UNIT 8 : THEORETICAL DISTRIBUTION I

UNIT STRUCTURE

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8.1 LEARNING OBJECTIVES

After going through this unit, you will able to-

- bernoulli trial
- binomial (and bernoulli) distribution
- poisson distribution.

8.2 INTRODUCTION

In this unit, we will study theoretical discrete univariate (of only one variable) distribution. It gives us a law according to which different values of the random variable are distributed with specified probabilities. We will discuss two main distributions, viz., Binomial distribution, and poisson distribution.

8.3 BERNOULLI TRIALS

A Bernoulli trial (named after Jacob Bernoulli 1654-1705) is an experiment satisfying the following conditions.

1. There are only two possible outcomes, viz., success or failure (occurrence of an event is ‘success’. non-occurrence is ‘failure’.)
2. The prabability of a success (or failure) remains same in each trial.

3. Trials must be independent.
4. Number of trials must be finite.
5. The sum of probabilities of succes and failure is 1.
6. The probability of succes vis-a-vis failure is not very low.

8.4 BINOMIAL (AND BERNOULLI) DISTRIBUTION

Let there be n Bernoulli trials. Let the random variable X denote the number of successes in these trials. Let the probability of getting a success in a single trial be p, and that of getting a failure be q so that p + q = 1. Then $P(X=r) = {}^nC_r\, p^r\, q^{n-r}$

Proof : Let a success be denoted by S, and a failure be denoted by F.

Number of ways of getting r successes in n trials = ${}^nC_r$

$\therefore$ $P(X = r) = {}^nC_r$. P(SSS . . . . . S and FFF . . . . . F)
(r times, (n - r) times)

$= {}^nC_r$. P (S) P (S) . . . . P(S) . P(F) P(F) . . . . P(F)
(r times, (n - r) times)

$\therefore$

$= {}^nC_r\, p^r\, q^{n-r}$ (P(S) = p, P(F) = q)

Note :

1. $P(x = 0) = {}^nC_0\, p^0\, q^{n-0} = q^n$

   $P(x = 1) = {}^nC_1\, p\, q^{n-1}$

   $P(x = 2) = {}^nC_2\, p^2\, q^{n-2}$

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   $P(x = n) = {}^nC_n\, p^n\, q^{n-n} = p^n$

   The distribution can be expressed as

X	0	1	2	. . . . .	n
P(X)	q^n	${}^nC_1\, pq^{n-1}$	${}^nC_2\, p^2q^{n-2}$	. . . . .	p^n

2. The distribution is called Binomial distribution since the probabilities of 0, 1, 2, ..., n successes, viz. q^n, ${}^nC_1\, pq^{n-1}$, ${}^nC_2\, p^2q^{n-2}$, ..., p^n, are the successive terms of the binomial expression $(q + p)^n$.
3. Sum of the probabilities

$= q^n + {}^nC_1\, pq^{n-1} + {}^nC_2\, p^2 q^{n-2} + \ldots . + p^n$

$= (q + p)^n = 1$

4. X can take only the integral values, viz,. 0, 1, 2, ... , n. Therefore, Binomial distribution is a discrete distribution.
5. Any random variable which follows Binomial distribution is called a binomial variate.
6. We have

   $P(X = r) = {}^nC_r\, p^r\, q^{n-r}$

   Thus, P(X = r) depends only on n, and p(or q). These constants are called parameters. Thus, Binomial distribution has two parameters.

Remark : The Bernoulli distribution is a special case of the binomial distribution where n = 1.

Example 1 : Find the probability distribution of the number of heads in 6 tosses of a coin.

Solution : If a coin is tossed, S = { H, T}

P (getting a head) ,

P (not getting a head) $= 1 - \frac{1}{2} = \frac{1}{2}$

Let X be the random variable denoting the number of heads.

In 6 trials, there may be 0 or 1 or 2 or 3 or 4 or 5 or 6 heads.

$$P(X = 0) = {}^6C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{6-0} = \frac{1}{64}$$

$$P(X = 1) = {}^6C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{6-1} = \frac{3}{32}$$

$$P(X = 2) = {}^6C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{6-2} = \frac{15}{64}$$

$$P(X = 3) = {}^6C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{6-3} = \frac{5}{16}$$

$$P(X = 4) = {}^6C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{6-4} = \frac{15}{64}$$

$$P(X = 5) = {}^6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{6-5} = \frac{3}{32}$$

$$P(X=6) = {}^6C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{6-6} = \frac{1}{64}$$

$\therefore$ the probability distribution is

X	0	1	2	3	4	5	6
P(X)		$\frac{3}{32}$	$\frac{15}{64}$	$\frac{5}{16}$	$\frac{15}{64}$	$\frac{3}{32}$	$\frac{1}{64}$

Note : Sum of the probabilities

$$= \frac{1}{64} + \frac{3}{32} + \frac{15}{64} + \frac{5}{16} + \frac{15}{64} + \frac{3}{32} + \frac{1}{64} = 1$$

Example 2 : A die is thrown 3 times. Find the probability of getting at least one six.

Solution : If a die is thrown, S = { 1, 2, 3, 4, 5, 6}

$\therefore$ P (getting a six)

P (not getting a six) $= 1 - \frac{1}{6} = \frac{5}{6}$

Let X be the random variable denoting the number of sixes.

In 3 trials, there may be 0, or 1, or 2, or 3 sixes.

$\therefore$ the probability of getting at least 1 six

= P(X = 1) + P(X = 2) + P(X = 3)

$$= \left(3 \times \frac{1}{6} \times \frac{25}{36}\right) + \left(3 \times \frac{1}{36} \times \frac{5}{6}\right) + \left(1 \times \frac{1}{216} \times 1\right)$$

$$= \frac{25}{72} + \frac{5}{72} + \frac{1}{216} = \frac{75+15+1}{216} = \frac{91}{216}$$

Mean of a Binomial Distribution :

Let a random variable X take the values x_1, x_2, x_3, ..., x_n with probabilities p_1, p_2, ..., p_n respectively. We know that the mean μ is defined by

Now, let us consider the binomial distribution.

$P(X = r) = {}^{n}C_r\, p^r\, q^{n-r}, \quad r = 0, 1, 2, 3, \ldots., n.$

$$\therefore \mu = \sum_{r=0}^{n} r \,.\, P(X = r)$$

$$= \sum_{r=0}^{n} r \,.\, {}^{n}C_r\, p^r q^{n-r}$$

$$= 0 + 1.{}^{n}C_1 pq^{n-1} + 2^{n}C_2 p^2 q^{n-2} + 3^{n}C_3 p^3 q^{n-3} + \ldots\ldots + {}^{n}n \,.\, {}^{n}C_n p^n q^{n-n}$$

$$= 1.np.q^{n-1} + n(n-1)p^2 q^{n-2} + \frac{n(n-1)(n-2)}{2!} p^3 q^{n-3} + \ldots\ldots + np^n$$

$$= np\left[1.q^{n-1} + (n-1)pq^{n-2} + \frac{n(n-1)(n-2)}{2!} p^2 q^{n-3} + \ldots\ldots + p^{n-1} \right]$$

$$= np\left[{}^{n-1}C_0\, q^{n-1} + {}^{n-1}C_1\, q^{(n-1)-1}\, p + {}^{n-1}C_2\, q^{(n-1)-2} p2 + \ldots\ldots + {}^{n-1}C_{n-1}\, p^{n-1} \right]$$

$$= np\,(q + p)^{n-1}$$

$$= np, \quad \therefore q + p = 1$$

Variance of a Binomial Distribution

We know that the variance σ^2 is defined by

$$\sigma^2 = \sum_{i=1}^{n} r_i^2 p_i - \mu^2$$

So for the above binomial distribution,

$$\text{Variance} = \sigma^2 = \sum_{r=0}^{n} r^2 p(X = r) - \mu^2$$

$$= \sum_{r=0}^{n} r^2\, {}^{n}C_r p^r q^{n-r} - n^2 p^2$$

$$= \sum_{r=0}^{n} \left[r + r(r-1) \right] {}^{n}C_r p^r q^{n-r} - n^2 p^2$$

$$= \sum_{r=0}^{n} r \,.\, {}^{n}C_r p^r q^{n-r} + \sum_{r=0}^{n} r(r-1)^{n} C_r p^r q^{n-r} - n^2 p^2$$

$$= np + \sum_{r=2}^{n} r(r-1) \frac{n(n-1)}{r(r-1)}\, {}^{n-2}C_{r-2} p^2 p^{r-2} q^{n-r} - n^2 p^2$$

$$= np + n(n-1)p^2 \sum_{r=2}^{n} {}^{n-2}C_{r-2}\, p^{r-2} q^{n-r} - n^2 p^2$$

$$= np + n(n-1)p^2 - n^2 p^2$$

$$= np + n^2 p^2 - np^2 - n^2 p^2$$

$$= np - np^2$$

$$= np(1 - p)$$

Thus, variance $= \sigma^2 = npq$

$\therefore\ SD = \sigma = \sqrt{npq}$

Recurrence relation for a binomial distribution :

We know that

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$\therefore\ P(X = r+1) = {}^nC_{r+1} p^{r+1} q^{n-r-1}$$

$$\therefore \frac{P(X = r+1)}{P(X = r)} = \frac{{}^nC_{r+1} p^{r+1} q^{n-r-1}}{{}^nC_r p^r q^{n-r}}$$

$$= \frac{n!}{(r+1)!\,(n-r-1)!} \frac{r!(n-r)!}{n!} \cdot \frac{p}{q}$$

$$= \frac{n-r}{r+1} \cdot \frac{p}{q}$$

$$\therefore\ P(X = r+1) = \frac{(n-r}{(r+1)} \cdot \frac{p}{q} \cdot P(X = r)$$

Example 3 : The mean of a binomial distribution is 5, and the variance is 2.5. Find the distribution.

Solution : Let X be a binomial distribution with parameters n, and p.

We know that the mean is np, and the variance is npq, q = 1 – p.

$\therefore$ np = 5, and npq = 2.5

5q = 2.5

$\therefore$ p = 1 – q = 1 –

$\therefore$ n = 10

the distribution is

, $0 \le r \le 10$.

Example 4 : The mean of a binomial distribution is 4 and the variance is $\frac{4}{3}$. Find $P(X \ge 1)$.

Solution : We have np = 4, and npq = $\frac{4}{3}$

$$\therefore\ 4q = \frac{4}{3} \Rightarrow q = \frac{1}{3}$$

$$\therefore\ p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\therefore\ \frac{2n}{3} = 4 \quad \Rightarrow n = 6$$

Now $P(X \geq 1) = 1 - P(X = 0)$

$$= 1 - {}^{6}C_0 p^0 q^{6-0} = 1 - q^6 = 1 - \left(\frac{1}{3}\right)^6$$

$$= 1 - \frac{1}{729} = \frac{728}{729}$$

Property : For a binomial distribution, mean is greater than variance.

Proof : Let X be a binomial distribution with parameters n, and p.

Now, mean = np

$$> npq, \quad \because\ 0 < q < 1$$

= variance.

Example 5 : Can there be a binomial distribution with mean 5, and variance 7.5 ?

Solution : We know that in a binomial distribution, mean > variance.

Now, 5 < 7.5

$\therefore$ there cannot be binomial distribution with mean 5, and variance 7.5.



CHECK YOUR PROGRESS 1

Q 1: Find the probability distribution of the number of tails in 4 tosses of a coin.

Q 2: Find the probability distribution of the number of doublets in three throws of a pair of dice.

Q 3: A coin is flipped 8 times. Find the probability of getting at least 2 heads.

Q 4: Find the binomial distribution whose mean is 3, and the variance is 2.

Q 5: Find p for a binomial random variable X, if n = 6, and 9p(X = 4) = P(X = 2). Also, find the mean and the variance of the distribution.

Q 6: Let X be a binomial variate such that the mean is double the standard deviation. If P(X = 0) = , find the probability that X is at most 1.

Q 7: A coin is flipped 8 times. Find the probability of getting at least 7 heads.

Q 8: For a binomial distribution, the mean is 6 and the standard deviation is $\sqrt{2}$. Write down the distribution.

Q 9: The mean and variance of a binomial variate X with parameters n and p are 16 and 8. Find (i) P (X = 0), (ii) P (X = 1), (iii) $P(X \geq 2)$

Q 10: Are the following statements true ? Justify your answer.

(i) The mean of a binomial distribution is 6, and its standard deviation is 3.

(ii) The mean of a binomial distribution is 5, and its variance is 2.

$\frac{p}{16} \to 0$

8.5 POISSON DISTRIBUTION

(After the name of French mathematician simeon Denis Poisson, 1781-1840)

Poisson distribution is a limiting case of the binomial distribution under the following three conditions.

1. The number of trials n is indefinitely large, i.e., $n \to \infty$
2. The constant probability p of success for each trial is indefinitely small, i.e.,
3. np = m (say) is finite.

We have

$$P(r) = P(X = r) = {}^{n}C_{r} p^{r} q^{n-r}$$

We know that P(0) = q^n

$$\therefore \lim_{n\to\infty} p(0) = \lim_{n\to\infty} q^n$$

$$= \lim_{n\to\infty} (1-p)^n$$

$$= \lim_{n\to\infty} \left(1-\frac{m}{n}\right)^n, \qquad \text{from np = m}$$

$$= e^{-m}$$

Again, we know that $\frac{P(r+1)}{P(r)} = \frac{n-r}{r+1}.\frac{p}{q}$

$$\therefore \lim_{n\to\infty} \frac{P(r+1)}{P(r)} = \lim_{n\to\infty} \frac{n-r}{r+1}.\frac{p}{q}$$

$$= \lim_{n\to\infty} \frac{n-r}{r+1}.\frac{\frac{m}{n}}{1-\frac{m}{n}}$$

$$= \lim_{n\to\infty} \frac{1-\frac{r}{n}}{1-\frac{m}{n}}.\frac{m}{r+1} = \frac{m}{r+1}$$

Now, $\lim_{n\to\infty} P(r) = \lim_{n\to\infty} \frac{P(r)}{P(r-1)} . \lim_{n\to\infty} \frac{P(r-1)}{P(r-2)} . \ldots . \lim_{n\to\infty} \frac{P(1)}{P(0)} . \lim_{n\to\infty} P(0)$

$$= \frac{m}{r} . \frac{m}{r-1} . \ldots . \frac{m}{1} e^{-m}$$

$$= \frac{m^r e^{-m}}{r!}$$

Definition : A random variable X is said to follow a poisson distribution if it takes non-negative values with the probability given by

$$P(X=x)\begin{cases} = \frac{e^{-m}m^x}{x!}, & x=0, 1, 2, \ldots, \quad m>0 \\ = 0 \ otherwise \end{cases}$$

Note :

1. X can take only the integral values, viz., 0, 1, 2, ... Therefore, poisson distribution is a discrete distribution.
2. Poisson distribution has only one parameter, viz, m.

Some examples where poisson distribution may be applied :

1. Number of defective blades in a packet of 100 blades.
2. Number of printing mistakes at each page of a book.
3. Number of deaths from heart attack or snake bite.
4. Number of air accidents in India in one year.
5. Number of cars passing through a street-crossing during the busy hours of a day.

Mean of a poisson distribution :

Mean = μ

$$= \sum_{x=0}^{\infty} x \frac{e^{-m}\, m^x}{x!}$$

$$= \sum_{x=0}^{\infty} x \frac{e^{-m}\, m\, m^{x-1}}{x(x-1)!}$$

$$= m e^{-m} \sum_{x=1}^{\infty} \frac{m^{x-1}}{(x-1)!}$$

$$\equiv \sum_{x=0}^{\infty} x P(X = x)$$

$$= m e^{-m} \left[1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \ldots . \infty \right]$$

$$= m e^{-m} e^{m}$$

$$= m$$

Variance of a poisson distribution :

Variance

$$= \sum_{x=0}^{\infty} x^2 P(X = x) - \mu^2$$

$$= \sum_{x=0}^{\infty} x^2 \frac{e^{-m}\, m^x}{x!} - m^2$$

$$= \sum_{x=0}^{\infty} \left(x(x-1) \frac{e^{-m}\, m^x}{x!} + \frac{x e^{-m}\, m^x}{x!} \right) - m^2$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-m}\, m^x}{x(x-1)(x-2)!} + \sum_{x=0}^{\infty} \frac{x e^{-m}\, m^x}{x!} - m^2$$

$$= \sum_{x=0}^{\infty} \frac{e^{-m}\, m^2 m^{x-2}}{(x-2)!} + m - m^2$$

$$= e^{-m} m^2 \sum_{x=2}^{\infty} \frac{m^{x-2}}{(m-2)!} + m - m^2$$

$$= e^{-m} m^2 \left[1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \ldots\ldots\infty \right] + m - m^2$$

$$= e^{-m} m^2 e^m + m - m^2$$

$$= m^2 + m - m^2 = m$$

Note : Mean of a poisson distribution

= Variance

= m.

Recurrence relation for a poisson distribution

We have $P(X = x) = \frac{e^{-m} m^x}{x!},\ x = 0,1,2,\ldots\infty$

$$\therefore P(X = x+1) = \frac{e^{-m} m^{x+1}}{(x+1)!}$$

$$\therefore \frac{P(X = x+1)}{P(X = x)} = \frac{e^{-m} m^{x+1}}{(x+1)!} \frac{x!}{e^{-m} m^x}$$

$$= \frac{m}{x+1}$$

$$\therefore P(X = x+1) = \frac{m}{x+1} P(X = x)$$

Example 6 : A random variable X follows poisson distribution such that P(X = 0) = P(X = 1) = a. Find the mean of the distribution. Also find the value of a.

Solution : We have $P(X = x) = \frac{e^{-m} m^x}{x!}$

Now, P(X = 0) = P (X = 1), (given)

$$\therefore \frac{e^{-m} m^x}{0!} = \frac{e^{-m} m^1}{1!}$$

$$\therefore m = 1$$

i.e., mean = 1

Again P (X = 0) = a

or, $e^{-m} = a$

or, $e^{-1} = a$

i.e. $a = \frac{1}{e}$

Example 7 : A random variable X follows poisson distribution such that P(X = 1) = 2P(X = 2). Find P(X = 0), $P(X \geq 0)$.

Solution : P(X = 1) = 2P(X = 2), given

$$\therefore \frac{e^{-m}m^1}{1!} = 2\frac{e^{-m}m^2}{2!}$$

or, m = 1

$$\therefore P(X=0) = \frac{e^{-1}1^0}{0!} = e^{-1} = \frac{1}{e}$$

$$P(X \geq 0)$$

$$= P(X=0) + P(X=1) + P(X=2) + \ldots.$$

$$= e^{-1} + \frac{e^{-1}}{1!} + \frac{e^{-1}}{2!} + \frac{e^{-1}}{3!} + \frac{e^{-1}}{4!} + \ldots..$$

$$= e^{-1}\left(1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{3!} + \frac{1}{4!} + \ldots..\right)$$

$$= e^{-1}\ e$$

$$= e^0 = 1$$

Example 8 : 6 coins are tossed 6400 times. Use poisson distribution to find the approximate probability of getting 6 heads r times.

Solution : Here n = 6400

p = the probability of getting 6 heads in one throw of 6 coins

$$\therefore m = np = \frac{6400}{2^6}$$

$$\therefore P(X=r) = \frac{e^{-m}m^r}{r!}$$

$$= \frac{e^{-100}100^r}{r!}$$



CHECK YOUR PROGRESS 2

Q 1: A random variable X follows Poisson distribution such that P(X = 1) = P(X = 2). Find P(X = 3)

Q 2: A random variable X follows poisson distribution such that P(X = 2) = 3 P(X = 3). Find the probability that X is at least one.

Q 3: Examine if the following statement is right.

The mean of a poisson distribution is 3, and its standard deviation is 4.

Q 4: A random variable X follows poisson distribution such that P(X = 2) = 9P(X = 4). Find the mean of X.

Q 5: A random variable X follows poisson distribution such that P(X = 2) = P(X = 3). Find the variance of X.



8.6 LET US SUM UP

- A Bernoulli trial is an experiment satisfying the following conditions.
  1. There are only two possible outcomes, viz,. success or failure (occurrence of an event is 'success', non-occurrence is 'failure').
  2. The probability of a success (or failure) remains same in each trial.
  3. Trials must be independent.
  4. Number of trials must be finite.
  5. The sum of probabilities of success and failure is 1.
  6. The probability of success vis-a-vis failure is not very low.
- Let there be n Bernoulli trials. Let the random variable X denote the number of successes in these trials. Let the probability of getting a success in a single trial be p, and that of getting a failure be q so that p + q = 1. Then $P(X = r) = {}^{n}C_{r}\, p^{r}\, q^{n-r}$

  This is known as Binomial distribution.

- The bernoulli distribution is a special case of the binomial distribution where n = 1.
- Mean of a boniorial distribution is np, and the variance is npq.
- Poisson distribution is a limiting case of the binomial distribution under the following there conditions.
  1. The number of trials n is indefinitely large, i.e., $n \to \infty$.
  2. The constant probability p of success for each trial is indefinitely small, i.e., .
  3. np = m, (say), is finite.
- A random variable X is said to follow a poisson distribution if it takes non-negative values with the probability given by

$$P(X = x)\begin{cases} = \dfrac{e^{-m} m^x}{x!}, \; x = 0, 1, 2, \ldots, \quad m > 0 \\ = 0 \; otherwise \end{cases}$$

- Mean of a poisson distribution
  = variance
  = m

$p \to 0$



8.7 FURTHER READINGS

1) Rao. G. Shanker, Probability and Statistics, Universitis Press.
2) Statistics, R.S.N. Pillai and Bagavathi S. Chand and Company Ltd. New Delhi-110055.

8.8 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

Ans to Q No 1: If a coin is tossed, S = {H, T}

$\therefore$ P (getting a head)

P (not getting a head) $= 1 - \frac{1}{2} = \frac{1}{2}$

Let X be the random variable denoting the number of tails.

In 4 trials there may be 0 or 1 or 2 or 3 or 4 tails.

$$P(X=0) = {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{4-0} = \frac{1}{16}$$

$$P(X=1) = {}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{4-1} = \frac{1}{4}$$

$$P(X=2) = {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} = \frac{3}{8}$$

$$P(X=3) = {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{4-3} = \frac{1}{4}$$

$$P(X=4) = {}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{4-4} = \frac{1}{16}$$

$\therefore$ the probability distribution is

X	0	1	2	3	4
P(X)		$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Ans to Q No 2: [One of the meanings of the word doublet is as follows.

'Either of two dice, each of which, when thrown, has the same number of spots on the face lying uppermost.']

If a pair of dice is thrown, there will be 36 outcomes.

S = { (1, 2), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), ..., (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Thus n(s) = 36

Possible doublets are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6).

i.e., the number of doublets is 6.

$\therefore$ P(getting a doublet)

$\therefore$ P(not getting a doublet)

In 3 trials, there may be 0 or 1 or 2 or 3 doublets.

$$P(X=0)={}^3C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^3=\frac{125}{216}$$

$$P(X=1)={}^3C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2=\frac{25}{72}$$

$$P(X=2)={}^3C_2\left(\frac{1}{6}\right)^2\frac{5}{6}=\frac{5}{72}$$

$$P(X=3)={}^3C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^0=\frac{1}{216}$$

$\therefore$ the probability distribution is

X	0	1	2	3
P(X)		$\frac{25}{72}$	$\frac{5}{72}$	$\frac{1}{216}$

Ans to Q No 3: If a coin is flipped, S = {H, T}

$\therefore$ P(getting a head)

P (not getting a head) $=1-\frac{1}{2}=\frac{1}{2}$

Let X be the random variable denoting the number of heads.

$\therefore\ P(X=r)={}^nC_r p^r q^{n-r}$

$$={}^8C_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{8-r}={}^8C_r\left(\frac{1}{2}\right)^8$$

$\therefore$ P (getting at least 2 heads)

$$=1-[P(X=0)+P(X=1)]$$

$$= 1 - \left[{}^{8}C_0 \left(\frac{1}{2}\right)^8 + {}^{8}C_1 \left(\frac{1}{2}\right)^8 \right]$$

$$= 1 - \left[\frac{1}{256} + \frac{8}{256} \right]$$

$$= 1 - \frac{9}{256}$$

$$= \frac{247}{256}$$

Ans to Q No 4: Let X be the binomial variate.

We have mean = np = 3 (1)

variance = npq = 2 (2)

From (1) and (2), 3q = 2

$$\therefore\ q = \frac{2}{3}$$

$$\therefore\ p = 1 - q = 1 - \frac{2}{3} = \frac{1}{3}$$

Again, from (1) $\frac{n}{3} = 3$

$$\therefore\ n = 9$$

$\therefore$ the distribution is

, r = 0, 1, 2, 3, ... , 9.

Ans to Q No 5: We have

9p (x = 4) = p(x = 2)

or, 9 ${}^6C_4\ p^4\ q^{6-4}$ = ${}^6C_2\ p^2\ q^{6-2}$, (n = 6 given)

or, $9p^2 = q^2$, (${}^6C_4 = {}^6C_2$)

or, 3p = q

or, 3p = 1 – p

or, 4p = 1

$$\therefore\ p = \frac{1}{4} \qquad \therefore\ q = 1 - \frac{1}{4} = \frac{3}{4}$$

$\therefore$ mean

variance $= 6 \times \frac{1}{4} \times \frac{3}{2} = \frac{9}{4}$

Ans to Q No 6: We have

mean = 2 (standard deviation), (given)

or, $np = 2\sqrt{npq}$

or, $n^2 p^2 = 4npq$

or, np = 4q (1)

Again, $P(X = 0) = \frac{1}{16}$ (given)

$\therefore q^n = \frac{1}{16}$

or $q^n = \left(\frac{1}{2}\right)^4$ or $q^n = \left(\frac{1}{2}\right)^2$

$\therefore$ either or $q = \frac{1}{4},\ n = 2$

If $q = \frac{1}{2},\ n = 4$ then from (1)

$q = \frac{1}{2}, n = 4$ ${}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{4-0} + {}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{4-1}$

$4p = 4 \times \frac{1}{2}$ or $p = \frac{1}{2}$. $\therefore q = 1 - p = \frac{1}{2}$

If $q = \frac{1}{4},\ n = 2$ than from (1)

$2p = 4 \times \frac{1}{4}$ or 2p = 1 or $p = \frac{1}{2}$

$\therefore\ q = 1 - \frac{1}{2} = \frac{1}{2}$ which is not true.

$\therefore\ q = \frac{1}{4},\ n = 2$ is not admissible

$\therefore n = 4,\ q = \frac{1}{4},\ p = \frac{1}{2}$

P (X is at most 1)

= P(X = 0) + P(X = 1)

$= \left(\frac{1}{2}\right)^4 + 4 \times \left(\frac{1}{2}\right)^4$

$$= \frac{1}{16} + \frac{4}{16}$$

$$= \frac{5}{16}$$

Ans to Q No 7: If a coin is flipped, S = {H, T}

$\therefore$ P(getting a head)

P (not getting a head) $= 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore\ p = \frac{1}{2},\ q = \frac{1}{2}$

Let X be the random variable denoting the number of heads.

In 8 trials there may be 0 or 1 or 2 or 3 or 4 or 5 or 6 or 7 or 8 heads.

$\therefore$ P (getting at least 7 heads)

$$= {}^8C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{8-7} + {}^8C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{8-8}$$

$$= 8 \times \frac{1}{2^8} + \frac{1}{2^8}$$

$$= \frac{8}{256} + \frac{1}{256}$$

$$= \frac{9}{256}$$

Ans to Q No 8: We have np = 6 (1)

$$\sqrt{npq} = \sqrt{2}$$

$\therefore\ npq = 2$

or 6q = 2

$$\therefore q = \frac{1}{3}$$

$$\therefore p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$$

$\therefore$ from (1), or n = 9

$$\therefore P(X = r) = {}^9C_r\left(\frac{2}{3}\right)^r\left(\frac{1}{q}\right)^{q-r},$$ r = 0, 1, 2, 3, 4, 5, 6,7, 8, 9

Ans to Q No 9: We have np = 16, (1)

npw = 8 (2)

$\therefore 16q = 8$ or $q = \frac{1}{2}$

$\therefore\ p = 1 - q = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore$ from (1), or n = 32

(i) $P(X = 0) = {}^{32}C_0 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^{32-0}$

$= \frac{1}{2^{32}}$

(ii) $P(X = 1) = {}^{32}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{32-1}$

$= 32 \cdot \frac{1}{2^{32}} = \frac{2^5}{2^{32}} = \frac{1}{2^{27}}$

(iii) $P(X \geq 2)$

$= 1 - [P(X = 0) + P(X = 1)]$

$= 1 - \left(\frac{1}{2^{32}} + \frac{1}{2^{27}}\right)$

$\therefore n = 16$

$P(X = x) = \frac{e^{-m} m^x}{x!}$

Ans to Q No 10: (i) Mean = 6, SD = 3

$\therefore$ variance = 9

We know that for a binomial distribution mean > variance

In this case mean = 6, variance = 9

the statement is not true.

(ii) Mean = 5

Variance = 2

We know that for a binomial distribution, mean > variance.

In this case mean = 5, variance = 2.

the statement is true.

Check Your Progress 2

Ans to Q No 1: We know that, for a poisson distribution,

Now P(X=1) = P(X=2), (given)

$\therefore$ m = 2

Ans to Q No 2: We have

$$P(X = x) = \frac{e^{-m} m^x}{x!}$$

P (x = 2) = 3 P (x = 3)

or, $\frac{e^{-m} m^2}{2!} = 3 \cdot \frac{e^{-m} m^3}{3!}$

or, m = 1

$\therefore P(X \geq x)$

$$= 1 - \frac{e^{-1} 1^0}{0!}$$

$$= 1^{-e} - 1 = 1 - \frac{1}{e} = \frac{e-1}{e} = 0.632$$

Ans to Q No 3: We know that the mean of a poisson distribution = variance.

$\therefore$ the given statement is not true.

Ans to Q No 4: We have

Now P(x = 2) = 9 P(x = 4)

or, $= \frac{e^{-m} m^2}{2!} = 9 \cdot \frac{e^{-m} m^4}{4!}$

or, $\frac{m^2}{2} = \frac{3m^4}{8}$

or, $\frac{3m^2}{4} = 1$

or, $m^2 = \frac{4}{3}$

$\therefore m = \frac{2}{\sqrt{3}}$

$\therefore$ mean =

Ans to Q No 5: We have

P(x = 2) = P(x = 3)

or, $\frac{e^{-m} m^2}{2!} = \frac{e^{-m} m^3}{3!}$

or, m = 3

$\therefore$ variance = m = 3



8.9 MODEL QUESTIONS

Q 1: Name the discrete probability distribution for which

(i) mean is greater than the variance,

(ii) mean is equal to the variance.

$P(X=2) = P(X=1)$

Q 2: Derive poisson distribution as a limiting form of a binomial distribution.

Q 3: 8 coins are thrown simultaneously. Find the probability of obtaining at least 6 heads.

Q 4: Let X be a binomial variate such that mean = 1, variance = 0.75. Find (i) P(x = 0), (ii) P (x > 0).

Q 5: Write down the conditions under which a binomial distribution tends to the poisson distribution.

Q 6: Let X be a poisson variate such that .

Find P(x = 0)

Q 7: If the sum of the mean and variance of a binomial distribution for 5 trials is 1.8, find the distribution.

Q 8: The mean and variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively. FInd $P(X \geq 1)$.

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