UNIT 7 : RANDOM VARIABLES AND PROBABILITY DISTRIBUTION

UNIT STRUCTURE

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7.1 LEARNING OBJECTIVES

After going through this unit, you will able to-

- random variables
- discrete random variables
- continuous random variables
- probability distribution of random variables.

7.2 INTRODUCTION

Let 3 coins be tossed. One may be interested to know the number of heads obtained. Similarly, Let a pair of dice be thrown. One may be interested to know thrown. One may be interested to know the sum of points. In each of the cases a real number as associated with the outcome. A rule that assigns a real number to each outcome is called a random variable.

7.3 RANDOM VARIABLES

Let S be a sample space (Collection of outcomes) associated with a given random experiment.

Let $\omega \in S$ i.e is an outcome.

A rule X that associates **each** **to a unique** real number X( ) is called a random variable. Thus, a random variable is a real-valued function X which assigns a **unique** real number X( ) to **each** .

Let 3 coins be flipped.

Let S be the sample space associated with this random experiment. Then.

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Let us consider the function

X : , where IR is the set of real numbers such that.

X(w) = number of heads in w, for all

X (HHH) = 3, X(HHT) = 2, X(HTH) = 2, X(THH) = 2

X(HTT) = 1, X(THT) = 1, X(TTH) = 1, X(TTT) = 0

Here, X is a random variables

Clearly, range (X) = {0, 1, 2, 3}

7.3.1 Discrete Random Variable

A random variable which can take only finite or countable infinite number of values is called a discrete rancom variable.

The example given above is discrete random variable. In this case the random variable X takes four values, viz, 0, 1, 2, 3.

7.3.2 Continuours Random Variable

A random variable which can take any value between two given limits in called a continuous random variable.

Let the maximum life of electric bulbs be 2000 hours. The continuous random variable is

7.4 PROBABILITY DISTRIBUTION OF RANDOM VARIABLES

If the values of a random variable together with the corresponding probablities are given, then this description is called a probability distribution of the random variable.

Let a random variable X take the values x_1, x_2, ..., x_n with probabilities p_1, p_2, ... , p_n respectively. Then the probability distribution of X is given by

X	x_1	x_2	–	– – – – – x_n
P(x)	p_1	p_2	–	– – – – – – – p_n

Note : The above probability distribution of X is defined only when

(a) each $p_i \geq 0$

(b) $\sum_{i=1}^{n} p_i = 1$

Example 1: There coins are tossed simultaneously. Let X be the random variable such that.

X(w) = number of heads in w.

Write down the probability distribution of X.

Solution : Let S be the sample space.

Then

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

We have n(S) = 8

Here X = number of heads in tossing 3 coins.

Since the number of heads may be 0, 1, 2 or 3, therefore the possible values of X are 0, 1, 2, 3.

∴ P(X = 0) = P (getting no head)

= P(TTT) =

P(X=1) = P (getting I head)

= P(HTT or THT or TTH) = $\frac{3}{8}$

P(X=2) = P(getting 2 heads)

= P(HHT or HTH or THH} = $\frac{3}{8}$

P(X=3) = P(getting 3 heads)

= P(HHH) = $\frac{1}{8}$

$\therefore$ the probability distribution is

X	0	1	2	3
P(X)		$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Note : Here each $P_i > 0$

Also $P_1 + P_2 + P_3 + P_4 = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1$

Example 2 : A die is tossed. Let X be the random variable defined by

$$X = \begin{cases} 1, \text{ if the outcome is an even number} \\ 0, \text{if the outcome is an odd number} \end{cases}$$

Write down the probability distribution of X.

Solution: Let S be the sample space.

$\therefore$ S = {1, 2, 3, 4, 5, 6}

$\frac{1}{8}\frac{3}{6} = \frac{1}{2}$

Here X takes the values 0, 1.

P(X = 0) = P (getting an odd number)

P(X = 1) = P (getting an even number)

$= \frac{3}{6} = \frac{1}{2}$

$\therefore$ the probability distribution is

X	0	1
P(X)		$\frac{1}{2}$



CHECK YOUR PROGRESS 1

Q 1: Given an example of a discrete random variable.

Q 2: Give an example of a continuous random variable.

Q 3: Two coins are tossed simultaneously. Let X be the random variable such that
X(w) = number of heads in w. Write down the probability distribution of X.

Q 4: A die is tossed twice. 'Getting an even number on a toss' is considered a success. Find the probability distribution of number of successes.

Q 5: A die is tossed twice. 'Getting a number greater than 4' is considered a success. Find the probability distribution of number of successes.

Q 6: Two cards are drawn simultaneously from a well-shuffled pack of 52 cards. Find the probability distribution of number of queens.

Q 7: Two cards are drawn withoud replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of face cards.

Q 8: 20% of the bulbs produced by a factory are defective. Find the probability distribution of the number of defective bulbs in a sample of 4 bulbs chosen at random.

7.4.1 Mean and Variance of Random Variables

Let a random variable X take the values $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$ respectively such that each $p_i \geq 0$, and $\sum_{i=1}^{n} p_i = 1$

The mean of x, denoted by (), is defined by

$$\mu = \frac{\sum_{i=1}^{n} x_i p_i}{\sum_{i=1}^{n} p_i} = \sum_{i=1}^{n} x_i p_i, \because \sum_{i=1}^{n} p_i = 1$$

The mean of x is also called expected value of X. It is denoted by E(X). Thus,

$$\mu = E(X) = \sum_{i=1}^{n} x_i p_i$$

The variance of X, denoted by x_2, is given by $x^2 = \Sigma x_i^2 p_i - \mu^2$

The standard deviation of X, denoted by a, is the positive square root of variance.

Example 3: Find the mean and the standard devation for the following probability distribution.

X	1	3	5
P(X)	0.3	0.4	0.3

Solution : Mean μ = 1 x 0.3 + 2 x 0.4 + 5 x 0.3

= 0.3 + 1.2 + 1.5

Now $\Sigma x_i^2 p_i = 1 \times 0.3 + 9 \times 0.4 + 25 \times 0.3$

= 0.3 + 3.6 + 7.5

= 11.4

$\therefore\ SD = \alpha = \sqrt{11.4 - 9} = \sqrt{2.4}$

$\therefore a = \alpha = \sqrt{\Sigma x_i^2 p_i - \mu^2}$ SD

Example 4 : A random variable X has the following probability distribution

X	0	1	2	3
P(X)	a	3a	3a	a

Find the value of a. Also find the mean.

Solution : We know that $\Sigma p_i = 1$

$\therefore$ a + 3a + 3a + a = 1

or 8a = 1

$$a = \frac{1}{8}$$

2nd part : Mean $\mu = \Sigma x_i p_i$

$$= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

$$= \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = \frac{3}{2}$$

Example 5 : A random variable X has the following probability distribution.

X	0	1	2	3	4
P(X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Find the mean of X. Also find (i) P (X > 3), (ii) P(0 < X < 3)

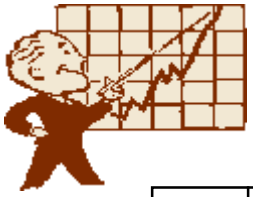
Solution : Mean $\mu = 0\times\frac{1}{16}+1\times\frac{1}{4}+2\times\frac{3}{8}+3\times\frac{1}{4}+4\times\frac{1}{16}$

$$=\frac{1}{4}+\frac{3}{4}+\frac{3}{4}+\frac{1}{4}=2$$

(i) $P(X>3)=P(X=4)=\frac{1}{16}$

(ii) $P(0<X<3)=P(X=1)+P(X=2)$

$$=\frac{1}{4}+\frac{3}{8}=\frac{2+3}{8}=\frac{5}{8}$$



CHECK YOUR PROGRESS 2

Q 1: A random variable X has the following probability distribution.

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

Find the mean and the standard deviation.

Q 2: Two cards are drawn simultaneously from a well-shuffled pacck of 52 cards. Find the mean and the standard deviation of the number of aces.



7.5 LET US SUM UP

- Let S be a sample space associated with a given random experiment. A rule X that associates each $\omega \in S$ to a unique real number X(w) is called a random variable.
- A random variable which can take only finite or countably infinite number of values is called a discrete random variable.

- A random variable which can take any value between two given limits is called a continuous random variable.
- If the values of a random variable together with the corresponding probabilities are given, then this description is called a probability distribution of the random variable.
- Let a random variable X take the values $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$ respectively. The mean of x, denoted by , is defined by

  The variance of X, denoted by α^2, is given by $\alpha^2 = \Sigma x_i^2 p_i - \mu^2$

  The standard deviation of X, denoted by a, is the positive square root of variance.



7.6 FURTHER READINGS

μ

$\mu = \sum_{i=1}^{n} x_i p_i$

1) Business Mathematics, Qazi Zameeruddin, V K Khanna, and S K Bhambri, Vikas Publishing House Pvt Ltd, Second Reprint, 1995.
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7.7 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

Ans to Q No 1: An example of a discrete random variable :

Three coins are tossed simultaneously. Let X be the the random

variable such that X(w) = number of heads in w.

In this case the random variable X takes from values, viz, 0, 1, 2, 3. So, it is a discrete random variable.

Ans to Q No 2: An example of a continuous random variable :

Let the maximum life of electric bulbs be 2000 hours. The continuous random variable is $x(w) = \{x : 0 \le x \le 2000\}$

Ans to Q No 3: Let S be the sample space.

$\therefore$ S = {HH, HT, TH, TT}

We have n(S) = 4

Here X = number of heads in tossing 2 coins.

Since the number of heads may be 0, 1, or 2, therefore the possible values of X are 0, 1, 2.

P(X = 0) = P (getting no head)

= P (TT) =

P (x = 1) = P (getting 1 head)

= P (HT or TH) $= \frac{2}{4} = \frac{1}{2}$

P (x = 2) = P (getting 2 heads)

= P (HH) = $\frac{1}{4}$

$\therefore$ the probability distribution is

X	0	1	2
P(X)		$\frac{1}{2}$	$\frac{1}{4}$

Ans to Q No 4: Let the random variable, i.e., the number of successes be X.

On flipping a die twice, the number of successes can be 0, 1, or 2. Thus, X can take values 0, 1, 2.

P(X = 0) = P (no success in both flips)

= P (no success in 1st flip). P(no success in 2nd flip)

= $\frac{3}{6} \cdot \frac{3}{6} = \frac{1}{4}$

P(X = 1) = P(success in 1st toss and no-success in 2nd toss) or

P(no-success in 1st toss and success in 2nd toss)

$= \frac{3}{6} \cdot \frac{3}{6} + \frac{3}{6} \cdot \frac{3}{6} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

P(X = 2) = P (success in both the flips)

= P(success in 1st flip). P(success in 2nd flip)

$= \frac{3}{6} \cdot \frac{3}{6} = \frac{1}{4}$

$\therefore$ the probability distribution is

X	0	1	2
P(X)		$\frac{1}{2}$	$\frac{1}{4}$

Ans to Q No 5: Let the random variable, i.e., the number of successes be X.

One flipping a die twice, the number of successes can be 0, 1, or 2.

$\therefore$ X can take values 0, 1, 2.

P(X = 0) = P (no success in both flips)

= P(no success in 1st flip)

= P(no success in 2nd flip)

$= \frac{4}{6} \cdot \frac{2}{6} + \frac{2}{6} \cdot \frac{4}{6} = \frac{4}{9}$ = P(getting a number 4)

= P (getting a number 4)

P(X = 1) = P(success in 1st flip and no-success in 2nd slip) or P (no-success in 1st flip and success in 2nd slip)

= P(getting a number 4 and getting a number > 4) + P(getting a number > 4, and getting a number a number >4, and getting a number 4)

P(X = 2) = P (success in both the flips)

= P(success in 1st flip). P(success in 2nd flip)

$= \frac{2}{6} \cdot \frac{2}{6} = \frac{1}{9}$

$\therefore$ the probability distribution is

X	0	1	2
P(X)		$\frac{4}{9}$	$\frac{1}{9}$

Ans to Q No 6: [There are 4 queens in a pack of 52 cards.]

Let the random variable, i.e., the number of queens be X.

On drawing 2 cards, the number of queens can be 0, 1, or 2.

∴ X can take values 0, 1, 2.

P(X = 0) = P(getting no queen)

= P(2 non-queens out of 48)

P(X = 1) = P(getting 1 queen and 1 non-queen)

$$= \frac{{}^{4}C_{1} \cdot {}^{48}C_{1}}{{}^{52}C_{2}} = \frac{32}{221}$$

P(X = 2) = P(getting 2 queens)

$$= \frac{{}^{4}C_{2}}{{}^{52}C_{2}} = \frac{1}{221}$$

∴ the probability distribution is

X	0	1	2
P(X)		$\frac{32}{221}$	$\frac{1}{221}$

Ans to Q No 7: [The jack, queen, and the king are the face cards. There are 4 jacks, 4 queens, and 4 kings. Thus, there are altogether 12 face cards.]

Let the random variable, i.e., the number of faces be X.

On drawing 2 cards, the number of faces can be 0, 1, or 2.

∴ X can take values 0, 1, 2.

P(X = 0) = P (getting no face cards)

= P (2 non-face cards out of 40)

P(X = 1) = P(getting 1 face card and 1 non-face card

$$= \frac{{}^{12}C_{1} \cdot {}^{40}C_{1}}{{}^{52}C_{2}} = \frac{80}{221}$$

P(X = 2) = P (getting 2 face cards)

$$= \frac{^{12}C_2}{^{52}C_2} = \frac{11}{221}$$

∴ the probability distribution is

X	0	1	2
P(X)		$\frac{80}{221}$	$\frac{11}{221}$

Ans to Q No 8: Let the number of bulbs be 100.

∴ number of defective bulbs is 20 and the number of non-defective bulbs is 80.

Let the random variable, i.e., the number of defective bulbs be X.

On choosing 4 bulbs, the number of defective bulbs can be 0, 1, 2, 3 or 4.

X can take values 0, 1, 2, 3, 4.

P(X = 0) = P(Choosing no defective bulbs)

$$= \frac{^{80}C_4}{^{100}C_4}$$

P(X = 1) = P(Choosing 1 defective bulb and 3 non-defective bulbs)

$$= \frac{^{20}C_1 \times ^{80}C_1}{^{100}C_4}$$

P(X = 2) = P(Choosing 2 defective bulbs and 2 non-defective bulbs)

$$= \frac{^{20}C_2 \times ^{80}C_2}{^{100}C_4}$$

P(X = 3) = P(Choosing 1 defective bulb and 3 non-defective bulbs)

$$= \frac{^{20}C_3 \times ^{80}C_1}{^{100}C_4}$$

P(X = 4) = P(Choosing 4 defective bulbs)

$$= \frac{^{20}C_4}{^{100}C_4}$$

∴ the probability distribution is

X	0	1	2	3	4
P(X)		$\frac{^{20}C_1 \times ^{80}C_3}{^{100}C_4}$	$\frac{^{20}C_2 \times ^{80}C_2}{^{100}C_4}$	$\frac{^{20}C_3 \times ^{80}C_2}{^{100}C_4}$	$\frac{^{20}C_4}{^{100}C_4}$

Check Your Progress 2

Ans to Q No 1: The probability distribution is

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{10}$

Mean $\mu = \Sigma x_i p_i$

$$= \frac{1}{2} + \frac{3}{5} + \frac{1}{10} = \frac{5+6+1}{10}$$

$$= \frac{12}{10} = 1.2$$

$$SD = \alpha = \sqrt{\Sigma x_i^2 p_i - \mu^2}$$

Now $\Sigma x_i^2 p_i$

$$= 0 \times \frac{1}{6} + 1 \times \frac{1}{2} + 4 \times \frac{3}{10} + 9 \times \frac{1}{30}$$

$$= \frac{1}{2} + \frac{6}{5} + \frac{3}{10} = \frac{5+12+3}{10} = \frac{18}{10} = 1.8$$

$$\therefore SD = \alpha = \sqrt{1.8 - 1.44} = \sqrt{0.36} = 0.6$$

Ans to Q No 2: [There are 4 aces in a pack of 52 cards]

Let the random variable, i.e., the number of aces be X.

On drawing 2 cards, the number of aces canbe 0, 1, or 2.

$\therefore$ X can take values 0, 1, 2

P(X = 0) = P(getting no ace)

= P (2 non-aces out of 48)

P(X = 1) = P(getting 1 ace and 1 non-ace)

$$= \frac{{}^4C_1 \cdot {}^{48}C_1}{{}^{52}C_2} = \frac{32}{221}$$

P(X = 2) = P(getting 2 aces)

$$= \frac{{}^4C_1}{{}^{52}C_2} = \frac{1}{221}$$

$\therefore$ the probability distribution is

X	0	1	2
P(X)		$\frac{32}{221}$	$\frac{1}{221}$

Mean $\mu = \Sigma x_i p_i = 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 2 \times \frac{1}{221}$

$$= \frac{32}{221} + \frac{2}{221} = \frac{34}{221} = \frac{2}{13}$$

$$SD = \alpha = \sqrt{\Sigma x_i p_i - \mu^2}$$

Now $\Sigma x_i p_i$

$$= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 4 \times \frac{1}{221} = \frac{32}{221} + \frac{4}{221}$$

$$= \frac{36}{221}$$

$$\therefore SD = \alpha = \sqrt{\frac{36}{221} - \frac{4}{221}}$$

$\frac{188}{221}$



7.8 MODEL QUESTIONS

Q 1: A die is thrown. Let X be the number of points thrown. Write down the probability distribution and find the mean.

Q 2: Find the mean, and the standard deviation of the number of heads in two tosses of a coin.

Q 3: Find the mean, and the standard deviation of the number of heads in three tosses of a coin.

Q 4: A die is tossed twice. 'Getting a number greater than 3' is considered a success. Find the probability distribution of number of successes. Also find the mean and the standard deviation of number of successes.

Q 5: Find the probability distribution of the number of doublets in four throws of a pain of dice.

[Meaning of the word 'doublet' in this context is as follows.

'Either of two dice, each of which, when thrown, has the same number of spots on the face lying uppermost.'

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