# UNIT 9: ELEMENTARY FORMAL DEDUCTIONS

## UNIT STRUCTURE


9.1 Learning Objectives

9.2 Introduction

9.3 Elementary Formal Deduction

9.4 Simple Introductory Deduction

9.5 The Strategy for Constructing Formal Deduction

9.6 Certain Examples of Formal deduction

9.7 Let us Sum up

9.8 Further Reading

9.9 Answers to Check Your Progress

9.9 Model Questions


## 9.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- explain elementary formal deduction
- illustrate simple introductory deduction
- discuss the strategy for constructing formal deduction
- explain certain examples of formal deduction.

## 9.2 INTRODUCTION

The primary concern of deductive logic is validity. In deductive logic arguments are ultimately classified into two types– valid arguments and invalid arguments. An argument is valid when the conclusion of the argument is fully justified by its premises.

We have already come across certain basic rules of inference. These basic rules of inference may also be understood as elementary valid argument forms. If a particular argument is in such an elementary valid argument form then the argument is demonstrated to be valid. For example the following is the rule known as Modus Ponens (M.P.)–

$p \supset q$

$p$

$\therefore$ $q$

Let us look at the following argument–

$(A.B) \supset R$

$A.B$

$\therefore$ $R$

This argument conforms to the structure of modus ponens. Therefore, it is a valid argument. Longer and more complex deductive arguments may be given to us. We may discover that these arguments consist of series of steps in which every step conforms to particular rule of inference. By finding out these steps one after another and by referring to the specific rules of inference conformed by these steps we can explain that the conclusion of the argument follows from the premises of the argument. Thereby, we can establish that the argument is valid.

## 9.3 ELEMENTARY FORMAL DEDUCTION

An elementary formal deduction consists of a series of steps. Here, we first write down the premises. Generally the conclusion is written in the right hand side. Then premises are combined and recombined in such a way that a series of steps is constructed. In combining and recombining the premises are constantly refer to the rules of inference. We are completely guided and logically controlled by the rules of inference. Ultimately the series of steps ends in the conclusion of the argument. Every step in this step is valid. Thus this series of arguments which is constructed is a series of valid arguments. Ultimately we arrive at the conclusion. The process demonstrates that the conclusion is logically validly derivable from the premises. The entire series of continuous steps from the premises to the conclusion is known as a formal deduction. When the formal deductions are not extremely logically complicated then we call then elementary formal deductions.

Alternately the process of constructing such a formal deduction is known as constructing a derivation. When a formal deduction successfully shows that the conclusion of an argument logically follows from the premises then we have constructed a demonstration or a proof validity of the argument.

## 9.4 SIMPLE INTRODUCTORY DEDUCTIONS

We may begin with arguments that are simple and then we may move on to arguments of greater complexity.

Let us first take up the following arguments.

R.S /∴ R v T

This argument may be shown to be valid by using just two rules of inference. One is simplification (simp.) and the other is addition (add.)

Let us construct the formal deduction of the above argument.

1) R.S / ∴ R v T
2) R From 1 Simp.
3) R v T From 2 Add.

Again let us take the following argument–

$A \supset B$

$C \supset D$

$\sim B \vee \sim D$ / ∴ $\sim A \vee \sim C$

This argument can be shown to be valid by using just rules of inference. They are conjunction (conj) and destructive dilemma (D.D.). The complete formal deduction for this argument may be constructed as follows–

1) $A \supset B$
2) $C \supset D$
3) $\sim B \vee \sim D$ / ∴ $\sim A \vee \sim C$
4) $(A \supset B).(CD)$ From 1 and 2 by Conj.
5) $\sim A \vee \sim C$ From 3 and 5 by D.D.

There is no mechanical process to be followed while constructing formal deductions. We do not know beforehand which rule of inference is to be used first, which rule of inference is to be used next and so on. There is no set formula. Every argument will require its particular formal deduction. Every particular formal deduction will have its own specific steps. The steps

of one formal deduction of one argument will not be exactly identical with the steps of another deduction of another argument. Every argument will call for its specific deduction. The strategy of one formal deduction will not be exactly identical with the strategy found in another deduction.

## 9.5 THE STRATEGY FOR CONSTRUCTING FORMAL DEDUCTION

When we attempt to construct the formal deduction of an argument then first we have to carefully look at the premises of the argument and also at the conclusion. We are to think, then, from the end of the conclusion. How this conclusion can come out of the premises? This will be our guiding question.

If there are certain premises then we may as– can we combine two or more premises and thereby construct a road towards the conclusion? Again if a compound statement is there as premise then we may see whether it can be broken down into constituents by using some rules such as the rule of simplification. If there is a new proposition in the conclusion that was not present in the premises then we may be rest assured that somehow the rule of deduction has to be applied.

There are just a few thumb rules. The most important point is that if an argument is given then we have to think backwards from the conclusion.

## 9.6 CERTAIN EXAMPLES OF FORMAL DEDUCTION

a) 1) $\sim J \supset N$
   2) $J \supset L \mathbin{/} \therefore L \vee N$
   3) $\sim L \supset \sim J$ Form 2 by Trans.
   4) $\sim L \supset N$ From 3 and 1 by H.S.
   5) $\sim\sim L \vee N$ From 4 by Impl.
   6) $L \vee N$ From 5 by D.N.

In constructing this formal deduction first we have employed the rule of transposition on the second premise. The rule of transposition is the following–

$(p \supset q) \equiv (\sim q \supset \sim p)$

Similarly, $J \supset L$ is equivalent to $\sim L \supset \sim J$. Next we have employed the rule of hypothetical syllogism on propositions 3 and 1. The rule of hypothetical syllogism is the following–

$p \supset q$

$q \supset r$

$\therefore \; p \supset r$

By combining the lines 3 and 1 and by employing the rule of hypothetical syllogism we get the following–

$\sim L \supset \sim J$

$\sim J \supset N$

$\therefore \; \sim L \supset N$

The conclusion is presented in the fourth line. By applying the rule of material implication upon the fourth line we get the following–

$\sim\sim L \; V \; N$

This is presented as the fifth line. By employing the rule of double negation on this line we may get the following line–

$L \; V \; N$

This is our sixth line of the formal deduction and it is also the conclusion. We have thereby demonstrated this conclusion, through a series of steps, follows from the premises. Thereby the argument is established to be a valid argument.

b) $R \supset S$

$R.T \; / \; \therefore S$

1) $R \supset S$
2) $R.T \; / \; \therefore S$
3) R From 2 by Simp.
4) S From 1 and 3 by M.P.

c) $P \; V \; (R.T)$

$\sim P$

$R \supset S \; / \; \therefore S$

1) $P \; V \; (R.T)$
2) $\sim P$

3) R ⊃ S / S

4) R.T From 1 and 2 by D.S.

5) R From 4 by Simp.

6) S From 3 and 5 by M.P.

d) P ⊃ R

~R.T

(~P.T) ⊃ S / ∴ S V L

1) P ⊃ R

2) ~R.T

3) (~P.T) ⊃ S / ∴ S V L

4) ~R From 2 by Simp.

5) ~P From 1 and 5 by M.T.

6) T.~R From 2 by Com.

7) T From 6 by Simp.

8) ~P.T From 5 and 7 by Conj.

9) S From 3 and 8 by M.P.

10) S V L From 9 by Add.

e) S ⊃ T

S V T / ∴ T

1) S ⊃ T

2) S V T / ∴ T

3) T V S From 2 by Com.

4) ~T ⊃ S From 3 by Impl.

5) ~T ⊃ T From 4 and 1 by H.S.

6) ~~T V T From 5 by Impl.

7) T V T From 6 by D.N.

8) T From 7 by Taut.

f) (F ⊃ R).(L ⊃ U)

(R.U) ⊃ C

~C /∴ ~F v ~L

1) (F ⊃ R).(L ⊃ U)

2) (R.U) ⊃ C

3) ~C /∴ ~F V ~L

4) ~(R.U) From 2 and 3 by M.T.

5) ~R V ~U From 4 by De.M.

6) ~F V ~L From 1 and 5 by D.D.



### CHECK YOUR PROGRESS

**Q.1:** What is the primary concern of deductive logic?

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**Q.2:** What is the other name of the basic rules of inference?

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**Q.3:** What is the elementary formal deduction?

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**Q.4:** How can a valid argument be established?

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**Q.5:** Find out the validity of the following argument:

a) $D \supset A$
$A \supset (B \supset C) / \therefore (D.B) \supset C$

b) $A \vee (B.C)$
$(A \vee B) \supset D / \therefore D \vee E$

c) $C / \therefore D \supset C$

d) $R \vee S$
$\sim R.T$
$S \supset L / \therefore L \vee K$



## 9.7 LET US SUM UP

- The formal deduction of an argument consists of a series of steps through which it is established that the conclusion of the argument follows from the premises.

- In constructing formal deductions we take the help of the rules of inference.
- There is no fixed mechanical device by using which formal deductions are constructed. Generally we have to think backward from the conclusion and to discover the steps which will lead us from the premises to the conclusion.
- If we can show through a series of valid steps that the conclusion of an argument follows from the premises then the argument is established to be a valid argument.



## 9.8 FURTHER READING

1) Copi, Irving M. (2006); *Symbolic Logic* (Fifth Edition); New Delhi: Prentice Hall of India Pvt. Ltd.
2) Copi. Irving M.; Cohen, Carl; Jetli, Priyedarshi & Prabhakar, Monika (2006); *Introduction to Logic* (Twelfth Edition); Delhi: Dorling Kindersley (India) Pvt. Ltd.
3) Nolt, John; Rohatyn, Dennis Hille (2004); *Theory and Problems of Logic* (Second Edition)*;* New Delhi: Tata McGraw-Hill.



## 9.9 ANSWERS TO CHECK YOUR PROGRESS

**Ans. to Q. No. 1:** Validity of the argument is the primary concern of deductive logic.

**Ans. to Q. No. 2:** The basic rules of inference are also known as elementary valid argument forms.

**Ans. to Q. No. 3**: When the formal deductions are not extremely logically complicated then, these rules can be regarded as elementary formal deduction.

**Ans. to Q. No. 4:** If it can be shown that through a series of valid steps that the conclusion of an argument follows from the premises then the argument is established to be a valid argument.

**Ans. to Q. No. 5:** a) 1) $D \supset A$

2) $A \supset (B \supset C) / \therefore (D.B) \supset C$

3) $D \supset (B \supset C)$ From 1 and 2 by H.S.

4) $(D.B) \supset C$ From 3 by Exp.

b) 1) $A \vee (B.C)$

2) $(A \vee B) \supset D / \therefore D \vee E$

3) $(A \vee B).(A \vee C)$ From 1 by Dist.

4) $(A \vee B)$ From 3 by Simp.

5) D From 2 and 4 by M.P.

6) $D \vee E$ From 5 by Add.

c) 1) $C / \therefore D \supset C$

2) $C \vee \sim D$ From 1 by Add.

3) $\sim D \vee C$ From 2 by Com.

4) $D \supset C$ From 3 by Impl.

d) 1) $R \vee S$

2) $\sim R.T$

3) $S \supset L / \therefore L \vee K$

4) $\sim R$ From 2 by Simp.

5) S From 1 and 4 by D.S.

6) L From 3 and 5 by M.P.

7) $L \vee K$ From 6 by Add.



## 9.10 MODEL QUESTIONS

**A)** Construct formal deduction of the following arguments:

1) $M \supset N$

$N \supset O$

$(M \supset 0).(N \supset P)$

$(M \supset P) \supset Q / \therefore Q$

2) $J \supset K$

$J \vee (K \vee \sim M)$

$\sim K / \therefore \sim M.\sim K$

3) $E \supset (F . \sim G)$
   $(F \vee G) \supset H$
   $E / \therefore H$
4) $\sim(P.Q)$
   $Q \vee (R \supset S)$
   $P.\sim S / \therefore R \supset (R.S)$
5) $(G \vee X) \supset (E.K)$
   $(K \supset L)$
   $\sim L / \therefore \sim G$
6) $\sim H \vee R$
   $H / \therefore R \vee T$
7) $W \vee D$
   $W \supset F$
   $\sim F / \therefore D$
8) $(E.F) \vee (G \supset H)$
   $I \supset G$
   $\sim(E.F) / \therefore I \supset H$
9) $T \supset \sim(U \supset V) / \therefore T \supset U$
10) $(G \vee X) \supset (E.K)$
    $K \supset L$
    $\sim L / \therefore \sim G$
11) $(A \vee B) \supset (C.D)$
    $\sim C / \therefore \sim B$
12) $J \vee \sim K$
    $K \vee (L \supset J)$
    $\sim J / \therefore L \supset J$
13) $F \vee (G \vee H)$
    $(G \supset I).(H \supset J)$
    $(I \vee J).(F \vee H)$
    $\sim F / \therefore H$

**B) Long Questions:**

**Q.1:** What is formal deduction? Explain with illustrations.

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