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Micro Economics- II

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ECONOMICS

BLOCK : 1



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COURSE INTRODUCTION

Micro Economics II is the first course of second semester of the M.A. in Economics programme of this University.This is the second course deals with Micro Economics as we have already studied about the Micro Economics in the course of first semester of this programme as Micro Economics I.So this course basically covers the topics of Micro Economics which are not covered in the first semester course and enables the learners with various new knowledge and ideas of Micro Economics. This course comprises of 15 units and has been divided into two blocks. The first block comprises seven units and the second block comprises eight units.

BLOCK INTRODUCTION

This is the first block of the course and comprises of seven units. The first and second unit shall discuss the markets and welfare.The first unit deals with first fundamental theorem of welfare economics,second theorem of welfare economics,concept of social optimum and equity-efficiency Trade-off. The second unit deals with market failure and the different causes of market failure, concepts of public good and externality. Apart from it this unit also explain the importance of property rights and how we can internalise externality through assigning property rights. The third unit deliberates on basic ideas and concepts of game theory to solve the problems occur frequently within economics.The fourth unit will introduce you about the concept of factor pricing, marginal productivity theory of distribution,equilibrium under perfect competition and imperfect competition in the factor market, modern theory of distribution and product exhaustion problem. The fifth unit explains the inter- temporal choice of investment decisions,computation of NPV and IRR and their relevance to choice of investment project, consumption as well as production decision under inter-temporal choice pattern.The sixth unit will help you to acquire knowlege about the individual behaviour towards risk, various types of risk an individual can possess, importance of the expected utility theorem,mean variance analysis and portfolio selection in the risk analysis. The seventh unit deals with the concept of real and nominal wages, marginal productivity theory of wages and determination of wages under imperfect market forms. This unit will also help you to understand the exploitation of labour and the role of trade union.

While going through a unit, you will notice some along-side boxes, which have been included to help you to know some of the difficult, unseen terms. Again, we have included some relevant concepts in “LET US KNOW” along with the text. And, at the end of each section, you will get “CHECK YOUR PROGRESS” questions. These have been designed to self-check your progress of study. It will be better if you solve the problems put in these boxes immediately after you go through the sections of the units and then match your answers with “ANSWERS TO CHECK YOUR PROGRESS” given at the end of each unit.

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UNIT 1: MARKETS AND WELFARE-I

UNIT STRUCTURE

1.1 Learning Objectives

1.2 Introduction

1.3 First Fundamental Theorem of Welfare Economics

1.4 Second Theorem of Welfare Economics

1.5 Concept of Social Optimum

1.6 Equity-Efficiency Trade-off

1.7 Let Us Sum Up

1.8 Further Reading

1.9 Answers to Check Your Progress

1.10 Model Questions

1.1 LEARNING OBJECTIVES

After going through this unit, you will be able to-

- learn how a general equilibrium attained in a perfectly competitive market form can satisfy maximum social welfare or economic efficiency or Pareto optimality
- learn total and necessary or marginal conditions and total order conditions of Pareto optimality in lieu of perfect competition
- understand the concept of social optimum
- understand the concept of equity and efficiency
- find out why there exists a trade-off between equity and efficiency.

1.2 INTRODUCTION

Welfare economics is normative in nature. By evaluating social efficiency of a particular policy, normative economics or welfare economics prescribes a policy option which is economically as well as socially desirable. There is a possibility of welfare losses which is undesirable and desirable welfare gains due to allocation of resources or policy changes. This welfare losses or gain is the outcome of interdependence among various economic

agents of an economy. For example, a change in the price of one factor can easily change the price of other factors. This interrelationship among various parts of the economy means that certain particular change in one part of the economy affects the resource allocation in all other parts of it. Therefore, when one's welfare increases, it may decrease the welfare of others which leads to economic inefficiency in any change in allocation of resources or goods.

In this context, Italian economist Vilfredo Pareto (1848-1923) laid down three marginal conditions so that economic efficiency can be achieved. According to Pareto for the attainment of economic efficiency the marginal conditions to be satisfied in an economy are: (a) Efficiency in allocation of distribution of goods (Consumption efficiency); (b) Efficiency in allocation of resources (Product efficiency) and; (c) Efficiency in allocation of product mix (Simultaneous efficiency of production and consumption).

In this unit we will discuss how a general equilibrium attained in a perfectly competitive market forms can satisfy all the marginal conditions of Pareto optimality so that it can ensure economic efficiency. Further we will discuss the second welfare theorem and emphasize on the separation of the concept of issue of efficiency from issue of equity by redistributing the initial endowments. In this unit we will also discuss the concept of social optimum in terms of equality between Marginal Cost (MC) and Marginal Benefit (MB) and Marginal Social Cost (MSC) and Marginal Social Benefit (MSB).



CHECK YOUR PROGRESS

Q.1: Distinguish between positive and normative economics. (Answer in about 20 words)

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Q.2: What are the three marginal conditions of Pareto optimal? (Answer in about 10 words)

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Q.3: Define Pareto optimality. (Answer in about 20 words)

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1.3 THE FIRST THEOREM OF WELFARE ECONOMICS

We know that Pareto efficiency has three marginal conditions:

- Efficient allocation of distribution of goods (Consumption efficiency)
- Efficient allocation of resources (Product efficiency)
- Efficient allocation of product mix (Simultaneous efficiency of production and consumption)

To attain Pareto efficiency in an economy these basic conditions must be satisfied. The general equilibrium attained in perfectly competitive market is the most ideal condition in which the above mentioned marginal conditions are satisfied and lead to maximum social welfare or economic efficiency in the sense of Pareto optimality. This is known as first or fundamental theorem of welfare economics. By assuming perfect competition, where there is no individual influence in buyer and seller market, free play of self interest and without government intervention in economic activity we can discuss how perfectly competitive market equilibrium satisfy all basic conditions of Pareto optimality. Let us discuss how all marginal conditions of Pareto optimality that are fully satisfied by general equilibrium are attained under perfect competition.

- **Perfect Competition and Optimal Distribution of Goods or Efficiency in Consumption**

With regard to the distribution of goods among the consumers, one important marginal condition of Pareto efficiency is the marginal rate of substitution (MRS) which is equal for all the consumers. To prove it, let us assume an economy under the following conditions:

- Two consumers A and B along with their budget constraint/limit are always trying to maximize their level of satisfaction from consumption of goods X and Y.
- There exists perfect competition so that the price of goods X and Y are given and same for both consumer A and B.

Now, given the conditions mentioned above consumer A will maximize his satisfaction when:

$MRSxy^A = Px/Py$ ..... (i)

Similarly, consumer B will attain equilibrium or maximize satisfaction when:

$MRSxy^B = Px/Py$ ..... (ii)

Since we assumed that perfect competition is prevailing in the economy, the price ratio (Py/Px) must be equal for both individual A and B. Therefore, the MRS of individual A in equation (i) and MRS of individual B in equation (ii) can be written as:

$$MRSxy^A = MRSxy^B$$

This equation implies that there is efficiency in distribution of the goods X and Y between consumer A and B under perfect competition conditions. Here, MRS *x* for *y* is the same for consumer A and B. Thus, efficiency in consumption has been attained.

The condition of efficiency in consumption can be easily understood with the help of Edgeworth Box diagram. In Figure 1.1 OA and OB represents utility corner for consumer A and consumer B and they consumed goods X and Y. The consumers A and B have their own respective indifference curves which are convex to the origin. IA_1, IA_2, and IA_3 are the respective indifference curves for consumer A. The indifference curve IA_3 represents higher level of satisfaction over IA_1 and IA_2 indifference curves for consumer A. The indifference curves for consumer B is denoted by IB_1, IB_2, and IB_3. In case of the level of satisfaction for consumer B due to efficient distribution of goods, higher level indifference curve represents higher level of satisfaction. CC is the contract curve passing through points of tangency of the indifference curves of consumer A and B. On CC contract curve MRS between the two goods X and Y remain same for both the consumers A and B. In the figure

before distribution of the goods, the initial endowments of A and B is at point K. At point K, the MRS_{XY} for consumer A and B are not equal.

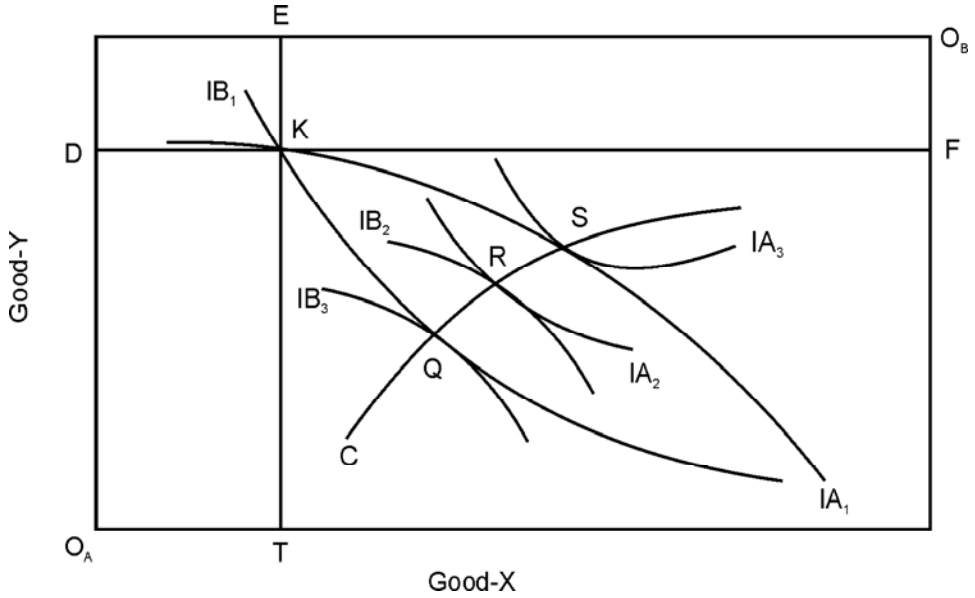


Fig. 1.1: Efficiency in Consumption

Now through exchange, the initial endowment is shifted from point K to point S on the contract curve CC. At point S, the indifference curve of consumer B remains the same IB_1 with initial endowment point K. In this distribution of goods, the movement from point K to S neither better off nor worse off. On the other hand, at point S, the indifference curve IA_1 of consumer A moves to IA_3 indifference curve. This movement from lower indifference curve to higher indifference curve that is point from K to S represents Pareto improvement. So, at point S, consumer A is better off without affecting consumer B's endowment. Similarly, the movement from initial endowment point K to Q implies consumer B is better off without affecting the endowments of consumer A.



ACTIVITY 1.1

Mr. X has 3 apples and 12 bananas. On the other hand, Mr. Y has 8 apples and 4 bananas. With these endowments, Mr. X's Marginal Rate of Substitution (MRS) of apple for banana is 5 and Mr. Y's MRS is equal to 3. If Mr.

X gives 4 bananas to Mr. Y for 1 apple then will Mr. Y get involved in this trade or not?

- **Perfect Competition and Optimal Allocation of Factors between Firms**

The marginal condition for Pareto optimality relates to optimal allocation of factors among the production of various goods. To achieve this condition, the marginal rate of technical substitution (MRTS) of factors must be equal among the firms.

Now, assume a firm working under perfect competition conditions will employ two factors labour (L) and capital (K) with given constant prices. The firm will attain equilibrium or able to minimize cost where the firm's given iso-quant is tangent to its corresponding iso-cost line. Here, the iso-quant represents MRTS between labour (L) and capital (K) and iso-cost line expresses the ratio of prices of L and K that is (P_L/P_K).

Thus, under perfect competition the firm/producer A will minimize cost where MRTS between labour and capital equate with the price ratio of both factors. It can be expressed as

$$MRTS_{LK}^{A} = P_L/P_K \text{ ——————————(i)}$$

Likewise, producer B will attain equilibrium or minimize cost at

$$MRTS_{LK}^{B} = P_L/P_K \text{ ——————————(ii)}$$

As we know both the producers are working under perfect competition, hence the price ratio (P_L/P_K) will be same. Thus, we ensure that the general equilibrium attend in perfect competition satisfy the marginal conditions of Pareto efficiency by proving equality between $MRTS_{LK}^{A} = MRTS_{LK}^{B}$.

As like efficiency in consumption we can explain efficiency in production with the help Edgeworth Box diagram. In panel A of the figure 1.2. we assume two goods X and Y that can be produced by two factors of production labour (L) and capital (K).

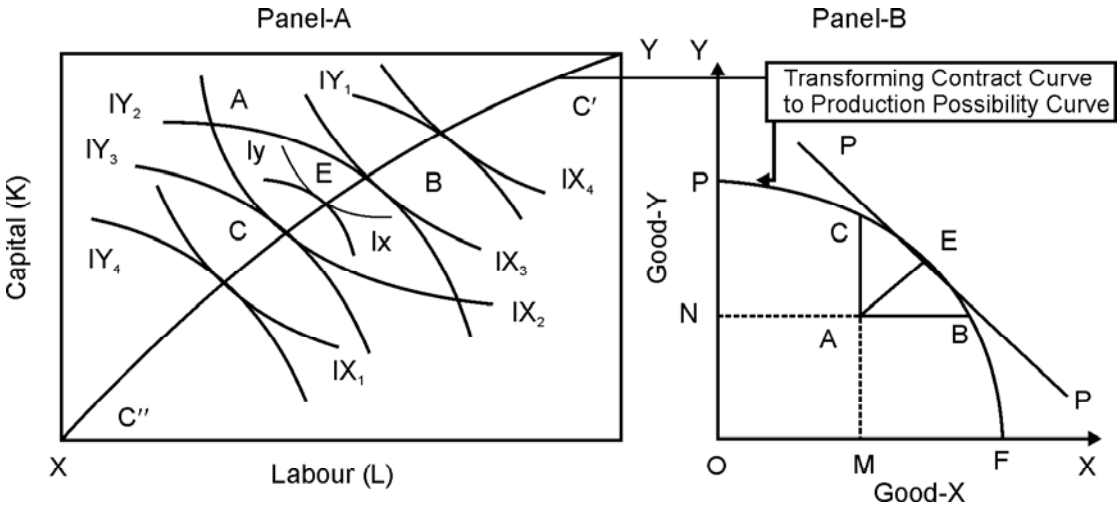


Fig. 1.2: Efficiency in Production and Derivation of Production Possibility Frontier

We can have various iso-quant such as IX_1, IX_2, IX_3 and IX_4 for good X. And for good Y we have IY_1, IY_2, IY_3 and IY_4. The rightward shift from iso-quant IX_1 to IX_4 indicates increase in production of good X. and move from IY_1 to IY_4 indicates more production of good Y. Now by allocating factor endowments if we move from point A to B and C there will be increase in production without reducing the production of other goods. So, the point A is inefficient. The point B is Pareto efficient because there is upward shift in iso-quant from IX_2 to IX_3. In this shift due to allocation of resources production of X will increase without reducing the production of Y. The point C is also Pareto efficient. At point C there is increase in production of goods Y without reducing goods X.

There are different tangency points between iso-quant of goods X and Y. These tangency points indicate efficient level of production. Now by joining all efficient points we can get contract curve of production. From this Edgeworth contract curve, we can derive production possibility frontier in panel B of figure 1.2. The PF is the production possibility frontier or production possibility curve. Every point of production possibility frontier represents production efficiency. Any side inside the production possibility frontier will be Pareto inefficient. The point A represents Pareto inefficient because without reducing either goods X or Y there is increase in goods X at point B. The

movement from point A to point B indicates Pareto efficient. At this point there is an increase in production of goods X without reducing goods Y. As like point B move from point A to C indicates increase in production of goods Y without reducing goods X. Point C also represents Pareto efficient.

- **Perfect Competition and Allocative Efficiency (Optimum Direction of Production)**

This condition requires the equality between MRT (Marginal Rate of Product Transformation) in production and MRS in consumption for every pair of commodities and for individuals. In the case of a very simple economy composed of only two commodities X and Y and two individuals A and B, Pareto optimality requires that

$$MRT_{XY} = MRSxy^{A} = MRSxy^{B}$$

Again, this condition can be ensured under perfect competition. We know a firm under perfect competition in equilibrium point produces output to that extent where its marginal cost is equal to the price of the commodity. Thus, for firms are in perfect competition, $MC_X=P_X$ and $MC_Y=P_Y$ where MC_X and MC_Y marginal costs of production of commodities X and Y respectively and P_X and P_Y are prices of commodities X and Y. Therefore, in perfect competition the equilibrium will set at such quantities as

$$MC_X/ MC_Y = P_X/ P_Y$$

We know that the ratio of marginal cost shows the number of units of Y that must be given up in order to expand the output of Y by one unit. This, however, is the definition of marginal rate of transformation and a firm working under perfect competition is as follows:

$MRT_{XY}=MC_X/MC_Y=P_X/P_Y$ ..... (i)

We already have that each consumer's MRS_{XY} is equal to price ratio (P_X/P_Y) under perfect competition:

$MRS_{XY}=P_X/P_Y$ ..... (ii)

As a result, we have $MRS_{XY}=MRT_{XY}$, which satisfy the product substitution condition or allocative efficiency or optimal product mix by operation of price system in a perfectly competitive market.

The above discussion ensures that all first order or marginal conditions required for the attainment of Pareto-optimality is possible to achieve if all the markets are perfectly competitive. This is called fundamental theorem of welfare economics.



CHECK YOUR PROGRESS

Q.4: What is meant by contract curve? (Answer in about 10 words).

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Q.5: What is the full form of MRS and MRTS? (Answer in about 10 words)

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Q.6: What are the assumptions required to full-fill all the marginal conditions of Pareto optimality? (Answer in about 20 words)

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1.4 SECOND THEOREM OF WELFARE ECONOMICS

The second welfare theorem tells us that when indifference curves are convex to their origin, every efficient allocation at any point on the contract curve represents competitive equilibrium depending on the initial allocation of an endowment. At competitive equilibrium a suitable redistribution of endowments can ensure social welfare in an economy.

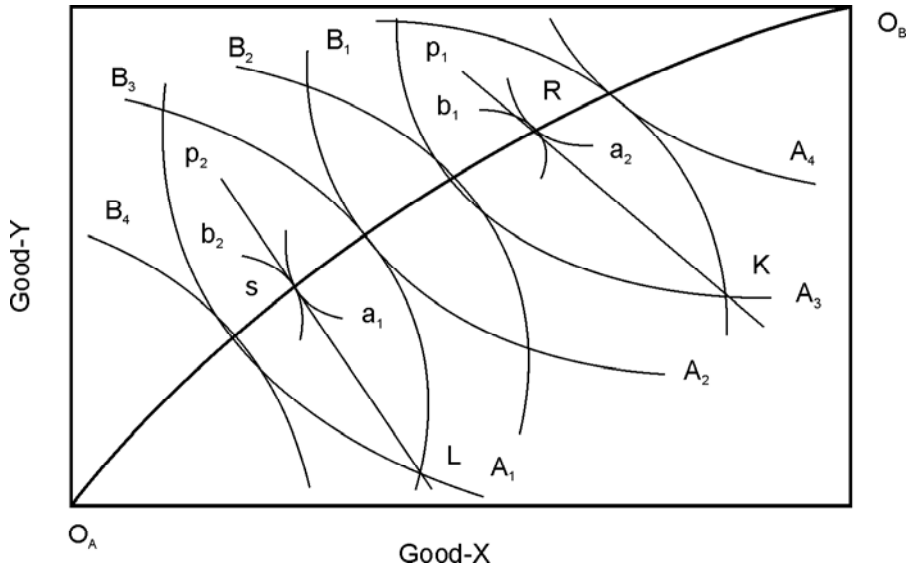


Fig. 1.3: Edgeworth Box Diagram to Prove Second Theorem of Welfare Economics

When preferences are convex, at Pareto efficient allocation, the bundles preferred by person A and bundles preferred by person B are disjoint. Therefore, if both agents have convex preferences, then we can draw a straight line between two sets of preferred bundles that separate one from others. The slope of this line gives us relative prices and any endowment that puts both agents on this line lead to final market equilibrium being the original Pareto efficient allocation. This can be shown in figure 1.3. Suppose the initial distribution between goods X and goods Y for two individuals A and B is at point K. Now due to existence of perfect competition, this can lead to the determination of price ratio of two goods as shown by price line P_1 so that by exchanging the goods X and Y both the individuals A and B can reach at point R. The point R represents higher level of welfare and Pareto optimal because at point R the new indifference curve a_2 of individual A and b_1 of individual B lies on contract curve and within the indifference curve A_3 of individual A and B_1 of individual B. Similarly, if initial distribution of goods that is real income is at point L, then perfect competition can lead to the determination of price ratio of goods as shown by price line P_2 so that by exchanging both the individuals can reach at point S. The point S represents higher level of welfare and Pareto optimal because at point S

the new indifference curve a_1 of individual A and b_2 of individual B lies on contract curve and within the indifference curve A_1 of individual A and B_3 of individual B.



LET US KNOW

Case where second welfare theorem does not hold

In case of non-convexity of preferences, the second welfare theorem does not hold. For example, out of two individuals, individual A has non-convex preferences. Then point like E can never be competitive market equilibrium. E represents Pareto efficient because here $MRS_{XY}^{A}=MRS_{XY}^{B}$ on indifference curve I_A of individual A and I_B of individual B. But at point E there is no set of prices at which both A and B will simultaneously want to consume. Suppose if we assumed that price set as a slope of the common tangent at E. Then individual B would prefer combinations of two goods X and Y at point E where his highest possible indifference is tangent to budget line. A, on the other hand, due to non-convex preferences choose bundle T on higher indifference curve IA_1 instead of bundle E on indifference curve I_A.

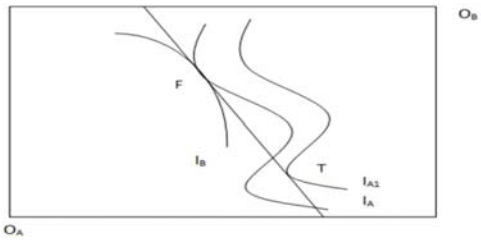


Fig.: 1.4 Non-convex Preferences



CHECK YOUR PROGRESS

Q.7: What is the shape of contract curve in the second theorem of welfare economics?

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1.5 CONCEPT OF SOCIAL OPTIMUM

We know that when demand is equal to supply in perfectly competitive market conditions, there is optimum production and exchange in an economy. The optimum production and exchange from consumers and producers' point of view represents the equality between price (P) and marginal cost (MC). The price is the amount that consumers pay for various units of goods. The price paid by a consumer is equivalent to marginal utility or marginal benefit that the consumers obtain from consuming various amounts of the goods. A consumer obtains consumer surplus when he is willing to pay over and above market price. Thus, the consumer surplus represents the net gain of utility or marginal benefit or well-being obtain from consuming the goods.

As like consumer surplus, a producer at P=MC when optimum production and exchange of goods take place as a producer from their respective marginal cost or supply curve earn producer surplus. The producer surplus is the excess of market price at which producers sell the quantity of a commodity over and above the minimum price at which they would be willing to supply it.

The surplus gained by both the producer and consumer is known as economic surplus. This is one of the most beneficial outcomes of the operation of perfectly competitive market that results in allocative efficiency. This allocative efficiency of perfectly competitive market implies the mutually beneficial exchange between consumers and producers so that both are better off and maximise social welfare due to participation in exchange. Allocative efficiency in perfect competition enables equality between marginal cost of producers and marginal benefit of consumers at a given price. To understand this, consider figure 1.4 of Panel-A, where competitive equilibrium is attained by producing OQ quantity of the good. The OQ output represents optimum output because at OQ output, marginal cost is equal with marginal benefit. If the level of production is below OQ then it will lower total economic surplus tough marginal cost which is lower than

marginal benefit (MC<MB). On the other hand, if the quantity is greater than OQ, i.e. more than optimum, it will lead to loss of total economic surplus and increased marginal cost in comparison to marginal benefit (MC>MB).

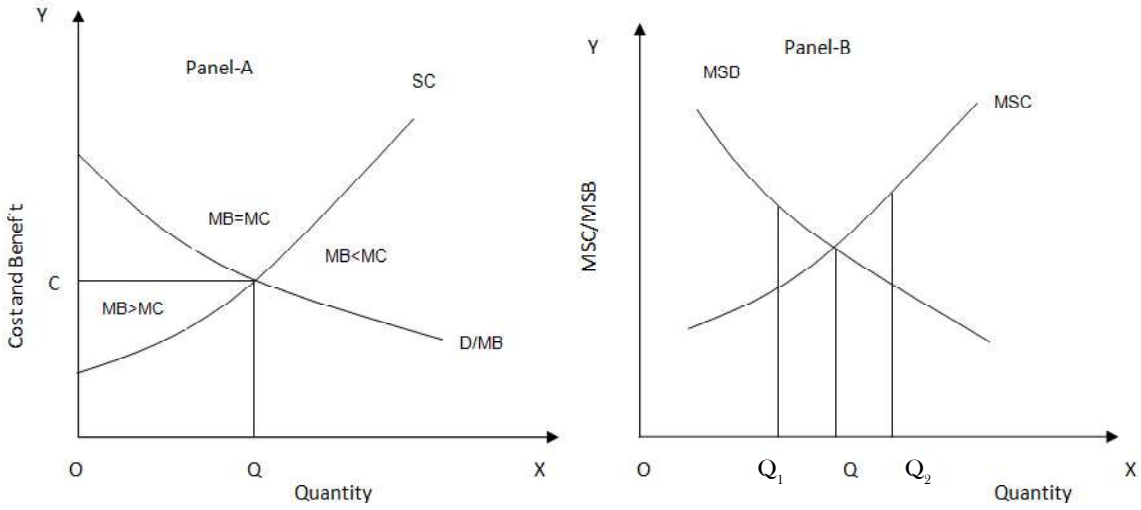


Fig. 1.4: Optimum Quantity

The attainment of social optimal output is simply not only equality between marginal cost and marginal benefit discussed above. Pigou, for the first time, in his book *Wealth and Welfare* discusses that even existence of perfect competition cannot guarantee the achievement of Pareto optimum. There is always presence of externalities. Any product has external effects or externalities. The externalities could be both positive and negative. For optimal distributions of resources in society, both the external costs and benefits as well as internal costs and benefits are taken into account. Social efficiency occurs when Marginal Social Benefit (MSB) is equal with Marginal Social Cost (MSC).

Marginal Social Benefit (MSB) consists of the following:

MPB = Marginal Private Benefit

MEB = Marginal External Benefit

And MSB = MPB + MEB

Also, Marginal Social Cost (MSC) consists of the following:

MPC= Marginal Private Cost

MEC= Marginal External Cost

And MSC = MPC + MEC

For a single level of output in the presence of externalities when MSB = MSC, the socially optimal output is found. In the figure 1.5 of Panel-B, it indicates that as long as MSB > MSC, production should be expanded because benefit exceeds additional cost. Similarly, if MSB < MSC, then production should be decreased. At OQ level of output, social optimum is obtained.

1.6 EQUITY-EFFICIENCY TRADE-OFF

The concept of equity refers to an allocation if no individual prefers another individual's consumption bundle to his own. For example, if individual 'A' prefers B's bundle of goods to his own, it will be termed as A envies B and vice versa. Thus, an equitable allocation means that there is no individual who envies someone else's bundle to his own. The issue of equity in distribution is logically separable from the issue of efficiency in allocation. We already examined that the possibility of different efficient allocations of commodities and resources under perfectly competitive market conditions. A competitive equilibrium leads to a Pareto efficient allocation that may or may not be equitable. As we know any point on the contract curve represents competitive equilibrium depending on the initial allocation. The competitive equilibrium exists in all intermediate points on contract curve, i.e. the second theorem of welfare which enables us to examine that there is a trade-off between equity and efficiency.

The concept of trade-off between equity and efficiency can be discussed with the help of figure 1.5. In this figure AZ curve is the production possibility curve (PPC) which is also called the transformation curve. The slope of AZ curve represents marginal rate of transformation (MRT) between output X and Y. As we know MRT=-dY/dX which implies that to gain small increase in output X is the result of sacrifice of output Y. At each point on the AZ curve the different combinations of X and Y can be distributed between individual A and individual B in various ways. Let us assume that

point E is efficient production on AZ curve in the sense that an increase in the production of good X requires a sacrifice of the good Y. The availability for exchange of quantities of X and Y will be determine by point E.

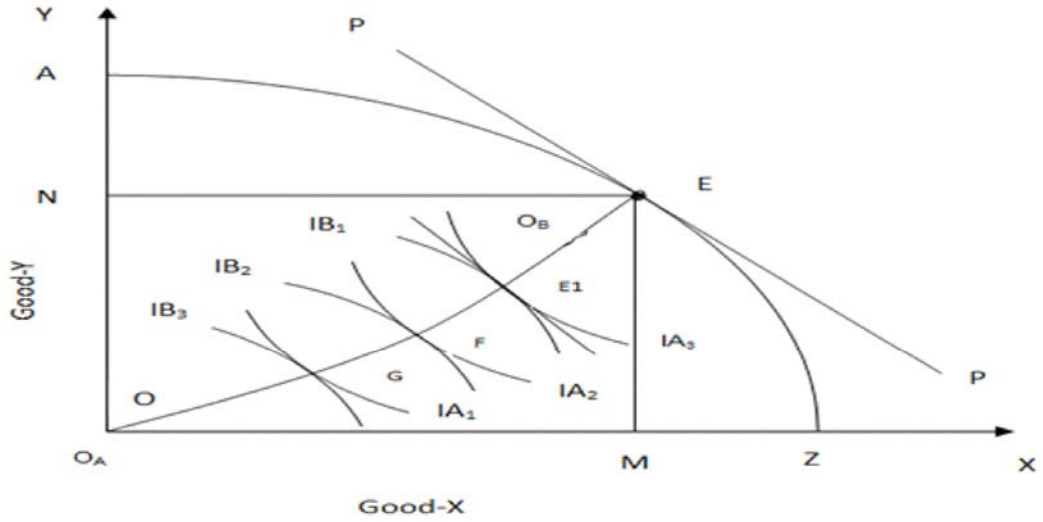


Fig. 1.5: Equity Efficiency Trade-off

Thus, we can construct an Edgeworth Box depicting the exchange possibilities for individuals A and B. At any point along the exchange contract curve, OE shows efficiency in exchange. As for example, the initial allocation gave all X and Y to B. At O_B and B would have no reason to enter into exchange. Similarly, at point O_A individual A in competitive equilibrium consume all goods X and Y. So, in that case individual A has no need to enter into exchange. Since efficient allocations are not necessarily equitable, society must rely to some extent on the government to achieve equity goals by redistributing income or goods among households. Thus, the second welfare theorem says that in order to distribute goods differently the state can tax one consumer on the basis of value of his endowment and transfer the money to others. For example, the revenue generated from progressive taxes are used to finance different public services such as health or medical care so that the poorer section can get benefit. Thus, the second welfare theorem tells us that redistribution need not be in conflict with economic efficiency.

Hence, according to the second welfare theorem lump sum taxation would be non-distortionary. It is required to lump sum suitable redistribution

among individuals to achieve Pareto optimality. However, redistribution of income in any society may discourage work. For example, the impositions of taxes discourage to work more and also negative impacts on consumption and production. In this context, as a policy planner, it is very difficult to make choices between equity goals and efficiency. So, the second welfare theorem ensures that there is usually trade-off between efficiency and equity.



LET US KNOW

We know that the locus of all efficient allocation of resources is Edgeworth contract curve. From this contract curve we can derive utility possibility frontier (UPF) and production possibility frontier (PPF). UPF and PPF represent various combinations of utility where consumption and production efficiency criteria are satisfied. Any point inside UPF or PPF will be inefficient because in that case it will be possible to make some one better off without making someone worse off.

Now, incorporating Bergson Samuelson Social welfare function we can easily understand the concept of social optimum and the trade-off between equity and efficiency.

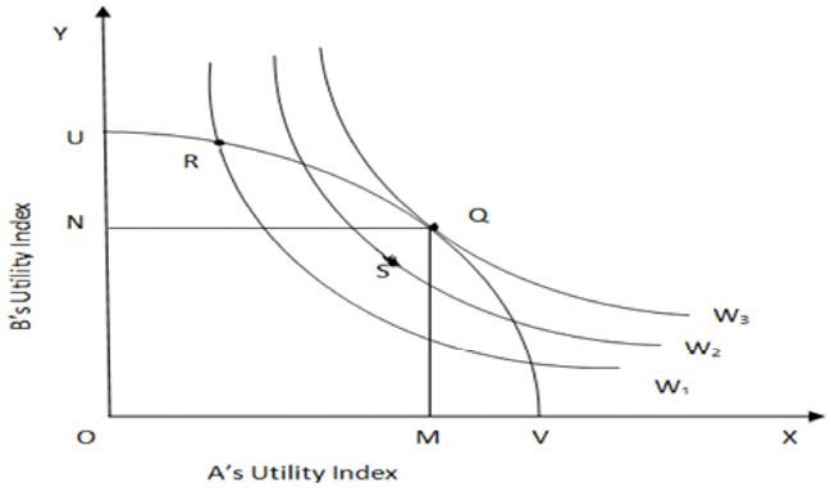


Fig. 1.6: Concept of Social Optimum and Equity Efficiency Trade-off

In the figure 1.6 on utility possibility frontier UV the social indifference curve W_3 of individual A and B tangent at point E. The

point E is known as the point of constrained bliss. Given the factor endowment and technology point E represents unique distribution of goods between both the individuals. In case of production it also represents unique combination of factors employed to produce a particular goods. Thus, the point constrained bliss represents optimum distribution as well as optimum level of production.

Point R of W_1 social indifference curve is lying on utility possibility frontier and representing economic efficiency. Point S of W_2 social indifference curve is economically inefficient because it is inside the utility possibility frontier. But, W_2 is above W_1 social indifference curve and representing more utility. So as per society's point of view point S is equitable distribution but not point R. But, for the sake of social interest sometimes inefficiency in resource allocation is accepted. Thus, we can conclude that there exists trade-off between equity and efficiency.



1.7 LET US SUM UP

- To establish the first theorem of welfare economics we need to sketch a general equilibrium model of an economy. In general equilibrium we assumed that there is efficiency in consumption, production and allocative efficiency. These conditions are known as marginal conditions of Pareto optimality. It is proved that in the presence of perfect competition all the marginal conditions are satisfied. Under the operation of perfect competition, the fulfillment of marginal conditions of Pareto optimality is known as first theorem of welfare economics.
- During the discussion of the first theorem with the help of Edgeworth box diagram, it is noted that indifference curves are convex to the origin and contract curve is concave to the origin. From contract curve we can derive production possibility curve.
- Convex preferences and efficiency on each point of concave contract

curve satisfy the total conditions of Pareto optimality. The contract curve or production possibility curve is concave to the origin due to diminishing returns in production or increasing opportunity cost. Opportunity cost increase because production differs for different goods.

- The competitive equilibrium exists on all intermediate points on contract curve which enables us to examine that there is a trade-off between equity and efficiency. The concept of equity-efficiency trade-off can easily be understood with the help of Bergson Samuelson social welfare function.
- The concept of social optimum indicates not just equality between MB=MC. There is presence of externalities. Thus optimum quantity means MSB=MSC.



1.8 FURTHER READING

1) Ahuja, H.L. (2017). *Advanced Economic Theory, Microeconomic Analysis*. New Delhi: S. Chand.

2) Koutsoyiannis, A. (1979). *Modern Microeconomics*. London: Macmillan Education Limited.

3) Madala, G.S. and Miller, Ellen. (2018). *Microeconomics Theory and Applications*. McGraw Hill Education (India) Pvt. Ltd.

4) Pindyck, Robert and Rubinfeld, Daniel (2019). *Microeconomics.* India: Pearson.



1.9 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q. No. 1: Positive economics deals with cause and effect relationships and normative economics deals with what ought to be, i.e. what is good or bad for society.

Ans to Q. No. 2: The three marginal conditions of Pareto optimality are-

- Efficiency in consumption;
- Efficiency in production and
- Efficiency in allocation.

Ans to Q. No. 3: Pareto optimality is that efficient allocation of resources where it will not be possible to make some members of society better off without making some members of society worse off.

Ans to Q. No. 4: Contract curve is the locus of all efficient combinations of resources in production and efficient distribution of goods in consumption. It is the tangency point between the indifference curves in consumption and iso-quants curves in production. From contract curve we can derive production possibility frontier and utility possibility frontier. Slope of PPF is MRPT and it measures the rate at which economy can transform one good for the other.

Ans to Q. No. 5: The full form of MRS is Marginal Rate of Substitution and the full form of MRTS is Marginal Rate of Technical Substitution.

Ans to Q. No. 6: The assumptions required to fulfill all the marginal conditions of Pareto optimality are-

- All individuals and firms in the economy are price takers;
- An individual trying to maximize utility and producer trying to maximize profits is subject to budget constraint and production constraint and
- Assumption of self-interest in operation of perfect competition.

Ans to Q. No. 7: The shape of contract curve is concave to the origin.



1.10 MODEL QUESTIONS

A) Short Questions (Answer each question in about 150 words).

Q.1: What is meant by economic efficiency? Does there exist economic efficiency and equity?

Q.2: What will happen if the preferences are non-convex in Pareto optimality? Explain.

Long Questions (Answer each question in about 300-500 words).

Q.1: All the three conditions for economic efficiency are satisfied only when perfect competitive markets are in equilibrium. Explain.

Q.2: Discuss the concept of social optimum.

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UNIT 2: MARKETS AND WELFARE-II

UNIT STRUCTURE

2.1 Learning Objectives

2.2 Introduction

2.3 Market Imperfection – Monopoly – Non-Linear Pricing

2.4 Externalities

- 2.4.1 Positive Externality
- 2.4.2 Negative Externality

2.5 Public Goods

2.6 Property Rights and Coase Theorem

2.7 Let Us Sum Up

2.8 Further Reading

2.9 Answers to Check Your Progress

2.10 Model Questions

2.1 LEARNING OBJECTIVES

After going through this unit, you will be able to-

- explain the market failure and its different causes
- know the concepts of public good and externality
- understand the property rights and how we can internalise externality through assigning property rights.

2.2 INTRODUCTION

A market is defined as an institution where consumers and producers of a well-defined commodity interact, either directly or otherwise. The market provides for an in-built mechanism where a complexity of forces acts, leading to an efficient allocation of resources. The buyers and sellers who comprise of the two ends of the market mechanism for exchange, acts with independent motives of utility maximisation and profit maximisation respectively. They further act under their own sets of constraints in decision making. The resultant market model comprises of simultaneously acting

forces of demand and supply where the equilibrium output and prices of the commodity exchanged are determined. However, in the individual pursuit of self-interest, the decisions of the buyers and sellers may fail to achieve the social optimum output. This is market failure. Market failure exists when the said equilibrium is not Pareto efficient. Accordingly, the allocation of resources is not optimal.

Market failure occurs under a variety of causes like asymmetric information, imperfect markets, public goods, externalities, etc. In case of the presence of externalities in production or consumption, goods may be over produced or under produced than the social optimum. Public goods are another instance of market failure for the existence of non-rivalry in consumption and non-excludability. Again, lack of property rights leads to market failure as there is over exploitation of the respective property. Even for common property resources this is true despite the exclusion of non-members, the problem being referred to as the 'Tragedy of Commons'. The present chapter deals at length with some of the causes of market failure.

2.3 MARKET IMPERFECTION – MONOPOLY – NON-LINEAR PRICING

Market imperfections are said to exists when either of the characteristics of a competitive market fails to occur. These as we know are – infinite numbers of buyers and sellers, homogeneous product or services, free entry or exit of firms, perfect information about the product or services and the prices. However, perfect competition is a myth in the real world and what exists is imperfect competition. Market imperfections occur as in monopoly, monopolistic competition, monopsony and oligopoly. Under such market imperfections the price and output differ from the social optimum. The case of monopoly welfare loss is illustrated in the present chapter. Market imperfections in the underdeveloped countries in the form of rigid social structure, immobility of factors, price rigidity ultimately leads to underutilisation of the resources. Market imperfections also occur in the form of differences in the social and private estimates of cost and benefits leading to under or over production of output. Sometimes, there occurs

incomplete information about the exchanged goods that makes optimum production or consumption difficult.

A monopoly is an imperfect market that restricts output in an attempt to maximise profit. A monopoly market form is another instance showing market failure. A monopolist produces too little and charges a higher price for his output leading to net welfare loss to the society which is in contrast to a competitive market. This is discussed with the help of the diagram 2.1. In the diagram, MR is the monopolist marginal revenue curve, AR is his average revenue curve and MC the marginal cost curve.

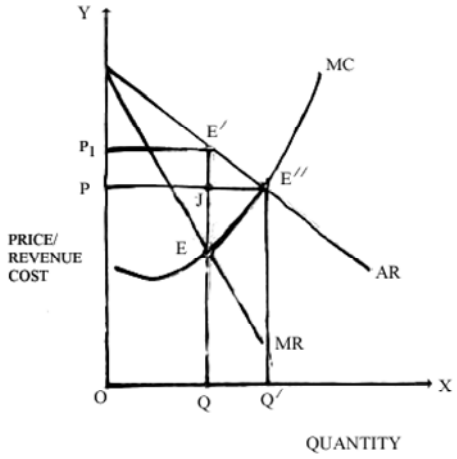


Fig. 2.1 : Price-output determination in monopoly

The profit-maximising equilibrium is at E where MC=MR. The monopolist determines its price at E′′. The competitive price would have been however determined at E′′. We, therefore, find that compared to competitive price the monopoly price is high. Again, the competitive output would be OQ′ but the monopolist produces only OQ. There is reduced consumer surplus to the extent of the area $P_1E'E''P$. But part of this is pocketed by the monopolist as higher prices to the extent of $P_1E'JP$. The area of the two triangles " E′E′′J and " E′′JE are the dead weight loss to the society arising out of the monopoly. The existence of deadweight loss is an indication of market failure. Monopoly form of market is further criticised of not remaining competitive over time in the absence of competition and for charging higher prices even beyond the cost in an attempt to maximise profits.

A monopolist in his urge to increase profit, discriminates the price charged. Price discrimination is of various types. In the first-degree price discrimination, a nomenclature followed by Prof. Pigou, the demand curve coincides with the marginal revenue curve. So, this implies that the competitive output is similar to the monopoly output. This perfect price discrimination is believed to lead to an efficient allocation of resources. The second-degree price discrimination or non-linear pricing is said to exist when the monopolist charges different prices for different ranges of output. For example, magazine subscribers are to pay more for the first year of subscription and lower prices for further years of subscription. Price discrimination is sometimes preferred from the society's point of view as in case of some goods production would not have been possible and the fulfilment of the want of some goods by the lower income class would not have been possible if higher price were not charged from those who could pay. Price discrimination is said to increase welfare if the total output produced by the monopolist increases in the urge to earn more profits. If the total output however, does not rise, price discrimination leads to loss in welfare as there is misallocation of resources through restriction in output.



CHECK YOUR PROGRESS

Q.1: Choose the correct one

Market failure occurs when there is-

a) Market imperfections b) Externality in production

c) None of these d) Both a and b

.....

Q.2: Is the monopoly output socially optimum? Why? (Answer in about 30 words)

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Q.3: What is nonlinear pricing? (Answer in about 10 words)

.....

.....

2.4 EXTERNALITIES

Externalities are the result of economic behaviour of individuals or firms that affects the production or consumption of a third party. For example, an industry may be dumping waste to a river that raises the cost of fishing for fisherman living on the river bank. In any activity of production or consumption if the private and social cost and benefit differ then it is a case for externalities. A.C. Pigou in his book *Wealth and Welfare (1912)* first systematically dealt with this concept of externalities. Externalities are called spill over effects or neighbourhood effects which do not get reflected in the price of the goods. Externalities may be either benefitting to the third party or may cause harm. This spill over effects do not enter the respective private marginal cost or benefits. Accordingly, externality forms another instance of market failure causing mis-allocation of resources and that production and consumption differs from the optimum. They are an instance of market imperfections where price is not offered for benefits or damage caused to others.

Externalities are of two types – positive and negative. The beneficial spill over effects are termed as positive externalities and negative externalities. Depending upon whether the externalities pertain to production or consumption, we differentiate between the following four types of externalities.

- **Positive externality in consumption:** The consumption of a good which enhances the welfare of others is called a positive externality in consumption.
- **Negative externality in consumption:** When the consumption activity of one reduces the welfare of others it is called a negative externality in consumption.
- **Positive externality in production:** Sometimes some production activity renders benefits to other firms or individuals. These benefits accrue to the later in the form of reduced average costs of production and is called positive externality in production.

- **Negative externality in production:** When the production activity affects others adversely it is a case of negative externality in production.

As discussed above, externalities occur due to the difference between private and social cost and benefits. A discussion on externalities would be incomplete without understanding this discrepancy and analysing how this can be met to yield optimum levels of production or consumption. This is included in the following sections. One way to internalise the externalities is by levying taxes and subsidies to yield socially optimum output. Now, we know, that the marginal social cost or benefit is derived by adding up the private and external cost or benefit. In the following discussion the notations MSC, MPC, MEC, MSB, MPB, and MEB implies the following.

MSC-Marginal Social Cost

MPC-Marginal Private Cost

MEC-Marginal External Cost

MSB- Marginal Social Benefit

MPB- Marginal Private Benefit

MEB- Marginal External Benefit

Thus, we have; MSC= MPC+MEC; and MSB=MPB+MEB.

2.4.1 Positive Externality

The term positive here implies profitable, beneficial or pleasant spill over effects of any activity for which no price is appropriated. It is an effect external to the actual producer (or consumer) which benefits other producers (or consumers).

- ***Positive externality in production:*** An example of this type of externality is that of a bee hive near an apple orchard which helps increase production of apples as bees help enhanced pollination. As there are external benefits, the marginal social cost is less than the marginal private cost, i.e. MSC<MPC. Accordingly, if market forces are left alone the output will be less than what is socially optimal. This is shown in diagram 2.2.

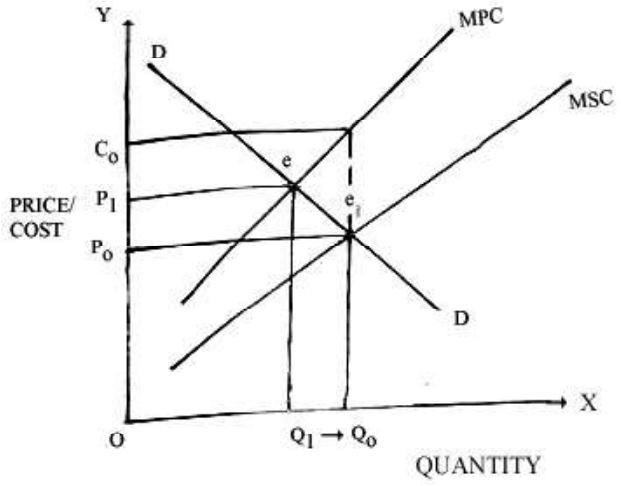


Fig. 2.2 Positive Externality in Production

The demand curve DD shows the marginal social (and private) benefits as there are no externalities in consumption, which cuts the MPC curve at e. OQ_1 output is produced corresponding to e. But the social optimum output is OQ_0 which corresponds to the intersection of the demand curve to the MSC curve at e_1. At OQ_0 level of output producers get P_0 price but their cost is C_0. One way to raise the production to OQ_0 is to pay the producers a subsidy for Q_1Q_0 to the extent C_0-P_0. It is also suggested that the beneficiaries of the externality could be made to pay for raising production to the optimal level.

- ***Positive externality in consumption:*** An example for this may be that, if a person gets vaccinated for some contagious disease, he/she causes welfare to the entire neighbourhood by preventing the spread of the disease. Assuming that there are no externalities in production, the market supply curve reflects both the marginal social or private cost. However, due to existence of externalities in consumption the marginal social benefit is greater than the demand (or marginal private benefit).

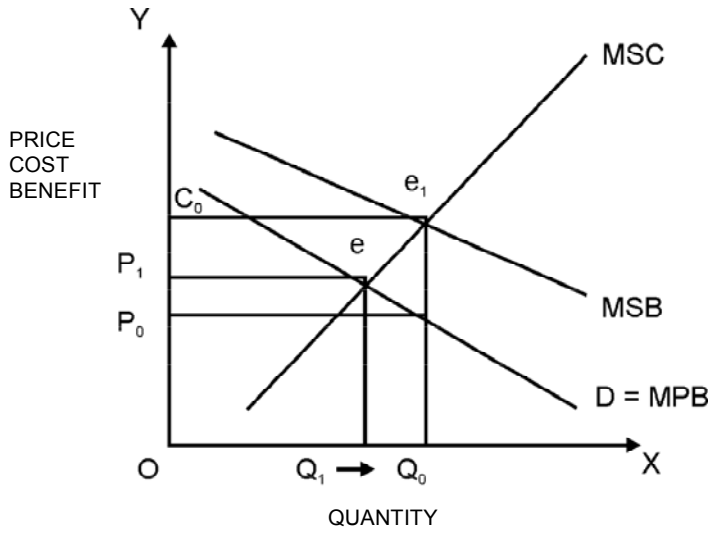


Fig. 2.3 Positive externality in consumption

Accordingly, in figure 2.3 we have MSB curve lying above the MPB or demand curve. The output produced is only OQ_1 as against the social optimum OQ_0. If OQ_0 is produced the cost is C_0 which is to be paid to the producers. But the market price will be P_0 which will be paid by the consumers. Therefore the consumers' needs to be subsidised by the amount C_0P_0 to raise their consumption.

2.4.2 Negative Externality

Negative externalities are those activities that reduces the welfare or harms individuals, firms or the environment at large for which the responsible party incurs nothing. For example, industries causing air or water pollution, noise pollution or causing harmful radiations, etc.

- ***Negative externality in production:*** An example of this is an industry dumping untreated waste to a river that raises the cost of fishing for nearby fisherman. As in figure 2.4, the MSC curve lies above the MPC implying that due to negative externalities the social cost of production is greater than the estimated private cost. Thus, the market production OQ_1 is more than the social optimum OQ_o. Thus, there is an excess production to the extent

Q_1Q_0. The private cost C_0 against the equilibrium is less than the optimum price of P_0 where a social optimum output OQ_o is produced. In this case, the producer of the negative externality can be taxed to the extent P_0C_0.

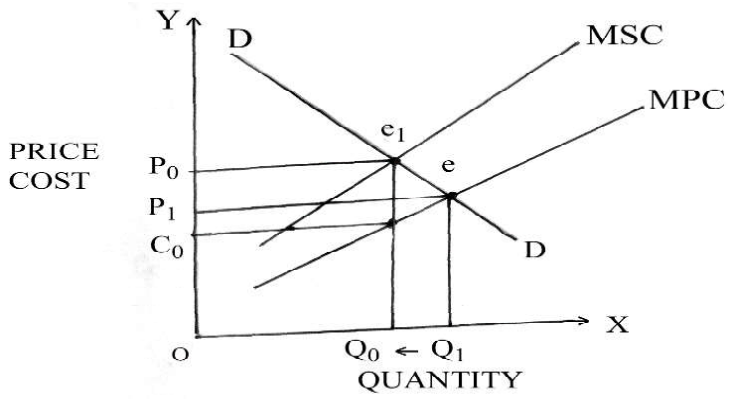


Fig. 2.4 Negative externality in production

- ***Negative externality in consumption*:** An example of this is playing of loud music during some festival in a neighbourhood by some, that cause annoyance to others. In case of negative externalities in consumption, the marginal social benefit is less than the marginal private benefit. The optimum output is OQ_o which is less than the OQ_1. This is shown in figure 2.5. The optimum price is P_0 which is greater than the cost C_0. The difference P_0C_0 can be raised as tax from the consumers which may be used to compensate the victims of the damage.

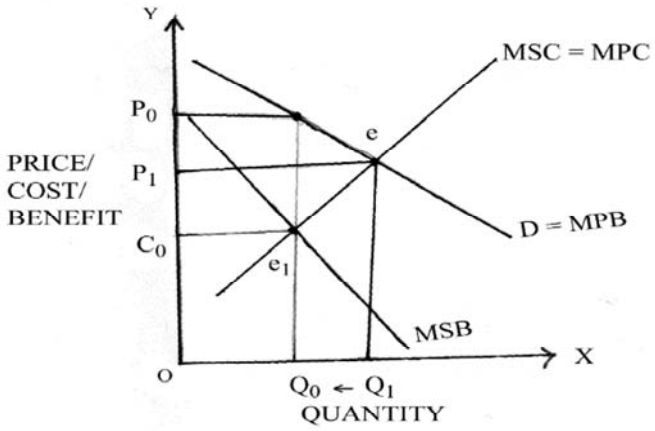


Fig. 2.5 Negative externality in consumption



CHECK YOUR PROGRESS

Q.4: Name one production activity that involves an external benefit.

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Q.5: Name one consumption activity that involves an external cost.

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2.5 PUBLIC GOODS

Public goods are another instance of the aforesaid market failure. Public goods are defined as goods characterised by non-excludability and non-rivalry. Any good will be excludable if it is possible to exclude individuals from using it. In case of public goods, it is either very expensive or technically impossible to exclude anyone from benefitting from the good. A classic example cited for non-excludability is that of a lighthouse. No ships can be stopped from using the services of a lighthouse. But we can certainly exclude people from watching a movie if they do not purchase the tickets. On the other hand, rivalry exist if the consumption of a good by one reduces the availability for others. However, for public goods consumption does not reduce its availability for others. For example, while a cake will be unavailable to others if consumed by some, the lighthouse utility will be the same for all ships. The following table 2.1 showing the classification of goods according to excludability and rivalry elicits the difference between public goods and private goods. This classification is irrespective of the nature of the physical attributes of goods.

A pure public good is both non-rival and non-excludable. For example, the lighthouse, national defence, environmental quality, etc. A pure private good is on the contrary, excludable and rival. Goods may be excludable yet non-rival. Public roads though are pure public goods yet toll roads can exclude non payers. Again, goods can be rival but non-excludable, example, forests, oil fields, ground water, etc.

Non-excludability, non-rivalry (Pure public good)	**Non-excludability, rivalry**
Excludability, non-rivalry	**Excludability, rivalry (Pure private good)**

Table 2. 1: Classification of goods on the basis of rivalry and excludability

Public goods do not provide any incentives to the private producers as they cannot be priced for being non-excludable. Again, the marginal cost of producing the goods being 0, it leads to the free rider problem. That is, once such good is provided no one wants to pay for it. The market failure in case of public goods leads economists to suggest that they be provided by the government.



CHECK YOUR PROGRESS

Q.6: Choose the correct one

Public goods are characterised by-

a. Rivalry b. Divisibility

c. Both rivalry and externality d. Externality

Q.7: Define public goods. And write its characteristics. (Answer in about 50 words)

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2.6 PROPERTY RIGHTS AND COASE THEOREM

Ronald H. Coase argued that in the presence of externalities it is not necessary that the government should intervene to provide the optimum output with subsidies or taxes. There is an administrative cost involved with such measures, adopted by the government and if this cost is higher than the social benefits from such government intervention then the government intervention in the same will not raise welfare. Coase, hence

suggested a private costless bargaining approach instead. He put forward the idea of assigning 'property rights' in this context.

Ronald Coase theorem can precisely be stated as that if property rights to a contested resource is assigned to any one of the concerned parties then negotiations between the two parties will lead to the attainment of a socially optimum equilibrium and a socially optimum level of output will be produced. This according to Coase will then be more effective way to internalise the negative externalities than government imposition of taxes or subsidies. The Coase theorem is based on the following assumptions.

- The numbers of affected parties are few.
- There is perfect information.
- There are no transaction costs.
- There exists a costless way to enforce agreements.
- The property rights must be well-defined.

Let us now consider the case of a river. Suppose there are two parties involved – first, an industry which dumps untreated waste into the river and second, the fishermen of a nearby village who depends on the river for their livelihood. For the Coase theorem it is necessary that property rights to the contested resource, here the river, be clearly defined. It is irrelevant whether the property rights are assigned to the polluter or the victims of the damage. Coase argued that the negotiations between the two parties will lead to socially optimum output and the externality will be internalised.

Bargaining with property rights to the fishermen: Let us assume that the property rights are assigned to the fishers. Here, the industry owner has to compensate the fishers for polluting the river. In fig. 2.6 we discuss how the externality gets internalised. The marginal social benefit (MSB) is here equal to the marginal private benefit (MPB) as there is no externality of consumption. But due to negative externality of production the marginal social cost curve (MSC) is above the marginal private cost (MPC) estimated by the industry.

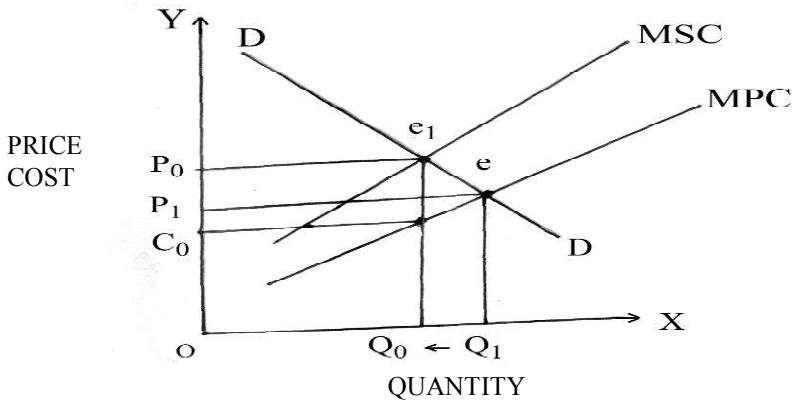


Fig. 2.6 Assigning property rights to victim(s)

As in the above figure 2.6 Q_0 is the socially optimum output but the industry produces Q_1. Now if the fishermen have the right to the river then the industry has to compensate for the pollution it causes. This compensation would be MSC-MPC per unit of pollution caused. Thus, including the compensation amount in estimating the cost the industry no longer considers MPC but MSC curve. Thus, the optimal output Q_0 is produced.

Bargaining with property rights to the industry: On the other hand, if the property rights were to be assigned to the industry then the fishermen have to bribe the industry to lessen their production and hence the pollution level. The idea here is not to consider pollution as an ideal but to explain the role of assigning property rights to either of the parties. The case is illustrated with figure 2.7 again.

Here, the industry has the right to dump waste to the river. Q_0 is the social optimum output. However, the industry will produce only the profit maximising output of Q_1. The fishermen need to bribe the industry to the extent MSC-MPC per unit of output produced. The industry will include this compensation from them and their cost curve will no longer be MPC but MSC (i.e. the opportunity cost of the increased production is included). Thus, Q_0 of output will be produced.

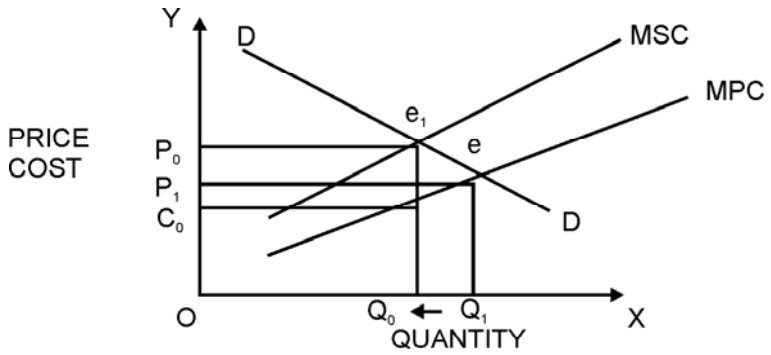


Fig. 2.7 Assigning property rights to polluter

The Coase theorem is based on some very restricting assumptions. Firstly, in reality the number of affected parties in either side of the market is innumerable. In our example the number of industries located near the river may be very high and also the beneficiaries of the resource are also usually high. Here it will be not possible to assign property rights. Moreover, parties will not be able to arrive at any consensus after negotiations. The assumptions of no transaction cost and income effects are also restrictive.



CHECK YOUR PROGRESS

Q.8: Choose the correct one

Coase theorem associated with-

a) Pricing of public goods

b) Solution to the problem of externalities c) Both a and c

d) None of the above



2.7 LET US SUM UP

- Market failure occurs when the freely functioning market mechanism fails to attain optimum output and their results in mis-allocation of resources.
- Market imperfections, divergence between social and private cost and benefit, non-rivalry, excludability, are the various instances where market failure occurs.
- Market imperfections leads to over or under production of the output and prices differ from the optimum level.

- Socially optimal output occurs at the point where social cost and benefits are equal. If production and consumption involve externality then the output determined is not optimum.
- For internalising the externality government intervention through taxes and subsidies becomes necessary.
- Public goods are characterised by the properties of non-excludability and non-rivalry.
- Coase argued that presence of externalities does not necessarily imply government intervention. The same can be dealt through mere assigning of property rights. It is however insignificant as to whom the property rights are assigned.



2.8 FURTHER READING

1) Madalla, G.S. Miller, E. (2018). *Microeconomics, theory and Applications*. McGraw Hill.

2) Dwivedi, D.N. (2008). *Microeconomics, theory and Applications. Pearson Education.*



2.9 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q. No. 1: d)

Ans to Q. No. 2: No, monopoly output is not socially optimum. A monopolist produces at the point where his marginal cost equals marginal revenue. Thus, his output is less than the social optimum.

Ans to Q. No. 3: Nonlinear pricing or second-degree price determination means charging different prices for different quantities of the good.

Ans to Q. No. 4: When massive plantation projects like timber, etc. enhances social fertility which helps farmers in agriculture.

Ans to Q. No. 5: Playing of loud music in festivals creates disturbances for neighbours.

Ans to Q. No. 6: c)

Ans to Q. No. 7: Public goods are those goods that are characterised by non-rivalry and externality. For example, a light house. Non-rivalry means that a good do not reduce in availability to others when consumed by some. Non excludability means any good cannot be prevented from being used by anybody or it is very expensive to do so.

Ans to Q. No. 8: b)



2.10 MODEL QUESTIONS

A) Short Questions (Answer each question in about 150 words).

Q.1: What is meant by non-rivalry?

Q.2: What is meant by excludability?

Q.3: State Coase theorem.

Q.4: Cite an example of non-excludable good.

Q.5: Mention one way to internalise externality.

B) Essay-type Questions (Answer each question in about 300-500 words)

Q.1: What is meant by market failure? Discuss its various causes.

Q.2: What is meant by externality? Discuss its various types.

Q.3: Explain with an example how we can internalise the positive externality in consumption through giving subsidies.

Q.4: Give the assumptions of Coase theorem.

Q.5: Discuss how property rights allocated to the polluter help in achieving a socially optimal output.

*** ***** ***

UNIT 3: GAME THEORY

UNIT STRUCTURE

3.1 Learning Objectives

3.2 Introduction

3.3 Some Preliminary Definitions and Terminologies

3.4 Types of Game

3.5 Two Person Zero Sum Game

3.5.1 The Maximin Minimax Principle

3.6 The Prisoner's Dilemma Game

3.7 Nash Equilibrium

3.8 Saddle Point

3.8.1 Rules for Determining a Saddle Point

3.9 Pure Strategy and Mixed Strategy

3.10 Dominant Strategy

3.10.1 Rules of Dominance Principle

3.11 Game Without Saddle Point (Mixed Strategy)

3.11.1 Algebric Method

3.11.2 Graphical Method

3.12 Limitation of Game Theory

3.13 Let Us sum Up

3.14 Further Reading

3.15 Answers To Check Your Progress

3.16 Model Questions

3.1 LEARNING OBJECTIVES

After going through this unit, you will be able to-

- understand the basic ideas of game theory
- apply the various concepts of game theory to solve the problems occur frequently within economics, as well as in the rest of the world
- obtain saddle point with pure and mixed strategies.

3.2 INTRODUCTION

Game theory offers many insights into oligopolistic interdependence and strategic behaviour of the firms. The interdependence of firms in oligopolistic markets and the inherent uncertainty about competitors' reactions to any course of action adopted by a firm cannot be analysed effectively by the traditional tools of economic theory.

The first systematic attempt of application of theory of games to economics was Van Neumann's and Morgenstern's 'Theory of Games and Economic Behaviour' published in 1944. Since that time numerous economists have developed theories of oligopolistic behaviour based on the theory of games.

In general, game theory is concerned with the choice of an optimal strategy in conflict situations. Specifically, game theory can help an oligopolist choose the course of action (e.g., the best price to charge) that maximises its benefits or profits after considering all possible reactions of its competitors. Game theory is a tool that has been created to help us study decision-making in situations in which there is a mixture of conflict and cooperation.

3.3 SOME PRELIMINARY DEFINITIONS AND TERMINOLOGIES

Before going into the technical aspects of game theory, let us first learn about the terminologies used in this theory.

- **Game :** A game is a competitive activity involving skill, chance and endurance on the part of two or more persons, who play according to rules, regulations and conventions known to all players. A game is described in terms of the players, the rules of the game, the payoffs of the game, and the information that players have about the details of the game. These elements are common to all conflict situations, and they constitute the fundamental characteristics of a game.
- **Player :** The decision-making unit is called a player, who may be a single person, a firm, or an organisation.

- **Strategy :** A strategy is a complete specification of what actions a player will take under each contingency in the playing of a game. In other words, a strategy is one participant's plan of action adopted in the light of his own belief about the reactions of his rival players.Strategy may be of two types-pure strategy and mixed strategy.(This is explained in details in the later part of the unit).

- **Payoff :** The payoff is the outcome or consequence of each strategy. The payoff of a stratogy is the payment it brings to the participant for any given counter strategy of the rival (s). It is usually expressed in terms of the profits or losses of the firm that we are examining as a result of the strategies adopted by the firm and the responses of the rivals to those strategies.

- **Payoff matrix :** A table that gives the payoffs from all the strategies open to the firm and the rival's responses is called the payoff matrix. For example, let us assume that there are two firms in a industry. Firm 1 has to choose among five strategies (A_1, A_2, ..., A_5) and the Firm 2 can react by adopting any one of the six strategies open to it (B_1, B_2, ... , B_6). Thus to each strategy of firm I, there are six possible counter strategies of firm II, and similarly to each strategy of firm II there are five counter strategy of the rival firm I. Thus the payoff matrix of each firm will include 5x6 = 30 payoffs, corresponding to the results of each possible combination of strategies selected by both rivals.

  The payoff matrix can be written as–

  Firm 1's payoff matrix

		Firm II					
		B_1	B_2	B_3	B_4	B_5	B_6
Firm I	A_1	π_{11}	π_{12}	π_{13}	π_{14}	π_{15}	π_{16}
	A_2	π_{21}	π_{22}	π_{23}	π_{24}	π_{25}	π_{26}
	A_3	π_{31}	π_{32}	π_{33}	π_{34}	π_{35}	π_{36}
	A_4	π_{41}	π_{42}	π_{43}	π_{44}	π_{45}	π_{46}
	A_5	π_{51}	π_{52}	π_{53}	π_{54}	π_{55}	π_{56}

Where π_{ij} (i = 1, 2, ... , 5; j = 1, 2, ..., 6) is the net gain or payoff to firm I when it adopts ith strategy and firm II adopts j^{th} strategy.

3.4 TYPES OF GAME

A game can be classified on the basis of the number of players, the number of strategies, the nature of payoff function and the nature of negotiation.

- Games classified by the number of players in game theory are termed as two-person game, if there are two players; three-person game, if there are three players and n-person game, when there are n-players.
- Games classified by number of strategies are called finite games and infinite games, depending on the strategies available to each of the players in the game. The game is finite if the strategies available to each player is limited and in case of infinite games, choice of strategies available to each player is unlimited.
- Games classified by the nature of the payoff function are called constant sum game, zero-sum game and non zero-sum game. Costant sum game is a situation or interaction in game theory where the sum of the payoffs of all players always adds up to the some constant figure for any particular outcome. Thus, one player's gain must be offest by another one or more players' loss. Zero sum game is a type of constant sum game. In the zero sum game, the payoff to all the players sum up to zero. On the other hand, non-zero sum games are games in which the sum of gains and losses by the players are sometimes more or less than what they began with.
- The games classified by the nature of negotiations are called cooperative games or non-cooperative games. A game is called cooperative if the players can negotiate binding contracts that allow them to implement and to enforce joint strategies. By contrast, in a non-cooperative game, the negotiation or enforcement of a binding contract is not possible.

3.5 TWO PERSON ZERO SUM GAME

The simplest and most highly developed part of game theory is the two-person zero-sum game. It is a special type of zero sum game which involves only two players, and where the gains made by one equals the loss incurred by the other. In general, if firm X has m-strategies and firm Y has n strategies, the payoff matrix will be-

$$\begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & ..... & \pi_{1n} \\ \pi_{21} & \pi_{22} & \pi_{23} & ..... & \pi_{2n} \\ \pi_{31} & \pi_{32} & \pi_{33} & ..... & \pi_{3n} \\ ..... & ..... & ..... & \pi_{ij} & .... \\ \pi_{m1} & \pi_{m2} & \pi_{m3} & ..... & \pi_{mn} \end{bmatrix}$$

Where, π_{ij} is X's net gain if X emplays the ith strategy and Y employs the j^{th} strategy. Since, the game is zero sum, the corresponding payoff of the firm Y is - π_{ij} .

The game will be a two person zero sum game if

$$\pi_{ij}^{X} + \pi_{ij}^{Y} = 0$$

Let us take an example where there exists two firms– firm X and firm Y. Firm X has choice of two strategies ($X_1$and X_2) and firm Y has a choice of three strategies (Y_1, Y_2 and Y_3) . The payoff matrix is given as –

Firm X's strategies

	Firm Y's strategies		
	Y_1	Y_2	Y_3
X_1	(20, – 20)	(10, – 10)	(40, – 40)
X_2	(–10, 10)	(0, 0)	(60, – 60)

If X employs his first strategy and Y employs his third, X's profit is 40, and Y's is - 40. If x employs his second strategy and Y employs his first, X's profit is -10 and Y's is 10.

There are two key assumptions about the behaviour of the two players in the Zero Sum Game. The first is that both the players are rational. The second is that both players are greedy, which means that they choose their strategies in their own interest to promote their own gain.

3.5.1 The maximin minimax principle

The maximin minimax principle is used for the selection of optimal strategy by the two player.

The duopolist's decision problem consists of choosing an optimal strategy. X desires the outcome (40) and Y wishes the outcome (–10). But the final outcome depends upon the strategies of both duopolists, and neither has the power to enforce his desires. If X selects his first strategy, Y might select his 2nd and hence the outcome would be 10 to X rather than 40. If Y selects his first strategy, X might select his first, and the outcome would be 20 to X rather than –10. The point which needs emphasis is that largest possible guaranteed payoff results from choosing a strategy that maximises the payoff only under the assumption that the opponent reveals his own strategy in advance.

The theory of games postulates behaviour patterns which allow the determination of equilibrium in these situations. X fears that Y might discover his choice of strategy and desires to 'play it safe'. That is X always assumes that whatever row is choosen, Y will choose the column maximising the returns to Y and there by minimise the returns to X. Thus, if X selects his i^{th} strategy, his minimum profit, and hence Y's maximum, is given by the smallest element in the ith row of the profit matrix :

$$\min_j \pi_{ij}$$

X desires to maximise his minimum expected profit. Therefore, X selects the strategy i for which $\min_j \pi_{ij}$ is the largest. X's expected payoff is therefore.

$$\max_i \min_j \pi_{ij} \quad \text{—— (1)}$$

In any circumtances, X's expected payoff cannot be smaller than eq^n (short form of equation) (i), it may be larger.

i.e. $\pi_{ij} \geq \max_i \min_j \pi_{ij}$

Similarly, Y also follows the same principle of playing safe. If Y employs his jth strategy, he fears that X may employ the strategy corresponding to the largest element in the jth column of the payoff matrix :

$$\min_{i} \pi_{ij}$$

Therefore, Y select the strategy j for which $\max_{i} \pi_{ij}$ is the smallest, and his expected payoff is given by–

$$\min_{j} \max_{i} \pi_{ij} \quad \text{—— (2)}$$

Where $\pi_{ij} \leq \min_{j} \max_{i} \pi_{ij}$

The decisions of the duopolists are consistent and equilibrium is achieved if

$$\max_{i} \min_{j} \pi_{ij} = \min_{j} \max_{i} \pi_{ij} = \pi_{ij}$$

In the above example

	Y_1	Y_2	Y_3	Row min	Maximin
X^1	(20, –20)	(10, –10)	(40, –40)	10	10
X^2	(–10, 10)	(0, 0)	(60, –60)	0	
Column max	20	10	60		
Minimax		10			

Hence in this game

max.min π_{ij} = min.max π_{ij}=10

The value of the game is 10 to firms X, which is also the saddle point.

3.6 THE PRISONER'S DILEMMA GAME

The firms working in oligopolistic markets make decisions in the face of uncertainty about how their rivals will react to their moves. The prisoner's dilemma is a standard example of a game analysed in game theory that shows why two completely rational individuals might not cooperate, even if it appears that it is in their best interests to do so. Model of prisoner's dilemma explains how rivals behaving selfishly act contrary to their mutual

or common interest. Let us explain prisoner's dilemma with an example which helps in understanding the behaviours of firms faced with uncertainity of their rivals' action.

Two members of a criminal gang are arrested for committing a certain crime. However, the evidence is not adequate to charge them with the crime unless either or both confess. Each prisoner is interrogeted in isolation from his companion with no means of communication with the other. It is promised to each one that no punishment will be given to him if he confesses; but a heavy punishment of going to jail for say, 20 yrs if he does not confess. Each prisoner is given the opportunity either to betray the other by testifying that the other committed the crime, or to cooperate with the other by remaining silent. The possible outcomes are :

- If neither confesses, they both will be set free
- If both confesses, they face reduced sentences for say 10 yrs.
- If one confesses and the other does not, the one who confesses will be set free while the one who does not confess will face the maximum jail sentence say, for 20 yrs and vice versa.

The payoff matrix of this problem can be describe in the form of the following table :

		Prisoner's B's strategy	
		Confess	Not to confess
Prisoner's A's strategy	Confess	(10, 10)	(0, 20)
	Not to confess	(20, 0)	(0, 0)

Given the lack of communication between the players and the uncertainity over the loyalty of the other prisoner, each prisoner prefer to adopt the strategy 'confess', so that each of them get 10 years of imprisonment; clearly a worse of situations as compared with the adoption of not to confess strategy by both prisoners. If they could rely on each other and had not confessed, they would have both been better off. The irony of prisoner's dilemma is that when each of two players acts selfishly and does not cooperate with the other (i.e., when confesses), they do worse than when they act unselfishly and cooperate together (i.e. when they are not confesses). Thus, each player following the rule 'expect the worst from

the other,' confesses and find himself in a worse situation than both of them needed to be in. The individually rational behaviour led to the inferior situation for both.

The concept of prisoner's dilemma was developed by Merrill Flood and Melvin Dresher and was formalised by Albert W. Tucker.

3.7 NASH EQUILIBRIUM

Nash equilibrium is named after its inventor, John Nash, an American mathematician. It is considered one of the most important concepts of game theory, which attempts to determine mathematically and logically the actions that participants of a game should take to secure the best outcomes for themselves.

In game theory, the nash equilibrium is a concept involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players and no player has anything to gain by changing only his own strategy unilaterally. In the nash equilibrium, each player's strategy is optimal when considering the decisions of other players. A nash equilibrium exists when there is no unilateral profitable deviation by any of the players involved. In other words, no player in the game would take a different action as long as every player remains the same. If each player has chosen a strategy, and no player can benefit by changing strategies while the other players keep theirs unchanged, then the current set of strategy choices and their corresponding payoffs constitutes a nash equilibrium.

The concept of nash equilibrium can be explain with the following payoff matrix :

Payoff table of player A

Player A's strategy	Player B'S strategy	
	Advertise	Don't Advertise
Advertise	4,3	5,1
Don't Advertise	2,5	6,2

The Nash equilibrium is a situation in which each player chooses an optimal strategy, given the strategy chosen by the other player. In the above

example, the high advertising strategy for firm A and firm B is the Nash equilibrium, because given the firm B chooses its strategy of advertising, the optimal strategy for firm A is also to advertise. Similarly, if B believes that A will choose advertise then B will also choose advertise which is his best response. The belief that company A and B will adopt (advertise, advertise) is self conformatory. Thus it is in their best interest to adopt this strategy profile until the other player changes his strategy.

Thus (advertise, advertise) on (4, 3) is a Nash equilibrium.

3.8 SADDLE POINT

A saddle point of a payoff matrix is that position where the maximum of row minimum co-incides the minimum of column maximum. The payoff at the saddle point is called the value of the game and is obviously equals to the maximin and minimax value of the game.

3.8.1 Rules for Determining a Saddle Point

Step 1 : Find out the row minima of each row and mark them with a circle.

Step 2 : Select the column maxima of each column and mark them with a square.

Step 3 : Find the maximin and minimax and if they are equal then the game has a saddle point. The value of the maximin and minimax is the value of the game.

We can also find out the saddle point by looking at the payoff matrix. If the element in the payoff matrix marked both by a circle and a square appears then, the position of that element is the saddle point of the payoff matrix.

Example 1: Find out the saddle point solution of the following games

(a)

		Player B			
		B_1	B_2	B_3	B_4
	A_1	48	88	16	23
Player A	A_2	25	3	7	93
	A_3	65	32	10	20

Solution: Player B

	B_1	B_2	B_3	B_4	Row minima	Maximin
A_1	48	[88]	(16)	23	16	16
A_2	25	(3)	7	[93]	3	
A_3	[65]	32	(10)	20	10	
Column maxima	65	88	16	93		
minimax			16			

Here, maximin = minimax

$\therefore$ We have a saddle point. The saddle point solution is $\mathbf{A_1B_3}$ and value of the game is 16. Optimal strategy for player A and player B are A_1 and B_3 respectively.

Example 2: Player B

		B_1	B_2	B_3
Player A	A_1	15	2	3
	A_2	6	5	7
	A_3	-7	4	0

Solution: Player B

		B_1	B_2	B_3	Row minima	Maximin
Player A	A_1	[15]	(2)	3	2	
	A_2	6	(5)	[7]	5	5
	A_3	[-7]	4	0	-7	
Coloumn maxima		15	5	7		
minimax			5			

Since,

maximin = minimax

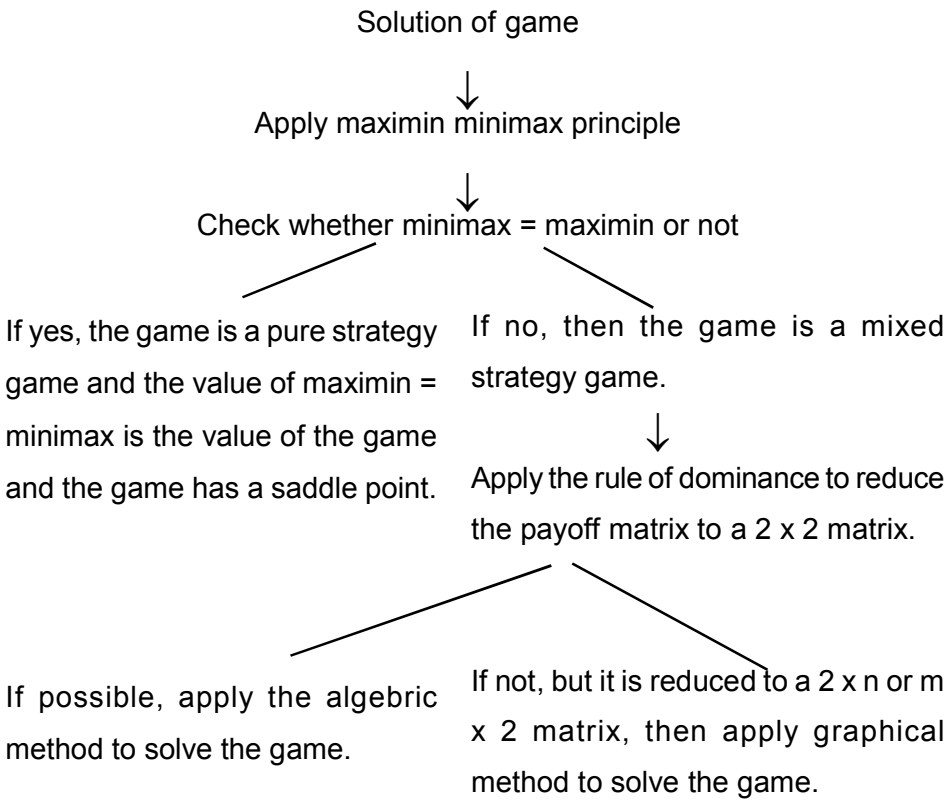
$\therefore$ The game has a saddle point. The saddle point solution is $\mathbf{A_2B_2}$ and the value of the game is 5.

3.9 PURE STRATEGY AND MIXED STRATEGY

Strategies may be of two types : pure and mixed

A pure strategy is when a player or individual chooses one particular course of action duting every play. In this case each player knows exactly what the other player is going to do and the objective of the player is to

maximise gains or minimise losses. On the other hand, mixed strategy is when individual chooses either of his actions with some probability. i.e., when the players use a combination of strategies and each player always keep on guessing to which course of action or strategy is to be selected by the other player than this is known as mixed strategy.



3.10 DOMINANT STRATEGY

In game theory, a dominant strategy is the course of action that results to the highest payoff for a player regardless of what the other player does. Not all players in all games have dominant strategies, but when they do, they can blindly follow them, since a dominant strategy is the optimal strategy unconditionally i.e., there is no dependence on the strategy the other player choses.

3.10.1 Rules of Dominance Principle

The principle of dominance states that if one strategy of a player dominates over the other strategy in all conditions then the later

strategy can be ignored. A strategy dominates over the other only if it is preferable over other in all conditions.

The rules of dominance principle are :

Rule 1: If each element of a column is greater than or equal to the corresponding element of other column in a payoff matrix, then the column with the greater element is dominated by the column with the smaller element. A logical player will never use the dominated strategy and so the column corresponding to this strategy can be eliminated from the payoff matrix.

Rule 2: If each element of a row is less than or equal to the corresponding elements of any others row in the payoff matrix, then the row with the smaller element is dominated by the row with the larger elements, and the dominated strategy can be deleted from the pay off matrix.

Rule 3: A strategy can also be eliminated if it is inferior to an average of two or more pure strategies.

The dominance principle in game theory problem is explained below with an example :

Player B

Player A	I	II	III	IV
I	3	2	4	0
II	3	4	2	4
III	4	2	4	0
IV	0	4	0	8

Here, all the elements of 1st row is dominated by IIIrd row, so delete 1st row from the payoff matrix. The payoff matrix became

B

A	I	II	III	IV
II	3	4	2	4
III	4	2	4	0
IV	0	4	0	8

For player B, all the element of 1st column is greater than or equal to IIIrd column, so column I can be eleminated.

B

$$A \quad \begin{matrix} & II & III & IV & \frac{III+IV}{2} \\ II & 4 & 2 & 4 & 3 \\ III & 2 & 4 & 0 & 2 \\ IV & 4 & 0 & 8 & 4 \end{matrix}$$

Again, all the elements of II[nd] column are greater than the corresponding average of III[rd] and IV[th] columns, and hence IInd column may be deleted from the payoff matrix. Thus, the new payoff matrix is

B

$$A \quad \begin{matrix} & III & IV \\ II & 2 & 4 \\ III & 4 & 0 \\ IV & 0 & 8 \end{matrix}$$

$$\begin{matrix} \frac{III+IV}{2} & 2 & 4 \end{matrix}$$

All the elements of II[nd] row are equal to the corresponding average of IIIrd and IV[th] rows. So the II[nd] row can be eleminated and obtain a reduced (2 x 2) payoff matrix.

B

$$A \quad \begin{matrix} & III & IV \\ III & 4 & 0 \\ IV & 0 & 8 \end{matrix}$$

Example 3: Reduce the following payoff matrix to a 2 x 2 matrix using the rules of dominance.

(a) Player B

$$\text{Player A} \quad \begin{matrix} & I & II & III \\ I & 4 & 5 & 6 \\ II & 6 & 5 & 8 \\ III & 8 & 2 & 7 \end{matrix}$$

Solution:

Player B

$$\text{Player A} \quad \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array} \begin{array}{c} \begin{array}{ccc} \text{I} & \text{II} & \text{III} \end{array} \\ \begin{bmatrix} 4 & 5 & 6 \\ 6 & 5 & 8 \\ 8 & 2 & 7 \end{bmatrix} \end{array}$$

All the elements of column 3 are greater than the corresponding elements of column 2, so column 3 can be deleted. The new payoff matrix is

Player B

$$\text{Player A} \quad \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array} \begin{array}{c} \begin{array}{cc} \text{I} & \text{II} \end{array} \\ \begin{bmatrix} 4 & 5 \\ 6 & 5 \\ 8 & 2 \end{bmatrix} \end{array}$$

Again, all the elements of row 1 are smaller than or equal to the corresponding elements of row 2, therefore row 1 can be deleted and obtain a reduced (2 x 2) payoff matrix.

Player B

$$\text{Player A} \quad \begin{array}{c} \\ \text{II} \\ \text{III} \end{array} \begin{array}{c} \begin{array}{cc} \text{I} & \text{II} \end{array} \\ \begin{bmatrix} 6 & 5 \\ 8 & 2 \end{bmatrix} \end{array}$$

Example 4: Reduce the following game by rule of dominance

Player B

$$\text{Player A} \quad \begin{bmatrix} 3 & 5 & 4 & 2 \\ 5 & 6 & 2 & 4 \\ 2 & 1 & 4 & 0 \\ 3 & 3 & 5 & 2 \end{bmatrix}$$

Solution : Given

Player B

$$\text{Player A} \quad \begin{bmatrix} 3 & 5 & 4 & 2 \\ 5 & 6 & 2 & 4 \\ 2 & 1 & 4 & 0 \\ 3 & 3 & 5 & 2 \end{bmatrix}$$

All elements of row 3 are smaller than or equal to the

corresponding elements of row 1, therefore row 3 can be eliminated. The payoff reduce to

$$\begin{array}{c} \\ \text{Player A} \end{array} \begin{array}{c} \text{Player B} \\ \begin{bmatrix} 3 & 5 & 4 & 2 \\ 5 & 6 & 2 & 4 \\ 3 & 3 & 5 & 2 \end{bmatrix} \end{array}$$

All element of column 2 are greater than or equal to the corresponding elements of column 1, therefore column 2 can be deleted.

$$\begin{array}{c} \\ \text{Player A} \end{array} \begin{array}{c} \text{Player B} \\ \begin{bmatrix} 3 & 4 & 2 \\ 5 & 2 & 4 \\ 3 & 5 & 2 \end{bmatrix} \end{array}$$

Again,

All elements of column 1 are greater than column 3, so column 1 can be eliminated.

$$\begin{array}{c} \\ \text{Player A} \end{array} \begin{array}{c} \text{Player B} \\ \begin{bmatrix} 4 & 2 \\ 2 & 4 \\ 5 & 2 \end{bmatrix} \end{array}$$

Finally, all elements of row 1 are smaller than or equal to the corresponding elements of row 3, therefore row 1 can be deleted and obtain a reduce (2 x 2) payoff matrix.

$$\begin{array}{c} \\ \text{Player A} \end{array} \begin{array}{c} \text{Player B} \\ \begin{bmatrix} 2 & 4 \\ 5 & 2 \end{bmatrix} \end{array}$$

Example 5: Determine which of the following two person zero sum games are strictly determinable and fair. Give the optimum strategies for each player in case of strictly determinable game.

A. B.

$$\begin{array}{c} \\ A \begin{matrix} A_1 \\ A_2 \end{matrix} \end{array} \begin{array}{c} \begin{matrix} B_1 & B_2 \end{matrix} \\ \begin{bmatrix} 8 & 0 \\ 5 & -2 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ A \begin{matrix} A_1 \\ A_2 \end{matrix} \end{array} \begin{array}{c} \begin{matrix} B_1 & B_2 \end{matrix} \\ \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} \end{array}$$

Solution A :

1. Given, the payoff matrix for player A is

		B			
		B_1	B_2	Row minima	maximin
A	A_1	[8]	(0)	0	0
	A_2	5	(−2)	−2	
Column maxima		8	0		
minimax			0		

$\because$ maximin = minimax = 0

$\therefore$ The game has a saddle point and the value of the game is zero.

It is a pure strategy game and it is strictly determinable and fair. Optimum strategies for player A and B are A_1 and B_2 respectively.

N.B. : In game theory, a strictly determined game is a two player zero sum game that has at least one Nash equilibrium with both players using pure strategies.

A game is called a fair game if the value of the game is zero and if it is non-zero than the game is called as an unfair game. Thus

$\begin{cases} \text{if maximin = minimax = 0} \Rightarrow \text{fair game} \\ \text{if maximin = minimax} \neq 0 \Rightarrow \text{unfair game} \end{cases}$

Solution B :

2. Given, the payoff matrix for player A is

		B			
		B_1	B_2	Row minima	maximin
A	A_1	[5]	(0)	0	0
	A_2	(0)	[4]	0	
Column maxims		5	4		
minimax			4		

$\because$ maximin $\neq$ minimax

$\therefore$ The game has no saddle point and it is not strictly determinable.

Example 6 : Determine the range of values of p and q that will make the payoff element a_{22} a saddle point for the game whose payoff matrix is given below :

Player B

$$\text{Player A} \quad \begin{matrix} & B_1 & B_2 & B_3 \\ A_1 & 2 & 4 & 7 \\ A_2 & 12 & 7 & q \\ A_3 & 4 & p & 10 \end{matrix}$$

Solution : Let us find out the maximin and minimax value of the payoff matrix ignoring the values of p and q.

Player B

$$\text{Player A} \quad \begin{matrix} & B_1 & B_2 & B_3 & \text{Row Minima} & \text{maximin} \\ A_1 & 2 & 4 & 7 & 2 & \\ A_2 & 12 & 7 & q & 7 & 7 \\ A_3 & 4 & p & 10 & 4 & \end{matrix}$$

Comuma maxima 12 7 10

minimax 7

$\because$ maximin = minimax

$\therefore$ The game exists a saddle point at the position a_{22}

Now, p must be less than or equal to 7 and q must be greater than or equal to 7 to make the payoff element a_{22} a saddle point.

Therefore, the required range of values of p and q is

$p \leq 7$ and $q \geq 7$



CHECK YOUR PROGRESS

Q.1: Find the saddle point (or points) and hence solve the following game.

(a)

Player B

$$\text{Player A} \quad \begin{matrix} & B_1 & B_2 & B_3 \\ A_1 & 15 & 2 & 3 \\ A_2 & 6 & 5 & 7 \\ A_3 & -7 & 4 & 0 \end{matrix}$$

(b)

Player B

$$\text{Player A} \quad \begin{matrix} & B_1 & B_2 & B_3 \\ A_1 & 0 & -4 & -3 \\ A_2 & 1 & 3 & 1 \\ A_3 & 1 & 5 & -1 \end{matrix}$$

Q 2: Reduce the following game by rules of dominance.

Player B

Player A

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{bmatrix} -20 & 40 & -10 & 60 \\ 15 & -5 & 25 & 70 \end{bmatrix} \end{array}$$

3.11 GAME WITHOUT SADDLE POINT (MIXED STRATEGY)

If maximin and minimax of a particular game is not equal then we do not obtain a saddle point. Under such circumtances, when given payoff matrix has no saddle point, a mixed strategy technique of solving the game is used. In such cases both the players must determine an optimal mixture of strategies to find a saddle or equilibrium point. The optimal mix for each player may be determined by assigning each strategy a probability of it being chosen. Thus these mixed strategies are probabilistic combinations of available better strategies and such games hence called probabilistic games.

In this chapter we will discuss the following two methods for the solutions of games with mixed strategies :

- Algebric method
- Graphical method

3.11.1 Algebric Method

A 2 x 2 payoff matrix where there is no saddle point can be solved by algebric method.

Let us assume that for a two person zero sum game player A and B each have two strategy A_1, A_2 and B_1, B_2 respectively. Again, let player A selects stratgies with probabilities P_1 and P_2 so that $P_1 + P_2 = 1$, Similarly, player A selects strategies with probabilities q_1 and q_2 where $q_1+q_2 = 1$. The payoff matrix of player A is

B's strategy

A's strategy

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{array}$$

Value of the game is

$$V = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

with

$$p_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$q_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

Example 7:

(i) Find the value of the game

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} \end{array}$$

Solution:

	B_1	B_2	Row minima	maximin
A_1	5	1	1	
A_2	3	4	3	3
Column maxima	5	4		
minimax			4	

$\because$ maximin $\neq$ minimax, therefore no saddle point exists.

We can solve by algebric method.

Let we assume that p_1 and p_2 be the probabilities with which player A chooses strategy A_1 and A_2.

Again q_1 and q_2 be the probabilities with which player B chooses strategy B_1 and B_2.

V is the value of the game.

$$P_1 = \frac{4 - 3}{(5 + 4) - (1 + 3)}$$

$$= \frac{1}{9 - 4} = \frac{1}{5}$$

$$P_2 = 1 - p_1 \quad [\because \ p_1 + p_2 = 1]$$

$$= 1 - \frac{1}{5} = \frac{4}{5}$$

$$q_1 = \frac{4 - 1}{(5 + 4) - (1 + 3)}$$

$$= \frac{3}{9-4}$$

$$= \frac{3}{5}$$

$$q_2 = 1 - q_1 \quad [\because \ q_1 + q_2 = 1]$$

$$= 1 - \frac{3}{5}$$

$$= \frac{2}{5}$$

$$V = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$= \frac{5.4 - 3.1}{(5+4) - (1+3)}$$

$$= \frac{20 - 3}{9 - 4}$$

$$= \frac{17}{5}$$

(ii) In Example 4 above, we reduce the given payoff matrix using dominance property to 2 x 2 payoff matrix

Player B

Player A $\begin{bmatrix} 2 & 4 \\ 5 & 2 \end{bmatrix}$

Since, maximin $\neq$ minimax, there is no saddle point, it shall, therefore, be solved using mixed strategy technique.

$$p_1 = \frac{2-5}{(2+2)-(4+5)} \quad = \frac{3}{5}$$

$$p_2 = 1 - p_1 \quad [\because \ p_1 + p_2 = 1]$$

$$= 1 - \frac{3}{5}$$

$$= \frac{2}{5}$$

$$q_1 = \frac{2-4}{(2+2)-(4+5)}$$

$$= \frac{2}{5}$$

$$q_2 = 1 - q_1 \quad [\because \ q_1 + q_2 = 1]$$

$$= 1 - \frac{2}{5}$$

$$= \frac{3}{5}$$

and the value of the game is

$$V = \frac{2.2 - 5.4}{(2+2) - (4+5)}$$

$$= \frac{4 - 20}{4 - 9}$$

$$= \frac{16}{5}$$

3.11.2 Graphical Method

The graphical method is used to solve the games whose payoff matrix has

- Two rows and n columns (2 x n)
- m rows and 2 columns (m x 2)

Let us consider the following 2 x n game

		Player B				
		B_1	B_2	.....	B_n	Probability
	A_1	a_{11}	a_{12}	.....	a_{1n}	p_1
Player A	A_2	a_{21}	a_{22}	.....	a_{2n}	$1- p_1$
Probability		q_1	q_2	.....	q_n	1

We again assume that the game has no saddle point. Player A chooses his strategies A_1 and A_2 with probabilities P_1 and P_2 ($P_1 + P_2=1$). Player B has n strategies $B_1, B_2, \ldots, B_n$ which can select with probabilities $q_1, q_2, \ldots q_n$ such that $q_1 + q_2 + \ldots + q_n = 1$.

The expected payoff for A for different strategy selection by player B is given by

B's strategy A's expected payoff (Ep)

B_1 $E_p^1 = a_{11}p_1 + a_{21}p_2 = a_{11}p_1 + a_{21}(1-p_1)$

B_2 $E_p^2 = a_{12}p_1 + a_{22}p_2 = a_{12}p_1 + a_{22}(1-p_1)$

- -
- -
- -
- -

B_n $E_p^n = a_{1n}p_1 + a_{2n}p_2 = a_{1n}p_1 + a_{2n}(1-p_1)$

According to the maximin criteria. Player A will choose to the values of P_1 and P_2 so as to maximise his minimum expected payoff. It can be done by plotting the straight lines representing the expected payoff of player A.

$$E_p^j = a_{ij}\, p_1 + a_{2j}\,(1-p_1)$$
$$= a_{ij}\, p_1 + a_{2j} - a_{2j}\, p_1$$
$$= (a_{ij} - a_{2j})\, p_1 + a_{2j},\ (j = 1, 2, \ldots, n)$$

The highest point on the lower envelope of these straight lines will give the maximum expected payoff to player A and also the maximum value of p_i.

The two lines passing through the maximin point identify the 2 critical moves or strategies of player B, which combined with the two strategies of A gives a 2 x 2 matrix. The algrabic method can now be applied to determined the optimum strategies of two players and also the value of the game.

The m x 2 games can also be solved in the same way but in such games minimax point is the lowest point on the upper envelope of straight lines.

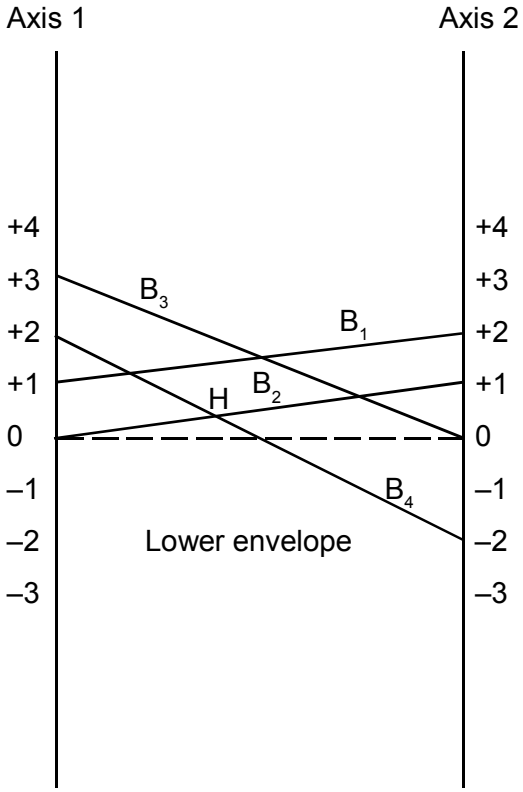
Example 6: Solve the following game graphically

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	2	1	0	–2
	A_2	1	0	3	2

Solution : Let P_1 is the probability of selecting strategy A_1 and P_2 ($P_2 = 1–P_1$) is the probability of selecting strategy A_2 by player A. The expected payoff of A for various strategies of player B are

B's strategy	Expected payoff of player A
B_1	$2p_1+1p_2 = 2p_1+1(1-p_1) = 2p_1+1-p_1 = p_1+1$
B_2	$1.p_1+0.p_2 = p_1$
B_3	$0.p_1+3p_2 = 3(1-p_1) = 3 - 3p_1$
B_4	$-2p_1+2p_2 = -2p_1+2(1-p_1) = -2p_1+2-2p_1 = -4p_1+2$

These expected payoff lines are row plotted as functions of P_1 as shown in the diagram.



The diagram shows the expected payoff of player A, the line B_1 joints the first payoff element 2 in the first column represented by + 2 on axis 2, and the second payoff element 1 in the first column represented by +1 on axis 1. Similarly, lines B_2, B_3 and B_4 joined the corresponding payoff element in the second, third and fourth columns. Since the player A wishes to maximise his minimum expected payoff, the highest point of intersection H on the lower envelope of the A's expected payoff equations represents the maximin expected value of the game for A. The lines B_2 and B_4 passing through H defines the two relevent moves or strategies for

player B and other two strategies B_1 and B_3 can be eliminated from the payoff matrix. The payoff matrix now becomes

		Player B	
		B_1	B_2
Player A	A_1	1	–2
	A_2	0	2

$$\therefore \quad p_1 = \frac{2-0}{(1+2)-(-2+0)}$$

$$= \frac{2}{3+2}$$

$$= \frac{2}{5}$$

$$p_2 = 1 - p_1$$

$$= 1 - \frac{2}{5}$$

$$= \frac{3}{5}$$

$$\text{Again, } \quad q_1 = \frac{2-(-2)}{(1+2)-(-2+0)}$$

$$= \frac{2+2}{3+2}$$

$$= \frac{4}{5}$$

$$q_2 = 1 - q_1$$

$$= 1 - \frac{4}{5}$$

$$= \frac{1}{5}$$

$$\text{And, } \quad v = \frac{1\times 2 - 0\times(-2)}{(1+2)-(-2+0)}$$

$$= \frac{2+0}{3+2}$$

$$= \frac{2}{5}$$

3.12 LIMITATION OF GAME THEORY

The limitations of the game theory are as follows-

- The assumption, that each player has the complete knowledge about his own and other player's payoff is not practical.
- The method of solution becomes complex with more than two players.
- The technique of solving games involving mixed strategies is very complicated.
- The assumption of both the players are equally wise and they behave in a rational way, is also not possible.



3.13 LET US SUM UP

- Game theory is a tool that has been created to help us study decision-making in situations in which there is a mixture of conflict and cooperation.
- A game can be classified on the basis of the number of players, the number of strategies, the nature of payoff function and the nature of negotiation
- A game may or may not have a saddle point.
- If maximin = minimax, then we have a saddle point and the game is a pure strategy game and if maximin and minimax of a particular game is not equal, then the game is a mixed strategy game.
- The value of the maximin = minimax is the value of the game.
- Algebric and graphical methods are used for the solutions of games with mixed strategies.



3.14 FURTHER READING

1) Madnani, G.M.K., (2003) Mathematical Economics: A Mathematical Approach to Microeconomic Theory. New Delhi: Oxford and IBH Publishing Co. Pvt. Ltd.
2) Koutsoyiannis, A., (1979) *Modern Microeconomics*. London: Macmillan. Education Ltd.

3) Swarup, K., Gupta, P. K. and Mohan, M., (2006) *Operations Research.* New Delhi : Sultan Chand and Sons.

4) Henderson, J. M. and Quant, R. E.. (2003) *Microeconomic Theory : A Mathematical Approach.* Mc Graw Hill International Editions.

5) Varian, H. R., (2006) *Intermediate Microeconomics: A Modern Approach.* New Delhi: EWP Pvt. Ltd.

3.15 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q. No. 1: (a)

Player B

Player A	B_1	B_2	B_3	Row minima	maximin
A_1	[15]	(2)	3	2	
A_2	6	[5]	[7]	5	5
A_3	(−7)	4	0	−7	
Column maxima	15	5	7		
minimax		5			

$\because$ maximin = minimax

$\therefore$ The game has a saddle point and value of the game is 5. Optimal strategy for player A and B are A_2 and B_2 respectively.

Ans to Q. No. 2: (b) Given,

Player B

Player A	B_1	B_2	B_3	Row minima	maximin
A_1	0	(−4)	−3	−4	
A_2	[1]	3	[1]	1	1
A_3	1	[5]	(−1)	−1	
Column maxima	1	5	1		
minimax		1			

Since, maximin = minimax, the game is a pure strategy game and value of the game is 1.

Here, we observed that there are two saddlepoints of the game.

The optimal strategy for player A is A_2 and the optimum strategy for player B are B_1 and B_3.

Ans to Q. No. 3: (a) Given,

Player B

$$\text{Player A}\begin{matrix} \\ A_1 \\ A_2 \end{matrix}\begin{bmatrix} B_1 & B_2 & B_3 & B_4 \\ -20 & 40 & -10 & 60 \\ 15 & -5 & 25 & 70 \end{bmatrix}$$

Each elements of the 3rd column is greater than the corresponding elements of 1st column and hence the 3rd column is dominated by 1st column. So, 3rd column can be deleted.

B

$$\begin{matrix} \\ A_1 \\ A_2 \end{matrix}\begin{bmatrix} B_1 & B_2 & B_4 \\ -20 & 40 & 60 \\ 15 & -5 & 70 \end{bmatrix}$$

Again, 4th column is dominated by 1st and 2nd columns and hence it can be eliminated. Therefore, the payoff matrix can be reduced to :

B

$$A\begin{matrix} \\ A_1 \\ A_2 \end{matrix}\begin{bmatrix} B_1 & B_2 \\ -20 & 40 \\ 15 & -5 \end{bmatrix}$$



3.16 MODEL QUESTIONS

Q.1: Find the optimal strategies and value of the game whose payoff matrix is

		Player B			
		B_1	B_2	B_3	B_4
	A_1	1	7	3	4
Player A	A_2	5	6	4	5
	A_3	7	2	0	3

Q.2: Find the saddle point and value of the game for the following games:

		Player B		
		B_1	B_2	B_3
Player A	A_1	15	0	–2

A_2	0	–15	–1
A_3	1	2	0

Q.3: For what value of λ, the game with following payoff matrix is strictly determinable?

Player B

$$\text{Player} \quad \begin{matrix} \\ A_1 \\ A_2 \\ A_3 \end{matrix} \begin{matrix} B_1 & B_2 & B_3 \\ \end{matrix} \begin{bmatrix} \lambda & 6 & 2 \\ -1 & \lambda & -7 \\ -2 & 4 & \lambda \end{bmatrix}$$

Q.4: Give the following payoff matrix of player A and total market share of profit is Rs. 10.

$$\begin{bmatrix} 2 & 8 & 1 \\ 4 & 3 & 9 \\ 5 & 6 & 7 \end{bmatrix}$$

(i) Find the nature of the game.

(ii) Find maximin and minimax payoffs.

(iii) Find the saddle point of the game.

(iv) Find the optimal strategies for the two players.

(v) Is possible to find payoff matrix for player B.

Q.5: Solve the following game and determine the value of the game :

$$\begin{matrix} \\ A \end{matrix} \begin{matrix} B \\ \end{matrix} \begin{bmatrix} 6 & -3 \\ -3 & 0 \end{bmatrix}$$

Q61: Use graphical method in solving the following game.

Player B

$$\text{Player A} \quad \begin{bmatrix} 6 & -3 & 7 \\ -3 & 0 & -6 \end{bmatrix}$$

*** ***** ***

UNIT 4: THE THEORY OF FACTOR PRICING

UNIT STRUCTURE

4.1 Learning Objectives

4.2 Introduction

4.3 Importance of Factor Pricing

4.4 Marginal Productivity Theory of Distribution

4.5 Equilibrium under Perfect Competition in the Factor Market

4.6 Equilibrium under Imperfect Competition in the Factor Market

4.7 Modern Theory of Distribution

4.8 Product Exhaustion Problem

4.9 Let Us Sum Up

4.10 Further Reading

4.11 Answer to Check Your Progress

4.12 Model Questions

4.1 LEARNING OBJECTIVES

After going through this unit, you will be able to—

- understand the importance of factor pricing
- describe the marginal productivity theory of distribution
- explain the equilibrium under perfect and imperfect competition in the factor market
- discuss the modern theory of distribution
- explain the product exhaustion problem.

4.2 INTRODUCTION

The theory of factor pricing is also known as the theory of distribution. In the theory of factor pricing we study how the inputs used for producing goods and services are rewarded with the aim of making economic profit. We already know that there are four factors of production i.e land, labour, capital and entrepreneurship. Output of an economy is produced by employing these four factors. After output is produced it is sold in the market

resulting in revenue generation. This revenue is distributed among the various factors. Hence this theory is also known as the theory of distribution. According to Chapman, "The economics of distribution accounts for the sharing of the wealth produced by a community among the agents, or the owners of the agents, which have been active in its production". In short we can say that the theory of factor pricing or the theory of distribution studies how rent of land, wage of labour, interest on capital and profit of entrepreneur is determined.

There are two aspects of distribution-personal distribution and functional distribution. Personal distribution refers to the distribution of national income among various individuals or persons in the society. It is a macroeconomic aspect. And functional distribution deals with the distribution of income in relation to the factors of production. It studies how the various factors of production are rewarded for their service in the production process.

In this chapter we will study how this revenue is distributed among various factors. We will also discuss about the equilibrium under perfect and imperfect competition in factor market. Along with these we will study the modern theory of distribution. The adding up problem or the product exhaustion theorem is discussed at the end of the unit.

4.3 IMPORTANCE OF FACTOR PRICING

Production of goods and services in an economy is the result of joint efforts of factors of production. Factor pricing is concerned with the analysis of how and in what manner the factors of production get remunerated in such production process for their contribution. The main reasons of studying the factor pricing are:

- Factor pricing determines the allocation of productive resources among the various alternative uses
- Optimum utilization of input factors crucially depends on factor price.
- Factor prices determine the share of various factors of production in national income.

4.4 MARGINAL PRODUCTIVITY THEORY OF DISTRIBUTION

The marginal productivity theory of distribution which is also known as the neo-classical theory of distribution, is an attempt on the part of the economists like J.B. Clark, Wickstead, Marshall and others, to evolve a general theory which will explain the determination of factor prices. In other words, it serves as a general theory of distribution in terms of which the price of a variable factor could be determined. According to this theory, under certain conditions, the variable factors are paid according to their marginal product. The theory is based on the following assumptions,

- Perfect competition prevails both in the factor and product market.
- The law of variable proportion prevails.
- There is full employment in the economy.
- State of technology remains unchanged.
- Profit maximization is the goal of the factor employing firm.

Now, before we explain the theory, let us understand the meaning of marginal productivity. Marginal productivity of a factor is the increase in total output as a result of employing an additional unit of the factor, quantity of other factors remaining the same. If we measure the increase in total output in physical terms i.e. in terms of physical units, we receive the marginal physical product (MPP) of a factor. Again, if we measure such increase in total output in money terms, we get the marginal revenue product (MRP). MRP is the increase in the total value of the product as a result of the employment of an additional unit of a variable factor, when quantity of other factors remaining the same. The marginal revenue product (MRP) of a factor can be obtained by multiplying the marginal revenue (MR) of the firm with the marginal physical product (MPP) of the variable factor. More specifically,

MRP = MPP x MR

A related concept of marginal productivity is the value of marginal product (VMP), which is obtained by multiplying the price (average revenue/

AR) of the product of the firm with the marginal physical product (MPP) of the variable factor. i.e.

VMP = MPP x Price (AR)

Under perfect competition, the demand curve of the product of an individual firm is perfectly elastic, so price (AR) and marginal revenue (MR) of the firm are equal. As a result, under perfect completion, marginal revenue product (MRP) and value of marginal product (VMP) of a variable factor are equal. i.e.

MRP = VMP

Now, let us explain the marginal productivity theory with the help of the following diagram-

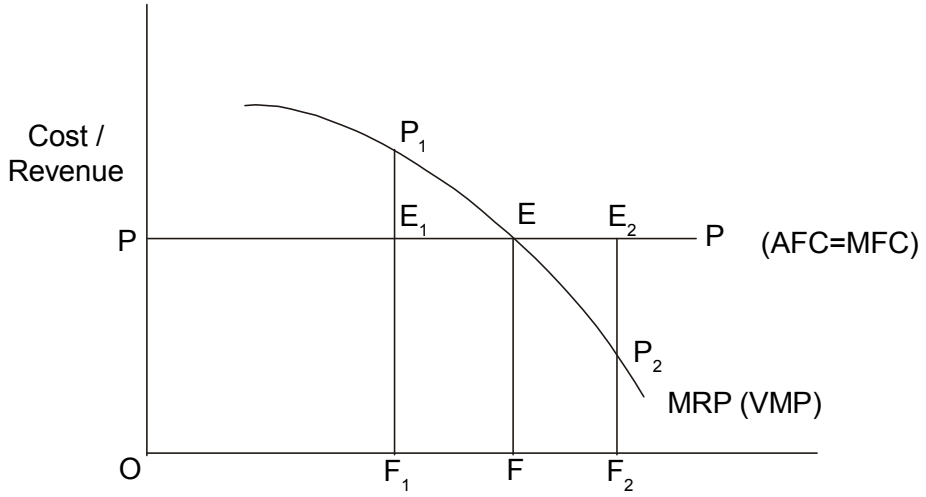


Fig. 4.1: Determination of equilibrium

In the above figure, we measure cost and revenue of labour in the Y-axis and units of factor in the X- axis. Since there is perfect competition in the factor market, the factor price will be given to the firm and the wage line PP is parallel to X axis indicating that at factor price OP unlimited amount of factor will be available to the firm. The factor price OP can also be considered as the cost for the entrepreneur. In actual terms, the price OP is the average factor cost of the variable factor and since there is perfect competition, it is also the marginal factor cost of the variable factor. Because of the operation

of the law of variable proportion, the MPP will be downward sloping. Also, since there is perfect competition in the product market the MR will remain constant, as a result of that MRP curve is also downward sloping and it coincide with VMP curve. The profit maximizing firm attains equilibrium by equating its MR and MC curve. In our diagram, at the point E_1, price of the labour is less than its marginal revenue, since MRP line lies above the price curve. So it will be profitable for the firm to increase the number of labour. E is the point where MRP is equal to the price of the labour. In other words, E is the point of equilibrium, where MC curve is intersecting the MRP curve from below. At equilibrium the firm is employing OF unit of the variable factor at OP price. Beyond this point price of the labour will be more than its marginal productivity, as shown by point E_2 in our diagram.

Criticisms:

The marginal productivity theory of distribution has been considered as a pillar in the neo-classical theory of income distribution. However, the theory suffers from certain serious weaknesses which are noted below :

- Unrealistic assumptions: The assumption of perfect competition is unrealistic both in the factor market and in the product market. Features like homogeneity of factors, perfect mobility, perfect knowledge, and non intervention of the state are not to be found in the real world.

  But Prof. Douglas pointed out that most of the assumptions are reasonably good description of actual long run market conditions and are thus generally valid over time.

- Unrealistic assumption of profit maximization : Another assumption of the theory is that the producer is motivated by profit maximization. This will be true only when the wages of a labour is equal to its marginal productivity. R. A. Lester, an American economist criticized the marginal productivity theory of distribution on the ground that entrepreneurs do not behave as a maximizer of profit and therefore wages of the real world will differ from the marginal productivity of labour.

  But economist like Machlup and Pen defend the theory. Pen said that, “It is not necessary for every entrepreneur to be able to find the

exact point of equilibrium, some will overshoot the equality of wage and marginal product; others will remain below it. However, they always tend towards equality".

- One sided theory: The theory discusses about the demand of the factors, and neglects the supply side of the factor . The theory does not explain how the given wage rate and given interest rate is determined. According to Samuelson, this theory cannot be applied to the factor market as a whole which requires a theory of both the demand for and supply of factors. Thus it is a one sided theory.
- Ignores the importance of technical progress: According to Hicks, this theory fails to throw light on the determination of the relative shares of the factors by neglecting the influence of technological change.



CHECK YOUR PROGRESS

Q.1: Name some economists who have rendered the marginal productivity theory of distribution?

.....

.....

Q.2: What is meant by MRP? (Answer in about 20 words)

.....

.....

Q.3: State the concept of marginal productivity. (Answer in about 20 words)

.....

.....

4.5 EQUILIBRIUM UNDER PERFECT COMPETITION IN THE FACTOR MARKET

The Equilibrium of the firm under perfect competition can be discussed by the marginal productivity theory of distribution. We know that according to the theory the firm will be in equilibrium when the marginal revenue

productivity (MRP) of any factor of the firm equals its marginal cost (MC). In case of a firm with perfect competition in the labour market, the equilibrium will be established when MRP of labour equals the MC of labour.

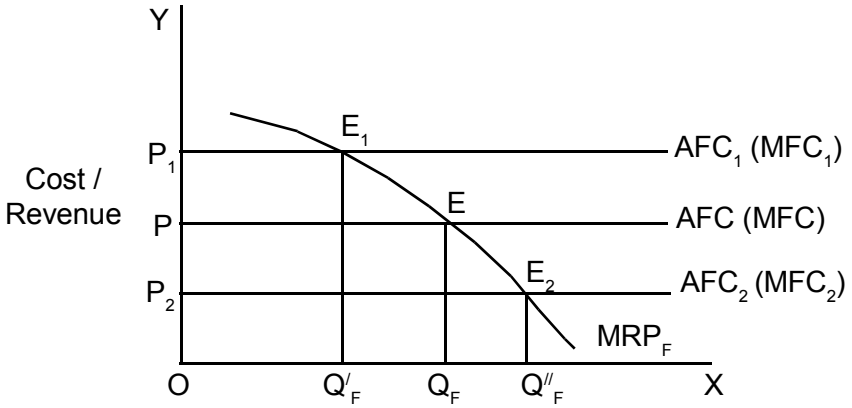


Fig. 4.2: Derivation of demand curve

Since we are dealing with the perfect competition, so the price of the variable factor is given to the firm by the market and in our diagram, we consider it to be OP. Again given the downward sloping MRP_F (the MRP curve of a factor), E is the pt. of equilibrium. In this situation, the firm employs OQ_F amount of the factor at OP price. Assume that the market given price of a factor increases to P_1, other things being equal ,the employment offered by the firm reduces to OQ'_F. Likewise if market given price of the factor goes down to OP_2, the employment offered by the firm increases to OQ''_F. Thus, if we join the equilibrium points such as E_1, E and E_2 , we get a curve which reflect the inverse relationship between the quantity of factor employed by the firm and the factor price. Therefore, we can consider such a line as the demand curve of the firm for the factor. In general the MRP curve of a variable factor is also the demand curve for the factor of the employing firm. We may note that the demand for a variable factor depends on the following–

- The price of the input. The higher the price of a factor lower is the demand for it.
- Marginal physical product of the factor.

- The price of the cammodity produced by the factor.
- The amount of other factors which are combined with a factor.
- The prices of other factors.
- The technological progress as it effects the MPP (Marginal Physical Product) of inputs and hence their demand.

In the above section, we discussed about the derivation of the demand curve of an individual firm for a variable factor. Now, we try to derive the market demand curve for a variable factor from the individual demand curves of different firms.But we cannot simply laterally sum up the individual demand curves of firms to get the market demand curve for a variable factor,as the changes in factor price will bring about changes in the marginal productivity of the factor. This is explained below with the help of the following figures -

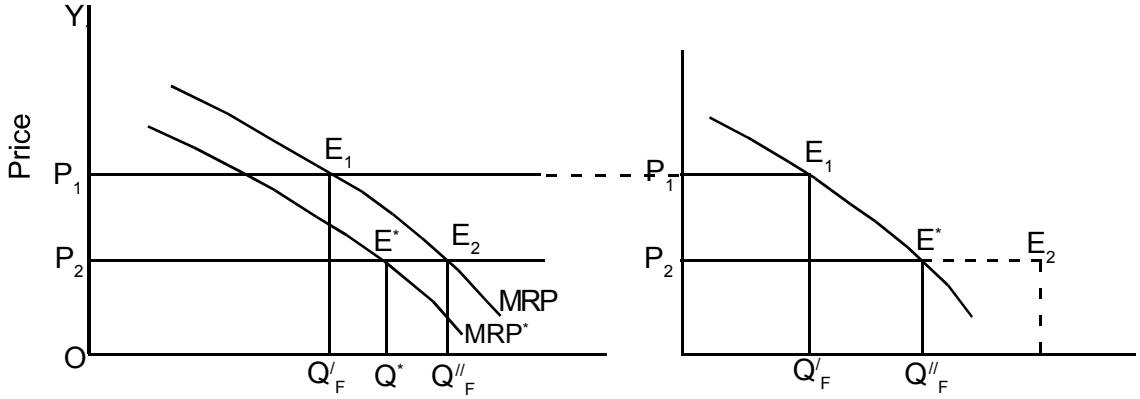


Fig. 4.3: Demand for a single firm **Fig. 4.4: Market demand for Labour**

It is clear in the diagram that, as the price of the variable factor decreases from P_1 to , the firm increases its employment from $Q^{/}F$ to $Q^{//}_F$. Thus the section E_1E_2 (in figure 4.3) constitutes its demand curve. But we may note that all other firms in the market will do the same. Consequently, overall employment of the factor will go up which will increase the production and hence the supply of the output produced by this factors. Given market demand for the product, the price of the product will fall. In other words, the MR will decrease which will cause MRP to shift downwards. Therefore

following the fall in price, the MRP curve will shift downward to a new position. In our figure 4.3, the new MRP curve is MRP^*. When the price of the variable factor falls to P_2, the firm will be in equilibrium not at point E_2 (on the original MRP curve) but at point E^* on the new MRP or demand curve MRP^*. At this new position the firm will employ OQ^* amount of the factor and not $OQ^{//}_F$. When we connect the points E_1 and E^* we get the actual demand curve of the firm of the variable factor where market adjustments have been taken into account. The market demand curve for the variable factor is derived simply by laterally summing up such individual demand curves as shown in figure 4.4. Summing horizontally over all firms, at different prices of the variable factor like P_1 and P_2 we obtain the total demand for the input as represented by the points E_1 and E^* respectively in figure 4.4. Joining this two points we get the market demand curve for the variable factor, which is obviously downwards sloping.It is important to note here that, point E_2 in figure 4.4, does not belong to the market demand curve for the variable factor, as it represents the demand for labour in the market with the price of the commodity unchanged. If we consider it to be one point of market demand curve, it will led to an overestimation of the demand for the variable factor.

In general, whether the variable factor is labour or any other factor, the supply curve in a market is positively sloped. If the variable factor happens to be labour then the main determinants of the market supply are–

- Price of labour
- Taste of consumers, which define their trade off between leisure and work.
- The size of the population.
- The labour force participation rate.
- Occupational, educational and geographic distribution of labour price.

Now, given the downward sloping market demand curve and positively sloped market supply curve of the variable factor, market equilibrium is acheived where both the curves intersect each other and at that particular point the price of the variable factor is also determined. As seen in the figure 4.5, positively sloped supply curve S_F intersects the demand curve D_F at E. Thus, at this point the demand for factor equals its supply. Hence,

E is the point of equilibrium. Corresponding to it we get OP as equilibrium factor price at which OF amount of employment will be offered.

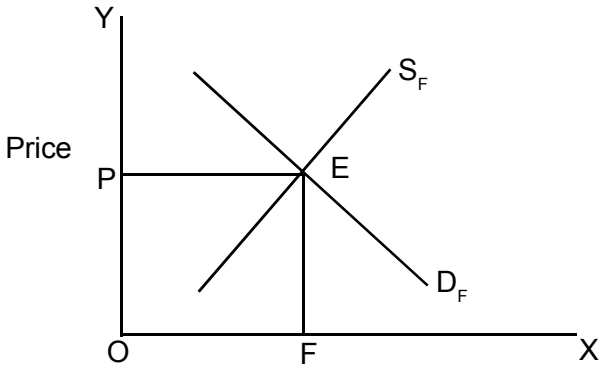


Fig. 4.5: Determination of Equilibrium

4.6 EQUILIBRIUM UNDER IMPERFECT COMPETITION IN THE FACTOR MARKET

We have already discussed about the equilibrium under perfect competition in the factor market. However, in the real world, factor market is imperfect. Here we are going to discuss about the equilibrium under imperfect competition. The same mechanism is deployed for the determination of equilibrium condition under imperfect competition(when there are imperfection in the commodity and factor market) as in case of perfectly competitive market i.e through the interaction of demand and supply. But in case of imperfect markets, the factors determining demand and supply are different.Since here, both the labour and product markets are involved, we get the following combination of imperfections in the market–

- Perfect competition in the product and imperfect competition in the factor market.
- Imperfect competition in the product market and perfect competition in the factor market.
- Imperfect competition both in product and factor markets.

- **Perfect competition in the product and imperfect competition in the factor market:**

Let us suppose that imperfections in the factor market takes the form of monopsony. In a monopsony factor market , there will be a single firm buying the services of the factor .The bargaining power of labour under this situation will be very weak and the firm can get more of factor services by offering higher price for it. Here, we are considering the case of a firm in the labour market. In the labour market the presence of a monopsony implies that Average Wage (AW) curve will be increasing and Marginal Wage (MW) curve will be above the AW curve as more and more workers are employed.

Again, since there are perfect competition in the product market, it implies that the price of the product will be given to the firm and it will remain constant. Therefore, MR curve will coincide with the AR curve and the marginal product of labour in revenue term i.e marginal revenue product of labour (MRP_L) will be equal to the value of marginal product of labour (VMP_L). The equilibrium in the market is shown in the following diagram :

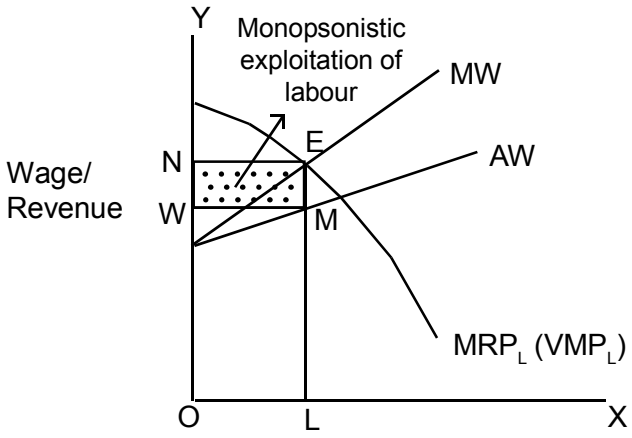


Fig. 4.6: Determination of Equilibrium in the Market

In the above figure 4.6 it is clear that the marginal wage curve is intersecting the MRP_L curve at E. It is important to note here that marginal wage curve and MRP_L curve are MC curve and MR curve respectively for the employment giving firm. So, E is the point of equilibrium. And at equilibrium, OL amount of labour is employed by the firm and OW will be the equilibrium wage rate. The total wage is given by the area OLMW (OL x OW), but as we

can see that the total value of marginal product of labour or the total value of the contribution made by the workers is given by the area OLEN (OL x ON). So it is clear from the above diagram that the workers are not paid according to their contribution, but less than that. The amount of difference between their contribution (OLEN) and payment received (OLMW) is given by the area WMEN. This is the area representing the exploitation of labour and this exploitation is possible due to the presence of monopsony in the factor market. Professer John Robinson termed it as monopsonistic exploitation of labour.

- **Imperfect competition in the product market and perfect competition in the factor market:**

Let us suppose that imperfections in the product market takes the form of monopoly.We know that in the monopoly structure of a market both the AR and MR curves are downward sloping and MR curve lies below the AR curve. Therefore, the VMP_L curve which is nothing but AR curve and MRP_L curve which is representing the MR curve will be both downward sloping and MRP_L will lie below VMP_L.

Again since there is perfect competition in the factor market, so the market determined wage will be given to the firm. Therefore, AW curve is a straight line and the MW curve will coincide with it.

The equilibrium in this situation is shown below–

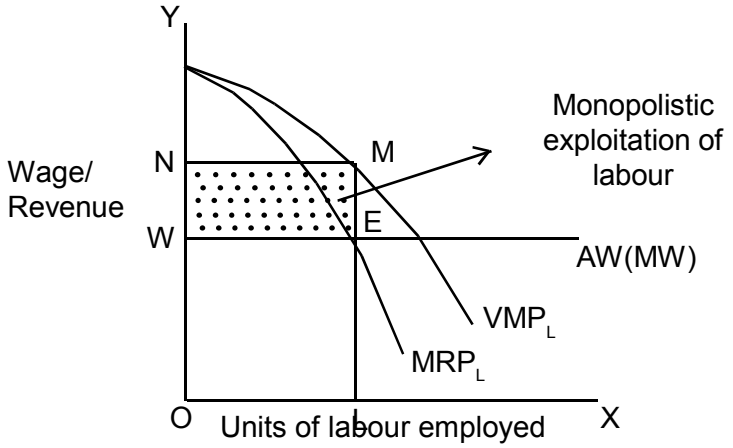


Fig. 4.7: Determination of Equilibrium in the Market

In the above figure 4.7, E is the point of equilibrium as at that particular point MW curve cut the MRP_L curve . And at equilibrium, the firm employs

OL units of labour at OW wage rate. Thus, the total payment made to the labour is given by the area OLEW(OL x OW). As, the value of marginal product of labour is ML(ON) per unit of labour, so the total contribution made by the labour employed is represented by the area OLMN. So it is clear from our diagram that employed workers are paid less than their contribution by the amount EMNW(OLMN - OLEW). This area clearly representing the exploitation of labour and since this exploitation is possible due to the presence of monopoly in the product market, so this is known as monopolistic exploitation of labour.

- **Imperfect competition both in product and factor markets:**

Let us suppose that imperfections in the product and factor markets take the form of monopoly and monopsony respectively. In this situation, because of presence of monopsony market structure, both the MW and AW curves are upward sloping and MW curve lies above the AW curve. Also, there is presence of monopoly market structure, so both VMP_L and MRP_L curves are downward sloping and MRP_L curve lies below the VMP_L curve. The equilibrium in this situation is shown below.

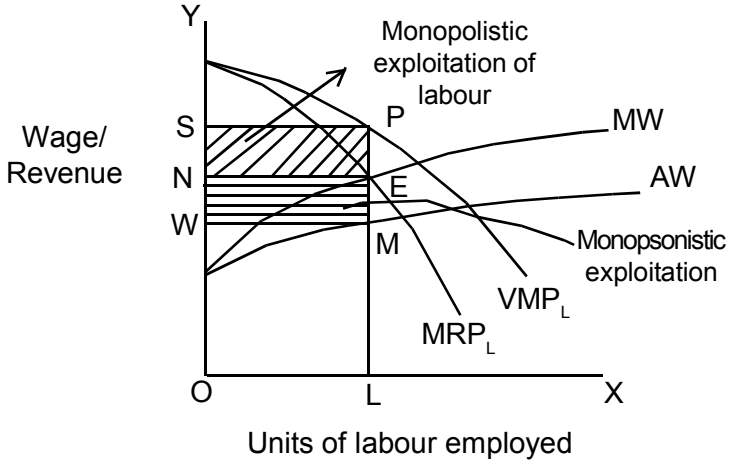


Fig. 4.8: Determination of Equilibrium in the Market

In the Figure 4.8 we see that the MW curve cuts the MRP_L curve from below at point E. Therefore, E is the point of equilibrium. Accordingly, the firm employs OL units of labour at OW wage rate. Thus the total payment made to employed labour is given by the area OLMW.

However, the value of marginal product of labour is given by the distance PL per unit of labour. Therefore, the total contribution made by the labour

employed is given by the area OLPS. It is important to note here that, the total contribution of labour (represented by the area OLPS) is more than what they actually receive (represented by the area OLMW).So it clearly reveals that the workers are deprived of an amount represented by the area WMPS in our diagram.This is exploitation of labour. We may note that the amount of WMPS can be divided into two parts- WMEN and NEPS. The first part i.e WMEN arises because of the presence of monopsonistic power in the factor market. So the first part is termed as monopsonistic exploitation of labour. Again, second part i.e NEPS arises because of the presence of monopoly power in the product market. So, it is termed as monopolistic exploitation of labour. So, we can conclude that, when there is imperfections both in the factor and product markets, there is double exploitation of labour.



CHECK YOUR PROGRESS

Q.4: State the condition of equilibrium of the firm under perfect competition according to the the marginal productivity theory of distribution. (Answer in about 20 words)

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Q.5: Can we simply laterally sum up the individual demand curves of firms to get the market demand curve for a variable factor under perfect competition? If not why? (Answer in about 40 words)

.....

.....

.....

Q.6: State True (T) or False (F):

(a) In the labour market the presence of a monopsony implies that Average Wage (AW) curve will be increasing and Marginal Wage (MW) curve will be above the AW curve as more and more workers are employed. (True/False)

(b) When there is imperfections both in the factor and product markets, there is double exploitation of labour. (True/False)

4.7 MODERN THEORY OF DISTRIBUTION

We have already studied the marginal productivity theory of distribution. This theory tells us about how many labour an employer will engage at a given wage rate in order to maximize the profit. But the theory does not explain how that wage rate is determined. The marginal productivity theory was also criticized on the ground that it is a one sided theory since it ignores the supply side. Hence this theory is not considered as an inadequate explanation for the determination of the factor prices. To overcome these disadvantages the modern theory of factor pricing has been developed which provides satisfactory explanation for the determination of the factor prices. The theory states that price of a factor is determined by demand for and supply of that particular factor. According to the modern theory of distribution, the factor price is determined at a point where demand for and supply of factors intersect each other.

Assumptions:

- Every producer tries to get maximum profit.
- There is no intervention of the state in determining the price of the factors.
- Perfect competition exists both in the factor and product market.

Determination of demand curve for the factor of production:

The demand for a factor of production is known as derived demand. The productivity of a factor refers to the amount of contribution made by the factors in the process of production. The demand of the employer for a factor depends on its marginal revenue productivity and the quantity of the factor that a firm will employ will depend on the prevailing wage rate. More labour will be employed if wages are low and less if wages are high. If the demand for good produced by the factor is more, than its demand will also increase and vice versa.

The following diagram will illustrate the position of a firm regarding the employment of the factor, say, labour.

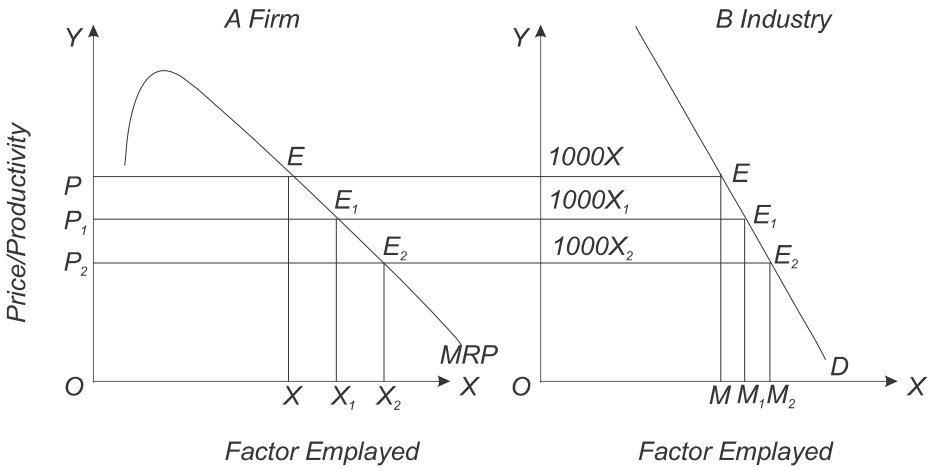


Fig. 4.8: Derivation of Demand Curve

When the price of the factor is OP, the firm is in equilibrium at the point E and the demand for the factor is OX; similarly, at OP_1 price, the demand is OX_1 and at OP_2 price rate the demand is OX_2. MRP is the demand curve for a factor of production for an individual firm. The demand curve of an industry is the sum total of the marginal productivity curve of all individual firms . DD is the demand curve for the industry. We assume that there are 100 firms in the industry. At OP price rate the demand of an individual firm is OX, but the demand of the whole industry at the same price is OM, which is equal to 100 OX. Similarly, at OP_1 price, the demand of factor for the whole industry is OM_1, which is equal to 100 OX_1 and at OP_2 price, the demand of factor for the entire industry is OM_2, which is equal to 100 OX_2.The demand curve of the entire industry is DD which is downward sloping to the right. This is because of the law of diminishing marginal productivity, which states that the more a factor is employed the lower is the marginal productivity.

Determination of Supply curve for the factor of production:

For the supply side, the supply curve of a factor depends on various factors such as size and composition of population, geographical distribution, education, labour efficiency and so on. In case of labour if the wage goes up, supply of labour will increase but demand for labour will decrease. Again if wage rate decreases, the supply of labour decreases but demand for labour increases. Hence, the supply curve of a factor rises from left to right upward.

Interaction of demand and supply:

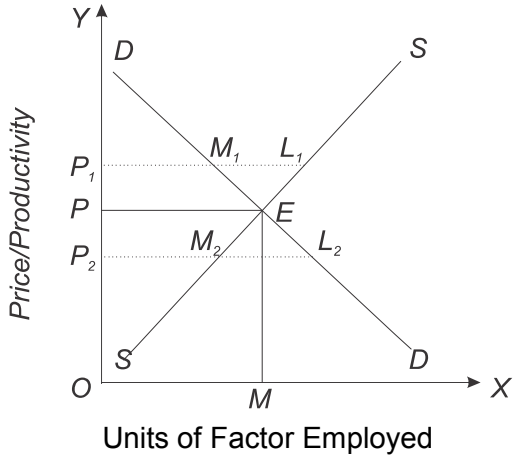


Fig. 4.9: Determination of Equilibrium

The price of the factor is established at the point E where demand and supply are in equilibrium. This equilibrium is at the point of intersection of the demand and supply curves. In the diagram they intersect at the point E, and the price of the factor thus determined will be OP. At OP_1 price level, demand for labour is P_1M_1 which is less than the supply P_1L_1 .In this case, the competition among the sellers will tend to bring down the price to OP. On the other hand, at OP_2 price level the demand for labour P_2L_2 is greater than supply P_2M_2, hence the price will tends to grow up to OP where demand and supply will be equal.

This is how the price of a factor of production in the factor market is determined by the interaction of the forces of demand and supply of the factors.

4.8 PRODUCT EXHAUSTION PROBLEM

The marginal productivity theory of distribution created a controversy among the economists regarding whether the total value of the product will be exhausted if factors are paid according to their marginal product. In other wards, the controversy was whether the total factor payments in the economy will add up to the value of the total output produced in the economy. And this is known as the adding up problem or product exhaustion problem.

Now, assuming that there are only two factors of production viz. labour (L) and capital (K) and Q is the quantity of output produced in the

economy in physical terms and P be its price, then the adding up problem can be expressed as,

P. Q = w.L + r.K ---------- (1)

Where w is price of labour & r is price of capital.

In equation (1) L.H.S. represents value of output the R.H.S. represents the total factor payments.

i.e. Value of output = Payment to the labour + Payment to the capital

Now, dividing both side of the equation (1) by PQ, we get,

$$\Rightarrow \frac{PQ}{PQ} = \frac{wL}{PQ} + \frac{rK}{PQ}$$

$\Rightarrow$ 1=wL/PQ+rK/PQ

$\Rightarrow$ 1 = share of L + share of K.

Therefore,the adding up problem also shows that the factor shares add to unity.

The marginal productivity theory of distribution tells us that each factor should be paid according to its marginal product. Accordingly, when all factors of production are remunerated according to their marginal product, the total product will be exactly exhausted without any surplus or deficit. If MPP_L is the marginal physical product of labour and MPP_k is the marginal physical product of capital, then according to adding up problem,

$Q = MPP_L.L + MPP_k.K$ ..... (2)

Leonhard Euler, a Swiss mathematician has shown that the equation (2) will hold if there is constant returns to scale in production or the production function is linearly homogeneous.. This proposition has came to be known as the Euler's theorem.

Now, let us prove or establish the Euler's theorem by using the case of a typical Cobb Douglas production function which is subject to constant returns to scale in production. The typical expression of a Cobb Douglas production function is

$$Q = AK^{\alpha}L^{\beta}(\alpha + \beta = 1) \quad \text{----------(3)}$$

where, Q – quantity; A – efficiency parameter; L – labour; K - capital and α ε β are co-efficients

Now, differenciating equation (3) with respect to L we get,

$$MPP_L = \frac{\delta Q}{\delta L} = \frac{\delta}{\delta L}(AL^{\alpha}K^{\beta})$$

$$= A\alpha L^{\alpha-1}K^{\beta}$$

Multiplying L with both side of the above equation we get,

$$L.MPP_L = A\alpha L.L^{\alpha-1}K^{\beta}$$

$$= \alpha A L^{\alpha} K^{\beta} \text{ ----------(4)}$$

Again, differenciating equation (3) with respect to K we get,

$$MPP_K = \frac{\delta Q}{\delta K} = \frac{\delta}{\delta K}(AL^{\alpha}K^{\beta})$$

$$= AL^{\alpha}\beta K^{\beta-1}$$

Multiplying K with both side of the above equation we get,

$$K.MPP_K = A\beta L^{\alpha}K^{\beta-1}.K$$

$$= \beta AL^{\alpha}.K^{\beta} \text{ ----------(5)}$$

Adding equation (4) and (5), we get,

$$\therefore L.MPP_L + K.MPP_K = (\alpha+\beta)AL^{\alpha}K^{\beta} = Q \quad (\text{Since } \alpha+\beta=1)$$

The above clearly establishes that the equation (2) holds.

This proves that if the production function is linearly homogeneous or homogeneous of degree one, then with Euler's theorem it is easy to demonstrate that when each factor is paid according to its marginal product, the total product will be fully exhausted.



CHECK YOUR PROGRESS

Q.7: What is meant by modern theory of distribution? (Answer in about 20 words)

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Q.8: Briefly explain the adding up problem. (Answer in about 50 words)

.....

.....

Q.9: State the Euler's theorem. (Answer in about 20 words)

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.....

Q.10: State True (T) or false (F):

(a) The adding up problem also requires that the factor shares add to unity. (True/False)

(B) According to adding up problem, Q = MPPL.L + MPPk.K. (True/False)



4.9 LET US SUM UP

- Production of goods and services in an economy is the result of joint efforts of factors of production. Factor pricing is concerned with the analysis of how and in what manner the factors of production get remunerated in such production process for their contribution.
- According to the marginal productivity theory, under certain conditions, the variable factors are paid according to their marginal product.
- Marginal revenue product (MRP) is the increase in the total value of the product as a result of the employment of an additional unit of a variable factor, when quantity of other factors remaining the same.
- Value of marginal product (VMP) is obtained by multiplying the price (average revenue/AR) of the product of the firm with the marginal physical product (MPP) of the variable factor.
- Under perfect completion : MRP = VMP
- The Equilibrium of the firm under perfect competition can be discussed by the marginal productivity theory of distribution.
- In case of imperfection the factors determining the demand and supply are different.Since here, both the labour and product markets are

involved, we get the following combination of imperfections in the market–

(a) Perfect competition in the product and imperfect competition in the factor market.

(b) Imperfect competition in the product market and perfect competition in the factor market.

(c) Imperfect competition both in product and factor market.

- The exploitation of labour is possible due to the presence of monopsony in the factor market and perfect competition in the product market, which is known as monopsonistic exploitation of labour.
- The exploitation of labour is possible due to the presence of monopoly in the product market and perfect competition in the factor market, which is known as monopolistic exploitation of labour.
- When there is imperfections both in the factor and product markets, there is double exploitation of labour.
- According to the modern theory of distribution the factor price is determined at a point where demand for and supply of factors intersect each other.
- If MPP_L is the marginal physical product of labour and MPP_K is the marginal physical product of capital, then according to adding up problem, $Q = MPP_L.L + MPP_K.K$



4.10 FURTHER READING

1) Ahuja, H. L. (2000). *Advance Economic Theory: Microeconomic Analysis.* New Delhi: S. Chand & Company Ltd.

2) Jhingan, M. L. (2005). *Microeconomic Theory.* Delhi: Vrinda Publications (P) Ltd.

3) Koutsoyiannis, A. (1979). *Modern Microeconomics.* London: Macmillan Press.

4) Maddala, G. S. & Ellen Miller (1989). *MicroEconomic Theory and Applications*. Singapore: Mcgraw-Hill.

5) Seth, M. L. (2004). *Refresher Course in Micro Economics.* Agra: Educational Publishers.



4.11 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q. No. 1: The marginal productivity theory of distribution was developed by host of economists like J.B. Clark, Wickstead, Marshall and others.

Ans to Q. No. 2: MRP is the increase in the total value of the product as a result of the employment of an additional unit of a variable factor, when quantity of other factors remaining the same

Ans to Q. No. 3: The term marginal productivity refers to the extra output gained by adding one unit of variable factor when all other factors of productions are remained constant.

Ans to Q. No. 4: According to the marginal productivity theory of distribution, the firm will be in equilibrium when the marginal revenue productivity (MRP) of any factor to the firm equals its marginal cost (MC).

Ans to Q. No. 5: No, we cannot simply laterally sum up the individual demand curves of firms to get the market demand curve for a variable factor under perfect competition, as the changes in factor price will bring about changes in the marginal productivity of the factor.

Ans to Q. No. 6: (a)True (b) True

Ans to Q. No. 7: The modern theory of distribution states how the price of the factor is determined through demand and supply of the factor of production.

Ans to Q. No. 8: The marginal productivity theory of distribution created a controversy among the economists regarding whether the total value of the product will be exhausted if factors are paid according to their marginal product. In other wards, the controversy was whether the total factor payments in the economy will add up to the value of the total output produced in the economy. And this is known as the adding up problem or product exhaustion problem.

Ans to Q. No. 9: According to the Euler's theorem if the production function is linearly homogeneous or homogeneous of degree one and each factor of production is paid according to its marginal product, then the total product will be fully exhausted.

Ans to Q. No. 10: (a) True (b) True



4.12 MODEL QUESTIONS

A) Short Questions (Answer each questions in about 150 words)

Q.1: State the importance of factor pricing.

Q.2: 'Each factor receives an income equal to its marginal product'. Comment.

Q.3: Define the marginal physical product of a factor of production.How is the marginal revenue product related to the marginal physical product?

Q.4: Distinguish between the value of marginal product and marginal revenue product of a factor. Under what conditions will they be equal?

Q.5: Briefly explain the product exhaustion problem.

B) Long Questions (Answer each questions in about 300 - 500 words)

Q.1: Critically explain the marginal productivity theory of distribution.

Q.2: Explain the equilibrium of the firm in the factor market under perfect competition.

Q.3: Explain the equilibrium of the firm in the factor market under imperfect competition.

Q.4: Explain the modern theory of distribution.

Q.5: 'When each factor is paid according to its marginal productivity, the total product is exhausted'. Examine critically.

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UNIT 5: INTER-TEMPORAL CHOICE

UNIT STRUCTURE

5.1 Learning Objectives

5.2 Introduction

5.3 The Concept of Present Value

- 5.3.1 Computation of Present Value
- 5.3.2 Present Value of an Income Stream
- 5.3.3 Net Present Value and Choice of Investment Project

5.4 Internal Rate of Return

5.5 Inter-Temporal Consumption Decisions

- 5.5.1 Investment in Human Capital

5.6 Inter-Temporal Production Decisions

- 5.6.1 The Behaviour of Market Price

5.7 Let Us Sum Up

5.8 Further Reading

5.9 Model Questions

5.10 Answers to Check Progress

5.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- explain the inter- temporal choice of investment decisions
- understand computation of NPV and IRR and their relevance to choice of investment project
- explain the individual consumption decisions of inter-temporal choice
- explain the production decision under inter-temporal choice pattern with behaviour of market price.

5.2 INTRODUCTION

When a firm decides whether to build a factory or purchase machines, it must compare the outlays it would have to make now with the additional

profit that the new capital will generate in the future. To make this comparison, it must address the following question: how much are future profits worth today? This problem does not arise while hiring labour or purchasing raw materials. To make these choices, the firm need only compare its current expenditure on the factor. Investment decisions involve comparing an outlay today with the profits that will be received in the future. Individuals also make decisions involving costs and benefits of investments occurring at different points of time, and the same principles apply. The answers to these investment and production decisions depend in part on the interest rate that one pays or receives when borrowing or lending money.

5.3 THE CONCEPT OF PRESENT VALUE

Present Value is the concept that states that an amount of money today is worth more than that same amount in the future**.** Money received today is valued more than money receivable tomorrow. Cash in hand is valued more because it gives a) liquidity and b) opportunity to invest it and earn return (interest) on it. This is called the time value of money. The time lag between the investment and the first return from the investment is called time lag. During this time lag, the investor losses interest on the expected incomes. This implies that a rupee received today is worth more than a rupee receivable at some future date. Or conversely, a rupee expected to be received one year hence is worth less than a rupee today. In the contest of the time value of money, the present value of a future income is lower than the value of the same amount received today.

5.3.1 Computation of Present Value

Suppose an amount of X_0 is invested for a period of one year at a compound interest rate. At the end of the year, the total receipt, say X_1 can be expressed as:

$$X_1 = (X_0 + r X_0) = X_0 (1+r) \quad \text{.....(i)}$$

Where r refers to the rate of interest.

Equation (i) shows that X_0 increases at the rate of (1+r) to take the value X_1 after one year. It implies that if X_1 is discounted at the same rate of interest, it gives its Present Value (PV). The formulae for computing the present value is give below:

$$\text{PV of } X_0 = \left(\frac{X_1}{1+r}\right) = X_1\left(\frac{1}{1+r}\right) \quad \text{.....(ii)}$$

In equation (ii), $\left(\frac{1}{1+r}\right)$ is the discount rate for one year. Given the rate of interest (the numerical value for r), any income receivable after one year can be discounted to its present value. For example, the present value of an income of Rs.500 expected after one year at 10 percent interest per annum (where r=0.10) can be calculated as

$$PV = 500\left(\frac{1}{1+0.10}\right) = 454.55$$

It means that at 10 percent interest rate, the present value of Rs.500 expected after one year is Rs.454.55. The discount rate (d_2) for an income receivable after 2 years will be $\left(\frac{1}{(1+r)^2}\right)$ and for an income receivable after 3 years, $d_3 = \left(\frac{1}{(1+r)^3}\right)$ and so on. The formulae for discount rate for the n^{th} year is given as

$$d_n = \frac{1}{(1+r)^n} \quad \text{.....(iii)}$$

The formulae for calculating Present Value (PV) of an amount receivable in the n^{th} year is given as

$$PV = X_n\left[\frac{1}{(1+r)^n}\right] \quad \text{..... (iv)}$$

Or, $PV = \frac{X_n}{(1+r)^n}$

5.3.2 Present Value of an Income Stream

The concept of present value is associated with the time value of money. The idea is that an amount of money today is worth more

than the same in the future. This happens due to that fact that prices of goods and services always go on rise and we have limited amount of income of the other. The formulae for calculating the Total Present Value (TPV) of an income stream of annual return (R) over n years is given as

$$TPV = \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \frac{R_3}{(1+r)^3} + \ldots\ldots + \frac{R_n}{(1+r)^n} \quad \text{.....(v)}$$

$$= \sum_{j=1}^{n} \frac{R_n}{(1+r)^n} \quad \text{.....(vi)}$$

For example, if a stream of cash flows (or the return) consists of + Rs.100 at the end of period one, and the interest rate is 5% (0.05) then the present value of the Cash Flow will be:

$$TPV = \frac{R_n}{(1+r)^n} = \frac{100}{(1+0.05)^1} = \frac{100}{1.05} = \text{Rs. } 95.24$$

5.3.3 Net Present Value and Choice of Investment Project

Present value adjusted for the cost of investment provides the basis of investment decisions. The investment decision is taken on the basis of net present value. It may be defined as the difference between the Present Value (PV) of an income stream and the cost of investment (C), that is

$$NPV = PV - C$$

$$= \sum_{j=1}^{n} R_n \left[\frac{1}{(1+r)^n}\right] - C$$

Where C is the total cost of investment without any recurring expenditure

The investment decision rules can be specified as below:

- If NPV>0, the project is acceptable
- If NPV=0, the project is accepted or rejected on non economic considerations

- If NPV<0, the project is rejected

If investment is a recurring expenditure, the Total Present Cost (TPC) for n years can be calculated in the same manner as present value of an income stream is calculated, that is

$$TPC = \sum_{j=1}^{n} \frac{C_n}{(1+r)^n} \quad \text{.....(vii)}$$

And then, $NPV = \sum_{j=1}^{n} \frac{R_n}{(1+r)^n} - \sum_{j=1}^{n} \frac{C_n}{(1+r)^n}$

$$= \sum_{j=1}^{n} \frac{R_n - C_n}{(1+r)^n} \quad \text{.....(viii)}$$

The investment decision rule in this case is the same as given above.



CHECK YOUR PROGRESS

Q.1: What is meant by NPV? (Answer in about 20 words)

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Q.2: Briefly explain how investment decisions are taken based on NPV. (Answer in about 40 words)

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5.4 INTERNAL RATE OF RETURN

Internal rate of return (IRR) refers to the percentage rate of return implicit in the flows of benefits and costs of projects. IRR is the discount rate at which the net present value of the future cash flows is equal to the initial investment, and it is also the discount rate at which the total present value of costs equals the total present value of the benefits. The Internal

rate of Return (IRR) is also called Marginal Efficiency of Investment (MEI). If a one year project costing Rs.100 million yields Rs.120 million at the end of the year, then its IRR (r) can be obtained as follows:

$$\frac{120 \text{ million}}{1+r} = \text{Rs. } 100 \text{ million}$$

$$= (1+r)\ 100 = 120$$

$$1+r = \frac{120}{100}$$

$$r = 1.2-1$$

$$r = 0.2$$

The IRR of this project is 0.20 or 20 percent. No other value of r can equate the NPV of the project with its cost.

The IRR or MEI is defined as 'the rate of interest or return which renders the discounted present value of its expected future marginal yields exactly equal to the investment cost of project'. In other words, 'IRR is the rate of return (r) at which the discounted present value receipts and expenditures are equal.' The IRR of a project yielding a stream of returns over n years and involving different investment costs can be obtained by using the formula given in the following equation:

$$\sum_{j=1}^{n} \frac{R_n}{(1+r)^n} = \sum_{j=1}^{n} \frac{C_n}{(1+r)^n}$$

This criterion is basically the same as Keynesian MEI which is theoretically superior to other criteria.

5.5 INTER-TEMPORAL CONSUMPTION DECISIONS

Consumers also face some situations when they purchase durable goods, such as the cars or major appliances. Suppose, one has to decide to buy a new car from the market; if s (he) keeps the car or six or seven years, most of the benefits will occur in the future. Therefore, s (he) must compare the future flow of net benefits from owning the car with the purchase price.

The main benefit from owning a car is the flow of transportation services it provides. The value of those services differs from consumer to

consumer. Let's assume that the consumer values the service at S per year and total operating expense (insurance, maintenance etc.) is E per year, that the car costs Rs.20000 and that after six years, its resale value will be Rs. 4000. The decision to buy the car can then be frames in terms of present value:

$$NPV = -2000 + (S-E) + \frac{(S-E)}{(1+R)} + \frac{(S-E)}{(1+R)^2}$$

$$+.....+ \frac{S-E}{(1+R)^6} + \frac{4000}{(1+R)^6}$$

Here, the discount rate is the opportunity cost of money. If the consumer already has Rs. 20000 and does not need a loan, the correct discount rate is the return that could be earned by investing the money in another asset- say, a savings account or a government bond. On the other hand, if the consumer is in debt, the discount rate would be the borrowing rate that he or she is already paying. Because this rate is likely to be much higher than the interest rate on a bond or savings account, the NPV of the investment will be smaller.



CHECK YOUR PROGRESS

Q.3: What is meant by IRR? (Answer in about 20 words)

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5.5.1 Investment in Human Capital

Human capital is the knowledge, skills and experience that make an individual more productive and thereby able to earn a higher income over a lifetime. Now, the question is how an individual should decide whether to invest in human capital or not. Here, net present value can be applied. For example, one needs to decide whether to invest in going to college for four years after high school or skip college and go to work instead. Let us analyse the situation purely on financial basis and ignore other factors. An important benefit of

college is the ability to earn a higher salary throughout your working life. In India, a college student graduate will on average earn about Rs.20000 per year more than a high school graduate. In practice, the salary differential is largest during the first 5 to 10 years following the college graduation and then becomes smaller. For simplicity, however, we will assume that this Rs.20000 salary differential persists for 20 years. In that case, the NPV of investing in a college education is

$$NPV = -40 - \frac{40}{1+R} + \frac{40}{(1+R)^2} - \frac{40}{(1+R)^3} + \frac{20}{(1+R)^4} + \ldots\ldots + \frac{20}{(1+R)^{23}}$$

We shall use real discount rate. In this case, a reasonable real discount rate would be about 5 percent. This rate would reflect the opportunity cost of money for many households- the return that could be made by investing in assets other than human capital. The NPV shall come out to be Rs. 66000. With 5 percent discount rate, investing in a college education is a good idea, at least as a purely financial matter.

5.6 INTER- TEMPORAL PRODUCTION DECISIONS

Production decision often has inter- temporal aspects- production today affects sales or costs in the future. Suppose, one of your rich friend gives you an oil well. The well contains 1000 barrels of oil that can be produced at a constant average and marginal cost of Rs.10 per barrel. Now the question is whether to produce oil today or save it for future. The decision depends not on the current profit level but on how fast you expect the price of oil to rise. Oil in the ground is like money in the bank. You should keep it in the ground only if it earns a return at least as high as the market interest rate. If you expect the price of oil to remain constant or rise very slowly, you would be better off extracting and selling all of it now and investing the proceeds. But, if you expect the price of oil to rise rapidly, you should leave it in the ground. The value of oil in your barrel of oil in your well is equal to the price of oil less the Rs.10 cost of extracting it. (This is the profit you can obtain by extracting and selling each barrel). This value must rise at least as fast as the rate of interest for you to keep the oil. Your production

decision rule is therefore: Keep all your oil if you expect its price less its extraction cost to rise faster than the rate of interest. Extract and sell all of it if you expect price less cost to rise at less than the rate of interest. What if you expect price less cost to rise at exactly the rate of interest? Then you would be indifferent between extracting the oil and leaving it in the ground. Letting, P_t be the price of oil this year, P_{t+1} the price next year, and c the cost of extraction, we can write this production rule as follows:

If $(P_{t+1} - C) > (1+R)(P_t - C)$, Keep the oil in the ground

If $(P_{t+1} - C) < (1+R)(P_t - C)$, sell all the oil now

If $(P_{t+1} - C) = (1+R)(P_t - C)$, makes no difference

Given our expectation about the growth rate of oil prices, we can use this rule to determine production. But the question arise how fast should we expect the market price of oil to rise.



CHECK YOUR PROGRESS

Q.4: Define human capital. (Answer in about 20 words)

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5.6.1 The Behaviour of Market Price

Suppose there are no OPEC cartels and the oil market consisted of many competitive producers with oil wells like our own. Then, one needs to determine how quickly oil prices are likely to rise by considering the production decisions of other producers. If other producers want to earn the highest possible return, then one needs to follow the production rule as stated above. This means that price less marginal cost must rise equally with the rate of interest. Suppose, price less cost is to rise faster than the rate of interest- in that case no one would sell any oil. Inevitably, this would drive up the current price. If, on the other hand, price less cost were to rise at a rate less than the rate of interest, everyone would try to sell all of their oil immediately, which would drive the current price down.



5.7 LET US SUM UP

- When a firm makes a capital investment, it spends money now in order to earn profits in the future. To decide whether the investment is worthwhile, the firm must determine the present value of future profits by discounting them.
- The discount rate that a firm uses to calculate the NPV for an investment should be the opportunity cost of capital – that is the return the firm could earn on a similar investment.
- Consumers are also faced with investment decisions that require the same kind of analysis as those of firms.
- When deciding whether to buy a durable good like a car or a major appliance, the consumer must consider the present value of future opportunity costs.
- Investments in human capital- the knowledge, skills and experience that make an individual more productive and thereby able to earn a higher income in the future- can be evaluated in much the same way as other investments.
- Investing in further education, for example, makes economic sense if the present value of the expected future increases in income exceeds the present value of the costs.



5.8 FURTHER READING

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3) Alexander, G. J., Sharpe, W. F. & Bailey, J. V. (2007). *Fundamentals of Investments.*, Singapore: Pearson Education.

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5.9 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q. No. 1: NPV is the difference between the Present Value (PV) of an income stream and the cost of investment (C), that is NPV = PV– C.

Ans to Q. No. 2: The investment decisions are taken based on the following conditions of NPV:

- If NPV>0, the project is acceptable
- If NPV=0, the project is accepted or rejected on non economic considerations
- If NPV<0, the project is rejected

Ans to Q. No. 3: IRR is the rate of return (r) at which the discounted present value receipts and expenditures are equal.

Ans to Q. No. 4: Human capital is the knowledge, skills and experience that make an individual more productive and thereby able to earn a higher income over a lifetime.



5.10 MODEL QUESTIONS

A) Short Questions (Answer each questions in about 150 words)

Q.1: Why cash in hand is valued more?

Q.2: What is meant by Present Value?

Q.3: Define time value of money.

Q.4: Briefly explain how present value is computed.

B) Easy type Questions (Answer each question in about 300-500 words)

Q.1: How the decisions are made on the basis of NPV? Explain

Q.2: Explain inter-temporal consumption decision with a suitable example.

Q.3: Explain inter-temporal production decision with a suitable example.

Q.4: Discuss how investment decisions are taken based on the different criterion.

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UNIT 6: CHOICE UNDER UNCERTAINTY

UNIT STRUCTURE

6.1 Learning Objectives

6.2 Introduction

6.3 Individual Behaviour Towards Risk

- 6.3.1 Risk-Averse
- 6.3.2 Risk-Loving
- 6.3.3 Risk-Neutral
- 6.3.4 Preference Axioms

6.4 Expected Utility and Certainty Equivalence Approaches

6.5 Mean-Variance Analysis

6.6 Portfolio Selection

6.7 Let Us Sum up

6.8 Further Reading

6.9 Answers to Check Your Progress

6.10 Model Questions

6.1 LEARNING OBJECTIVES

After going through this unit, you will be able to-

- learn about individual behaviour towards risk and various types of risk an individual can possess
- identify economic agents as being risk adverse, risk neutral and risk seeker/risk lover
- know the pivotal importance of the expected utility theorem and understand the important concepts like certainty equivalence
- understand the mean variance analysis and portfolio selection in the risk analysis.

6.2 INTRODUCTION

The behaviour of economic agents under conditions of certainty can be seen easily, i.e., the situation in which consequences of any choice

made was fully known beforehand. The traditional theory of consumer behaviour does not include an analysis of uncertain situation. Von Neumann and Morgenstern showed that under some circumstances it is possible to construct a set of numbers for a particular consumer that can be used to predict her choices in uncertain situations. However, there is a great controversy that has centred on the question of whether the resulting utility index is ordinal or, cardinal. It will be shown that Von Neumann – Morgenstern utilities possess at least some cardinal properties. Traditional consumer behaviour analysis is unrealistic in the sense that it assumes actions the consumer are followed by determinate consequences which are knowable in advance. For instance, all automobiles of the same model and produced in the same factory will not always have the same performance characteristics. As a result of random accidents in the production process, some substandard automobiles could be occasionally produced and sold. The consumer has no way of knowing ahead of time whether the particular automobile, which she purchased, is of standard quality or not. Let A represent the situation in which the consumer possesses a standard quality automobile and B be a situation in which she does not. Again, let there be C, in which she possesses a substandard automobile. Assume that the consumer prefers A to B and B to C. That is, not having a car is assumed preferable to owning a substandard one because of the nuisance and expense involved in its uptake. Present her with a choice between two alternatives: (1) she can maintain the status quo and have no car at all. This is a choice with certain outcome i.e., the probability of the outcome equals unity. (2) She can obtain a lottery ticket with a chance of winning either a satisfactory automobile (alternative A) or an unsatisfactory one (alternative C). The consumer may prefer to retain her income (or money) with certainty, or she may prefer the lottery ticket with dubious outcome, or she may be indifferent between them. Her decision will depend upon the chances of winning or losing in this particular lottery. If the probability of C is very high, she might prefer to retain her money with certainty; if the probability of A is very high, she might prefer the lottery ticket. The triplet of numbers (P, A, B) is used to denote a lottery offering outcome A with probability $0<P$.

Once we bring in choices to be made under uncertainty, the theoretical prescriptions by far remain an uncovered domain. As our income levels fluctuate, price we pay change or health conditions worsened, we need to model choices accounting for such possible events. Also, see that most of decisions we take are forward looking: planning a holiday trip or to buy insurance are examples. Decisions on these are usually taken on the basis of our beliefs about what is the optimal plan for present and future. Thus, these choices are made in the context of uncertainty. Consequently, there is a risk that the assumptions made in our plans may not materialise. Anticipating such eventualities we resort to contingencies and probabilities. That is, if we want a realistic model of choice, it would be necessary to include in our models the effects of uncertainties. This unit discusses certain important ideas pertaining to uncertainties.

6.3 INDIVIDUAL BEHAVIOUR TOWARDS RISK

Individuals can be grouped into three categories according to their attitude towards risk given by their respective utility functions. These categories are risk averse, risk loving and risk neutral. Consider the case of a coin toss, Rs.10 is earned if heads, and Rs.20 if tails. The expected value is (0.5 x 10) + (0.5 x 20) = Rs.15. Different risk attitudes are explained below.

6.3.1 Risk-Averse

If an individual's utility from the expected value of the lottery exceeds the expected utility from the lottery, he is categorised as risk-averse. Risk-averse behaviour is usually enraptured by a concave utility function, for instance, a logarithmic function. For the above coin toss experiment, we consider a risk-averse person whose utility function is of the form U (E) = log (E), where E is the outcome. The expected utility is 0.5 x log (10) + 0.5 x log (20). This equals to 1.15. The utility of the expected value of this coin toss on the other hand is log (15) = 1.176

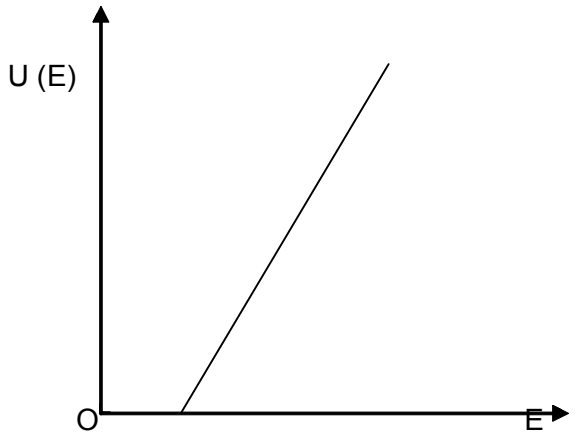


Fig. 6.1: Utility Function of a Risk-Averse Individual

6.3.2 Risk-Loving

If an individual's utility of the expected value of the lottery is less than his expected utility from the lottery, he is the risk-loving type. A convex utility function depicts risk-loving behaviour; an exponential function is a good example for this case. Let the utility function take the form $U(E) = E^2$. The expected utility is $0.5\times 102 + 0.5 \times 202 = 250$; it needs to be compared with the utility of the expected value of the coin toss which is $15^2 = 225$

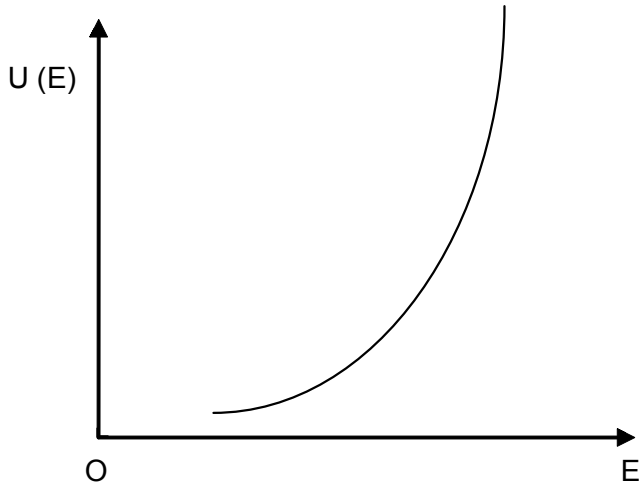


Fig. 6.2: Utility Function of a Risk-Loving Individual

6.3.3 Risk-Neutral

If an individual's utility of the expected value of a lottery is equal to his expected utility from the lottery, he is said to be risk-neutral.

Risk-neutral behaviour is captured by a linear utility function, for instance U (E) = 2E. The expected utility in this case is given by (0.5 x 2 x 10) + (0.5 x 2 x 20) = 30, this is equal to the utility of the expected value of the lottery itself, 2 x 15 = 30.

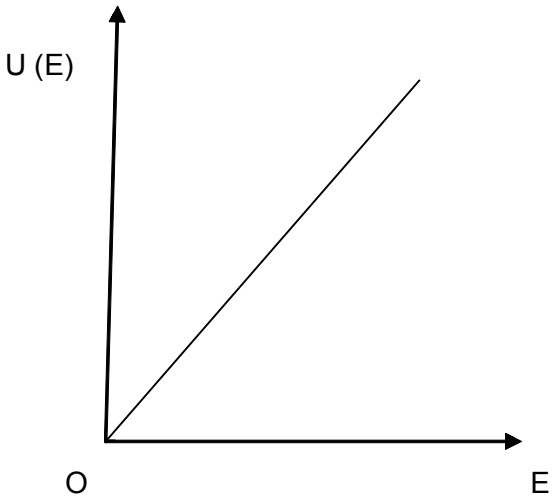


Fig. 6.3: Utility Function of a Risk-Neutral Individual

6.3.4 Preference Axioms

It is possible to construct a utility index which can be used to predict choice in uncertain situations if the consumer conforms to the following five axioms:

- **Axiom of Complete-ordering**: For the two alternatives A and B one of the following must be true: the consumer prefers A to B, she prefers B to A, or she is indifferent between them. The consumer's evaluation of alternatives is transitive: if she prefers A to B and B to C, she prefers A to C.
- **Axiom of Continuity:** Assume that A is preferred to B and B to C. The axiom asserts that there exists some probability P, $0<P<1$.
- **Axiom of Compound-lottery**: Let $L_1 = (P_1, A, B)$, and $L_2 = (P_2, L_3, L_4)$, where $L_3 = (P_3, A, B)$ and $L_4 = (P_4, A, B)$, be a compound lottery in which the prizes are lottery tickets. L_2 is equivalent to L_1 if $P_1 = P_2P_3 + (1-P_2) P_4$. Given L_2 the probability of obtaining L_3 is P_2. Consequently, the probability of obtaining A through L_2 is

P_2P_3. Similarly, the probability of obtaining L_4 is $(1-P_2)$, and the probability of obtaining A through L_4 is $(1- P_2) P_4$. The probability of obtaining A with L_2 is the sum of the two probabilities. The consumer evaluates lottery tickets only in terms of the probabilities of obtaining the prizes, and not in terms of how many times she is exposed to a chance mechanism.

These axioms are very general in nature, and it may be difficult to object to them on the grounds that they place unreasonable restrictions upon the consumer's behaviour. However, they rule out some types of plausible behaviour. Consider a person who derives satisfaction from the share of gambling. It is conceivable that there exists no P other than P=1 or P=0 for such a person, so that she is indifferent between outcome B with certainty and the uncertain prospect consisting of A and C; she will always prefer the "sure thing" to the dubious prospect. This type of behaviour is ruled out by the continuity axiom and the compound lottery axiom. The axioms have been developed for situations in which there are only two outcomes. Assuming that the pair-wise axioms hold, analysis is easily extended to cases with any number of outcomes. Let $L = (P_1,\ldots,P_n, A_1,\ldots,A_n)$ denote a lottery with n outcomes where $0<P_i<1$ is the probability of the outcome A_i and $\sum_{i=1}^{n} P_i = 1$



CHECK YOUR PROGRESS

Q.1: Distinguish between the categories - risk averse and risk loving. (Answer in about 40 words)

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Q.2: What is meant by the risk neutral? (Answer in about 30 words)

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6.4 THE EXPECTED UTILITY AND CERTAINTY EQUIVALENCE APPROACHES

Expected utility: Expected utility theory is used as a tool for analyzing situations where individuals make a decision without knowing which outcomes may result from that decision, i.e., decision making under uncertainty. It can be calculated by taking the weighted average of all possible outcomes under certain circumstances, with the weights being assigned by probability on the basis of occurrence of a particular event.

Let us take an utility function U that has the expected utility property (for any gamble) E with possible outcomes $\{e_1, e_2,...,e_n\}$ and probabilities P_1, $P_2,...,P_n$, respectively, then the expected utility is given by: $U(E) = P_1U(e_1) + P_2U(e_2) + ... + P_n U(e_n)$. An individual chooses one gamble over another on the basis of its expected utility.

Most of the people are unwilling to participate in fair game. For example, offer of participating in a gamble is which a person has even chance of winning or losing of any amount is a fair game. Daniel Bernoulli has provided an explanation to this case. According to him, a rational individual will take decisions under risky and uncertain situations on the basis of expected utility rather than expected monetary value. He further contended that marginal utility of money to the individual declines as he has extra money. Under such circumstances, most individuals will not play the game that is he will not make a bet.

Certainty Equivalence: The Certainty equivalent is a guaranteed return that someone would accept now, rather than taking a chance on a higher, but uncertain return in future. An individual is indifferent between certainty equivalent and the probable amount of money with a certain probability in an uncertain and risky situation such as in a lottery ticket or gamble. An example will make the point clearer. Let us take an example of a lottery ticket whose first prize is Rs.5000 with a probability of 0.6 of winning and 0.4 of losing (getting a consolidation prize of Rs.10). We ask the individual X as to how much sure sum of money and the above lottery ticket with the given prize and probabilities he would be indifferent. Suppose the individual reveals that

he is indifferent between Rs.3000 and the above lottery ticket with the aforesaid prize and probabilities. Then, in this case, certainty equivalent is Rs.3000 with which one can calculate the marginal utility of Rs.3000.



LET US KNOW

St. Petersburg Paradox

Let's consider the case, wherein, a fair coin will be tossed repeatedly until tails turns up. If tails turns up on the n^{th} toss, the individual will receive Rs.2^n, i.e. if tails turns up on the 6^{th} toss, the person will receive Rs.2^6 = Rs.48.

The expected value of the gamble can be calculated by the usual procedure, the probability of the coin turning up tails on the n^{th} toss equals $1/2^n$. The expected value of the gamble would be:

(1/2) x 2 + (1/4) x 4 + (1/8) x 8 + (1/16) x 16 + ... = 1 + 1 + 1 + 1 + ...

This is an infinite series and expected value is infinity. It would be hard to find anyone who would want to participate in such a gamble with infinite expected value. This situation has been termed as St. Petersburg Paradox.



CHECK YOUR PROGRESS

Q.3: How Daniel Bernoulli resolved the St. Petersburg Paradox? (Write in 70 words)

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6.5 MEAN-VARIANCE ANALYSIS

Usually individuals prefer more assets to less. However, with risk aversion, risk is seen as a variation in return. So we can measure risk

through variance and represent preferences such that individuals make choice by looking at mean and variance. There is one section of risk analysis, which indicates that individual's choice among assets is decided by the mean and variance of the return.

Let us specify the following elements of the problem for an individual decision maker under uncertainty:

a) A set of states (1,2,,,,,s,...S) of nature. A state of nature is a complete description of a possible environment.For example, the possible states of nature may be 'rain', 'cloud sky' and 'clear sky'. The possible states of nature may not refer to weather conditions at all. For example, one may be interested in the economy and define a particular state of nature (such as high inflation rate, low employment rate, high fiscal deficit). In the simplest case, 'Nature' chooses which state of nature actually materializes.

b) A set of acts (1,2,...,x.....X) available to the individual. For example, the acts may be 'buy an umbrella' or 'not to buy an umbrella'.

c) A consequence function c(x,s) showing outcomes under all possible combinations of acts and states. For example, weather might materialize together with the act of the individual will determine his/her payoffs.

Agents who use the expected value rule to choose between alternatives are focusing only on the mean outcome and ignoring the dispersion of the values around mean. The dispersion of the value represents the risk associated with an alternative. One measure of dispersion that is widely used is the variance V:

$V(x) = \sum p_x [c(x,s) - Ec]^2$

To calculate variance, let us find out the differences of each value from the mean value, square these differences, weigh them by respective probabilities and finally sum them up over all possible states of nature. Sometimes, we also use the square root of V, which is called the standard deviation (s.d)

Since V or s.d measures the risk of an alternative, one method of taking a decision is to use mean variance analysis, which employs both the mean and the variance. Given two risky alternatives A and B, the mean-variance analysis says:

1. If alternative A has a higher expected value and a lower variance than B, then A should be chosen over B.
2. If both A and B have the same expected value, then the one with the lower variance should be chosen.
3. If both have identical variances, then the one with higher expected value should be chosen.



CHECK YOUR PROGRESS

Q.4: What is the difference between Standard Deviation and Variance? (Answer in about 30 words)

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6.6 PORTFOLIO SELECTION

Individuals try to reduce risk by diversification by which they choose a portfolio of assets. To examine the choice of an optimum portfolio, budget frontier helps us. A budget frontier represents the combinations of risk and return that are obtainable with the given available funds from mixed portfolios of any two assets, say, shares of Aditya Birla capital and Kotak industries limited. Suppose the expected returns from these assets are 20 percent and 10 percent respectively. If a portion W_i of the given funds are invested in Aditya Birla Capital and the remaining funds W_t are invested in Kotak industries limited, the expected return of the portfolio of these two assets is given by:

$r_p = W_i r_i + W_t r_t$ ..... (1)

Where,

r_p = expected return of the portfolios of two assets

r_i = the expected return of investment in Aditya Birla Capital

r_t = the expected return of investment in Kotak industries limited

W_i = the proportion of the given funds invested in Aditya Birla Capital

W_t = the remaining portion of the given funds invested in Kotak industries limited

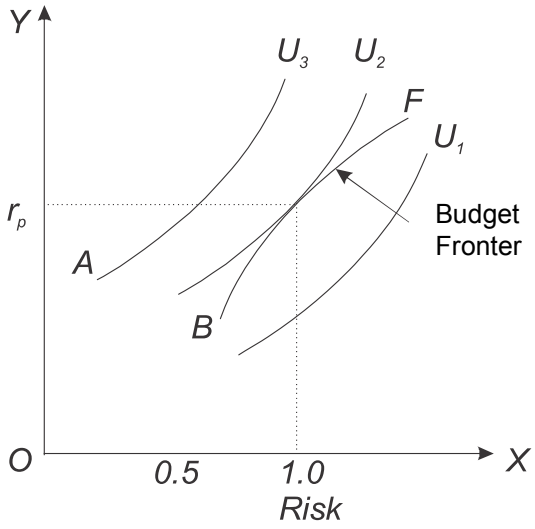


Fig. 6.4 : Portfolios Selection

Note that $W_i + W_t = 1$ and different portfolios of two assets involve different degrees of risk (σ) and also yield different returns. In the above fig. 6.4, BF shows the budget line which is the combinations or portfolios of two assets obtainable from the given funds. The budget frontier BF is tangent to the trade-off function curve U_2 at point E which represents the optimum portfolio of two assets that yield a return of and involve a risk (σ) of 1.0. The combination of two assets represented by E is an optimally diversified portfolio containing a mix of two assets. Suppose the individual has spent 40 percent of its investible funds on shares of Aditya Birla Capital and the remaining 60 percent on shares of Kotak industries limited. The rate of return on the portfolio can be calculated as follows:

$r_p = W_i r_i + W_t r_t$

$= 0.40 \times 20 + 0.60 \times 10$

$= 8+6 = 14$

Thus, the rate of return of the portfolio of the two assets is 14 percent in the above case.



CHECK YOUR PROGRESS

Q.5: Define Budget Frontier. (Answer in about 30 words)

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6.7 LET US SUM UP

- There are three groups of individuals on the basis of risk taking. They are- risk lover, risk neutral and risk averse.
- Their utility function varies on the basis of their relative risk behaviours. With wealth as the single argument of the utility function, the utility of the expected value of the outcome of an uncertain situation exceeds the expected utility of the outcome for a risk-averse consumer; i.e., her utility function is strictly concave. Similarly, risk lovers and risk neutrals have strictly convex and linear utility functions respectively.
- The mean variance analysis is a mathematical framework for assembling a portfolio of assets such that the expected return is maximised for a given level of risk.
- It is a formalisation and extension of diversification in investing, the idea that owning different kinds of financial assets is less risky than owning only one type. Its key insight is that an asset's risk and return should not be assessed by itself, but by now it contributes to a portfolio's overall risk and return. It uses variance of asset prices as a proxy for risk.
- Certainty equivalent is the amount of guaranteed cash that a person would accept instead of taking the risk of receiving a larger amount at a later date.
- The difference between the certainty equivalent and the amount an organisation must pay investors for the use of their money is thus risk differential. For example, when a start up company must pay investors a 15% return when the yield on U.S Treasury insurance is 2%, this means that investors must be paid the 13% differential because they perceive the investment to be a risky one.
- The certainty equivalent varies by investor. Since each person has a different tolerance for risk. For example, a person approaching retirement is more likely to have a high certainty equivalent, since he is less willing to put his retirement funds at risk.



6.8 FURTHER READING

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6.9 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q. No. 1: Risk averter is a type of individual who prefers a certain given income to a risky job with the same expected income. On the other hand, risk lover or risk seeker person prefers a risky outcome with the same expected income as a certain income.

Ans to Q. No. 2: If an individual's utility of the expected value of a lottery is equal to his expected utility from the lottery, he is said to be risk-neutral. Risk-neutral behaviour is captured by a linear utility function, for instance U (E) = 2E.

Ans to Q. No. 3: St. Petersburg paradox refers to the problem why most people are unwilling to participate in a fair game or bet. Daniel Bernoulli has provided a convincing explanation of the said behaviour. According to him, a rational individual will take decisions under risky and uncertain situations on the basis of expected utility rather than expected monetary value. He advocated that marginal utility of money to the individual declines

as he has more of it. Since, the individual behaves on the basis of expected utility from the extra money if he wins the game and the marginal utility of money to him declines as he has extra money, most individuals will not play the game. It means the individual will not make fair bet. In this way, Bernoulli resolved St. Petersburg paradox.

Ans to Q. No. 4: The Variance of a data-set measures the mathematical dispersion of the data relative to the mean. On the other hand, standard deviation as the square root of the variance gives a value is in the same units as the original values.

Ans to Q. No. 5: A budget frontier represents the combinations of risk and return that are obtainable with the given available funds from mixed portfolios of any two assets.



6.10 MODEL QUESTIONS

A) Short Questions(Answer each question in about 150 words)

Q.1: Discuss the individual attitude towards risk with suitable diagram.

Q.2: How individual is better off with a mixed investment? Examine in the light of portfolio investment approach

Q.3: If there are two risky alternatives, what are the important conclusions that can be made from Mean Variance analysis on the basis of expected value and respective variances?

B) Long Questions (Answer each question in about 300-500 words)

Q.1: Explain Expected utility and certainty equivalence with suitable example.

Q.2: How mean variance analysis is used in the choice under uncertainty? Explain.

Q.3: Diagrammatically explain Portfolio selection process associated with investment in risky assets.

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UNIT 7: THEORY OF WAGES

UNIT STRUCTURE

7.1 Learning Objectives

7.2 Introduction

7.3 Nominal Wages Versus Real Wages

7.4 Subsistence Theory of Wages

7.5 Marginal Productivity Theory of Wages - Its Limitations

7.6 Wages under Imperfect Competition

7.7 Exploitation of Labour and Trade Union

7.8 Wage Differential

7.9 Let Us Sum Up

7.10 Further Reading

7.11 Answers to Check Your Progress

7.12 Model Questions

7.1 LEARNING OBJECTIVES

After going through this unit, you will be able to-

- know what are real and nominal wages
- explain the marginal productivity theory of wages
- understand the determination of wages under imperfect market forms
- discuss the exploitation of labour and the role of trade union
- understand the causes of wage differentials.

7.2 INTRODUCTION

Wage is the income of the labour engaged in the process of production. It is the amount paid by any employer to a labour under contract for the services rendered by the labour. Wages may be in cash or in kind. Wages may be paid at definite time intervals, like monthly, weekly, daily, etc or may be paid per work done. Wages differ across people and across occupations, resulting in horizontal and vertical differences in wages. The

present chapter discusses at length as to how wage paid to a labour is determined under different market forms.

7.3 NOMINAL WAGES VERSUS REAL WAGES

Money wages or nominal wages is the amount of money paid to a worker for his services. It is the amount of wages in monetary terms. However, real wages measure the purchasing power of the wage received by a worker. It means the necessities, comforts and luxury that a labour can command over, in return of his services. It is a true indicator of a workers' prosperity or the standard of living he has access to, rather than the money wages. Accordingly, real wages depend on a number of factors like the nature and conditions of the work done. If there are included favourable, agreeable, pleasant facilities of lunch, recreation etc. real wages are good. In high paying but risky jobs, the real wages will be less. Again, the price level determines the purchasing power of money. Thus, it determines the real wages of a worker's money income. The additional facilities or extra incomes that a worker receives over his income increase his real wage.



CHECK YOUR PROGRESS

Q.1: Define real wages. (Answer in about 20 words)

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7.4 SUBSISTENCE THEORY OF WAGES

Different theories were put forward by different economist at different times to determine the wages of workers. One such theory is the subsistence theory of wages. Subsistence wages means the minimum wages needed for the physical sustenance of the workers. As per the classicist, it is the natural level of wages. The exponents of this theory of wages were the Physiocrats and the theory was further accepted by the German Economists who called it the 'Iron law of wages'. The theory considers wages to be determined as the price of any commodity in the long run. In the long run

price equals cost. Accordingly, the wages in the long run are nothing but the cost of production of labour. This theory is based on the following assumptions of long run, that in the long run subsistence wages prevail and that subsistence wages remain constant for long time.

The theory of subsistence wages states that wages tend to remain at the subsistence or bare minimum level. Even if wages rise due to rise in demand there will be a rise in population. This shall increase a competition for resources for which wages will shrink to subsistence level. Alternatively, if wages fall labours will starve. Thus, population and hence labour supply will fall. This will raise wages again to the subsistence level.

The theory has been criticized on many grounds. Firstly, rise in wages enhances standard of living for which population tends to decrease. The theory advocates a homogeneous price across all labour. This is not true. Again, it is based on the supply-side of labour. It is based on long run and do not answer to wage determination in a point of time. The theory may be true for underdeveloped countries and with rapid population growth. However, it shows us that wages cannot or should not fall beyond a minimum level.



CHECK YOUR PROGRESS

Q.2: What is meant by subsistence wages? (Answer in about 20 words)

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7.5 MARGINAL PRODUCTIVITY THEORY OF WAGES- ITS LIMITATIONS

The marginal productivity theory of wages determines the price of labour, an important factor of production, according to their marginal physical product. Marginal physical product or the marginal product of labour (MP_L) is the change in the total output when an additional unit of labour is employed. The essence of this theory is that a firm employs a factor (here,

labour) because of its productivity or the contribution it makes to the total output. Hence, the remuneration to the factor will also depend upon the same. Higher the productivity, higher is the income and so on.

The marginal productivity theory is based on the following assumptions:

- There exists perfect competition in the product market.
- There is perfect competition in the factor market.
- All units of labour are homogeneous.
- There is full employment.
- The law of variable proportions operates.

The marginal productivity theory consists in the determination of wages in the same way as the price of any other good, i.e. from the demand and supply schedules. Therefore, the first priority is to understand the nature of the demand and supply of labour to the individual firm and industry. Since there exists perfect competition in the factor market, the supply curve of labour to the individual firm is a horizontal straight line parallel to the X axis at the ongoing OW wage. This is shown in fig. 7.1. Now, a firm being a profit maximiser will hire labour as long as the additional unit of labour will add more to the total revenue than to the cost. Thus, the demand for labour for the individual firm will be the value of marginal product curve of labour (VMP_L). VMP_L curve shows the value of the output produced by an additional unit of labour employed and is given by MP_L*Price of the product.

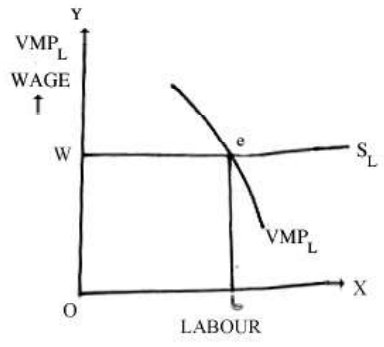


Fig: 7.1 : Equilibrium of a firm

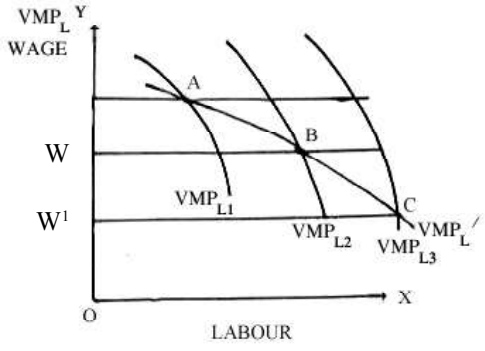


Fig: 7. 2 : Derivation of VMP_L curve

Thus, a firm will be in equilibrium at e, as in fig.7.1 where the VMP_L cuts the labour supply curve. Accordingly, in equilibrium the firm employs OL of labour at OW wage. To the left of e, labour cost less than the output produced and to the right of e cost of labour rises more than the value of the output. So, profit is maximum at e. The above discussion was based on the assumption that labour is the only variable factor. However, if all the factors of production are variable than the demand curve of labour is no longer the VMP_L curve. The demand for labour curve in this case will be a new curve obtained by joining the points of equilibrium of different wages and the corresponding VMP_L curves. This is the curve VMP_L shown in fig. 7.2. This too is a downward sloping curve. When wage falls from OW to OW^I and so on the VMP_L curve slopes down the original VMP_L curve to the right because of an outward shift in the marginal product of labour. This is because when wage falls there is not only an increase in output because with same total expenditure the firm can purchase more of labour and capital than before but also that the expenditure of the firm will increase in the context to raise profit. Thus, the MP_L will shift outward with increased amounts of all the factor inputs. For different wages we can generate points such as A, B, C etc. as in fig. 7. 2 and joining those points we can arrive at the demand for labour curve of the firm. This is sometimes referred to as the long run demand for labour curve as well.

For the industry, the equilibrium will be at the point where the market demand and supply curves will intersect. The market supply curve of labour is considered to be positively sloped as with increased wages more labour will be attracted. The market demand curve of labour is a downward sloping curve but it is not a simple horizontal summation of the individual VMP_L curves. This is because when wages fall all firms intend to demand more labour. Increased employment enhances output which further reduces price of the output. In the determination of the $VMP_{L,}$ price of the output is an important component. Accordingly, as wage falls VMP_L falls. This is shown in fig. 7. 3.

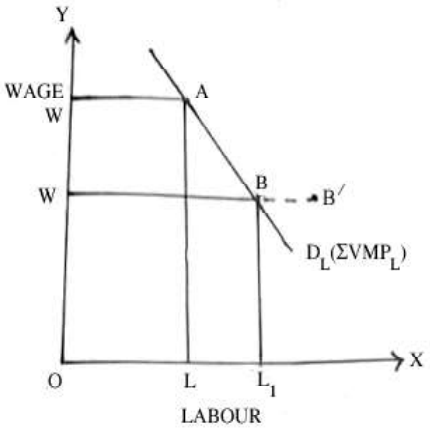


Fig: 7. 3 : Market demand curve

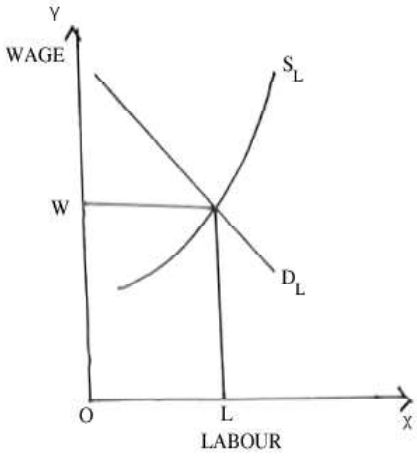


Fig: 7. 4 : Market equilibrium

If the change in output price is ignored, we will have a point B/ on the market demand for labour curve. But the appropriate market demand will be AB as shown in the figure 7.3. In fig. 7.4, we have the market demand curve of labour D_L and the market supply curve of labour S_L. The two intersects at point e which is the point of equilibrium. Thus, OW wage and OL employment is determined at equilibrium.

The limitations of this theory are discussed as below:

- The theory is based on the unrealistic assumption that all units of labour are homogeneous; there exists perfect competition; there is perfect labour mobility; there exists constant returns to scale.
- The theory is criticised on the ground that it does not determine the wage. Simply the firm equates the value of marginal product with the given wage.
- Another criticism against the theory is that it is biased on the demand side of the factor and ignores the effect of supply of the factor on its price.
- This theory considers the wage the reward of the value of marginal product but in reality, the two affect each other.
- Further the theory assumes that labour units are perfectly divisible while in reality, it is not so.



CHECK YOUR PROGRESS

Q.3: What is meant by Value of Marginal Product? (Answer in about 20 words)

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Q.4: Why is the supply of labour to a firm perfectly elastic for a competitive factor market? (Answer in about 30 words)

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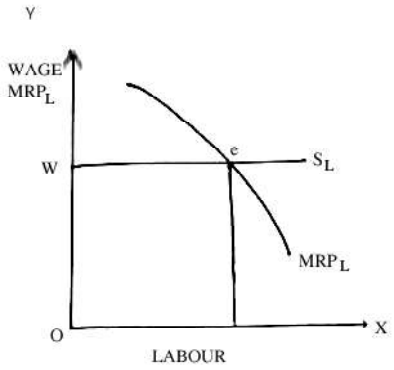
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7.6 WAGES UNDER IMPERFECT COMPETITION

The following analysis in only but an extension of the above discussed marginal productivity theory of wages. The above analysis assumed that there exists perfect competition in both the factor and product market. If this assumption is relaxed then there will be ensuing changes in the demand and supply of labour. This will affect the wage determined under conditions of equilibrium.

Model A: Monopolistic power in the product market and perfect competition in the factor market

Let us initially assume that labour is the only variable factor. Since the labour market is perfect the wage rate is given for the firm and the supply of labour is perfectly elastic to the individual firm. However, in the firm having monopoly power in the product market, its marginal product curve MR will lie below the average revenue curve AR. Here, the demand curve of labour shall be the marginal revenue product of labour curve, i.e. the MRP_L curve of labour (MRP_L=MR×MP_L, marginal product curve of labour). Since, MP_L and MR falls as output expands and price falls so the MRP_L will also fall.



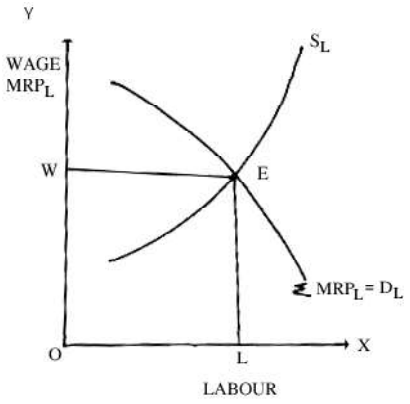


Fig: 7.5 : Equilibrium of firm **Fig. 7.6 : Market equilibrium**

The monopoly firm is in equilibrium at point e with OL employment and OW wage as in fig. 7.5. To the left of e, the further employment of labour adds more to the revenue of the firm than the expenses on labour. Secondly, to the right of e further employment of labour becomes more expensive than the change in total revenue. Thus, the monopolist using one variable factor labour is in equilibrium by employing labour till wage rate equals MRP_L. When more than one variable factor is used the demand curve of labour is no longer the MRP_L but a new curve obtained by joining the points of intersection between the shifting MRP_L curves and different wages at which the firm is in equilibrium. This is the same as the analysis contained in the above section. The market price of labour will be determined by the market demand D_L and supply curves S_L. While the market supply curve of labour is unchanged as in the above section the market demand for labour is obtained by aggregating the demand for labour curves of all the individual firms. In so doing, we must consider the decrease in price when wages fall and the corresponding fall in the marginal revenue product. The market equilibrium is shown in fig. 7.6.



CHECK YOUR PROGRESS

Q.5: Why is the demand for labour curve the marginal revenue curve is found in an imperfect product market? (Answer in about 20 words)

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Model B: Monopolistic power in the product market and monopsonist power in the factor market

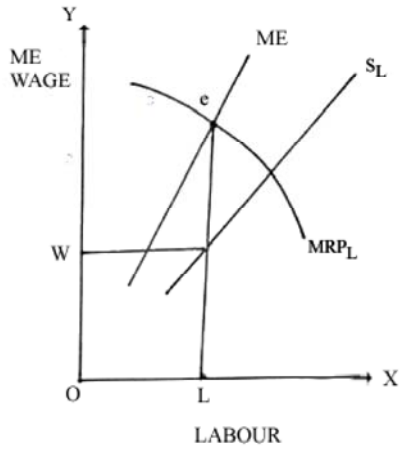


Fig. 7.7 : Market equilibrium

In this context, the demand curve of labour will be the same MRP_L curve as the individual firm is the same as in the previous section. However, the supply curve to the individual firm will change. As the firm is a monopsony, the supply of labour curve S_L will have a positive slope. This implies that the firm has to pay higher wages to enhance employment. As wage increase with additional employment the marginal expense on labour curve ME_L will be above the supply of labour curve as in fig. 7.7. The point of equilibrium will be point e where ME curve intersects the MRP_L curve. At e, the marginal expense of labour which is the marginal cost will be equal to the MRP_L. At equilibrium, OW wage is determined and OL labour is employed.



CHECK YOUR PROGRESS

Q.6: What is meant by marginal expense on labour? (Answer in about 10 words)

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7.7 EXPLOITATION OF LABOUR AND TRADE UNION

The existence of imperfections in market leads to exploitation of labour. To explain the inherent concepts, we may refer to the figures 7.8 and 7.9 In fig. 7.8, we may note that the monopolist firm is in equilibrium at point e with OW wages. The firm has paid to the labour its MRP and not the VMP. This causes wages to be short by the extent ee/. This is termed as the monopolist exploitation.

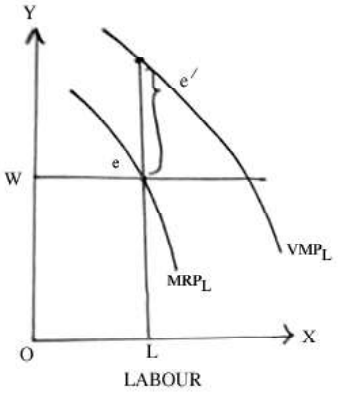


Fig. 7.8 : Equilibrium of monopolist firm

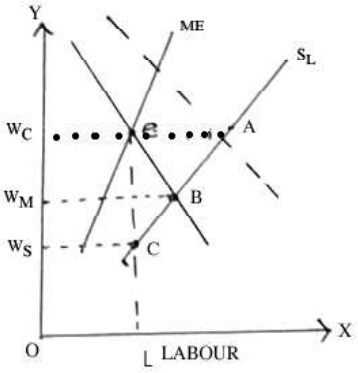


Fig. 7.9 : Equilibrium of firm in bilateral monopoly

As in fig. 7.8 a monopsonist firm pays wage not only less than the VMP_L but also less than the MRP_L. The perfectly competitive factor and product market equilibrium is point A in fig. 7.8. So, W_C wage is determined. Under monopolistic product market the equilibrium is at point B and W_m wage is determined which less than WC. Wage is lower and so also the level of employment. Finally, under monopsonist factor market, equilibrium is at C with further lower wage W_s and lower employment.

In the above analysis labour were not unionised. If, however it is so we will have a case of bilateral monopoly. The bilateral monopoly model is indeterminate with the final outcome lying in the bargaining skills of the trade or labour union and the firms (who join together to form a monopsony). The analysis is contained in the fig. 7.9.

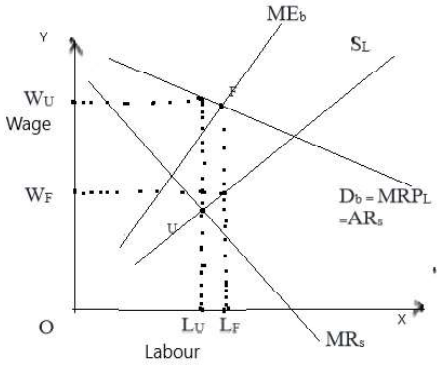


Fig. 7.10 : Equilibrium of monopsonist firm

For the monopsonist firm, the demand curve of labour is MRP_L (demand curve of buyer D_b) which is his average revenue curve and the supply of labour curve is S_L which shows his average revenue. Thus, his equilibrium will be point F where MRP_L cuts the marginal expense curve the ME_b curve. Accordingly, OL_F labour will be employed at OW_F wage. But the monopolist labour union views MRP_L as the average revenue curve (AR_S). The marginal supply curve of seller therefore is the MR_s which lies below AR_s. S_L is viewed by them as the marginal cost. So, their equilibrium is at point U. They expect OW_U wage for OL_U employment. Which wage will hold in the end is indeterminate and rest on the bargaining power of the two parties.



CHECK YOUR PROGRESS

Q.7: What is meant by bilateral monopoly? (Answer in about 10 words)

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7.8 WAGE DIFFERENTIAL

The analysis contained in the above sections consider that all units of labour are homogeneous. The theory as in the above, is therefore, determinant of a unified wage across all labour types. But in reality, all labour is not homogeneous. A doctor, a teacher, a writer is all different. In

economics, we consider both mental service or physical service as labour. The wages of each type of workers is different. This difference in wages across labour is called wage differential. The existence of wage differential does not make wage theories invalid. It is not one market for labour but many and there may be many within a market each having its own set of demand and supply forces. Different factors act causing wage differentials. They may be classified into four broad categories. These are discussed below.

- **Workers differ in abilities, skill and training:** Some works need special skills while others do not. If there is no wage differential then no one will be attracted to spend time and resources to gain the skill. When high skill is involved, the demand is high against unskilled labourers for the productivity of the later is low. The supply is high for the later as no training and resources thereof is required. So, the wages of skilled labour are high. Besides, biologically the abilities of labours may be different.
- **Nature of jobs, job requirements and the working environment differs between different types of jobs:** This therefore necessitates that we compensate people who engage in harsh, unpleasant, risky jobs. This is despite that the skills are homogeneous across jobs or the demand is same. Only that the supply to these works are less because of the nature of work involved and so one. Thus, wages are more. This type of wage differential compensates for the differences in the job characteristics.
- **Labour market conditions:** Rigidity in social norms, racial discriminations, gender biasedness, etc. causes wages for special sections of the society to be less. For example, women, people of lower class, etc. have suffered from such exploitations.
- **Value of output:** The price of output influences the wages of the people engaged in the production of the output.



7.9 LET US SUM UP

- Wage is the price of labour for the services, mental or physical, rendered by him.
- Real wage is a measure of the luxuries, comforts and necessities that a labour can command in return of his services.
- Subsistence wage are the minimum wages that is necessary for the sustenance of the labour. The theory of subsistence wages suggests that in the long run it is the sustenance wages that prevail.
- Under perfect competition in the product market wage is given for a firm and the firm can choose to employ any labour at the given wage.
- The demand for labour in a competitive product market is the value of marginal product for labour curve and for an imperfect product market is the marginal product curve.
- The demand and supply of a factor (labour) determines its price, wages as per the marginal productivity theory of wages.
- The wages and the level of employment are lower under market imperfections as compared to a perfectly competitive market. The existence of trade unions or labour unions can influence wages determination at a higher wage rate.
- The wages in the real world differs for different labour types and are not homogeneous across labour. This is called wage differential.



7.10 FURTHER READING

1) Koutsoyiannis, A. (1979). *Modern Microeconomics.* McMillan Press Limited.
2) Dwivedi D.N. (2008). *Microeconomics, theory and Applications.* Pearson Education.

7.11 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q. No. 1: Real wages are the command over luxuries, comforts and necessities that a worker has from his income. It measures the standard of living a worker can get access to.

Ans to Q. No. 2: Subsistence wages means the minimum level of wages that is required for the sustenance of life of a worker.

Ans to Q. No. 3: Value of marginal product is the value of the total output produced by an additional unit of labour. Mathematically it is marginal product of labour X price of the output.

Ans to Q. No. 4: In a perfectly competitive factor market, the wage is given for an individual firm. It can employ any labour at this wage. Thus, the supply of labour curve is perfectly elastic at the given wage.

Ans to Q. No. 5: A firm will employ labour till the cost of an additional unit adds more to the output than to the cost which means till wages equal the marginal revenue product. Marginal revenue product is marginal product X marginal revenue. Thus, the MRP_L is the demand for labour curve.

Ans to Q. No. 6: Marginal expense is the difference between the expenditures of a firm at successive higher levels of employment.

Ans to Q. No. 7: Bilateral monopoly arises when a single seller faces a single buyer in a market for exchange of any good or service.

7.12 MODEL QUESTIONS

A) Short Questions (Answer each question in about 150 words)

Q.1: Differentiate between nominal and real wages.

Q.2: What is meant by marginal product of labour?

Q.3: Define value of marginal product of labour and marginal revenue product of labour.

Q.4: What is the shape of the demand for labour curve in a perfectly competitive factor and product market?

Q.5: What is meant by wage differential?

B) Easy type Questions (Answer each question in about 300-500 words)

Q.1: Mention the criticisms against the subsistence theory of wages.

Q.2: Discuss the wage determination under perfect competition in factor and product market.

Q.3: Discuss the wage determination in a perfectly competitive factor market but an imperfect product market.

Q.4: Explain the concept of monopsonist exploitation.

Q.5: Discuss the process of collective bargaining under bilateral monopoly.

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1) Approximately how many hours did you spend for studying the units in the course? ☐☐☐

2) Please give your opinions (by ✓ mark) to the following items based on your reading of the block:

Sl. No.	Statements	Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
I)	The SLMs of the course fulfil the learning objectives set out in the beginning of each unit					
II)	The units of the SLM could satisfy my academic needs and expectations					
III)	The Unit writers have excellent knowledge about the course contents					
IV)	Language and contents of the units were presented at a level which I could readily understand					
V)	Ample opportunity for participation in the activities provided in the units					
VI)	Used enough Illustrations (Diagrams, tables etc.) for conceptual clarity					
VII)	Quality of content is engaging, relevant, and up-to-date					
VIII)	The self check questions are very helpful					
IX)	The Possible/model questions and the answers to check my progress have benefited me a lot					

Additional Comments: *(Please feel free to provide your open comments)*

1) Which aspects of the SLM, according to you, worked well?

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2) What sort of changes/improvements do you feel KKHSOU could implement to improve the overall quality of the SLM?

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Thank you for taking the time to complete this form.