55 | 55 | 55 | 110 | 110 | 55 | 110 | 55 | 110 |
| 110 | 110 | 110 | 110 | 110 | 110 | 110 | 110 | 110 |
| | | | | | | | | |

| . | 1 | 2 | 5 | 10 | 11 | 22 | 55 | 110 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 |
| 5 | 1 | 1 | 5 | 5 | 1 | 1 | 5 | 5 |
| 10 | 1 | 2 | 5 | 10 | 1 | 2 | 5 | 10 |
| 11 | 1 | 1 | 1 | 1 | 11 | 11 | 11 | 11 |
| 22 | 1 | 2 | 1 | 2 | 11 | 22 | 11 | 22 |
| 55 | 1 | 1 | 5 | 5 | 11 | 11 | 55 | 55 |
| 110 | 1 | 2 | 5 | 10 | 11 | 22 | 55 | 110 |
| | | | | | | | | |

| / | 1 | 2 | 5 | 10 | 11 | 22 | 55 | 110 |
|---|---|---|---|---|---|---|---|---|
| | 110 | 55 | 22 | 11 | 10 | 5 | 2 | 1 |

From the above tables, we see that + is a binary operation on B. .

is a binary operation on B, and $'$ is a unary operation on B.

Again 1 (i) a+b = l.c.m. of a, b

= l. c. m. of b, a

= b+a, $\forall a, b \in B$

(ii) a, b = g.c.d. of a, b

= g.c.d. of b, a

= b.a, $\forall a, b \in B$

2. From the tables, we have

(i) a + (b.c) = (a+b).(a+c)

(ii) a.(b+c) = (a.b) + (a.c), $\forall a, b, c \in B$

3. (i) a+1 = l .c.m of a, 1

= a, $\forall a \in B$

(ii) a.110 = g.c.d. of a, 110

= a, $\forall a \in B$

4. (i) $a+a^{/}$ = l.c.m of a, $a^{/}$

= l.c.m. of $a, \frac{110}{a}$

= 110, $\forall a \in B$

(ii) $a.a^{/}$ = g.c.d. of a, $a^{/}$

= g.c.d. of $a, \frac{110}{a}$

= 1, $\forall a \in B$

$\therefore$ (B, + , . , / ) is a Boolean Algebra; where 1 is the zero element, and 110 is the unit element.



## CHECK YOUR PROGRESS

**Q 1:** Let X ={1}. Show that the set P(x) = {{1}, $\phi$ } is a Boolean Algebra under operations of union, intersection, and complementation in P(x).

**Q 2:** Let X = {1, 2, 3}, and Let P(X) be the power set of X. Show that P(X) is a Boolean Algebra under operations of union, intersection, and complementation in P(X).

**Q 3:** Let B = {1, 2, 3, 5, 6, 10, 30} be the set of all positive divisors of 30. Define

a+b = l.c.m. of a, b; $a, b \in B$

a.b = g.c.d. of a, b

$a^{/} = \frac{30}{a}$

Show that (B, +, . , / ) is a Boolean Algebra

**Q 4:** Let B = {1, 2, 5, 7, 10, 14, 35, 70} be the set of all positive

divisors of 70. Define

a+b = l.c.m of a, b, $a, b \in B$

a.b = g.c.d. of a, b

$a^{/} = \frac{70}{a}$

Show that (B, +, . , / ) is a Boolean Algebra.

**Q 5:** Give an example of a Boolean Algebra with 16 elements.

**Q 6:** Let B = {0, a, b, 1}. Define +, . , / in B as follows.

| + | 0 | a | b | 1 |
|---|---|---|---|---|
| 0 | 0 | a | b | 1 |
| a | a | a | 1 | 1 |
| b | b | 1 | b | 1 |
| 1 | 1 | 1 | 1 | 1 |

| . | 0 | a | b | 1 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| a | 0 | a | 0 | a |
| b | 0 | 0 | b | b |
| 1 | 0 | a | b | 1 |

| / | 0 | a | b | 1 |
|---|---|---|---|---|
| | 1 | b | a | 0 |

Show that (B, +, . , / ) is a Boolean Algebra.

**Q 7:** Let B = { 1, 2, 4, 5, 10, 20} be the set of all positive divisors of 20. Define

a+b = l.c.m. of a, b $\forall a, b \in B$

a.b = g.c.d. of a, b $\forall a, b \in B$

$a^{/} = \frac{20}{a}, \ \forall a \in B$

Examine if (B, +, . , / ) is a Boolean Algebra.

## 14.4 BASIC LAWS OF BOOLEAN ALGEBRA

Let (B, +, . , / ) be a Boolean Algebra. Let 0 be the zero element, and 1 be the unit element of B.

**1. Idempotent Laws.**

(i) a+a = a, (ii) a.a = a, $\forall a \in B$

**Proof :** (i) a = a+0

= a + (a.a$^{/}$)

= (a+a).(a+a$^{/}$), by associative law

= (a+a).1

= a+a

i.e. a+a = a

(ii) a = a.1

= a.(a + $a^{/}$)

= (a.a) + (a.$a^{/}$), by associative law

= (a.a) + 0

= a.a

**Note :** The result (ii) is a dual of the result (i), and vice versa. The result (ii) can be obtained from (i) by replacing + by . , . by + , 0 by 1, 1 by 0.

**2. Boundedness Laws**

(i) a+1 = 1, (ii) a.0 = 0, $\forall\ a \in B$

**Proof :** (i) a+1 = (a+1) . 1

= (a+1).(a+$a^{/}$)

= a+(1.$a^{/}$), by associative law

= a+$a^{/}$, (1.$a^{/}$ = $a^{/}$.1 = a)

= 1

(ii) a.0 = (a.0) + 0

= (a.0) + (a.$a^{/}$)

= a.(0+$a^{/}$), by associative law

= a.$a^{/}$, (0+$a^{/}$ = $a^{/}$+0 = $a^{/}$)

= 0

**3. Absorption Laws**

(i) a+(a.b) = a, (ii) a.(a+b) = a, $\forall a, b \in B$

**Proof :** (i) a+(a.b) = (a.1) + (a.b)

= a.(1 + b)

= a.1, (1+b = b+1 = 1, by boundedness law)

= a

(ii) a.(a+b) = (a+0).(a+b)

= a+(0.b)

= a+0, (0.b = b.0 = 0, by boundedness law)

=a

**4. Associative Laws**

(i) (a+b)+c = a + (b+c)

(ii) (a.b).c = a.(b.c)

**Proof :** (i) Let L = (a+b) + c

R = a + (b+c)

Now a.L = a.((a+b)+c)

= a.(a+b) + a.c, by distributive law

= a + (a.c), by absorption law

= a, by absorption law

a . R = a.(a+(b+c))

= (a.a)+(a.(b+c)), by distributive law

= a+(a.(b+c)), by idempotent law

= a, by absorption law

$\therefore$ a.L = a.R

Again $a'$.L =$a'$.((a+b) + c)

= $a'$.(a+b) + ($a'$.c), by distributive law

= ($a'$.a) + ($a'$.b) + ($a'$.c), by distributive law

= 0 + ($a'$.b) + ($a'$.c)

= ($a'$.b) + ($a'$.c)

= $a'$.(b+c)

$a'$.R = $a'$.(a + (b+c))

= ($a'$.a) + $a'$.(b+c), by distributive law

= 0 + $a'$.(b+c)

= $a'$.(b+c)

$\therefore$ $a'$.L = $a'$.R

Now L = 1.L

= (a+$a'$).L

= (a.L) + ($a'$.L)

= (a.R) + ($a'$.R)

= (a+$a'$).R

= 1.R

= R

$\therefore$ L = R

i. e. (a+b) + c = a+(b+c)

(ii) Let L = (a.b).c

R = a. (b.c)

Now a + L = a + ((a.b).c)

= (a + (a.b)).(a+c), by distributive law

= a.(a+c), by absorption law

= a, by absorption law

a + R = a +(a.(b.c))

= (a+a).(a+(b.c)), by distributive law

= a.(a+(b.c)), by idempotent law

= a, by absorption law

$\therefore$ a + L = a+R

Again $a'$+L = $a'$ + (a.b).c)

= ($a'$ + (a.b)).($a'$+c), by distributive law

= (($a'$+a).($a'$+b)).($a'$+c), by distributive law

= 1.($a'$+b).($a'$+c)

= ($a'$+b).($a'$+c)

= $a'$ + (b.c)

$a'$ + R = $a'$ +(a.(b.c))

= ($a'$+a).($a'$ + (b.c)), by distributive law

= 1.($a'$ + (b.c))

= $a'$ + (b.c)

$\therefore$ $a'$ + L = $a'$ + R

Now L = 0 + L

= (a.$a'$) + L

= (a+L).($a'$+L)

= (a+R).($a'$+R)

= (a.$a'$) + R

= 0 + R

= R

$\therefore$ L = R

i.e. (a.b).c = a.(b.c)

**5. Involution Law :** $\left(a'\right)' = a$

**Proof :** By definition of complement, we have

a+$a'$ =1, and a.$a'$ = 0

i. e. $a'+a = 1$, and $a'.a = 0$, by commutative law

$\therefore$ by uniquences of complement, a is the complement of $a'$, i. c., $(a')' = a$.

**6. De Morgan's Laws**

(i) $(a+b)' = a'.b'$, (ii) $(a.b)' = a'+b'$

**Proof :** (i) $(a+b) + (a'.b') = b+a + (a'.b')$, by commutative law

$= b +(a+a').(a+b')$, by distributive law

$= b+1.(a+b')$

$= b + (a+b')$

$= b + (b'+a)$, by commutative law

$= (b+b') +a$, by associative law

$= 1+a$

$= 1$, by boundedness law

$(a + b).(a'.b') = ((a+b).a').b'$, by associative law

$= ((a.a') + (b.a')).b'$, by distributive law

$= (0 + (b.a')).b'$

$= (b.a').b'$

$= b.(a'.b')$, by associative law

$= b.(b'.a')$, by commutative law

$= (b.b').a'$, by associative law

$= 0.a'$

$= 0$

Thus, we see that $a'.b'$ is the complement of a+b,

i. e. $(a+b)' = a'.b'$

[Augustus De Morgan (1806 – 1871) was a famous mathematician.

He was born in Madurai, India]

Corollary (i) $a+b = (a'.b')'$, (ii) $a.b = (a'+b')'$

**Proof :** (i) $(a'.b')' = (a')'+(b')'$, by De morgan's law

$= a+b$, by involution law

(ii) $(a'+b') = (a')'.(b')'$, by De morgan's law

$= a.b$, by involution law

**7.** (i) If $b+a = c+a$, and $b+a' = c+a'$, then $b = c$.

(ii) If $b.a = c.a$, and $b.a' = c.a'$, then $b = c$.

**Proof :** (i) $b = b+0$

$= b + (a.a^{/})$

$= (b+a).(b+a^{/})$, by distributive law

$= (c+a).(c+a^{/})$, (given)

$= c+(a.a^{/})$, by distributive law

$= c+0$

$= c.$

(ii) $b = b.1$

$= b.(a+a^{/})$

$= (b.a) + (b.a^{/})$, by distributive law

$= (c.a) + (c.a^{/})$, given

$= c.(a + a^{/})$

$= c.1$

$= c$

8. (i) $a+b^{/} = 1$ if and only if $a+b = a$

(ii) $a.b^{/} = 0$ if and only if $a.b = a$

**Proof :** (i) Let $a+b^{/} = 1$

Now $a + b = (a+b) . 1$

$= (a+b).(a+b^{/})$, given

$= (a+(b.b^{/})$, by distributive law

$= a+0$

$= a$

Conversely, Let $a+b = a$

Now $a+b^{/} = 1.(a+b^{/})$

$= (a+a^{/}).(a+b^{/})$

$= a+(a^{/}.b^{/})$, by distributive law

$= a+(a+b)^{/}$, by De morgan's law

$= a+a^{/}$, given

$= 1.$

(ii) Let $a.b^{/} = 0$

Now $a.b = a.b + 0$

$= a.b+a.b^{/}$, given

$= a.(b+b^{/})$, by distributive law

$= a . 1$

= a

Conversely, let a.b = a

Now $a.b^{/} = 0 + a.b^{/}$

$= (a.a^{/}) + (a.b^{/})$

$= a.(a^{/}+b^{/})$, by distributive law

$= a.(a.b)^{/}$, by De Mergan's law

$= a.a^{/}$, given

= 0.

9. (i) $a+(a^{/}.b) = a+b$

(ii) $a.(a^{/}+b) = a.b$

**Proof :** (i) $a+(a^{/}.b) = (a+a^{/}).(a+b)$, by distributive law

$= 1.(a+b)$

$= a+b$

(ii) $a.(a^{/}+b) = (a.a^{/}) + (a.b)$

$= 0 + (a.b)$

$= a.b$

**Example 4:** Let B be a Boolean Algebra.

Let a, b $\in$ B. Show that

$a.(a.b)^{/} = a.b^{/}$

**Solution :** $a.(a.b)^{/}$

$= a.(a^{/}+b^{/})$, by De Morgan's law

$= (a.a^{/}) + (a.b^{/})$

$= 0 + (a.b^{/})$

$= a.b^{/}$

**Example 5:** Let B be a Boolean Algebra, and a, b, c $\in$ B. Show that

$(a+b).(b+c).(c+a) = a.b + b.c + c.a$

**Solution :** (a+b).(b+c).(c+a)

= (a+b).(b.c + b.a + c.c + c.a)

= (a+b).(b.c + b.a + c + c.a)

= a.(b.c) + a.(b.a) + a.c + a.(c.a)+ b.(b.c) + b.(b.a) + b.c + b.(c.a)

= a.b.c.+ a.(a.b) + a.c + a.(a.c) + (b.b).c + (b.b).a + b.c + a.b.c

= (a.b.c + a.b.c) + (a.a).b + a.c + (a.a).c + b.c + b.a + b.c)

= a.b.c + a.b + a.c + a.c + b.c + b.a + b.c

= a.b.c + (a.b + a.b) + (a.c + a.c) + (b.c + b.c)

= a.b.c + a.b + a.c + b.c

= a.b. (c+1) + a.c + b.c

= a.b.1 + c.a + b.c

= a.b + b.c + c.a

**Example 6 :** Let B be a Boolean Algebra, and a, b $\in$ B Prove that a+b = 0 if and only if a = 0 and b = 0 Write the dual of the above proposition and prove it

**Solution :** Let a = 0, and b = 0

$\therefore$ a+b = 0+0

= 0

Converselt, Let a+b = 0

Now a = a+0

= a+(b.b$^{/}$)

= (a+b).(a+b$^{/}$), by distributive law

= 0.(a+b$^{/}$), given

=0

b = b+0

= b + (a.a$^{/}$)

= (b+a).(b+a$^{/}$)

= (a+b).(b+a$^{/}$)

= 0.(b+a$^{/}$)

= 0

**2nd Part**

Dual of the above proposition is a.b = 1 $\Leftrightarrow$ a = 1, and b = 1

If a = 1, b = 1,

a.b = 1.1 = 1

**Conversely,** Let a.b = 1

Now a = a.1

= a.(b+b$^{/}$)

= (a.b) + (a.b$^{/}$)

= 1 + (a.b$^{/}$)

= 1

b = b.1

= b.(a+$a^{/}$)

= (b.a) + (b.$a^{/}$)

= (a.b) + (b.$a^{/}$)

= 1 + (b.$a^{/}$)

= 1

**Example 7 :** Let B be a Boolean Algebra and a, b, c$\in$B. Prove that the following expressions are equal.

(i) (a+b).($a^{/}$+c).(b+c)

(ii) a.c + $a^{/}$.b + b.c

(iii) (a+b).($a^{/}$+c)

(iv) a.c + $a^{/}$.b

**Solution :** (a+b).($a^{/}$+c).(b+c)

= (a+b).($a^{/}$.b + $a^{/}$.c + c.b + c.c)

= (a+b).($a^{/}$.b + $a^{/}$.c + c.b + c)

= a.$a^{/}$.b + a.$a^{/}$.c + a.c.b + a.c + b.$a^{/}$.b + b.$a^{/}$.c + b.c.b + b.c

= 0.b + 0.c + a.c.b + a.c + b.$a^{/}$ + b.$a^{/}$.c + b.c + b.c

= b.c.(a + $a^{/}$) + a.c + b.c + b.$a^{/}$

= b.c.1 + a.c + b.c + $a^{/}$.b

= b.c + a.c + b.c + $a^{/}$.b

= a.c + $a^{/}$.b + b.c (1)

Again, (a+b).($a^{/}$+c)

= a.$a^{/}$ + a.c + b.$a^{/}$ + b.c

= 0 + a.c + $a^{/}$.b + b.c

= a.c + $a^{/}$.b + b.c (2)

Again, a.c + $a^{/}$.b + b.c

= a.c + $a^{/}$.b + 1.b.c

= a.c + $a^{/}$.b + (a+$a^{/}$).b.c

= a.c + $a^{/}$.b + a.b.c + $a^{/}$.b.c

= ((a.c) + (a.c).b) + (($a^{/}$.b) + ($a^{/}$.b).c)

= a.c + $a^{/}$.b, by absortion law (3)

From (1), (2) and (3), we conclude that

(a+b).(($a^{/}$+c).(b+c)

= a.c + $a^{/}$.b + b.c

= (a+b).($a^{/}$+c)

= a.c + $a^{/}$.b



**CHECK YOUR PROGRESS**

**Q 8:** Prove that there cannot be a Boolean Algebra with 3 elements.

**Q 9:** Let B be a Boolean Algebra, and let a, b, c$\in$B. If a+c=a, b+c=b, show that (a.b) + c = a.b

**Q 10:** Let B be a Boolean Algebra. Let a, b $\in$ B. Show that a+(a+b)$^{/}$ = a+$b^{/}$.

**Q 11:** Let B be a Boolean Algebra, and let a, b$\in$B. Prove that the following properties are equivalent

(i) a+b=a, (ii) a+$b^{/}$=1, (iii) a.b=b

Write the dual of the above proposition and prove it.

**Q 12:** Let B be a Boolean Algebra, and let a, b, c$\in$B. Show that

[(abc+$a^{/}b^{/}$)$^{/}$ + bc]$^{/}$ = $a^{/}b^{/}$

[Here abc means a.b.c]

## 14.5 BOOLEAN EXPRESSION

Let us consider the Boolean Algebra B={0, 1}, where +, . , and $^{/}$ are defined by

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 1 |

| . | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

| / | 0 | 0 |
|---|---|---|
| | 1 | 0 |

A literal symbol that is used to represent an element of B is called a Boolean variable. Thus, if x denotes a Boolean variable, then x will have the value 0 or 1. If x is a Boolean variable, $x^{/}$ is also a Boolean variable.

Let x, y be two Boolean variables. The values of x, y may be given by the following table.

| x | y |
|---|---|

| | |
|---|---|
| 1 | 1 |
| 1 | 0 |
| 0 | 1 |
| 0 | 0 |

Let x, y, z be three Boolean variables. The values of x, y, z may be given by the following table.

| x | y | z |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

By a Boolean expression, we mean an expression that is construted by a finite set of symbols, each representing a Boolean variable, by applying the binary operations +, and . , and the unary operation /. Thus. $x.y + x^{/}$ is a Boolean expression of two variables. Similarly x.y+y.z+z.x is a Boolean expression of three variables.

We note that 0 and 1 are Boolean expressions. Any variable is a Boolean expression.

Corresponding to a Boolean expression, we can construct a table known as the truth table (or the assignment table or the value table).

We construct a truth table for the Boolean expression $xy+x^{/}$ (By xy we mean x.y).

| x | y | xy | $x^{/}$ | $xy+x^{/}$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 |

Two Boolean expressions of variables $x_1, x_2, \ldots, x_n$ are said to be

equal if they assume the same values for every assignment of values 0, 1 to the variables $x_1, x_2, \ldots, x_n$.

The equality of two Boolean expressions is denoted by " = ".

**Example 8:** Construct the truth table for the Boolean expression xy + xz of the variables x, y, z.

**Solution :**

| x | y | z | xy | xz | xy+xz |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |

**Example 9:** Construct the truth table for the Boolean expression $xy+x'y+xy'+x'y'$ of the variables x, y.

**Solution :**

| x | y | $x'$ | $y'$ | xy | $x'y$ | $xy'$ | $x'y'$ | $xy+x'y+xy'+x'y'$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |

**Example 10:** Prove that the Boolean expressions $(xy+x)'$ and $(x'+y')x'$ are equal.

**Solution :**

| x | y | xy | xy+x | $(xy+x)'$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 1 |

Again,

| x | y | $x'$ | $y'$ | $x'+y'$ | $(x'+y')x'$ |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 |

Looking at the last column of the above two tables, we conclude that $(xy+x)' = (x'+y')x'$.



**CHECK YOUR PROGRESS**

**Q 13:** Let $f(x, y, z) = xy + z'$ be a Boolean expression of three variables x, y, z. Find the values of f(1, 1, 0), f(0, 1, 1), f(0, 1, 0), f(1, 1, 1), f(0, 0, 0).

**Q 14:** Construct truth tables for the following Boolean expressions.

(i) $x + yx'$ (ii) $x(x+y')$

(iii) $xy + yz + zx$ (iv) $xy' + x(y'+z)$

**Q 15:** Examine if the following Boolean expressions are equal

(i) $(x+y)'$, $x'y'$

(ii) $(xy)'$, $x' + y'$

(iii) $x(y+z)$, $xy+xz$

(iv) $xz + (x'+y')'$, $x(y+z)$



## 14.6 LET US SUM UP

- Let B be a non-empty set equipped with two binary operations + and . , a unary operation /, and two distinct elements 0 and 1. Then B is called a Boolean Algebra if the following conditions are satisfied.

  1. (i) a+b = b+a, (ii) a.b = b.a, $\forall\ a, b \in B$

  2. (i) a+ (b.c) = (a+b).(a+c)

     (ii) a.(b+c) = (a.b)+(a.c), $\forall\ a, b, c \in B$

  3. (i) a+0 = a, (ii) a.1 = 1, $\forall\ a \in B$

(ii) $a+a^{/} = 1$, (ii) $a.a^{/} = 0$, $\forall\ a \in B$

0 is called the zero element.

1 ia called the unit element.

$a^{/}$ is the complement of a.

- In a Boolean Algebra, zero element is unique.
- In a Boolean Algebra, unit element is unique.
- In a Boolean Algebra, complement of each element is unique.
- In a Boolean Algebra B, $1^{/} = 0$, $0^{/} = 1$.
- Every statement and every theorem in a Boolean Algebra remains valid if the binary operations + and . , and the zero element 0 and the unit element 1 are interchanged. This principle is known as the principle of duality.
- In a Boolean Algebra B,

  1. (i) $a+a = a$, (ii) $a.a = a$, $\forall\ a \in B$
  2. (i) $a+1 = 1$, (ii) $a.0 = 0$
  3. (i) $a+(a.b) = a$, (ii) $a.(a+b) = a$
  4. (i) $(a+b)+c = a+(b+c)$
     (ii) $(a.b).c = a.(b.c)$,
  5. $(a^{/})^{/} = a$
  6. (i) $(a+b)^{/} = a^{/}.b^{/}$, (ii) $(a.b)^{/} = a^{/}+b^{/}$
  7. (i) If $b+a=c+a$, and $b+a^{/} = c+a^{/}$, then $b=c$
     (ii) If $b.a. = c.a$, and $b.a^{/} = c.a^{/}$, then $b=c$
  8. (i) $a+b^{/}=1$ if and only if $a+b = a$
     (ii) $a.b^{/} = 0$ if and only if $a.b = a$
  9. (i) $a+(a^{/}.b) = a+b$
     (ii) $a.(a^{/}+b) = a.b$

- A literal symbol that is used to represent an element of the Boolean Algebra B = {0, 1} is called a Boolean variable. In B, +, . , / are defined by

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 1 |

| . | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

| / | 0 | |
|---|---|---|
| | 1 | 0 |

- By a Boolean expression, we mean an expression that is constructed by a finite set of symbols, each representing a Boolean variable, by

applying the binary operations + and . , and the unary operation /.

- 0 and 1 are Boolean expressions.
- Any variable is a Boolean expression.
- Two Boolean expressions of variables $x_1, x_2, \ldots, x_n$ are said to be equal if they assume the same values for every assignment of values 0, 1 to the variables $x_1, x_2, \ldots, x_n$.

  The equality of two Boolean expressions is denoted by " = ".



## 14.7 ANSWERS TO CHECK YOUR PROGRESS

**Ans to Q No 1:** Here x = {1}

P(x) = {{1}, $\phi$ }

| $\cup$ | {1} | $\phi$ |
|---|---|---|
| {1} | {1} | {1} |
| $\phi$ | {1} | $\phi$ |

| $\cap$ | {1} | $\phi$ |
|---|---|---|
| {1} | {1} | $\phi$ |
| $\phi$ | $\phi$ | $\phi$ |

| | $\cap$ | / |
|---|---|---|
| | {1} | $\phi$ |
| | $\phi$ | {1} |

From the tables, we see that $\cup$ is a binary operation on P(x), $\cap$ is a binary operation on P(x), / is a unary operation on P(x).

Now, 1. (i) $A \cup B = B \cup A$

(ii) $A \cap B = B \cap A, \ \forall \ A, B \in P(x)$

2. (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C), \ \forall \ A, B, C \in P(x)$

3. (i) $A \cup \phi = A$

(ii) $A \cap \{1\} = \{1\}, \forall \ A \in P(x)$

4. (i) $A \cup A^{/} = \{1\}$

(ii) $A \cap A^{/} = \phi, \ \forall \ A \in P(x)$

$\therefore (P(x), \cup, \cap, /)$ is a Boolean Algebra, where $\phi$ is the zero element and {1} is the unit element.

**Ans to Q No 2:** Left as an exercise.

**Ans to Q No 3:** We construct the composition tables for +, . , / as follows.

| + | 1 | 2 | 3 | 5 | 6 | 10 | 15 | 30 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 5 | 6 | 10 | 15 | 30 |
| 2 | 2 | 2 | 6 | 10 | 6 | 10 | 30 | 30 |
| 3 | 3 | 6 | 3 | 15 | 6 | 30 | 15 | 30 |
| 5 | 5 | 10 | 15 | 5 | 30 | 10 | 15 | 30 |
| 6 | 6 | 6 | 6 | 30 | 6 | 30 | 30 | 30 |
| 10 | 10 | 10 | 30 | 10 | 30 | 10 | 30 | 30 |
| 15 | 15 | 30 | 15 | 15 | 30 | 30 | 15 | 30 |
| 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 |

| . | 1 | 2 | 3 | 5 | 6 | 10 | 15 | 30 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 2 |
| 3 | 1 | 1 | 3 | 1 | 3 | 1 | 3 | 3 |
| 5 | 1 | 1 | 1 | 5 | 1 | 5 | 5 | 5 |
| 6 | 1 | 2 | 3 | 1 | 6 | 2 | 3 | 6 |
| 10 | 1 | 2 | 1 | 5 | 3 | 5 | 15 | 15 |
| 15 | 1 | 1 | 3 | 5 | 3 | 5 | 15 | 15 |
| 30 | 1 | 2 | 3 | 5 | 3 | 10 | 15 | 30 |

| / | 1 | 2 | 3 | 5 | 6 | 10 | 15 | 30 |
|---|---|---|---|---|---|---|---|---|
| | 30 | 15 | 10 | 6 | 5 | 3 | 2 | 1 |

From the above tables, we see that + is a binary operation on B, . is a binary operation on B, and $'$ is a unary operation on B.

Again, 1. (i) a+b = l.c.m of a, b

= l.c.m of b, a

= b+a, $\forall\ a, b \in B$

(ii) a.b = g.c.d of a, b

= g.c.d of b, a

= b.a, $\forall\ a, b \in B$

2. From the tables, we have

(i) a+(b.c) = (a+b).(a+c)

(ii) a.(b+c) = (a.b)+(a.c), $\forall\ a, b, c \in B$

3. (i) a+1 = l.c.m of a, 1 = a, $\forall\ a \in B$

   (ii) a. 30 = g.c.d of a, 30 = 1, $\forall\ a \in B$

4. (i) $a+a' =$ l.c.m of a, $a'$

   = l.c.m of a, $\frac{30}{a}$

   = 30

   (ii) $a.a' =$ g.c.d of a, $a'$

   = g.c.d of a, $\frac{30}{a}$

   =1

   $\therefore$ (B, +, . , /) is a a Boolean Algebra, where 1 is the zero element, and 30 is the unit element.

**Ans to Q No 4:** Left as an exercise.

**Ans to Q No 5:** Let X = {1, 2, 3, 4}

Then the power set P(x) of x contains $2^4$ i.e. 16 elements.

The set P(x) is a Boolean Algebra under operations of union, intersection, and complementation in P(x).

**Ans to Q No 6:** Here B = {0, a, b, 1}

+, . , / are defined in B as follows

| + | 0 | a | b | 1 |
|---|---|---|---|---|
| 0 | 0 | a | b | 1 |
| a | a | a | 1 | 1 |
| b | b | 1 | b | 1 |
| 1 | 1 | 1 | 1 | 1 |

| . | 0 | a | b | 1 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| a | 0 | a | 0 | a |
| b | 0 | 0 | b | b |
| 1 | 0 | a | b | 1 |

| / | 0 | a | b | 1 |
|---|---|---|---|---|
| | 1 | b | a | 0 |

From the tables, we see that + is a binary operation on B, . is a binary operation on B, $'$ is a unary operation on B.

Now,

1. (i) + is commutative on B.

   (ii) . is commutative on B.

2. (i) + is distributive over . in B

   (ii) . is distributive over + in B

3. (i) a+0 = a

   (ii) a.1 = 1, $\forall\ a \in B$

4. (i) $a+a^{/} = 1$

(ii) $a.a^{/} = 0, \ \forall \ a \in B$

$\therefore$ (B, +, . , /) is a Boolean Algebra, where 0 is the zero element, and 1 is the unit element.

**Ans to Q No 7:** Here B = {1, 2, 4, 5, 10, 20} is the set of all positive divisors of 20.

We construct the composition tables as follows.

| + | 1 | 2 | 4 | 5 | 10 | 20 |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 4 | 5 | 10 | 20 |
| 2 | 2 | 2 | 4 | 10 | 10 | 20 |
| 4 | 4 | 4 | 4 | 20 | 20 | 20 |
| 5 | 5 | 10 | 20 | 5 | 10 | 20 |
| 10 | 10 | 10 | 20 | 10 | 10 | 20 |
| 20 | 20 | 20 | 20 | 20 | 20 | 20 |

| . | 1 | 2 | 4 | 5 | 10 | 20 |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 | 1 | 2 | 2 |
| 4 | 1 | 2 | 4 | 1 | 2 | 4 |
| 5 | 1 | 1 | 1 | 5 | 5 | 5 |
| 10 | 1 | 2 | 2 | 5 | 10 | 10 |
| 20 | 1 | 2 | 4 | 5 | 10 | 20 |

| / | 1 | 2 | 4 | 5 | 10 | 20 |
|---|---|---|---|---|---|---|
| | 20 | 10 | 5 | 4 | 2 | 1 |

From the tables, we see that + is a binary operation on B, . is a binary operation on B, / is a unary operation on B.

Now, 1 (i) a+b = l.c.m of a, b

= l.c.m of b, a

= b+a

(ii) a.b = g.c.d of a, b

= g.c.d of b, a

= b.a

2. From the tables, we have

(i) a+(b.c) = (a+b), (a+c)

(ii) a.(b+c) = (a.b) + (a.c), $\forall \ a, b, c \in B$

3. (i) a+1 = l.c.m of a, 1=a

(ii) a.20 = g.c.d of a, 20

= a, $\forall \ a \in B$

The conditions 4 (i), and 4 (ii) are not satisfied. For

$10+10^{/}$ = l.c.m of 10, $10^{/}$

= l.c.m of 10, 2=10

$\therefore$ B is not a Boolean Algebra.

**Note :** We state the following theorem without proof.

Let n > 1 be a natural number. Let B be the set of all positive divisors of n. We define +, . , / in B as follows.

a+b = l.c.m of a, b

a.b = g.c.d of a, b

$a' = \frac{n}{a}$

Then B is a Boolean Algebra if and only n is not divisible by any square intager.

In the above example, n = 20 and 20 is divisible by 4, which is a square integer.

**Ans to Q No 8:** If possible, let B be a Boolean Algebra with 3 elements.

Let B {0, 1, a}, where $a \neq 0,\ a \neq 1$

We know that $0' = 1$, $1' = 0$

$\therefore$ $a' = a$

Now a = a.a

= $a.a'$

= 0, which is a contradition

$\therefore$ there cannot be a Boolean Algebra with 3 elements.

**Ans to Q No 9:** (a.b) + c = c+(a.b), by commutative law

= (c+a).(c+b), by distributive law

= (a+c). (b+c), by commutative law

= a.b, (given)

**Ans to Q No 10:** $a+(a+b)' = a+(a'.b')$

$= (a+a').(a+b')$

$= 1.\ (a+b')$

$= a+b'$

**Ans to Q No 11:** First we prove that (i) $\Rightarrow$ (ii)

We have a+b = a

$\therefore$ $a+b' = 1.\ (a+b')$

$= (a+a').(a+b')$

$= a+(a'.b')$

$= a+(a+b)'$

$= a+a'$

$= 1$

Next we prove that (ii) $\Rightarrow$ (iii)

We have $a+b' = 1$

$\therefore a+b = (a+b).1$

$= (a+b).(a+b')$

$= a+(b.b')$

$= a+0$

$= a$

Now $a.b = (a+b).b$

$= b.(a+b)$

$= b.(b+a)$

$= b$, by absorption law

Next we prove that (iii) $\Rightarrow$ (i)

We have $a.b = b$

Now $a+b = a+ (a.b)$

$= a$, by absorption law

Thus, we see that the properties (i), (ii), and (iii) are equivalent.

**2nd Part :** The dual of the above proposition is

(i) $a.b = a$, (ii) $a.b' = 0$, (iii) $a+b = b$

It can be proved by replacing . by + , + by . , 1 by 0, and 0 by 1 in the first part.

**Ans to Q No 12:** $[(abc+a'b')' + bc]'$

$= (abc+a'b') (b'+c')$, by De Mergan's law

$= abc\ b' + abc\ c' + a'b'b'+a'b'c'$

$= ac\ 0 + ab\ 0 + a'b' + a'b'c'$, $[bb'=0, cc' = 0, b'b'=b']$

$= a'b' + a'b'c'$

$= a'b'\ I + a'b'c'$

$= a'b' (1+c')$

$= a'b'$, $1+c'=1$

**Ans to Q No 13:** $f(x, y, z) = xy+z'$

$\therefore f(1, 1, 0) = 1.1 + 0'$

$= 1+1$

$= 1$

$f(0, 1, 1) = 0.1 + 1'$

$= 0 + 0$

$= 0$

$f(0, 1, 0) = 0.1 + 0'$

$= 0+1$

$= 1$

$f(1, 1, 1) = 1.1.+1'$

$= 1+0$

$= 1$

$f(0, 0, 0) = 0.0 + 0'$

$= 0+1$

$= 1$

**Ans to Q No 14:**

(i)

| x | y | $x'$ | $yx'$ | $x+yx'$ |
|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 |

(ii)

| x | y | $y'$ | $x+y'$ | $x(x+y')'$ |
|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |

(iii)

| x | y | z | xy | yz | zx | xy+yz+zx |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(iv) Left as an exercise

**Ans to Q No 15:**

(i)

| x | y | x+y | $(x+y)'$ | $x'$ | $y'$ | $x'y'$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |

From column 4 and column 7, we conclude that $(x+y)' = x'y'$

(ii) Left as an exercise.

(iii) Left as an exercise.

(iv)

| x | y | z | xz | $x'$ | $y'$ | $x'+y'$ | $(x'+y')'$ | $xz+(x'+y')'$ | y+z | x(y+z) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |

From column 9 and column 11, we conclude that

$xz + (x'+y')' = x(y+z)$



## 14.8 FURTHER READINGS

1) Sen, M. K, Chakraborty B. C. : *Introduction to Discrete Mathematies* (2010), Books and Allied (P) Ltd, Kolkata 700010.

2) Dasgupta Asok : *Abstract and Linea Algebra* (2014), Asoke Prakasan, Kolkata 700007.



## 14.9 MODEL QUESTIONS

**Q 1:** Let X = {1, 2}. Let P (x) be the power set of X. Show that the set P(x) is a Boolean Algebra under union, intersection, and complementation in P(x).

**Q 2:** Let B = {1, 2, 5, 10, 17, 34, 85, 170} be the set of all positive divisors of 170. Define +, . , / in B as follows.

a+b = l.c.m of a, b

a.b = g.c.d of a, b

$a' = \frac{170}{a}$

Examine if (B, +, . , /) is a Boolean Algebra.

**Q 3:** Let B = {1, 2, 3, 4, 6, 8, 12, 24} be the set of all positive divisors of 24. Define +, . , / in B as follows.

a+b = l.c.m of a, b

a.b = g.c.d of a, b

$a' = \frac{24}{a}$

Examine if (B, +, . , /) is a Boolean Algebra.

**Q 4:** Let (B, +, . , /) be a Boolean Algebra, and let a, b $\in$ B. Express the following in simplied form

(i) $(a+b).(a'+b').(a'+b).(a+b')$

(ii) $(a.b)+(a'.b)+(a.b')+(a'.b')$

**Q 5:** Let (b, +, . , /) be a Boolean Algebra, and let a, b, c, d $\in$ B. Prove that a.c + b.d = (a+b).(b+c).(c+d).(d+a)

**Q 6:** Give an example of a Boolean Algebra that has infinite number of elements.

**Q 7:** Let (B, +, . , /) be a Boolean Algebra, and let a, b, c $\in$ B. Prove that $(a+b')(b+c')(c+a')=(a'+b)(b'+c)(c'+a)$

**Q 8:** Let $f(x, y, z) = x.y.z + x.y'.z + x'.y.x + x'.y'.z'$ be a Boolean expression of three variables x, y, z. Find the values of f(1, 1, 1), f(1, 1, 0), f(1, 0, 1), f(0, 1, 1), f(0, 0, 0).

**Q 9:** Construct truth tables for the following Boolean expressions

(i) $xy+x(x+y)$ (ii) $x'y+xy'+x(x+y')$ (iii) $xyz+xy'z+x'yz+xyz'$

**Q 10:** Examine if the following Boolean expressions are equal

(i) $(x+y+z)'$, $x'y'z'$

(ii) $(xyz)'$, $x'+y'+z'$

(iii) $x.(x+y)$, $x$

(iv) $(x+y)z'$, $xz'+yz'$

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