UNIT 8: RULES OF REPLACEMENTS

UNIT STRUCTURE

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8.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- explain why rules of replacement is required
- discuss solved examples by rules of replacements
- explain how their implication is different from rules of inference.

8.2 INTRODUCTION

In the previous unit, we have discussed about the rules of inference. But there are many valid truth-functional arguments whose validity, we cannot prove by using only the nine rules of inferences. As for example, in order to construct a formal proof validity for the following problem, rules of inferences are not enough.

$A \supset B$

$C \supset \sim B$

$\therefore A \supset \sim C$

To solve this kind of problem some additional rules are required.

In any Truth-functional compound statement, if a compound statement in its being replaced by another statement having the same truth-functions, then the truth value of the compound statement will remain unchanged. But only compound statements that concern us here are truth

functional compound statements. That is why we can add some more or additional principles of rules of inference that is the rules of replacement. With the help of these rules we can replace any component of a statement by any other statement *logically* equivalent to the component replaced. For example, using the principle of Double Negation (D.N.), which asserts that 'p' is logically equivalent to '~~p', we infer from A ⊃ ~~B any of the following–

i.e., A ⊃ B, ~~A ⊃ ~~B, ~~ (A ⊃ B) or A ⊃ ~~~~B

by rules of replacement to make the ten rules definite, we list ten tautologous logically true bi-conditionals with which it can be used. These bi-conditionals provide additional rules of inference to be used in providing the validity of extended arguments.

The process of replacements is very different form substitution. We substitute statements for statements variables, whereas we replace statements with other statements with other statements. In moving from a statement form to a substitution instance of it, or from an argument form to a substitution instance of it, we can substitute any statements for any statement variable, provided that if a statement is substituted for one occurrence of a statement variable it must be substituted for every other occurrence of that statement variable. But in moving from one statement to another way of replacement, we can replace a component of the first only by a statements certified to be logically equivalent to the component by one of the logical equivalences 10 to 19, and we can replace one occurrence of that component without having to replace any other occurrence of it.

8.3 RULES OF REPLACEMENT

It is already mentioned that there are ten rules of replacement. They are stated as follows:

11) De Morgan's Theorems (De. M.)
~(p.q) ≡ (~p v ~q)
~(p v q) ≡ (~p.~q)

12) Commutation (Com.)
(p v q) ≡ (q v p)
(p.q) ≡ (q.p)

13) Association (Assoc.)
[p v (q v r)] ≡ [(p v q) v r)]
[p.(q.r)] ≡ [(p.q).r]

14) Distribution (Dist.)
[p.(q v r)] ≡ [(p.q) v (p.r)]
[p v (q.r) ≡ [(p v q).(p v r)]

15) Double Negation (D.N.)

$p \equiv \sim\sim p$

16) Transposition (Trans.)

$(p \supset q) \equiv (\sim q \supset \sim p)$

17) Material Implication (Impl.)

$(p \supset q) \equiv (\sim p \vee q)$

18) Material Equivalence (Equiv.)

$(p \equiv q) \equiv [(p \supset q).(q \supset p)]$

$(p \equiv q) \equiv [(p.q) \vee (\sim q.\sim p)]$

19) Exportation (Exp.)

$[(p.q) \supset r] \equiv [p \supset (q \supset r)]$

20) Tautology (Taut.)

$p \equiv (p \vee p)$

$p \equiv (p.p)$

8.4 SOME SOLVED EXAMPLES OF RULES OF REPLACEMENT

A) 1) $(A \supset B).(C \supset D)$ / $\therefore (A \supset B).(\sim D \supset \sim C)$

2) $(A \supset B).(\sim D \supset \sim C)$ From 1 by Trans.

B) 1) $(E \supset F).(G \supset \sim H)$ / $\therefore (E \supset F).(G \supset \sim H)$

2) $(\sim E \vee F).(G \supset \sim H)$ From 1 by Equiv.

C) 1) $(\sim M \vee \sim N) \supset (0 \supset \sim\sim P)$ / $\therefore \sim (M.N) \supset (0 \supset \sim\sim P)$

2) $\sim(M.N) \supset (0 \supset \sim\sim P)$ From 1 by De.M.

D) 1) $(X \vee Y).(\sim X \vee \sim Y)$ / $\therefore [(X \vee Y).\sim X] \vee [(X \vee Y).\sim Y]$

2) $[(X \vee Y).\sim X] \vee [(X \vee Y).\sim Y]$ From1 by Dist.



CHECK YOUR PROGRESS

Q.1: How many rules are there in the rules of replacement?

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Q.2: Write down the rule of Double Negation.

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Q.3: Give an example of Tautology.

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Q.4: Write down the rule of DeMorgan Theorem.

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Q.5: How many truth-functions are used in the rule of material implication?

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Q.6: Construct a formal proof of validity by applying the rules of replacement.

$p \supset q$

$r \supset \sim q \quad / \therefore p \supset \sim r$

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8.5 LET US SUM UP

- In order to construct formal proof of validity, with rules of inference we need rules of replacement too.
- There are ten rules of replacements.
- In the rules of replacement one side of the statement can be replaced with the other side of the equivalent statement.
- Rules of replacement can be applied to a part of the statement too, which is not possible in case of rules of inference.



8.6 FURTHER READING

1) Copi, Irving M.; Cohen, Carl; Jetli, Priyedarshi; Jetli, Prabhakar M. (2006); *Introduction to Logic*; Dorling Kindersley (India) Pvt. Ltd.
2) Hurley, J. Patrick (2007); *Introduction to Logic*; Wadsworth.
3) Hausman, A.; Kahane, H. & Tidman, P. (2007); *Logic and Philosophy*; Wadsworth.
4) Jain, Krishna (1998); *A Textbook of Logic*; D. K. Printworld (P) World.



8.7 ANSWERS TO CHECK YOUR PROGRESS

Ans. to Q. No. 1: There are ten rules in the rules of replacement.

Ans. to Q. No. 2: The rule of Double Negation is: $p \equiv \sim\sim p$

Ans. to Q. No. 3: The example of tautology is as follows:

The rain comes tomorrow or the rain comes tomorrow

Ans. to Q. No. 4: The rule of DeMorgan's theorem is:

$\sim(p.q) \equiv (\sim p \vee \sim q)$

$\sim(p \vee q) \equiv (\sim p.\sim q)$

Ans. to Q. No. 5: Three truth-functions are used in the rule of material implication. They are: implicative function, disjunctive function and the negative function.

Ans. to Q. No. 6: 1. $p \supset q$

2. $r \supset \sim q$ / $\therefore p \supset \sim r$
3. $\sim\sim q \supset \sim r$ from 2 by Trans.
4. $q \supset \sim r$ From 3 by D.N.
5. $p \supset \sim r$



8.8 MODEL QUESTIONS

A) Very Short Questions:

Q.1: Define rules of replacement.

Q.2: How rules of replacements are different from rules of inference?

Q.3: What is the logical value of association?

Q.4: Rules of replacements are the rules of deduction—Is this statement true?

Q.5: Is substitution and replacement are same?

Q.6: Write down the logical value of Material Equivalence?

B) Construct formal proof of validity for each of the following argument forms:

Q.1: $A \supset B$

$C \supset \sim B$ / $\therefore A \supset \sim C$

Q.2: $(D.E) \supset F$

$(D \supset F) \supset G$ / $\therefore E \supset G$

Q.3: $A \supset B$
$B \supset C$
$C \supset A$
$A \supset {\sim}C$ / $\therefore$ ~A.~C

Q.4: $(H \vee I) \supset [J.(K.L)]$
I / $\therefore$ J.K

Q.5: $W \supset X$
$\sim Y \supset {\sim}X$ / $\therefore W \supset Y$

Q.6: $(L \supset M).(N \supset M)$
$L \vee N$ / $\therefore M$

Q.7: $(D.{\sim}E) \supset F$
$\sim(E \vee F)$ / $\therefore$ ~D

Q.8: $Z \supset A$
$Z \vee A$ / $\therefore A$

Q.9: $(N.O) \supset P$
$\sim (P \supset Q) \supset Q$ / $\therefore N \supset Q$

Q.10: C / $\therefore D \supset C$

Q.11: $H.(I.J)$ / $\therefore J.(I.H)$

Q.12: $O \supset P$
$P \supset {\sim}P$ / $\therefore$ ~O

Q.13: $(M \supset N).(\sim O \vee P)$
$M \vee O$ / $\therefore N \vee P$

Q.14: $(Z \vee A) \vee B$
~A / $\therefore Z \vee B$

Q.15: $E \vee (F.G)$ / $\therefore E \vee G$

Q.16: $T \supset U$
$\sim (U \vee T)$ / $\therefore$ ~T

Q.17: $A \supset B$
$\sim A \supset C$
$\sim(B \vee C)$ / $\therefore D$

Q.18: $B \equiv C$
$\sim D \equiv {\sim}C$
$D \supset (E \vee F)$
~E / $\therefore$ ~F

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