UNIT 7: RULES OF INFERENCE

UNIT STRUCTURE

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7.1 LEARNING OBJECTIVES

After going through this unit, you will be to:

- discuss the rules of inference
- discuss some examples of rules of inference.

7.2 INTRODUCTION

This unit not only introduces to you the rules of inference but also the uses of the rules in case of formal proof of validity. It also provides us to go through some solved examples of the rules of inference in order to know how the rules of inference are applied to test the validity of arguments.

7.3 RULES OF INFERENCE

In the preceding chapter, you have already come to know the formal proof of validity which discusses the concepts like strategy for deduction, rules of inference, rules of replacement, differences between rules of inference and rules of replacement, test of formal proof, general suggestion for formal deduction etc. Therefore, this unit emphasizes only on the application of the rules of inference.

There are nine rules of inference. These rules are considered self-evident and are therefore valid without any proof. On the basis of these

rules we can determine or test the validity or invalidity of arguments. As we can derive a conclusion on the basis of these self-evident rules, so the method of deriving a conclusion from the rules is called deductive method.

Rules of inference are:

1) Modus Ponens (M.P.): $p \supset q$

p

$\therefore q$

This rule means affirmation of the antecedent and on that basis consequent is affirmed.

We can take an example of valid argument by applying this rule:

If Madhu is intelligent, he will be able to pass in the examination Madhu is intelligent.

Therefore, he will be able to pass in the examination.

This argument can be symbolised as valid argument form of the rule Modus ponens.

i) $p \supset q$

p

$\therefore q$

There are also other valid arguments forms of modus ponens. They are:

ii) $\sim p \supset \sim q$

$\sim p$

$\therefore \sim q$

2) Modus Tollens: $p \supset q$

$\sim q$

$\therefore \sim p$

This argument means the denial of the consequent and on that basis the antecedent is denied.

We can take an example of valid argument form by applying this rule. The argument is: If he wins in the match, he will give us a party

He will not give us a party.

$\therefore$ He does not win the match.

The symbolic form of this argument is: $p \supset q$

$\sim q$

$\therefore \sim p.$

Apart from this there are also other valid argument forms of modus ponens. They are:

i) $\sim p.\sim q$ ii) $(p.q) \supset r$

$\sim p$ $p.q$

$\therefore \sim q$ $\therefore \sim r$

This symbolic form is considered valid argument form of modus tollens. There are also other valid argument forms of modus tollens:

i) $p \supset \sim q$ ii) $p \supset (q\ r)$

q $\sim(q.r)$

$\therefore \sim p$ $\therefore \sim p$

3) Disjunctive Syllogism: $p \vee q$

$\sim p$

$\therefore q$

This argument holds that if one of the options or disjuncts is denied in the minor premise, the other option or disjunct is accepted in the conclusion. We can take real example of disjunctive syllogism.

Either He will win or defeat in the match.

He will not win in the match.

Therefore, he will defeat in the match. This real example can be symbolized in this way: $p \vee q$

$\sim p$

$\therefore q.$

Besides, there are also other valid forms of disjunctive syllogism. They are:

i) $\sim p \vee q$ ii) $\sim p \vee \sim q$ iii) $(p.q) \vee r$

p p $\sim(p.q)$

$\therefore q$ $\therefore q$ $\therefore r$

4) Hypothetical Syllogism: $p \supset q$

$q \supset r$

$\therefore p \supset r$

This rule means that both the premises are conditional statements and they have one common statement. The common statement is known as antecedent in one premise and other is consequent in another premise. We can take real example of hypothetical syllogism in order to better understand its logical form. As for instance:

If Madhu is intelligent, he will be able to pass in the examination

If he passes the examination, he will be able to get a job

Therefore, If Madhu is intelligent, he will be able to get a job

This example can be symbolized in the following way:

$p \supset q$

$q \supset r$

$\therefore\ p \supset r.$

This is a valid form of hypothetical syllogism. There are also other valid forms of hypothetical syllogism. They are respectively:

i)	ii)	iii)
$q \supset r$	$p \supset \sim q$	$\sim p \supset \sim q$
$p \supset q$	$\sim q \supset r$	$\sim q \supset \sim r$
$\therefore\ p \supset r$	$\therefore\ p \supset r$	$\therefore\ p \supset r$

5) Conjunction:

p

q

$\therefore\ p.q$

This rule means that the first two premises are joined together by the word 'and'.

6) Simplification:

$p.q$

$\therefore\ p$

This rule states that if the two premises are joined by the word 'and' or by the symbol '.', the first variable or conjunct can be inferred as the conclusion. The other valid forms of simplification are:

i)	ii)
$\sim p.q$	$(p \vee q).r$
$\therefore\ p$	$\therefore\ p \vee q$

7) Addition:

p

$\therefore\ p \vee q$

This rule means that we can add variable or variables to the premises by the symbol 'v'. The other valid forms of syllogism are:

i) p ii) p iii) p

∴ p v (q.r) ∴ p v ~q ∴ p v (q v (p.r))

8) Absorption (Abs): p ⊃ q

∴ p ⊃ (p.q)

According to this rule, consequent carries antecedent in the conclusion.

9) Constructive Dilemma (C.D.): (p ⊃ q)(r ⊃ s)

p v r

∴ q v s

This rule states that there are two conditional propositions. And the major premise possesses two different antecedents, therefore both denotes different consequent. We can take a concrete example:

If a man is honest, he must be respected; and if he is a social worker, he must get reward. A man is either an honest man or a social worker.

∴ Either he must be respected or he must get reward.

This example can be symbolized as–

(p ⊃ q).(r ⊃ s)

p v r

∴ q v s

This is a valid logical form of constructive dilemma.

The other valid form of constructive dilemma is:

(~p ⊃ q).(~r ⊃ s)

~p v ~r

∴ q v s

10) Destructive Dilemma (D.D.): (p ⊃ q).(r ⊃ s)

~q v ~s

∴ ~p v ~r

We can take a concrete example in order to understand its logical form. If a man is honest, he must be respected; and if he is a social worker, he must get reward. A man is either not respected or does not get reward.

∴ Either he is not honest or not a social worker.

This example can be symbolized as–

$(p \supset q)(r \supset s)$

$\sim q \vee \sim s$

∴ $\sim p \vee \sim r$. This is a valid form of destructive dilemma.

7.4 SOLVED EXAMPLES OF RULES OF INFERENCE

There are some solved examples of formal proof of validity given in this section in order to understand how the rules of inference are applied to test the validity of arguments.

A) 1) $(p \supset q)$
   2) $(q \supset s)$
   3) $p \ / \therefore s$
   4) $p \supset s$ 1st and 2nd by H.S.
   5) s 4th and 3rd by M.P.

B) 1) $(p \vee q) \supset r$
   2) $s \vee p$
   3) $\sim s \ / \therefore r$
   4) p 2st and 3nd by DS
   5) $p \vee q$ 4 addition
   6) r 1, 5 Modus ponens

C) 1) $(p \supset q)$
   2) $(r \supset s)$
   3) $p \vee r$
   4) $(q \vee s) \supset t \ / \therefore t$
   5) $(q \vee s)$ 1, 2 C.D.
   6) t 4, 5 M.P.

D) 1) $(p \supset q)$
   2) $(q \supset r)$
   3) $\sim p \supset (s \vee t)$
   4) $\sim r \ / \therefore s \vee t$
   5) $p \supset r$ 1, 2 H.S.

6) ~p 5, 4 M.T.

7) s v t 3, 6 M.P.

E) 1) ~p ⊃ (I v E)

2) p.r

3) ~I / ∴ E

4) p 2nd sim

5) I v E 1st and 4th by D.S.

6) E 5th and 3rd by D.S.

F) 1) p ⊃ ~q

2) ~q ⊃ r

3) r ⊃ s

4) (p.s) ⊃ u / ∴ p⊃u

5) p ⊃ r 1st and 2nd by H.S.

6) p ⊃ s 5th and 3rd by H.S.

7) p ⊃ (p.s) 6th by Abs.

8) p ⊃ q 7th and 4th by H.S.

G) If rains come in time, farmers will be happy. If farmers are happy, there will be good crops. Rains come in time. Therefore, there will be good crops.

This real example can be symbolized in this way:

1) R ⊃ F

2) F ⊃ C

3) R / ∴ C

4) R ⊃ C 1, 2 H.S.

5) C 4, 3 M.P.



CHECK YOUR PROGRESS

Q.1: What is deductive method?

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Q.2: Write the symbolic form of:

a) Modus Tollens

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b) Modus Ponens

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c) Constructive Dilemma

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d) Destructive dilemma

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e) Simplication

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f) Conjunction

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Q.3: What do you mean by the rule of modus ponens?

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Q.4: What do you mean by the rule of disjunctive syllogism?

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Q.5: Write a valid form of modus ponens.

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Q.6: Construct a formal proof of validity by applying the rules of inference.

i) $p \supset q$ ii) $q \supset r$ iii) $p \ / \therefore r$

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7.5 LET US SUM UP

- Rules of inference are considered self-evident and are therefore valid without any proof. On the basis of these rules we can determine or test the validity or invalidity of arguments. That is why these rules are known as deductive method.
- Modus ponens means affirmation of the antecedent and on that basis consequent is affirmed.
- Modus tollens means the denial of the consequent and on that basis the antecedent is denied.
- This argument holds that if one of the options or disjuncts is denied in the minor premise, the other option or disjunct is accepted in the conclusion.
- Hypothetical syllogism means that both the premises are conditional statements and they have one common statement. The common statement is known as antecedent in one premise and other is consequent in another premise.



7.6 FURTHER READING

1) Chhanda, Chakraborti (2006); *Logic: Informal, Symbolic and Inductive*; New Delhi: Prentice– Hall of India Private Limited.
2) Copi, Irving M. (1979); *Symbolic Logic* (fifth edition); Callier Macmillan International Editions.
3) Copi, Cohen (1995); *Introduction to Logic* (Ninth Edition); New Delhi: Prentice– Hall of India Private Limited.
4) Jain, Krishna (1998); *A Textbook of Logic*; D. K. Printworld.
5) Munshi, R.C. (1991); *Hand Book of Logic*; Calcutta: Baikuntha Book House.
6) Singh, S. Shyam Kishore (2000); Modern Logic (Volume one); Lamyana Press.

7.7 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q. No. 1: Rules of inference are considered self-evident and are therefore valid without any proof. On the basis of these rules we can determine or test the validity or invalidity of arguments. That is why these rules are known as deductive method.

Ans. to Q. No. 2:

a) Modus Ponens (M.P.)

$p \supset q$

p

$\therefore q$

b) Modus Tollens (M.T.)

$p \supset q$

$\sim q$

$\therefore \sim p$

c) Constructive Dilemma (C.D.)

$(p \supset q) (r \supset s)$

$p \vee r$

$\therefore q \vee s$

d) Destructive Dilemma

$(p \supset q) (r \supset s)$

$\sim q \vee \sim s$

$\therefore \sim p \vee \sim r$

e) Simplification (Simp.)

$p.q$

$\therefore p$

f) Conjunction (Conj.)

p

q

Ans. to Q. No. 3: Modus ponens means affirmation of the antecedent and on that basis consequent is affirmed.

Ans. to Q. No. 4: This argument holds that if one of the options or disjuncts is denied in the minor premise, the other option or disjunct is accepted in the conclusion.

Ans. to Q. No. 5: $\sim p.\sim q$

$\sim p$

$\therefore \sim q$

Ans. to Q. No. 6:

i) $p \supset q$

ii) $q \supset r$

iii) $p \quad \therefore r$

iv) $p \supset r$ i) and ii) H.S.

v) r iv) and v) M.P.



7.8 MODEL QUESTIONS

A) Very short questions:

Q.1: What do you mean by method of deduction?

Q.2: State the logical form of modus ponens.

Q.3: What do you mean by the rule of disjunctive syllogism?

Q.4: State the meaning of modus tollens.

Q.5: Write the logical form of hypothetical syllogism.

Q.6: Give an concrete example of modus ponens.

B) Construct formal proof of validity for each of the following argument forms:

Q.1: p. p /∴ p

Q.2: p / pVq

Q.3: i) ~p ⊃ q
ii) ~p /∴ q

Q.4: i) p ⊃ q
ii) q ⊃ r
iii) ~ r /∴ ~ p

Q.5: i) p ⊃ (q v r)
ii) s ⊃ p
iii) s /∴ (q v r)

Q.6: i) p ⊃ q
ii) q ⊃ s
iii) p /∴ q

Q.7: i) p ⊃ q
ii) q ⊃ r
iii) r ⊃ s /∴ (p ⊃ s)

Q.8: i) s ⊃ h
ii) h ⊃ ~ l
iii) l /∴ s

Q.9: i) p ⊃ ~q
ii) ~q ⊃ r
iii) r ⊃ s
iv) (p.s) ⊃ u /∴ (p ⊃ u)

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