UNIT 3 : MEASURES OF CENTRAL TENDENCY

UNIT STRUCTURE

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3.1 LEARNING OBJECTIVES

After going through this unit, you will able to learn -

- arithmetic mean
- geometric mean
- harmonic mean
- median
- mode
- quartile

3.2 INTRODUCTION

By a measure of central tendency, we mean ‘a summary measure that attempts to describe a whole set of date with single value that represents the middle or centre of its distribution. There are five measures of central tendency, viz, arithmetic mean (or simply mean), geometric mean, harmonic mean, median, and mode. These measures are of common use. We will discuss these measures in this unit.

3.3 ARITHMETIC MEAN

By the word mean, we mean arithmetic mean.

I. Mean for ungrouped data

Let there be n observations $x_1, x_2, x_3, \ldots\ldots, x_n$. The mean of these observations, denoted by $\bar{x}$, is defined by

$$\bar{x} = \frac{x_1 + x_2 + \ldots\ldots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Note : $\sum$ is the eighteenth letter (capital) of the Greek alphabet. It is pronounced as 'sigma'.

Example 1 : The heights of 8 persons in centimetres are 155, 161, 164, 166, 167, 169, 171, 177. Find the mean height.

Solution : $\bar{x} = \frac{155+161+164+166+167+169+171+177}{8}$

$= \frac{1330}{8}$ = 166.25

$\bar{x}$

$\therefore$ Mean height = 166.25 cm

II. Mean for discrete frequency distribution

Let there be n observations $x_1, x_2, \ldots., x_n$ occurring with frequencies $f_1, f_2, \ldots., f_n$ respectively. The mean of these observations, denoted by , is defined by

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \ldots\ldots + f_nx_n}{f_1 + f_2 + \ldots\ldots + f_n}$$

$$= \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_2} = \frac{\sum_{i=1}^{n} f_i x_i}{N}, \quad \text{Where } N = \sum_{i=1}^{n} f_i$$

Example 2 : Find the mean of the following frequency distribution.

x_i	3	5	7	9	11	13
f_i	6	8	15	25	8	4

Solution : $\bar{x} = \frac{6\times3+8\times5+15\times7+25\times9+8\times11+4\times13}{6+8+15+25+8+4}$

$$= \frac{18+40+105+225+88+52}{6+8+15+25+8+4}$$

$$= \frac{528}{66} = 8$$

III. Mean for grouped or continuous frequency distribution

Mean for grouped or continuous frequency distribution is given by

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{\sum_{i=1}^{n} f_i x_i}{N}$$

where x_i is taken as the mid-value of the corresponding class.

Example 3 : Find the mean for the following frequency distribution.

Marks	0–10	10–20	20–30	30–40	40–50	50–60
No. of students	15	17	19	27	19	12

Solution : [It is a continuous frequency distribution. Upper limits of each class are excluded from the respective class. For example 10 is not included in the class 0–10; it is included in the class 10–20.]

Class interval	Frequency f_i	Mid-value x_i	$f_i x_i$
0–10	15	5	75
10–20	17	15	255
20–30	19	25	475
30–40	27	35	945
40–50	19	45	855
50–60	12	55	660

$$N = \sum f_i = 109, \quad \sum f_i x_i = 3265$$

$$\therefore \bar{x} = \frac{\sum f_i x_i}{N} = \frac{3265}{109}$$

= 29.954 (approx)

Note : In the above example f_i's and x_i's are large. So, $f_i x_i$'s are larger. In order to avoid big calculations, we use a short method to obtain the mean.

Short Method.

In this method, the mean $\bar{x}$ is given by

$$\bar{x} = a + \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - a)\ ,$$

Where a is an arbitrary number, called the assumed mean.

In case of grouped or continuous frequency distribution having equal class-interval a shorter method is used. In this case, we put

$d_i = \frac{x_i - a}{h}$, where h is the width of the class-interval

The mean is given by

$$\bar{x} = a + h\frac{\sum f_i d_i}{N}$$

Example 4 : Find the mean of the frequency distribution given in Example 3 by short method.

Solution : We take a = 35. h = 10

Class interval	Frequency f_i	Mid-value x_i	$d_i = \frac{x_i - 35}{10}$	$f_i d_i$
0–10	15	5	–3	–45
10–20	17	15	–2	–34
20–30	19	25	–1	–19
30–40	27	35	0	0
40–50	19	45	1	19
50–60	12	55	2	24

$N = \sum f_i = 109$

$\sum f_i d_i = -55$

$$\therefore\ \bar{x} = 35 + 10 \times \left(\frac{-55}{109}\right)$$

$$= 35 - \frac{550}{109}$$

= 35 – 5.0459

= 29.9541

Propertice of Arithmetic Mean

1. The algebraic sum of the deviations of a set of values from their arithmetic mean is zero.

Proof : Let there be n observations $x_1, x_2, \ldots, x_n$ occurring with frequencies

$f_1, f_2, \ldots\ldots, f_n$ respectively. Then the mean $\bar{x}$ is given by

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \ldots\ldots + f_nx_n}{f_a + f_2 + \ldots\ldots + f_n}$$

$$= \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1} f_i} = \frac{\sum_{i=1}^{n} f_i x_i}{N}, \qquad \text{where } N = \sum_{i=1}^{n} f$$

It is required to prove that $\sum_{i=1}^{n} f_i(x_i - \bar{x}) = 0$

Now, $\sum_{i=1}^{n} f_i(x_i - \bar{x}) = 0$

$$= f_1(x_1 - x) + f_2(x_2 - \bar{x}) + \ldots\ldots + f_n(x_n - \bar{x})$$

$$= (f_1x_1 - f_2x_2 + \ldots\ldots + f_nx_n) - (f_1 + f_2 + \ldots\ldots + f_n) - \bar{x})$$

$$= \sum_{i=1}^{n} f_i x_i - \sum_{i=1}^{n} f_i \bar{x}$$

$$= N\bar{x} - N\bar{x}$$

= 0

2. The sum of the squares of the deviations of a set of values is minimum when taken about mean.

Proof : Let there be n observations $x_1, x_2, \ldots\ldots x_n$ occurring with frequencies $f_1, f_2, \ldots\ldots, f_n$ respectively. The mean is given by

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \ldots\ldots + f_nx_n}{f_1 + f_2 + \ldots\ldots + f_n}$$

$$= \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = = \frac{\sum_{i=1}^{n} f_i x_i}{N_i}, \qquad N = \sum_{i=1}^{n} f_i$$

It is required to prove that $\sum_{i=1}^{n} f_i(x_i - a)^2$ is minimum when $a = \bar{x}$, where a is an arbitrary number.

Let

$$= f_1(x_1 - a)^2 + f_2(x_2 - a)^2 + \ldots\ldots + f_n(x_n - a)^2$$

$$= f_1(x_1 - \bar{x} + \bar{x} - a)^2 + f_2(x_2 - \bar{x} + \bar{x} - a)^2 + \ldots\ldots + f_n(x_n - \bar{x} + \bar{x} - a)^2$$

$$= f_1[(x_1 - \bar{x})^2 + (\bar{x} - a)^2 + 2(x_1 - x)(\bar{x} - a)^2]$$

$$+ f_2[(x_2 - \bar{x})^2 + (\bar{x} - a)^2 + 2(x_2 - \bar{x})(\bar{x} - a)] + \ldots\ldots\ldots\ldots$$

$$+ f_n[(x_n - \bar{x})^2 + (\bar{x} - a)^2 + 2(x_n - \bar{x})(\bar{x} - a)]$$

$$= f_1(x_1 - \bar{x})^2 + f_2(x_2 - \bar{x})^2 + \ldots\ldots + f_n(x_n - \bar{x})^2 + (\bar{x} - a)^2(f_1 + f_2 + \ldots\ldots + f_n)$$
$$+ 2(\bar{x} - a) + [f_1(x_1 - \bar{x}) + f_2(x_2 - \bar{x}) + \ldots\ldots + f_n(x_n - \bar{x}_n)$$
$$+ 2(\bar{x} - a) + [f_1(x_1 - \bar{x}) + f_2(x_2 - \bar{x}) + \ldots\ldots + f_n(x_n - \bar{x}_n)$$
$$= \sum_{i=1}^{n} f_i(x_i - \bar{x})^2 + (\bar{x} - a)^2 \sum_{i=1}^{n} f_i + 2(\bar{x} - a) \sum_{i=1}^{n} f_i + 2(x_i - \bar{x})$$

We know that $\sum_{i=1}^{n} f_i = N$ and $\sum_{i=1}^{n} f_i(x_i - \bar{x})^2 = o$ (property 1)

$$\therefore\ x = \sum_{i=1}^{n} f_i(x_i - a)^2 = \sum_{i=1}^{n} f_i(x_i - \bar{x})^2 + (\bar{x} - a)^2 N$$

If $a = \bar{x}$,

$\therefore$ z is minimum if and the minimum value is .

3.4 GEOMETRIC MEAN

Let there be n observations $x_1, x_2, \ldots\ldots, x_n$; where x_i's are non-zero and positive. The geometric mean of these observations, denoted by G, is defined by

$$G = (x_1 x_2 \ldots . x_n)^{\frac{1}{4}}$$

$$\therefore \log G = \log(x_1 x_2 \ldots . x_n)^{\frac{1}{4}}$$

$$= \frac{1}{n}\log(x_1 x_2 \ldots . x_n)$$

$$= \frac{1}{n}\log[\log x_1 + \log x_2 + \ldots . + \log x_n] \qquad (1)$$

$\therefore$ G = antilog

Again, let there be n observations $x_1, x_2, \ldots\ldots, x_n$ occurring with frequencies $f_1, f_2, \ldots\ldots, f_n$ respectively. The geometric mean of these observations, denoted by G, is defined by

$$G = \left(x_1^{f_1} x_2^{f_2} \ldots . x_n^{fn}\right)^{\frac{1}{N}}, \qquad \text{where } N = \sum_{i=1}^{n} f_i$$

$$\therefore \log G = \log\left(x_1^{f_1} x_2^{f_2} \ldots . x_n^{fn}\right)^{\frac{1}{N}}$$

$$= \frac{1}{N}\log\left(x_1^{f_1} x_2^{f_2} \ldots . x_n^{fn}\right)$$

$$= \frac{1}{N}\left[\log x_1^{f_1} + \log x_2^{f_2} + \ldots . + \log x_n^{fn}\right]$$

$$= \frac{1}{N}\left[f_1 \log x_1 + f_2 \log x_2 + \ldots . + f_n \log x_n\right] \qquad (2)$$

$\therefore$ G = antilog

From (1) and (2) we see that the logarithm of the geometric mean of n observations is equal to the arithmetic mean of the logarithms of the observations.

3.5 HARMONIC MEAN

Let there be n observations $x_1, x_2, \ldots . , x_n$, where $x_i^{/}$ are non-zero.

The harmonic mean of these observations, denoted by H, is defined by

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots . + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

Again, let there be is observations $x_i, x_2, \ldots . , x_n$ occurring with frequencies $f_1, f_2, \ldots . ., f_n$ respectively. The harmonic mean of these observations, denoted by H, is defined by

$$H = \frac{N}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \ldots . + \frac{f_n}{x_n}} \qquad \text{where } N = \sum_{i=1}^{n} f_i$$

Example 5 : Calculate the arithmetic mean (AM), the geometric mean (GM), and the harmonic mean of 7, 8, 49, and show that AM > GM > HM.

Solution : $AM = \frac{7+8+49}{3} = \frac{64}{3} = 21.3$ (approx)

$$GM = \left(7 \times 8 \times 49\right)^{\frac{1}{3}} = \left(7 \times 2^3 \times 7^2\right)^{\frac{1}{3}} = \left(2^3 \times 7^3\right)^{\frac{1}{3}} = 2 \times 7 = 14$$

$$HM = \frac{3}{\frac{1}{7} + \frac{1}{8} + \ldots . + \frac{1}{49}} = \frac{3}{\frac{113}{392}} = 3 \times \frac{392}{113} = 10.41 \text{ (approx)}$$

Clearly AM > GM > HM



CHECK YOUR PROGRESS 1

Q 1: Find the mean of the following observations 14.9, 10.8, 12.3, 23.3

Q 2: Find the mean from the following data

Marks	5	10	15	20	25	30	35	40
No. of students	5	7	9	10	8	6	3	2

Q 3: Find the mean height from the following frequency distribution.

Height (in cm)	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	42	44	58	35	26	15

Q 4: Calculate the arithmetic mean (AM), the geometric mean (GM), the harmonic mean (HM) of 7, 27, 49, and hence show that AM>GM>HM.

Q 5: Find the average speed if the first km is travelled at 10 km an hour, the next km at 8 km an hour, and the third km at 4 km an hour.

Q 6: Find the mean of the squares of the first n natural numbers.

Q 7: The numbers 3.2, 5.8, 7.9, and 4.5 have frequences x, (x + 2), (x – 3), and (x + 6) respectively. If their mean is 4.876; find the value of x.

3.6 MEDIAN

By ‘median’ of a distribution, we mean the valus of the variable which divides the distribution into two equal parts.

I. Median for ungrouped data

Let there be n observations

First we arrange the observations in ascending (or descending) order

of magnitude.

(a) Let n be odd.

In this case, the $\left(\frac{n+1}{2}\right)$th term is the median, i.e. median = $\left(\frac{n+1}{2}\right)$th term.

(b) Let n be even.

In this case, the mean of the two middle terms, i.e., the mean of the $\frac{n}{2}$th and $\left(\frac{n}{2}+1\right)$th terms is the median.

Note : If n is even, any value lying between the two middle values can be taken as median. It is a convention to take the mean of the two middle terms as the median.

Example 6 : Find the median of the following observations.

5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27.

Solution : We arrange the observations in ascending order of magnitude as follows. 0, 5, 11, 19, 21, 27, 30, 36, 42, 50, 52.

There are 11 observations (odd number of observations).

$\therefore$ Median = th i.e., 6^{th} observation = 27

Example 7 : Find the median of the following observations

6, 10, 4, 3, 9, 11, 22, 18.

Solution : We arrange the observations in ascending order of magnitude as follows 3, 4, 6, 9, 10, 11, 18, 22.

There are 8 observations (even number of observatons).

$\therefore$ Median = Mean of the th and $\left(\frac{8}{2}+1\right)$th observations

= Mean of the 4th and 5th observations

$= \frac{9+10}{2} = \frac{19}{2} = 9.5$

II. Median for discrete frequency distribution

Let there be n observations $x_1, x_2, \ldots, x_n$ with frequencies $f_1, f_2, \ldots, f_n$ respectively. In this case median is obtained by considering the cumulative frequencies.

First we find $N = \sum_{i=1}^{n} f$. Then we obtain $\frac{N}{2}$. We obtain the cumulative frequency (c.f.) just greater than $\frac{N}{2}$. The corresponding value of x is the median.

Example 8 : Find the median for the following frequency distribution.

x_i	20	9	25	50	40	80
f_i	6	4	16	7	8	2

Solution :

x_i	f_i	cumulative frequency (c.f.) (in ascending order)
9	4	4
29	6	10
25	16	26
40	8	34
50	7	41
80	2	43

$N = \Sigma f_i = 43$, $\frac{N}{2} = 43 = 21.5$

The c.f. just greater than 21.5 in 25. The corresponding value of x is 25. wedian = 25.

III. Median for grouped or continueres frequency distribution

Median for grouped or continuous frequency distribution is given by

Median = $L + \frac{\frac{N}{2} - c}{f} \times h$

Where L = the lower limit of the median class,

f = the frequency of the median class,

c = the c.f. of the class preceding the median class,

h = the width of the median class.

By ‘median class’, we mean the class corresponding to the c.f. just greater than $\frac{N}{2}$, where $N = \Sigma f_i$

Example 9 : Find the median for the following frequency distribution.

Class interval	20-30	30-40	40-50	50-60	60-70
Frequency	5	10	20	5	3

Solution :

Class intervel	Frequency	c.f
20-30	5	5
30-40	10	15
40-50	20	35
50-60	5	40
60-70	3	43

Here $N = \Sigma f_i = 43$, $\frac{N}{2} = \frac{43}{2} = 21.5$

The class corresponding to the c.f. just greater than 21.5 is 40-50.

$\therefore$ Median class : 40 – 50

L = 40, f = 20, c = 15, h = 20

Median

$$= 40 + \frac{21.5 - 15}{20} \times 10$$

$$= 40 + \frac{6.5}{2}$$

= 40 + 3.25

= 43.25



CHECK YOUR PROGRESS 2

Q 1: Find the median of the following observations.

25, 30, 21, 55, 47, 10, 15, 17, 45, 35

Q 2: Find the median height from the following frequency distribution.

Heights (in cm)	150	155	158	160	164	168
No. of students	10	14	8	15	7	11

Q 3: Find the median for the following frequency distribution.

Class-interval	0-5	5-10	10-15	15-20	20-25
frequency	7	10	16	32	24

Q 4: The median of the observations 8, 11, 13, 15, x+1, x+3, 30, 35, 40, 43 arranged in ascending order is 22. Find x.

Q 5: Find the median of the following distribution.

Wages (in Rs.)	2200-2300	2300-2400	2400-2500	2500-2600	2600-2700
No. of workers	3	5	20	10	5

3.7 MODE

Mode is 'the most frequently occuning value in a distribution.'

Let us consider the following observations.

6, 5, 3, 4, 3, 7, 8, 5, 9, 5, 4.

Here the most frequently occuning value is 5.

∴ Mode of the distribution is 5.

Note : If there are more than one value each with the highest frequency, than the mode is not defined.

II. Mode for discrete frequency distribution

In case of discrete frequency distribution, mode is the value of the variable corresponding to the maximum frequency.

Let us consider the following example.

x_i	1	2	3	4	5	6	7
f_i	10	15	20	35	16	3	1

Here, maximum frequency = 35, and the corresponding value of x is 4. mode = 4.

This method is not applicable in any one (or more) of the following cases.

1. The maximum frequency is repeated.
2. The maximum frequency occurs at the very beginning or at the end of the distribution.
3. There are irregularties in the distribution.

In such cases, mode is obtained by the method of grouping. We will not discuss this method.

III. Mode for grouped or continuous frequency distribution :

Mode for grouped or continuous frequency distribution is given by

Mode ,

Where L = the lower limit of the modal class,

h = the width of the modal class.

f_1 = the frequency of the class preceding the modal class.

f_2 = the frequency of the class succeeding the modal class.

f_m = the frequency of the modal class.

By 'modal class', we mean the class having maximum frequency or the class in which the mode lies.

Example : Find the mode for the following distribution.

Class-interval	0-5	5-10	10-15	15-20	20-25
Frequency	7	10	16	32	24

Solution : Here, the maximum frequency is 32.

$\therefore$ the modal class is 15-20.

L = 15, fm = 32, f_1 = 16, f_2 = 24, h = 5

Mode

$$= 15 + \frac{32-16}{2\times 32-16-24}\times 5$$

$$= 15 + \frac{16}{24} \times 5$$

$$= 15 + \frac{2}{3} \times 5 \quad = 15 + \frac{10}{3} \quad = 18.3$$

Empirical relation between mean, median and mode

A distribution in which mean = median = mode is called a symmetrical distribution. If mean, median and mode are nor equal, the distribution is said to be asymmetrical or skewed. In moderately asymmetrical distribution, there exists a relationship between mean, median and mode. Statistician karl pearson expressed the relationship as

Mode = 3 median – 2 mean.

or, 25.5 = 3 x 24.25 – 2 mean

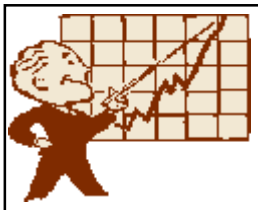
or, 2 mean = 72.75 – 25.5

or, 2 mean = 47.35

$\therefore$ mean

$= \frac{47.25}{2}$

= 23.625



CHECK YOUR PROGRESS 3

Q 1: Find the mode of the following distribution

Heights (in cm)	0-10	10-20	20-30	30-40	40-50	50-60
No. of plants	42	44	58	35	26	15

Q 2: If mode = 16, mean = 20.2, find the median.

Q 3: If the values of mode and mean are 60, and 66 respectively, find the value of the median.

Q 4: If mode = 50.04, mean = 45, what is median?

Q 5: Find the mean, median and mode of the following observations

1, 2, 2, 3, 4, 7, 9.

3.8 QUARTILE

Quartiles are those value of the variable which divide the sequence of observations into four equal parts. Each part contains equal number of items. The first point is known as first quartile (Q_1), the second point is known as second quartle (Q_2, median), the third point is known as third quantile (Q_3).

Computation of quantiles

I. Quartiles for ungrouped data

Let there be n observations– $x_1, x_2, \ldots, x_n$.

We arrange the observations in ascending order of magnitude. Let the total number of observations be N. Then

Q_1 = Value of the $\left(\frac{N+1}{4}\right)$th observation.

Q_2 = Value of the $\left(\frac{N+1}{2}\right)$th observation, if n is odd

= mean of the values of $\frac{N}{2}$ th and $\left(\frac{N+1}{2}\right)$th observations, if n is even.

Q_3 = Value of the $3\left(\frac{N+1}{4}\right)$th observation.

II. Quartiles for grouped or continuous frequency distribution.

We compute the cumulative frequency (c.f.) of the given distribution

Let $N = \sum_{i=1}^{n} f_i$

We compute $\frac{iN}{4}$, where i = 1 for Q_1, i = 2 for Q_2 (or median), i = 3 for Q_3.

We find the cumulative frequency just greater than $\frac{iN}{4}$ and the corresponding class. The corresponding class is called the quartile class.

We use the following formula to compute Q_1, Q_2, and Q_3.

$$Q_i = L + \frac{\frac{iN}{4} - c}{f} \times h$$

i = 1, 2, 3

Where L = the lower limit of the quartile class,

f = the frequency of the quartile class,

c = the c.f. of the class preceding the quartile class,

h = the width of the quartile class.



3.9 LET US SUM UP

- Let there be n observations x_1, x_2, ..., x_n. The mean of these n observations, denoted by $\bar{x}$, is defined by

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{n}$$

Let there be n observations x_1, x_2, ..., x_n occurring with frequencies f_1, f_2, ..., f_n respectively. The mean of these observations is defined by

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \ldots + f_n x_n}{f_1 + f_2 + \ldots + f_n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{\sum_{i=1}^{n} f_i x_i}{n}$$

Mean for grouped or continuous frequency distribution is given by

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{N}, \qquad N = \sum_{i=1}^{n} f$$

where x_i taken as the wid-value of the corresponding class.

- By 'median' of a distribution, we mean the value of the variable which divides the distribution into two equal parts.

We arrange the observations in ascending (or descending) order of magnitude. If there are n observations and n is odd, the $\left(\frac{N+1}{2}\right)$th term is the median. If n is mean, the mean of the $\frac{n}{2}$th and $\left(\frac{n}{2}+1\right)$th terms is the median.

Median for grouped or continuous.

frequency distribution is given by

$$\text{Median} = L + \frac{\frac{N}{2} - c}{f} \times h$$

Where L = the lower limit of the median class,

f = the frequency of the median class,

c = the c.f. of the class preceding the median class,

h = the width of the median class.

By 'median class', we mean the class corresponding to the c.f. just greater than $\frac{n}{2}$, where $N = \Sigma f_i$

Mode is 'the most frequently occurring value in a distribution'.

Mode for grouped or continuous frequency distribution is given by

$$\text{Mode} = L + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

Where L = the lower limit of the modal class,

h = the width of the modal class,

f_1 = the frequency of the class preceding the modal class.

f_2 = the frequency of the class succeeding the modal class,

f_m = the frequency of the modal class.

By 'modal class', we mean the class having maximum frequency or the class in which the mode lies.



3.10 FURTHER READINGS

1) Business Mathematics, Qazi Zameeruddin, V K Khanna, and S K Bhambri, Vikas Publishing House Pvt Ltd, Second Reprint, 1995.
2) Agarwal, D. R. (2006). Business Statistics. Delhi, Vrinda Publications.
3) Gupta S.C. (1994). Fundamentals of statistics. New Delhi, Himalayan Publishing House.
4) Rajagopalan, S. P. & Sattanthan R. (2009).Business Statistics and Operations Research. New Delhi, Tata McGraw-Hill
5) Sharma, J. K. (2007). Business Statistics. New Delhi. Pearson Education Ltd.

6) Verma, A. P. (2007). Business Statistics. Guwahat, Asian Books Private Limited.

3.11 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

Ans to Q No 1: $\bar{x} = \frac{14.9 + 10.8 + 12.3 + 23.3}{4} = \frac{61.3}{4} = 15.325$

Ans to Q No 2:

$$\bar{x} = \frac{5\times5 + 7\times10 + 9\times15 + 10\times20 + 8\times25 + 6\times30 + 3\times35 + 2\times40}{5+7+9+10+8+6+3+2}$$

$$= \frac{25 + 70 + 135 + 200 + 200 + 180 + 105 + 80}{50}$$

$$= \frac{995}{50} = 19.9$$

Ans to Q No 3:

Class interval	frequency (f_i)	Mid-value (x_i)	$f_i x_i$
0-10	42	5	210
10-20	44	15	660
20-30	58	25	1450
30-40	35	35	1225
40-50	26	45	1170
50-60	15	55	825

$N = \Sigma f_i = 220$, $\Sigma f_i x_i = 5540$

$\therefore \bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{5540}{220}$ = 25.18 (approx)

Ans to Q No 4: $AM = \frac{7 + 27 + 49}{3} = \frac{83}{3}$ = 27.67 (approx)

$$GM = (7 + 27 + 49)^{\frac{1}{3}} = (7 + 3^3 + 7^2)^{\frac{1}{3}} = (3^3 + 7^3)^{\frac{1}{3}} = 3\times7 = 21$$

$$HM = \frac{3}{\frac{1}{7} + \frac{1}{27} + \frac{1}{49}} = \frac{3}{\frac{265}{1323}} = 3\times\frac{1323}{265}$$

$\frac{3969}{265}$ = 14.98 (approx)

$\therefore$ AM > GM > HM

Ans to Q No 5: 1st km is travelled in hour

2nd km is travelled in $\frac{1}{8}$ hour

3rd km is travelled in $\frac{1}{4}$ hour

$\therefore$ The distance (3 km) is travelled in hour

$\therefore$ average speed km/hour

$$= \frac{3}{\frac{19}{40}} \text{ km/hour}$$

$$= 3 \times \frac{40}{19} \text{ km/hour}$$

= 6.316 km/hour (approx)

Note : Average speed is the harmonic mean of 10, 8, 4.

Ans to Q No 6: $\bar{x} = \frac{1^2 + 2^2 + 3^2 + \ldots\ldots + n^2}{n}$

$$= \frac{n(n+1)(2n+1)}{6n}$$

$$= \frac{(n+1)(2n+1)}{6}$$

Ans to Q No 7: $4.876 = \frac{(3.2)x + (5.8)(x+2) + (7.9)(x-3) + (4.5)(x+6)}{x + x + 2 + x - 3 + x + 6}$

or, $4.876 = \frac{(21.4)x + 11.6 - 23.7 + 27}{4x + 5}$

or, (4.876) (4x + 5) = (21.4)x + 14.9

or, (19.504)x + 24.38 = (21.4)x + 14.9

or, (21.4)x – (19.504)x = 24.38–14.9

or, (1.896)x = 9.48

$$\therefore\ x = \frac{9.48}{1.896} = 5$$

Check Your Progress 2

Ans to Q No 1: We arrange the observations in ascending order of magnitude as follows.

10, 15, 17,21, 25, 30, 35, 45, 47, 55

There are 10 observations (even number of observations).

$\therefore$ Median = mean of the th and $\left(\frac{10}{2}+1\right)$ th observations

= mean of the 5th and 6th observations

$$= \frac{25+30}{2} = \frac{55}{2} = 27.5$$

Ans to Q No 2: Try yourself

Ans to Q No 3: Try yourself

Ans to Q No 4: The observations in ascending order are 8, 11, 13, 15, x+1, x+3, 30, 35, 40, 43.

There are 10 observations

$\therefore$ median = mean of the 5th and 6th observations

$$= \frac{x+1+x+3}{2} = \frac{2x+4}{2}$$

= x + 2.

Now, median = 22 (given)

$\therefore$ x + 2 = 22

x = 20

Ans to Q No 5: Try yourself

Check Your Progress 3

Ans to Q No 1: Try yourself

Ans to Q No 2: Try yourself

Ans to Q No 3: Try yourself

Ans to Q No 4: Mode = 50.04, mean = 45 (given)

We know that

mode = 3 median – 2 mean

or, 50.04 = 3 median – 2 x 45

or, 3 median = 50.04 + 90

or, 3 median = 140.04

Median = = 46.68

Ans to Q No 5: The observations are

1, 2, 2, 3, 4, 7, 9

mean $= \frac{1+2+2+3+4+7+9}{7} = \frac{28}{7} = 4$

There are 7 observations.

median $= \left(\frac{7+1}{2}\right)$ th observation

= 4th observation

= 3

mode = 2

Note : mode < median < mean.



3.12 MODEL QUESTIONS

Q 1: Let there be n observations 1, 2, 3, ..., n with frequencies 1, 2, 3, ..., n respectively. Find the arithmetic mean.

Q 2: Find the arithmetic mean (AM), the geometric mean (GM), and the harmonic mean (HM) of 2, 4, 16 and 32. Hence show that AM > GM > HM.

Q 3: The algebraic sum of the deviations of 20 observations measured from 30 is 2. Find the mean of these observations.

Q 4: The mean height of 15 students is 154 cm. It was found later on that while calculating the mean, the reading 155 cm was wrongly read as 125 cm. Find the correct mean height.

Q 5: Find the mean, the median, the mode from the following distribution.

Class-interval	0-10	10-20	20-30	30-40	40-50
frequency	3	10	17	7	6

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