UNIT 6: CONDITIONAL PROBABILITY

UNIT STRUCTURE

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6.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- define conditional probability
- know multiplication theorem of probability
- define law of probability
- know Baye's theorem

6.2 INTRODUCTION

In unit 5, we discussed about probability and some important theorems related to probability. In this unit, we will discuss the concept of conditional probability. We will also discuss dependent and independent events. Multiplication theorems for dependent and independent events will also be discussed in this unit. Finally, we will discuss the law of probability and baye's theorem.

6.3 CONDITIONAL PROBABILITY

In unit 5, we have discussed the methods of finding the probability of events. Supose, we have two events from the same sample space in a

random experiment, does the occurrence of one of the events affect the probability of the other event? Let us see this situation with the following examples

1. Consider a random experiment of throwing a die.

Let S be the sample space and A be the event of of getting an odd numbers.

$\therefore$ and $A = \{1,3,5\}$

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

Suppose we consider another event B such that B be the event of getting the numbers greater than 3. Then $B = \{4,5,6\}$.

When we say that B has already occurred,the event of getting ' numbers is less than or equal to 3'is not possible. Now, the number of elements of of the sample space is reduced to 3 i.e $\{4,5,6\}$. Out of these three points, two points are even number $\{4,6\}$.

Thus, given that if the event B has already occurred, then $P(A) = \frac{2}{3}$. Hence, the probability of A given that B has already occurred is denoted by $P\left(\frac{A}{B}\right)$.

$\underline{S} \pm \frac{}{4} \{1,2,3,4,5,6\}$

2. Consider the experiment of tossing three fair coins. The sample space of the experiment is $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$. Let E be the event 'at least two tails appear' and F be the event 'first coin shows head'.

Now, suppose we are given that the first coin shows head, i.e. F occurs, then we have to find the probability of occurrence of E. With the information of occurrence of F, we are sure that the cases in which first coin does not result into a head should not be considered while finding the probability of E. This information reduces our sample space from the set S to its subset F for the event E. In other words, the additional information really amounts to telling us that the situation may be considered as being that of a new random experiment for which the sample space consists of all those outcomes only which are favourable to the occurrence of the event F. Now, the sample point of F which is favourable to event E is HTT. Thus, Probability of E considering F as the sample space .

Probability of E given that the event F has occurred $= \frac{1}{4}$

This probability of the event E is called the *conditional probability of E given that F has already occurred*, and is denoted by P (E|F).

3. Consider the random experiment of drawing a card from a deck of 52 cards. Suppose we draw a queen in successive draws. In first draw the probability of drawing a queen is $\frac{4}{52}$. If the card is not replaced before the second draw, then the event of getting queen in the second draw depends on the first event. Now, there will be only 3 queens. The probability of getting a queen in second draw will also be $\frac{3}{52}$.

In the above examples,we observe that the occurance of one event depends on another event which has already happend. Hence,an event E is said to be a dependent event if the occurnce of A depends on another event F which has already happend. It is denoted by $P(E \mid F)$.

The above examples leads to the definition of conditional probability as :

Let E and F be two events defined on a sample space S. Let $P(F) > 0$,then the conditional probability of E given that F has already occurred denoted by $P(E \mid F)$ and mathematically written as :

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}, P(F) > 0$$

Similarly, $P(F \mid E) = \frac{P(E \cap F)}{P(E)}, P(E) > 0$

Note : (1) $P(E \mid F)$ and $P(F \mid E)$ can also be expressed as follows :

$$P(E \mid F) = \frac{n(E \cap F)}{n(F)} \quad \text{and} \quad P(F \mid E) = \frac{n(E \cap F)}{n(E)}.$$

(2) $P(E \mid F)$ is read as probability of event E given event F.

Laws of Conditional probability :

(a) If E_1 and E_2 be any two events of a sample space S and A be any event of S such that $P(A) \neq 0$, then

$$P(E_1 \cup E_2 \mid A) = P(E_1 \mid A) + P(E_2 \mid A) - P(E_1 \cap E_2 \mid A)$$

We have

$$= \frac{P(E_1 \cap A) \cup P(E_2 \cap A)}{P(A)}$$

$$= \frac{P(E_1 \cap A) + P(E_2 \cap A) - P(E_1 \cap E_2 \cap A)}{P(A)}$$

$$= \frac{P(E_1 \cap A)}{P(A)} + \frac{P(E_2 \cap A)}{P(A)} - \frac{P(E_1 \cap E_2 \cap A)}{P(A)}$$

$$= P(E_1 \mid A) + P(E_2 \mid A) - P(E_1 \cap E_2 \mid A)$$

(b)

We have $P(S \mid F) = 1$

$$\Rightarrow P(E' \cup E \mid F) = 1$$

$$\Rightarrow P(E' \mid F) + P(E \mid F) = 1$$

$$\Rightarrow P(E' \mid F) = 1 - P(E \mid F)$$

Independent events

$P(E' \mid F) = 1 - P(E \mid F)$ $P(E_1 \cup E_2 \mid A) = \frac{P((E_1 \cap E_2) \cap A)}{P(A)}$ Events are said to be independent if the occurrence of one does not affect the others. In the experiment of tossing a fair coin, the occurrence of the event 'head' in the first toss is independent of the occurrence of the event ' head' in the second toss and third toss.

Example 1 : A card is chosen at random from an ordinary deck of 52 cards.

Let E be the event that the cards is an ace and F the event that the card is a club. Then E and F are independent.

Dependent events:

If the occurrence of one event influences the occurrence of the other, then the second event is said to be dependent on the first. In the above example, if we do not replace the first ball drawn, this will change the composition of balls in the bag while making the second draw and therefore the event of 'drawing a red ball' in the second will depend on event (first ball is red or white) occurring in first draw. Similarly, if a person draw a card from a full pack and does not replace it, the result of the draw made afterwards will be dependent on the first draw.

Example : Two fair dice are tossed. Let E be the event that the first die is a 3, F the event that the sum is 6 and G the event that sum is 7. Then E and F are dependent, but E and G are independent.

Note: If the events A and B are independent, that is the probability of occurrence of any one of them $P(E \mid F) = P(E)$ and $P(F \mid E) = P(F)$.

6.4 MULTIPLICATION THEOREM ON PROBABILITIES

Now, we discuss multiplication theorem on probabilities for both independent and dependent events.

Multiplication theorem on probabilities for independent events :

If two events E and F are independent, the probability that both of them occur is equal to the product of their individual probabilities. i.e $P(E \cap F) = P(E)P(F)$.

Note: (1)The theorem can be extended to three or more independent events. If E, F,G……. be independent events, then

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(2) If E and F are independent then the complements of E and F are also independent. i.e

Multiplication theorem for dependent events: If E and F be two dependent events, i.e the occurrence of one event is affected by the occurrence of the other event, then the probability that both A and B will occur is

$$P(E \cap F) = P(E)P(F \mid E).$$

Note: In the case of three events E, F, G

i.e., the probability of occurrence of E, F and G is equal to the probability of E times the probability of F given that E has occurred, times the probability of G given that both E and F have occurred.

Ilustrative Examples :

Example1: Let E and F are two events,such that $P(E) = 0.8$, $P(F) = 0.6$, $P(E \cap F) = 0.5$. Find the value of (i) $P(E \cup F)$ (ii) (iii) $P(E \mid F)$.

Solution : (i) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

(ii) $= \frac{0.5}{0.8} = \frac{5}{8}$

(iii) $P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{0.5}{0.6} = \frac{5}{6}$.

Example 2 : If $P(E) = 0.5, P(F) = 0.3$ and $P(E \cap F) = 0.15$. Find $P(E \mid F)$.

Solution : By definition of conditional probability

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$

$$= \frac{0.15}{0.3}$$

$$= 0.5$$

Example 3 : If find $P(E \cap F)$.

Solution : By definition of conditional probability

$$P(E \mid E) = \frac{P(E \cap E)}{P(E)}$$

$$\Rightarrow P(F \cap E) = P(E).P(F \mid E)$$

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Example 4 : Find the chance of drawing 2 white balls in succession from a bag containing 5 red and 7 white balls, the balls drawn not being replaced.

Solution : Let E be the event that ball drawn is white in the first draw and F be the event that ball drawn is white in the second draw.

Here $, P(F \mid E) = \frac{6}{11}$

$$\therefore P(F \cap E) = \frac{7}{12} . \frac{6}{11} = \frac{7}{22} .$$

Example 5 : In an examination 30% of the students have failed in Mathematics, 20% of the students have failed in chemistry and 10 % have failed in both Mathematics and Chemistry. A student is selected at random

(i) What is the probability that the students has failed in Mathematics if it is known that he has failed in Chemistry ?

Solution : Let E be the event of failing in Mathematics.

$$\therefore P(E) = 30\% = 0.3$$

Let F be the event of failing in Chemistry

$$\therefore P(F) = 20\% = 0.2$$

Let $E \cap F$ be the event of failing in both subjects.

The probability that the student has failed in Mathematics given that he has failed in Chemistry is

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{0.1}{0.2}$$

$$= 0.5 \cdot$$

Example 5 : A bag contains 5 white and 3 black balls.Two balls are drawn at random one after the other without replacement.Find the probability that both balls are black.

Solution : Probability of drawing a black ball in the first attempt is

Probability of drawing the second black ball given that the first ball drawn is black

$$P(F \mid E) = \frac{2}{5+2} = \frac{2}{7}$$

$\therefore$ The probability that both balls drawn are black is given by

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CHECK YOUR PROGRESS -1

Q 1: Two fair coins are tossed. Given that at least one of them is heads, what is the probability that both of them are heads?

Q 2: Suppose that a box contains 8 red balls and 4 white balls. We randomly draw 3 balls from the box without replacement. Find

the probability that all 3 are red.

Q 3: A die is rolled, If the outcome is an odd number. What is the probability that it is prime?

Q 4: A family has two children.Find the probability that both children are girls given that at least one of them is a girl.

6.5 TOTAL PROBABILITY

Theorem of total probability : If E_1 and E_2 are mutually exclusive and exhaustive events and $P(E_1) \neq 0$, $P(E_2) \neq 0$, then for any event A of S,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)$$

Proof : Since and E_2 are mutually exclusive and exhaustive events, we have $E_1 \cup E_2 = S$ and $E_1 \cap E_2 = \phi$.

$\therefore A = A \cap S$

.... (1)

Now,

$= \phi$

and are mutually exclusive and exhaustive events.

From (1),we have

[multiplication theorem of probability]

If , E_2, E_3 are mutually exclusive and exhaustive events and $P(E_1) \neq 0$, $P(E_2) \neq 0$, $P(E_3) \neq 0$ then for any event A of S,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2) + P(E_3)P(A \mid E_3)$$

6.6 BAYE'S THEOREM

The concept of conditional probability discussed earlier takes into account information about the occurrence of one event to predict the probability of another event. This concept can be extended to revise probabilities based on new information and to determine the probability that

a particular effect was due to specific cause. The procedure for revising these probabilities is known as Bayes theorem. The Principle was given by Thomas Bayes in 1763. By this principle, assuming certain prior probabilities, the posteriori probabilities are obtained.

Bayes' Theorem or Rule

Statement : Let be a set of n mutually exclusive and collectively exhaustive events and $P(E_1), P(E_2), P(E_3), \cdots, P(E_n)$ are their corresponding probabilities. If A is another event such that P(A) is not zero and the priori probabilities $P(A \mid E_i)$ i =1,2…, n are also known. Then

$$P(E_i \mid A) = \frac{P(E_i)P(A \mid E_i)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2) + \cdots + P(E_n)P(A \mid E_n)}$$

i.e., $$P(E_i \mid A) = \frac{P(E_i)P(A \mid E_i)}{\sum_{i=1}^{n} P(E_i)P(A \mid E_i)}$$

Proof : Let S be the sample space of the random experiment.

The events $E_1, E_2, E_3, \cdots, E_n$ being exhaustive.

$$\therefore S = E_1 \cup E_2 \cup \cdots \cup E_n$$

$$A = A \cap S$$

$$= (A \cap E_1) \cup (A \cap E_2) \cup \cdots (A \cap E_n) \quad \text{[Distributive law]}$$

$$\Rightarrow P(A) = P(A \cap E_1) + P(A \cap E_2) + \cdots + P(A \cap E_n)$$

$$= P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2) + \cdots + P(E_n)P(A \mid E_n)$$

$$= \sum_{i=1}^{n} P(E_i)P(A \mid E_i) \quad \text{.... (1)}$$

Now, $P(A \cap E_i) = P(A)P(E_i \mid A)$

$$\Rightarrow P(E_i \mid A) = \frac{P(A \cap E_i)}{P(A)}$$

$$= \frac{P(E_i)P(A \mid E_i)}{\sum_{i=1}^{n} P(E_i)P(A \mid E_i)} \quad \text{[using (1)]}$$

Note : (1) The probabilities $P(E_1), P(E_2), \cdots, P(E_n)$ are called as the 'a prori probabilites'.

(2) The probabilities $P(E_1 \mid A), P(E_2 \mid A), \cdots, P(E_n \mid A)$ are called'posteriori probabilities'.

Example 1 : A purse contains 6 silver coins and 3 gold coins.Another purse contains 4 silver coins and 5 gold coins.A purse is selected at random and a coin is drawn from it.What is the probability that it is a silver coin.

Solution : Let Let E = Event of selecting first purse

F = Event of selecting the second purse

A= Event of getting a silver coin

$$\therefore P(E) = \frac{1}{2} \text{ and } P(F) = \frac{1}{2}$$

Also, $P(E \mid A) = \frac{6}{9} = \frac{2}{3}$ and $P(F \mid A) = \frac{4}{9}$

$\therefore$ Required probability

$$= \frac{P(E)P(A \mid E)}{P(E)P(A \mid E) + P(F)P(A \mid F)} = \frac{\frac{1}{2} \times \frac{6}{9}}{\frac{1}{2} \times \frac{6}{9} + \frac{1}{2} \times \frac{4}{9}} = \frac{10}{18} = \frac{5}{9}$$

Example 2 : A box contains two coins. One coin is heads on both sides and the other is heads on one side and tails on the other. One coin is selected from the box at random and the face of one side is observed. If the face is heads, what is the probability that the other side is heads ?

Solution : Let E = Event of selecting two headed coin

F = Event of selecting the other coin (one head and one tail)

A= Event of getting the second side also head.

Now, we have to find $P(E \mid A)$.

Here, $P(E) = P(F) = \frac{1}{2}$

$$P(A \mid E) = 1, P(A \mid F) = \frac{1}{2}$$

By Baye's theorem, we have

$$P(E \mid A) = \frac{P(E)P(A \mid E)}{P(E)P(A \mid E) + P(F)P(A \mid F)}$$

$$= \frac{\frac{1}{2}\times 1}{\frac{1}{2}\times 1+\frac{1}{2}\times\frac{1}{2}}$$

$$= \frac{2}{3}.$$

Example 3 : Assume that a factory has two machines. Past records show that machine I produces 20% of the items and machine II produces 80% of the items. Further, it shows that 6% of the items produced by machine I are defective and only 1 % of the items produced by machine II were defective. If a defective item is drawn at random,what is the probability that it was produced by machine I ?

Solution : Let E_1, E_2 be the events of drawing an item produced by machines I and II repectively. Let A denote the event of drawing a defective item.

We have $P(E_1) = \frac{20}{100} = \frac{1}{5}$, $P(E_2) = \frac{80}{100} = \frac{4}{5}$

Also, we have $P(A \mid E_1) = \frac{6}{100}$, $P(A \mid E_2) = \frac{1}{100}$

From Bayes' theorem, the required probability is

$$P(E_1 \mid A) = \frac{P(E_1).P(A \mid E_1)}{P(E_1)P(A \mid E_1)+P(E_2)P(A \mid E_2)}$$

$$= \frac{\frac{1}{5}.\frac{6}{100}}{\frac{1}{5}.\frac{6}{100}+\frac{4}{5}.\frac{1}{100}} = \frac{6}{6+4} = 0.6.$$

Example4 : A bolt is manufactured by 3 machines A,B and C.A turns out twice as many items as B and machines B and C produce equal number of items. 2 % of bolts produced by A and B are defective and 4 % of bolts produced by C are defective.All bolts are put into one stock pile and 1 is chosen from this pile.What is the probability that it is defective ?

Solution : Let E_1, E_2, E_3 be the events of bolts manufactured by machine A,B,C repectively.

Here we have to apply total theorem on probability.

B and C produce equal no. of bolts and A produces two times that of B.

$\therefore$

$$P(E_2) = 25\% = \frac{25}{100} = 0.25$$

$$P(E_3) = 25\% = \frac{25}{100} = 0.25$$

Let D be the event of getting defective bolt.

$$P(D \mid E_1) = \frac{2}{100} = 0.02$$

$$P(D \mid E_2) = \frac{2}{100} = 0.02$$

$$P(D \mid E_3) = \frac{4}{100} = 0.04$$

By total theorem on probability.

$$P(D) = P(E_1)P(D \mid E_1) + P(E_2)P(D \mid E_2) + P(E_3)P(D \mid E_3)$$
$$= 0.5 \times 0.02 + 0.25 \times 0.02 + 0.25 \times 0.04$$

$$P(E_1) = 50\% = \frac{50}{100} = 0.50$$

Example 5 : A bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls.One ball is drawn at random at random from one of the bags and is found to be red,find the probability that it is drawn from bag B.

Solution : Here given that the ball drawn is red and we have to find the probability of the ball coming from bag B.

Let be the event of selecting bag A.

$\therefore$

Let E_2 be the event of selecting bag B.

$\therefore$

Let D be the event of selecting red ball.

Probability of drawing red ball from bag A is

$$P(D \mid E_1) = \frac{3}{5}$$

Similarly, probability of drawing red ball from bag B is

$$P(D \mid E_2) = \frac{5}{9}$$

We have to find the red ball was drawn from bag B i.e., $P(E_2 \mid D)$

By Baye's Theorem

$$P(E_2 \mid D) = \frac{P(E_2)P(D \mid E_2)}{P(E_1)P(D \mid E_1) + P(E_2)P(D \mid E_2)}$$

$$= \frac{\frac{1}{2}.\frac{5}{9}}{\frac{1}{2}.\frac{3}{5} + \frac{1}{2}.\frac{5}{9}}$$

$$= \frac{\frac{5}{18}}{\frac{3}{10} + \frac{5}{18}}$$

$$= \frac{\frac{5}{18}}{\frac{27+25}{90}}$$

$$= \frac{\frac{5}{18}}{\frac{52}{90}}$$

$$= \frac{25}{52}$$

Example 6 : In a bolt factory machines A, B, C manufacture repectively 25%, 35%, 40% of the total. Of their output 5%, 4%, 2% are defective bolts. A bolt is drawn at random and found to be defective. What is the probability that it was manufactured by machine B ?

Solution : Let E_1, E_2, E_3 be the events of bolts manufactured by machine A, B, C repectively.

$$P(E_1) = 25\% = \frac{25}{100} = 0.25$$

$$P(E_2) = 35\% = \frac{35}{100} = 0.35$$

$$P(E_3) = 40\% = \frac{40}{100} = 0.40$$

Let D be the event of drawing a defective bolt.

$\therefore$ [defective from 'A']

$$P(D \mid E_2) = \frac{4}{100} = 0.04$$ [defective from 'B']

$$P(D \mid E_3) = \frac{2}{100} = 0.02$$ [defective from 'C']

Now, we have to find the probability of defective bolt manufactured by machine B. i.e., $P(E_2 \mid D)$.

By Baye's Theorem :

$$P(E_2 \mid D) = \frac{P(E_2)P(D \mid E_2)}{P(E_1)P(D \mid E_1) + P(E_2)P(D \mid E_2) + P(E_3)P(D \mid E_3)}$$

$$= \frac{0.35 \times 0.04}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.4 \times 0.02}$$

$$P(D \mid E_1) = \frac{5}{100} = 0.05$$



CHECK YOUR PROGRESS -2

Q 1: Assume that a factory has two machines. Past records show that machine I produces 20% of the items and machine II produces 80% of the items. Further, it shows that 6% of the items produced by machine I are defective and 1% of the items produced by machine II are defective. If a defective item is drawn at random, what is the probability that it was produced by machine I ?

Q 2: Three machines A, B, C produce repectively 60%, 30%,10% of the total number of items of a factory. The percentage of respective outputs of these machines are repectively 2%, 3% and 4%. An item is selected at random and is found to be defective.Find the probability that the item was produced by machine C.

Q 3: A bag X contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y.

Q 4: Urn A contains 3 red and 4 white balls and Urn B has 4 red and 5 white balls. One urn is selected at random and one ball is taken out of it. Thrown ball is red. What is the probability that it was taken from urn B ?

Q 5: A factory has three production lines I, II and III contributing 20%, 30 % and 50 %respectively, to its total output. The percentages of substandard items produced by lines I, II and III are, respectively, 15, 10 and 2. If an item chosen at random from the total output is found to be substandard, what is the probability that the item is from line I ?.



6.7 LET US SUM UP

- For two events E and F, the probability of **an** event **E** under the condition that F has already occumh is the **conditional probability P(E|F)** of **E** and is given by $P(E \mid F) = \frac{P(E \cap F)}{P(F)}$.
- Two events for which the occurrence of one has no influence on the occurrence or non-occurrence of the other are **independent** events, otherwise, they are **dependent.**
- If the events E and F are independent and $P(E) > 0, P(F) > 0$, then $P(E \cap F) = P(E)P(F)$ and $, P(F \mid E) = P(F)$.



6.8 FURTHER READINGS

1) Rao. G. Shanker, Probability and Statistics, Universitis Press.

2) Statistics, R.S.N. Pillai and Bagavathi S. Chand and Company Ltd. New Delhi-110055.



6.9 ANSWERS TO CHECK YOUR PROGRESS

CHECK YOUR PROGRESS -1

Ans to Q No 1: Let E be the event that both of them are heads and F the event that at least one of them is heads.

Then, $P(E)=\frac{3}{4}, P(E\cap F)=\frac{1}{4}, P(F)=\frac{1}{4}$

$\therefore P(E\mid F)=\frac{P(E\cap F)}{P(F)}$

$=\frac{\frac{1}{4}}{\frac{3}{4}}$

$=\frac{1}{3}.$

By using the definition of conditional probability, we can easily get that

$P(E\cap F)=P(F)P(E\mid F)$.

Ans to Q No 2: For i= 1;2;3, let E_i be the event that thei-th ball is red.

Then, $P(E_1\cap E_2\cap E_3)=P(E_1)P(E_2\mid E_1)P(E_3\mid E_1E_2)$

$=\frac{8}{12}.\frac{7}{11}.\frac{6}{10}=\frac{14}{55}.$

Ans to Q No 3: When a die is rolled, the sample is $S=\{1,2,3,4,5,6\}$.

Let $E=$ Event of getting an odd number

= Event of getting a prime number

Then,

$\therefore$, $P(F)=\frac{3}{6}=\frac{1}{2}$ and $P(E\cap F)=\frac{1}{3}$.

P(getting a prime already when an odd number)

= P(getting a prime number | getting an odd number)

$=P(E\mid F)$

$=\frac{\frac{1}{3}}{\frac{1}{2}}=\frac{2}{3}$.

Ans to Q No 4: The sample space S is given by

$S=\{(b,b),(b,g),(g,b),(g,g)\}$

Let E : Event that both children are girls

F : Event that at least one of the child is a girl.

Then E and F

$$, P(E\cap F)=\frac{1}{4}$$

$\therefore$ Required probability is

$$=\frac{\frac{1}{4}}{\frac{3}{4}}$$

$$=\frac{1}{3}$$

CHECK YOU PROGRESS-2

Ans to Q No 1: Let E_1, E_2 be the events of drawing an item produced by machines I and II repectively.Let A denote the event of drawing a

defective item.

We have $P(E_1)=\frac{20}{100}=\frac{1}{5}, P(E_2)=\frac{80}{100}=\frac{4}{5}$

Also, we have $P(A \mid E_1)=\frac{6}{100}, P(A \mid E_2)=\frac{1}{100}$

From Bayes's Theorem, the required probability is

$$P(E_1 \mid A)=\frac{P(E_1)P(A \mid E_1)}{P(E_1)P(A \mid E_1)+P(E_2)P(A \mid E_2)}$$

$$=\frac{\frac{1}{5}.\frac{6}{100}}{\frac{1}{5}.\frac{6}{100}+\frac{4}{5}.\frac{1}{100}}=\frac{6}{6+4}=0.6.$$

Ans to Q No 2: Let E, F, G denote the events in which the items are selected by machines A, B, C repectively.

Then, we have $P(E)=\frac{60}{100}=0.6, P(F)=\frac{30}{100}=0.3, P(G)=\frac{10}{100}=0.1$

Let D denote the event in which the item selected at random is a defective item.Then

E_1

$$P(D \mid E)=\frac{2}{100}=0.02, P(D \mid F)=\frac{3}{100}=0.03, P(D \mid G)=\frac{4}{100}=0.04$$

Applying Bayes's theorem, we obtain

$$P(G \mid D)=\frac{P(G)P(D \mid G)}{P(E)P(D \mid E)+P(F)P(D \mid F)+P(G)P(D \mid G)}$$

$$=\frac{(0.1)(0.04)}{(0.6)(0.02)+(0.3)(0.03)+(0.1)(0.04)}$$

$$=\frac{0.004}{0.012+0.009+0.004}$$

$$=\frac{0.004}{0.025}$$

$$=0.16.$$

Ans to Q No 3: Let : the ball is drawn from bag X

E_2: the ball is drawn from bag Y

and A ; the ball is red.

Here $P(E_1)=P(E_2)=\frac{1}{2}$

Also, $P(A \mid E_1)$ = P(a red ball is drawn from bag X)

$$= \frac{3}{5}$$

$P(A \mid E_2)$ = P(a red ball is drawn from bag Y)

$$= \frac{5}{9}$$

By Baye's Theorem ,we have

$$P(E_2 \mid A) = \frac{P(E_1)P(A \mid E_1)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{5}{9}}{\frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{5}{9}} = \frac{25}{52}.$$

Ans to Q No 4: Required probability $= \frac{28}{45}$ [By Bayes's Theorem]

Ans to Q No 5: By Bayes formula

$$P(E_1 \mid A) = \frac{P(E_1)P(A \mid E_1)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)}$$

$$= \frac{0.2 \times 0.15}{0.2 \times 0.15 + 0.3 \times 0.10 + 0.5 \times 0.02}$$

$$= 0.069.$$



6.10 MODEL QUESTIONS

Q 1: E and F are two events for which $P(F) = \frac{1}{3}$ and $P(E \cap F) = \frac{1}{4}$ .Find (i) $P(E \mid F)$,(ii) $P(E \cup F)$ and (iii) .

Q 2: Three persons A, B, C supply raw mterials to a factory in the proportion $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ and $5\%, 6\%, 8\%$ of the materials supplied by them repectively are found to be defective. A material supplied is selected at random from the total supply. What are the probabilities that material was supplied by A, or by B or by C ?

Q 3: One of three identical coins is unbiased while the remaining two are biased with probabilities of getting head in a toss are $\frac{1}{3}$ and $\frac{2}{3}$. One of the coins selected randomly shows head when tossed. Find the probability of the coin to be unbiased.

Q 4: Boys from the districts of Kamrup, Nagaon and Jorhat come to a college in Assam in the proportions 50%, 30% and 20% repectively. 80%, 60% and 40% of the boys comingrepectively from the districts wear Dhooti. A boy selected randomly from all the boys of the college is found to wear Dhooti. What is the probability that he comes kamrup?

Q 5: Three trains X, Y, Z carry repectively 25%, 35%, 40% of the total passengers. Of the passengers carried by the tains 5%, 4% and 3% respectively are ladies. If a pssengers chosen at random out of all the passengers of the trains found to be a lady. Prove that the probability that she was carried by X, or Y or Z are repectively $\frac{25}{69}, \frac{28}{69}, \frac{16}{69}$.

Q 6: Three are three groups of children. The first group consists of 3 girls and 1 boy, the second consists of 2 girls and 2 boys and the third consists of 1 girl and 3 boys. One child is picked up at random from each group. Find the probability of selecting 1 girl and 2 boys.

Q 7: A bag contains 5 white and 7 red balls. If a ball is drawn twice at random successively, find the probability that one ball is white and the other is red.

Q 8: In a running competition, the probabilities of the two competitiors of winning the race is $\frac{1}{3}$ and $\frac{1}{5}$. What is the probability that both of them will not win ?

Q 9: One card is drawn from a well shuffled pack of 52 cards.Deduce the probability of drawing.

(i) a red card or an ace or both

(ii) a diamond or an ace or both.

(iii) king of diamond or king of heart or both.

Q 10: 4 men play with a pack of cards containing 52 cards. What is the probability that the 4 kings are held by only one of the players ?

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