UNIT 5 : PROBABILITY

UNIT STRUCTURE

5.1 Learning Objectives
5.2 Introduction
5.3 Basic Terminologies
5.4 Classical Definition of Probability
5.5 Axiomatic Approach to Probability
5.6 Some Theorems on Probability
5.7 Let Us Sum Up
5.8 Further Readings
5.9 Answers To Check Your Progress
5.10 Model Questions

5.1 LEARNING OBJECTIVES

After going through this unit, you will able to learn

- the meaning of probability
- basic terminologies of probability
- classical definition of probability
- axiomatic approach to probability
- some theorems on probability.

5.2 INTRODUCTION

'Probability is the measure of the likelihood that an event will occur.' In this unit, we will state the classical as well as the axiomatic definition of probability. We will also discuss some theorems on probability.

5.3 BASIC TERMINOLOGIES

We will discuss some basic terminologies which are required for definition of probability.

1. **Random experiment :** A random experiment is an action or an experiment which occurs under identical conditions. The result of a

random experiment is called an outcome.

Some examples of random experiment are given below.

(a) 'Tossing a coin' is a random experiment. If a fair coin is tossed, there are two possible outcomes, viz., head (H) and tail (T). Though we know that there are two possible outcomes, we cannot predict which one will occur.

(b) 'Throwing a die' is also a random experiment. A die is a solid cube, six faces of which are marked with 1, 2, 3, 4, 5, 6 dots. If a die is thrown, there are 6 possible outcomes, viz., 1, 2, 3, 4, 5, 6.

(c) 'Drawing a card from a well-shuffled pack of cards' is another example of random experiment. A pack of cards contains 52 cards in four suits, viz., Spades, Clubs, Hearts and Diamonds. Spades, and clubs are black-faced cards. On the other hand Hearts and Diamonds are red-faced cards. Each suit consists of 13 cards, of which nine cards are numbered from 2 to 10, an ace, a king, a queen, a jack (or knave). Thus, there are 4 aces, 4 kings, 4 queens and 4 jacks. Aces, kings, queens and jacks are called face cards.

2. **Trial :** The performance of a random experiment is called a trial.

3. **Event :** A collection of possible outcomes of an experiment is called an event.

4. **Equally likely events :** Events are said to be equally likely when there is no reason to expect any one of the events in preference over the others.

   In the random experiment of tossing a coin, the event of getting a **Head** and the event of getting a **Tail** are equally likely events.

5. **Mutually exclusive events :** Events are said to be mutually exclusive if the occurrence of one of them rules out the occurrence of all the others.

   In the random experiment of tossing a coin, the event of getting a **Head** and the event of getting a **Tail** are mutually exclusive events.

   In the random experiment of drawing a card, the event of getting a

king and the event of getting a heart are not mutually exclusive events. There is a king of hearts.

6. **Exhaustive events :** Two or more events are said to be exhaustive if there is a certain chance of occurrence of at least one of them when they are all considered together. It may be noted that exhaustive events may or may not be equally likely and mutually exclusive.
7. **Favourable events :** Outcomes which entail the occurence of an event is said to be favourable to the event. In the random experiment of throwing a die, the numbers 2, 3 and 5 are favourable to the appearance of a prime number.

5.4 CLASSICAL DEFINITION OF PROBABILITY

Let there be n equally likely, mutually exclusive and exhaustive outcomes and m of them are favourable to an event E. Then the probability of the occuvence of the event E, denoted by P(E), is defined by

$$P(E) = \frac{number\,of\ outcomes\ favomable\ to\,occurence\ of\ E}{number\,of\ all\ possible\,outcomes}$$

$$= \frac{m}{n}$$

Odds of an event : Let there be m outcomes favourable to an event E, and n outcomes unfavourable to E. Then

(i) odds in favour of $E = \frac{m}{n}$

(ii) odds against $E = \frac{n}{m}$

(iii) $P(E) = \frac{m}{m+n}$

Example 1 : The odds in favour of occurrence of an event are $\frac{3}{14}$. Find the probability of the occurrence of this event.

Solution : Number of favourable outcomes = 3

Number of unfavourable outcomes = 14

$\therefore$ Total number of outcomes = 3 + 14 = 17

the required probability

Example 2 : If the probability of the occurrence of an event is $\frac{6}{19}$, find the

(i) odds in favour of its occurrence,

(ii) odds against its occurrence.

Solution : Let the favourable outcomes be m and the unfavourable outcomes be n.

$\therefore$ the probability

But $\frac{m}{m+n} = \frac{6}{19}$, (given)

or 19m = 6m + 6n

or 13m = 6n

$\therefore \frac{m}{n} = \frac{6}{13}$

$\therefore$ (i) odds in favour

(ii) odds against $= \frac{n}{m} = \frac{13}{6}$

5.5 AXIOMATIC APPROACH TO PROBABILITY

The set of all possible outcomes in a random experiment is called a **sample space.** Generally it is denoted by S. Elements of a sample space are called sample points.

Two coins are flipped together. Then the outcomes are HH, HT, TH, TT. HH means head on the first coin as well as head on the second coin.HT means head on the first coin and tail on the second coin, and so on. Thus, S = {HH, HT, TH, TT}. Here the sample points are HH, HT, TH, TT, i.e. there are 4 sample points.

A subset of a sample space is called an event.

Definition : Let S be the sample space in a random experiment. Let E be an event associated with this random experiment. The probability of the

occurrence of the event E, denoted by P(E), is defined by

$$P(E) = \frac{n(E)}{n(S)}$$
$$= \frac{number\ of\ elements\ in\ E}{number\ of\ elements\ in\ S}$$

Note : $P(E) = \frac{n(E)}{n(S)} \geq 0, \because\ n(E) \geq 0$

Example 3: Two coins are tossed together. Find the probability of getting.

(i) one head,

(ii) two heads,

(iii) at least one head,

(iv) no head.

Solution : Let S be the sample space.

$\therefore$ S = {HH, HT, TH, TT}

(i) Let E_1 be the event of getting one head

E_1 = {HT, TH}

$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

(ii) Let E_2 be the event of getting 2 heads.

$\therefore$ E_2 = {HH}

(iii) Let E_3 be the event of getting at least one head.

$\therefore$ E_3 = {HT, TH, HH}

(v) Let E_4 be the event of getting no head.

$\therefore$ E_4 = {TT}

Now, we discuss some terminologies again.

Impossible event : Let S be a sample space in a random experiment.

Clearly $\phi \subseteq S$

is an event. It is called an **impossible event.**

Example : A die is thrown.

S = { 1, 2, 3, 4, 5, 6 }, the set of all outcomes.

Let E be the event of getting a number less than 1.

Clearly .

$\therefore$ E is an impossible event.

Note : Probability of an impossible event is zero.

Let S be a sample space in a random experiment

Let E be an impossible event, i.e.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{n(\phi)}{n(S)} = \frac{0}{n(S)} = 0$$

Sure event : Let S be a sample space in a random experiment.

Clearly $S \subseteq S$

S is an event. It is called a sure event.

Example : A die is thrown.

S = { 1, 2, 3, 4, 5, 6 }

Let E be the event of getting a number less than 7.

Clearly E = 5

E is a sure event.

Note : Probability of a sure event is one

Let S be a sample space in a random experiment.

Let E be a sure event, i.e., E = S

Elementary and Compound events : If a sample space contains one sample point only, then it is called an elementary (or simple) event.

An event which is not elementary is called a compound (or composite) event.

Mutually exclusive events : Two events E_1 and E_2 are said to be mutually exclusive if $E_1 \cap E_2 = \phi$

Note : (1) $E_1 \cap E_2$ is also written as E_1E_2.

(2) If $E_1, E_2, \ldots, E_n$ are events associated with a random experiment, then $E_1, E_2, \ldots, E_n$ are mutually exclusive if $E_i \cap E_j = \phi$, for $i \neq j$.

Thus, E_1, E_2, E_3 are mutualy exclusive if

Mutually exhaustive events : Two events E_1 and E_2 are said to be mutually exhaustive if $E_1 \cup E_2 = S$, where S is the sample space.

Note : (1) $E_1 \cup E_2$ can be denoted by $E_1 + E_2$ if $E_1 \cap E_2 = \phi$

(2) If $E_1, E_2, \ldots, E_n$ are events associated with a random experiment, then $E_1, E_2, \ldots, E_n$ are mutually exhaustive if $E_1 \cup E_2 \cup \ldots \cup E_n = S$ where S is the sample space.

Complementary event : Let S be a sample space, and $E \subseteq S$, i.e., E is an event. Clearly E^c is also an event, as . E^c is called the complementary event. Generally it is denoted by $\overline{E}$.

Note : If w is an outcome of a trial, and , we say that the event E has occurred. On the other hand, if , we say that the event E has not occurred

Clearly,

(i) the event E occurs if does not occur,

(ii) the event occurs if E does not occur.

Properties

I. Let S be a sample space. Let i.e., E_1, E_2 are two events. If E_1, E_2 are mutually exclusive and exhaustive events, then $P(E_1) + P(E_2) = 1$.

Proof : We have $E_1 \cap E_2 = \phi$

and $E_1 \cup E_2 = S$

Let n(S) = x, and $n(E_1)$ = y

$\therefore$ $n(E_2)$ = x – y,

Now $P(E_1) + P(E_2) = \frac{n(E_1)}{n(S)} + \frac{n(E_2)}{n(S)}$

$$= \frac{y}{x} + \frac{x-y}{x}$$

$$= \frac{y+x-y}{x} + \frac{x}{x} = 1$$

II. Let S be a sample space, and $E \subseteq S$, i.e., E is an event. Then , i.e. $P(\overline{E}) = 1 - P(E)$

Proof : We know that E and $\overline{E}$ are mutually exclusive and exhaustive events.

by the previous result,

i.e. $P(\overline{E}) = 1 - P(E)$

III. Let S be a sample sapce, and E_1, E_2 be two events such that $E_1 \subseteq E_2$. Then $P(E_1) \le P(E_2)$

Proof : We have $E_1 \subseteq E_2$

$\therefore\ n(E_1) \le n(E_2)$, ($E_1$, E_2 are finite)

or, $\frac{n(E_1)}{n(S)} \le \frac{n(E_2)}{n(S)}$

$\therefore\ P(E_1) \le P(E_2)$

IV. Let S be a sample space, and E be an event, i.e. $E \subseteq S$. Then

Proof : We know that

$\phi \subseteq E$ and

and

or, and

$\therefore\ 0 \le P(E) \le 1$

Example 4 : Two dice are thrown. Find the probability of getting a total of 9.

Solution : Let S be the sample space

$\therefore$ S = { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), ...

. . . . . .

. . . . . .

. . . . . .

. . . . . .

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

[Here (1, 1) means 1 in the first die, 1 in the second; (1, 2) means 1 in the first die, 2 in the second die, and so on]

n (S) = 6 x 6 = 36.

Let E be the event of getting a total of 9.

E = { (3, 6), 4, 5), (5, 4), (6, 3) }

n (E) = 4

Example 5 : A letter of the English alphabet is chosen at random. Find the probability that the letter so chosen.

(i) is a consonant,

(ii) precedes 0 and is a consonant (in alphabetical order),

(iii) follows 0 and is a consonant.

Solution : Let S be the sample space

$\therefore$ S = {a, b, c, ..., x, y, z}

Clearly n(S) = 26

(i) Let E_1 be the event of getting a consonant. We know that 21 consonants in the English alphabet.

(ii) Let E_2 be the event of getting a consonant that precedes 0.

$\therefore$ E_2 = {b, c, d, f, g, h, j, k, l, m, n}

$\therefore P(E_2) = \frac{n(E_2)}{n(S)} = \frac{11}{26}$

(iii) Let E_3 be the event of getting a consonant that follows 0.

$\therefore$ E_3 = {p, q, r, s, t, v, w, x, y, z}

$n(E_3) = 10$

Example 6 : 4 cards are drawn at random from a pack of well-shuffled pack of 52 cards. Find the probability that 2 are kings and 2 are queens.

Solution : 4 cards can be drawn from a pack of 52 cards in $^{52}C_4$ ways.

Let S be the sample space.

$\therefore$ $n(S) = {}^{52}C_4$

Let E be the event of getting 2 kings and 2 queens.

2 kings can be drawn in $^4C_2$ ways

2 queens can be drawn in $^4C_2$ ways

$\therefore$ 2 kings and 2 queens can be drawn in $^4C_2$ x $^4C_2$ ways.

$$\therefore P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{{}^{4}C_{2} \times {}^{4}C_{2}}{{}^{52}C_{4}}$$



CHECK YOUR PROGRESS 1

Q 1: Find the probability of getting a club from a well-shuffled pack of cards.

Q 2: A bag contains 6 red, 8 black, 10 yellow, and 1 green ball. Find the probability of drawing (i) one red ball, (ii) one black ball, (iii) one white ball.

Q 3: A coin is tossed 3 times in succession. Find the probability of getting (i) 2 heads, (ii) at least 2 heads.

Q 4: A number is chosen at randon from the first 150 natural numbers. Find the probability that the number chosen is a multiple of 5.

Q 5: A single letter is selected at random the word PROBABILITY. Find the probability that it is a vowel.

Q 6: Two dice are tossed simultaneously, Find the probability of getting a total (i) of 6, (ii) of 12, (iii) of 18.

Q 7: Two cards are drawn at random from a well-shuffled pack of 52 cards. Find the probability that one is a king and the other is a queen.

Q 8: Two cards are drawn at random from a well-shuffled pack of 52 cards. Find the probability of getting 2 hearts.

Q 9: A bag contains 4 white, 5 black, and 6 green balls. If 2 balls are drawn at random find the probability that none of them is black.

Q 10: A letter of the English alphabet is chosen at random. Find the probability that the letter so chosen precedes ‘h’ and is a vowel.

Q 11: What is the probability that a leap year selected at random will contain 53 Mondays?

5.6 SOME THEOREMS ON PROBABILITY

Addition Theorem of Probability

I. Let S be a sample space in a random experiment. Let E_1, E_2 be two events. Then

$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \text{ and } E_2)$

i.e., $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

Proof : $P(E_1 \cup E_2) = \frac{n(E_1 \cup E_2)}{n(S)}$

$$= \frac{n(E_1) + n(E_2) - (E_1 \cap E_2)}{n(S)}$$

$$= \frac{n(E_1)}{n(S)} + \frac{n(E_2)}{n(S)} - \frac{(E_1 \cap E_2)}{n(S)}$$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2$$

Deduction : If E_1, E_2 are mutually exclusive, then $E_1 \cap E_2 = \phi$ and $\therefore P(E_1 \cap E_2) = P(\phi) = 0$. Thus, for two mutually exclusive events E_1 and E_2,

$P(E_1 \cup E_2) = P(E_1) + (E_2)$

In this case we write

$P(E_1 + E_2) = P(E_1) + P(E_2)$

II. Let S be a sample space in a random experiment. Let E_1, E_2, E_3 be three events.

Then

$P(E_1 \text{ or } E_2 \text{ or } E_3)$

$= P(E_1 \cup E_2 \cup E_3)$

$= P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_3 \cap E_1) + P(E_1 \cap E_2 \cap E_3)$

Proof : We have

$P(E_1 \cup E_2 \cup E_3)$

$$= \frac{n(E_1 \cup E_2 \cup E_3)}{n(S)}$$

$$= \frac{n(E_1) + n(E_2) + n(E_3) - n(E_1 \cap E_2) - n(E_2 \cap E_3) - n(E_3 \cap E_1) + n(E_1 \cap E_2 \cap E_3)}{n(S)}$$

$$= \frac{n(E_1)}{n(S)} + \frac{n(E_2)}{n(S)} + \frac{n(E_3)}{n(S)} - \frac{n(E_1 \cap E_2)}{n(S)} - \frac{n(E_2 \cap E_3)}{n(S)} - \frac{n(E_3 \cap E_1)}{n(S)} + \frac{n(E_1 \cap E_2 \cap E_3)}{n(S)}$$

$$= P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_3 \cap E_1) + P(E_1 \cap E_2 \cap E_3)$$

Deduction : If E_1, E_2, E_3 are mutually exclusive events, then

$E_1 \cap E_2 = \phi,\ E_2 \cap E_3 = \phi,\ E_3 \cap E_1 = \phi,\ E_1 \cap E_2 \cap E_3 = \phi$

Thus, for three mutually exclusive events E_1, E_2 and E_3,

$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3)$

i.e., $P(E_1+E_2+E_3) = P(E_1) + P(E_2) + P(E_3)$

Similarly, if E_1, E_2, E_3, ..., E_n are mutually exclusive events.

$P(E_1 + E_2 + \ldots\ldots + E_n) = P(E_1) + P(E_2) + \ldots\ldots P(E_n)$.

Let E_1, E_2 be two events associated with a random experiment S. Then

(i) $P\left(\overline{E_1 \cup E_2}\right) = P\left(\overline{E_1} \cap \overline{E_2}\right)$

(ii) $P\left(\overline{E_1 \cap E_2}\right) = P\left(\overline{E_1} \cup \overline{E_2}\right)$

Proof : We have $E_1 \subseteq S,\ E_2 \subseteq S$

By De Mergan's Laws, we know that

$\left(E_1 \cup E_2\right)^c = E_1^{\,c} \cap E_2^{\,c}$

i.e., $\overline{E_1 \cup E_2} = \overline{E}_1 \cap \overline{E}_2$

$\therefore P\left(\overline{E_1 \cup E_2}\right) = P\left(\overline{E}_1 \cap \overline{E}_2\right)$

Also $\left(E_1 \cap E_2\right)^c = E_1^{\,c} \cup E_2^{\,c}$

i.e. $\left(\overline{E_1 \cap E_2}\right) = \overline{E}_1 \cap \overline{E}_2$

$\therefore\ P\left(\overline{E_1 \cap E_2}\right) = \left(\overline{E}_1 \cap \overline{E}_2\right)$

Example 7 : Two dice are tossed together. Find the probability of getting a doublet or a total of 8.

Solution : Let S be the sample space.

Clearly n(S) = 36 [See Example 4]

Let E_1 be the event of getting a doublet. [Doublet means 'either of two dice, each of which, when thrown, has the same number of spots on the face lying uppermost'.]

$\therefore$ E_1 = { (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) }

Let E_2 be the event of getting a total of 8.

E_2 = { (2, 6), (3, 5), (4, 4), (5, 3), (6, 2) }

Thus $n(E_1) = 6$, $n(E_2) = 5$, $n(E_1 \cap E_2) = 1$

$\therefore$ $P(E_1 \text{ or } E_2)$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{n(E_1)}{n(S)} + \frac{n(E_2)}{n(S)} - \frac{n(E_1 \cap E_2)}{n(S)}$$

$$= \frac{6}{36} + \frac{5}{36} - \frac{1}{36}$$

$$= \frac{6+5-1}{36}$$

$$= \frac{10}{36}$$

$$= \frac{5}{18}$$

Example 8 : Two dice are tossed. Find the probability of getting an odd number in the first die or a total of 8.

$E_1 P(E_1 \cup E_2) = P(3, 4), (5, 7)\}$

Solution : Let S be the sample space

$\therefore$ n(S) = 36

Let E_1 be the eventof getting an odd number in the first die.

$n(E_1) = 3 \times 6 = 18$

[There are 3 odd numbers, viz., 1, 3, 5. Any one of them will appear in the first die. Any one of 1, 2, 3, 4, 5, 6 will appear in the second die.]

Let E_2 be the event of getting a total of 8.

$E_2 = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

$n(E_2) = 5$

$$n(E_1 \cap E_2) = 2$$

$$\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{n(E_1)}{n(S)} + \frac{n(E_2)}{n(S)} - \frac{n(E_1 \cap E_2)}{n(S)}$$

$$= \frac{18}{36} + \frac{5}{36} - \frac{2}{36}$$

$$= \frac{18+5-2}{36} = \frac{21}{36} = \frac{7}{12}$$

Example 9 : Let E_1, E_2, E_3 be three mutually exclusive and exhaustive events associated with a random experiment. If $P(E_1) = \frac{3}{2}P(E_2)$ and $P(E_3) = \frac{1}{2}P(E_2)$, find $P(E_1)$, $P(E_2)$, and $P(E_3)$

Solution : Let S be the sample.

$\therefore E_1 \cup E_2 \cup E_3 = S$ ($\because$ E_1, E_2, E_3 are exhaustive events)

or, $P(E_1) + P(E_2) + P(E_3) = P(S)$ ($\because E_1$, E_2, E_3 are mutually exclusive events)

or,

or, $\left(\frac{3}{2}+1+\frac{1}{2}\right)P(E_2) = 1$

or, $3P(E_2) = 1$

$\therefore P(E_2) = \frac{1}{3}$

$\therefore P(E_1) = \frac{3}{2}.P(E_2) = \frac{3}{2} \cdot \frac{1}{3} = \frac{1}{2}$

$P(E_3) = \frac{1}{2}.P(E_2) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

Example 10 : Let E_1, E_2, be two events associated with a random experiment.

If $P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{3}$ and $P(E_1 \cap E_2) = \frac{1}{6}$, find

(i) $P\left(\overline{E_1 \cap E_2}\right)$, (ii) $P\left(\overline{E_1 \cup E_2}\right)$

Solution : (i) $P\left(\overline{E_1 \cap E_2}\right)$

$[P(\overline{E}) = 1 - P(E)]$

$= 1 - \frac{1}{6} = \frac{5}{6}$

(ii) $P\left(\overline{E_1 \cup E_2}\right)$

$= 1 - P(E_1 \cup E_2)$

$$= 1 - [P(E_1) + P(E_2) - P(E_1 \cap E_2)]$$

$$= 1 - \left(\frac{1}{2} + \frac{1}{3} - \frac{1}{6}\right) = 1 - \frac{2}{3} = \frac{1}{3}$$



CHECK YOUR PROGRESS 2

Q 1: A number is chosen at random from the first 150 natural numbers. Find the probability that the number so chosen is a multiple of 4 or a multiple of 5.

Q 2: A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability that the card so chosen is a spade or a king.

Q 3: Let E_1, E_2 be two events associated with a random experiment. If $P(E_1) = 0.3$, $P(E_2) = 0.2$, and $P(E_1 \cap E_2) = 0.1$, Find

(i) $P(\overline{E_1 \cap E_2})$, (ii) $P(\overline{E_1 \cup E_2})$

Q 4: Let E_1, E_2 be two mutually exclusive events associated with a random experiment. If P(not E_1) = 0.45 and P(E_1 or E_2) = 0.65, find P(E_2).

Q 5: Let E_1, E_2 be two events associated with a random experiment. If $P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{3}$, P(E_1 and E_2 < Find (i) P(E_1 or E_2),

(ii) P (neither E_1 nor E_2)

Q 6: Two dice are thrown together. Find the probability that the sum of the numbers on the two faces is neither divisible by 3 nor by 4.



5.7 LET US SUM UP

- A random experiment is an action or an experiment which occurs under identical conditions. The result of a random experiment is called an outcome. 'Tossing of a coin', 'Throwing a die', 'Drawing a card from a

well-shuffled pack of cards' are examples of random experiment.

- The set of all possible outcomes in a random experiment is called a sample space.
- A subset of a sample space is called an event.
- The probability of the occurence of the event E, denoted by P(E), is defined by $P(E) = \frac{n(E)}{n(S)}$

  where n(E) denotes the number of elements in the event E, and n(S) denotes the number of elements in the sample space S.
- Let S be a sample space in a random experiment. Let E_1, E_2 be two events. Then

  $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap_2)$



5.8 FURTHER READINGS

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5.9 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

Ans to Q No 1: Let S be the sample space.

One card can be drawn from a well-shuffled pack of 52 cards in $^{52}C_1$ ways i.e. 52 ways.

$\therefore$ n(S) = 52

Let E be the event of getting a club.

One club can be drawn in $<^{13}C_1$ ways, i.e., 13 ways.

n(E) = 13

Ans to Q No 2: Total number of balls = 6 + 8 + 10 = 24. One ball can be drawn in $^{24}C_1$ ways, i.e., 24 ways.

$\therefore$ $\therefore P(E_1) = \frac{n(E_2)}{n(S)} = \frac{13}{52} = \frac{1}{4}$ Let S be the sample space.

() n(S) = 24

(i) Let E_1 be the event of drawing a red ball.

One red ball can be drawn in $<^{6}C_1$ ways, i.e., 6 ways.

$\therefore$ $n(E_1) = 6$

(ii) Let E_2 be the event of drawing a black ball.

One black ball can be drawn in $<^{8}C_1$ ways, i.e., 8 ways.

$\therefore$ $n(E_2) = 8$

(iii) Let E_3 be the event of drawing a white ball.

There is no white ball in the bag.

$\therefore E_3 = \phi$

$\therefore P(E_3) = P(\phi) = 0$

Ans to Q No 3: Let S be the sample space.

$\therefore$ S = {HHH, HTH, HHT, THH, HTT, TTH, THT, TTT}

(i) Let E_1 be the event of getting 2 heads.

E_1 = { HTH, HHT, THH }

(ii) Let E_2 be the event of getting at least 2 heads.

$\therefore$ E_2 = { HTH, HHT, THH, HHH }

Ans to Q No 4: Let S be the sample space.

One number can be chosen from the first 150 natural numbers in $^{150}C_1$ ways, i.e., 150 ways.

$\therefore$ n(S) = 150

S = { 1, 2, 3, 4, ..., 149, 150 }

Let E be the event of choosing a multiple of 5.

E = { 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 135, 140, 145, 150 }

n(E) = 30

Ans to Q No 5: Let S be the sample space

$\therefore$ n(S) = 11

Let E be the event of getting a vowel

n(E) = 4

Ans to Q No 6: Let S be the sample space

$\therefore$ n(S) = 6 x 6 = 36

(i) Let E_1 be the event of getting a total of 6.

E_1 = { (1, 5), (2, 4), (3, 3), (4, 2), (5, 1) }

$n(E_1)$ = 5

(ii) Let E_2 be the event of getting a total of 12.

$\therefore$ E_2 {(6, 6)}

(iii) Let E_3 be the event of getting a total of 18.

Clearly $E_3 = \phi$, there cannot be total of 18.

$\therefore P(E_3) = P(\phi) = 0$

Ans to Q No 7: Two cards can be drawn in $^{52}C_2$ ways.

Let S be the sample space.

$$\therefore n(S) = {}^{52}C_2 = \frac{52!}{2!50!} = \frac{52 \times 51 \times 50!}{2 \times 1 \times 50!} = 26 \times 51$$

There are 4 kings, and 4 queens.

One king can be drawn in $^4C_1$, i.e., 4 ways.

One queen can be drawn in $^4C_1$, i.e., 4 ways.

Let E be the event of getting a king and a queen.

$\therefore$ n(E) = 4 x 4 = 16

$$\therefore P(E_1) = \frac{n(E_2)}{n(S)} = \frac{16}{26 \times 51} = \frac{8}{13 \times 51} = \frac{8}{663}$$

Ans to Q No 8: Let S be the sample space

$$\therefore n(S) = {}^{52}C_2 = 26 \times 51$$

Let E be event of getting 2 hearts.

$$\therefore n(S) = {}^{13}C_2 = \frac{13!}{2!11!} = \frac{13 \times 12 \times 11!}{2 \times 1 \times 11!} = 13 \times 6$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{13 \times 6}{26 \times 51} = \frac{1}{17}$$

Ans to Q No 9: Total number of bulls = 4 + 5 + 6 = 15

2 balls can be chosen from 15 balls in $^{15}C_1$ ways.

Let S be the sample space.

$$\therefore n(S) = {}^{15}C_2 = \frac{15!}{2!13!} = \frac{15 \times 14}{2 \times 1} = 15 \times 7 = 105$$

If none of the balls is black, then the two balls must be from white and green balls.

Total number of white and green balls = 4 + 6 =10

Let E be the event of getting 2 balls, none of which is black.

$$\therefore n(E) = {}^{10}C_2 = \frac{10!}{2!\,8!} = \frac{10 \times 9}{2 \times 1} = 45$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{45}{105} = \frac{3}{7}$$

Ans to Q No 10: Let S be the sample space.

$\therefore$ n(S) = 26

Let E be the event of getting a letter that precedes 'h' and is a vowel.

E = { a, e }

Ans to Q No 11: A leap year contains 366 days

366 days = 52 weeks + 2 days.

52 weeks contain 52 Mondays. So the 53rd Monday will be from the remaining 2 days.

The remaining 2 days will be any one of the following, – Sun and Mon, Mon and Tues, Tues and Wed, Wed and Thurs, Thurs and Fri, Fri and Sat, Sat and Sun. Out of these 7 possible outcomes, two outcomes are favomable (Sun and Mon, Mon and Tues)

$\therefore$ the required probability

Check Your Progress 2

Ans to Q No 1: Let S be the sample space.

$\therefore$ n(S) = 150

Let E_1 be the event of getting a multiple of 4.

$n(E_1) = 37$

Let E_2 be the event of getting a multiple of 5.

$n(E_2) = 30$

is the event of getting a multiple of 4 and of 5, i.e., of 20.

$\therefore\ n(E_1 \cap E_2) = 7$

Now, $P(E_1 \cup E_2)$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{n(E_1)}{n(S)} + \frac{n(E_2)}{n(S)} - \frac{n(E_1 \cap E_2)}{n(S)}$$

$$= \frac{37}{150} + \frac{30}{150} - \frac{7}{150}$$

$$= \frac{60}{150} = \frac{2}{5}$$

Ans to Q No 2: Let S be the sample space

$\therefore n(S) = {}^{52}C_1 = 52$

Let E_1 be the event of getting a spade.

$\therefore n(E_1) = {}^{13}C_1 = 13$

Let E_2 be the event of getting a king.

$\therefore n(E_2) = {}^{4}C_1 = 4$

$E_1 \cap E_2$ is the event of getting a spade and a king.

$\therefore n(E_1 \cap E_2) = 1$

$= P(E_1 \cup E_2)$ $\therefore$ P (getting a spade or a king)

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{n(E_1)}{n(S)} + \frac{n(E_2)}{n(S)} - \frac{n(E_1 \cap E_2)}{n(S)}$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Ans to Q No 3: We have $P(E_1) = 0.3$, $P(E_2) = 0.2$, $P(E_1 \cap E_2) = 0.1$

(i) $P\left(\overline{E_1 \cap E_2}\right) = 1 - P(E_1 \cap E_2) = 1 - 0.1 = 0.9$

(ii) $P\left(\overline{E_1 \cup E_2}\right) = 1 - P(E_1 \cup E_2)$

= 1 – [$P(E_1)$ + $P(E_2)$ – P()]

= 1 – 0.3 – 0.2 + 0.1 = 0.6

Ans to Q No 4: We have P(not E_1) = 0.45

or $P(\overline{E}_1) = 0.45$

or 1 – $P(E_1)$ = 0.45

$\therefore$ $P(E_1)$ = 1 – 0.45 = 0.55

E_1, E_2 are two mutually exclusive events.

$P(E_1 \text{ or } E_2) = P(\ ) = P(E_1) + P(E_2)$

$P(E_1) + P(E_2) = 0.65$

or, $0.55 + P(E_2) = 0.65$

$P(E_2) = 0.65 - 0.55 = 0.1$

Ans to Q No 5: We have

$P(E_1 \text{ and } E_2) = P(E_1 \cap E_2) = \frac{1}{6}$

(i) $P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

(ii) P(neither E_1 nor E_2)

$= P(\overline{E}_1 \text{ and } \overline{E}_2)$

$= P(\overline{E}_1 \cap \overline{E}_2)$

$= P\left(\overline{E_1 \cup E_2}\right)$

$= 1 - P(E_1 \cup E_2)$

$= 1 - \frac{2}{3} = \frac{1}{3}$

Ans to Q No 6: Let S be the sample space.

$\therefore n(S) = {}^6C_1 \times {}^6C_1 = 6 \times 6 = 36$

Let E_1 be the event of getting a sum that is divisible by 3.

$\therefore$ E_1 = { (1, 2), (1, 5), (2, 1), (2, 4), (3, 3), (3, 6), (4, 2), (4, 5), (5, 1), (5, 4), (6, 3), 6, 6) }

$n(E_1) = 12$.

Let E_2 be the event of getting a sum that is divisible by 4.

E_2 = {(1, 3), (2, 2), (2, 6), (3, 1), (3, 5), (4,4), (5, 3), (6, 2), (6, 6)}

$n(E_2) = 9$

is the event of getting a sum that is divisible by 3 and 4.

$\therefore E_1 \cap E_2 = \{(6, 6)\}, \quad \therefore n(E_1 \cap E_2) = 1$

P(neither E_1 nor E_2)

$= P(\overline{E}_1 \text{ and } \overline{E}_2)$

$= P(\overline{E}_1 \cap \overline{E}_2)$

$$= P\left(\overline{E_1 \cup E_2}\right)$$

$$= 1 - P(E_1 \cup E_2)$$

$$= 1 - [P(E_1) + P(E_2) - P(E_1 \cap E_2)]$$

$$= 1 - \left[\frac{12}{36} + \frac{9}{36} - \frac{1}{36}\right]$$

$$= 1 - \frac{5}{9}$$

$$= \frac{4}{9}$$



5.10 MODEL QUESTIONS

Q 1: In a simultancous toss of a coin and a die, what is the sample space?

Q 2: Find the probability of getting a tail in a throw of a coin.

Q 3: Three coins are tossed together, what is the probability of getting (i) one head, (ii) at least one head, (iii) two heads, (iv) at least two heads, (v) all heads.

Q 4: In a simultaneous throw of two dice, find the probability of getting a total of 9 or 11 ?

Q 5: What is the probability that an ordinary year (not leap year) has 53 Mondays ?

Q 6: An urn contains 15 balls numbering from 1 to 15 (no two balls are of the same number). What is the probability that a ball selected at random is a ball with number that is multiple of 3 or 4 ?

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