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SECURITY ANALYSIS AND PORTFOLIO MANAGEMENT

SEMESTER - 4

MASTER OF COMMERCE

(Finance & Accounting Specialisation)

BLOCK - 2

KRISHNA KANTA HANDIQUI STATE OPEN UNIVERSITY

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MASTER IN BUSINESS ADMINISTRATION

INVESTMENT MANAGEMENT
(FINANCE SPECIALISATION)
Block 2

DETAILED SYLLABUS

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UNIT 9: Valuation of Variable Income Securities or Equity Share Valuation **Page No. : 34-55**

Share valuation, Earnings valuation, Cash flow valuation, Asset valuation, Dividend-Discount model.

UNIT 10: Portfolio Management **Page No. :56-74**

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UNIT 14: Capital Market Theory and the Arbitrage Pricing Theory **Page No. : 137-161**

Capital Asset Pricing Model (CAPM), Concepts of Risk-free Asset, Risk-free Lending and Risk-free Borrowing, Efficient Set with Risk Free Lending and Borrowing, Leveraged Portfolio, Market Portfolio, Capital Market Line, The CAPM, Assumptions, Security Market Line, Limitations, Arbitrage Pricing Theory (APT)

COURSE INTRODUCTION

Investment Management provides a survey of the important areas of investments: valuation, the marketplace, fixed income instruments and markets, equity instruments and markets, derivative instruments, and a cross-section of special topics, such as international markets and mutual funds.

Investment is an important means for channelizing the savings into the development of economy. “Security Analysis and Portfolio Management course is structured into 8 units in such a manner that the learner takes a stage wise exposure to the concepts of investment management and also appreciates the instruments and the market place where the activity has to be practised.

The course has 15 units and is divided into two blocks: Block 1 and Block 2.

Block 1 deals with the introductory concepts of Investment- A Conceptual Framework,The Investment Enviornmnet,Risk and Return,Fundamental Analysis, Technical Analysis ,Efficient Market Theory and Valuation of Bonds

Block 2 concentrates on Valuation of variable Income Securities or Equity Share Valuation,Portfolio Management,Portfolio Construction, The Markowitz Model,Capital Asset Pricing Theory **,**Capital Market Theory and the Arbitrage Pricing Theory and Mutual Funds.

Each unit of these blocks includes some along-side boxes to help you know some of the difficult, unseen terms. Some “EXERCISES” have been included to help you apply your own thoughts. You may find some boxes marked with: “LET US KNOW”. These boxes will provide you with some additional interesting and relevant information. Again, you will get “CHECK YOUR PROGRESS” questions. These have been designed to self-check your progress of study. It will be helpful for you if you solve the problems put in these boxes immediately after you go through the sections of the units and then match your answers with “ANSWERS TO CHECK YOUR PROGRESS” given at the end of each unit. you in making your learning more active and efficient. And, at the end of each section, you will get “CHECK YOUR PROGRESS” questions. These have been designed to self-check

BLOCK INTRODUCTION:

This is the second block of the course "Security Analysis and Portfolio Management". The Block is divided into 7 units which are related to the differnt concepts of Invedtment Management.

This block comprises of the following seven units:

The eighth unit introduces us to Bond valuation:The Time Value Concept,The Present Value,The Bond Valuation Model,Bond Yield,Price-Yield Relationship,Bond market, The term structure of interest rate (yield curve),Riding the yield curve and Duration and Immunisation

The ninth unit introduces us to Share valuation, Earnings valuation,Cash flow valuation,Asset valuation and Dividend-Discount model

The tenth unit gives us a broad idea on the concepts of Inputs to Portfolio Management,Return and Risk Characteristics of Individual Assets, Expected Return and Risk of a Portfolio, Diversification of Risk, Portfolio Analysis and Selection,Correlation between Securities and its Impact on Portfolio Risk and Portfolio Selection

The eleventh unit gives us an idea on the Benefits of Portfolios, Approaches in Portfolio Construction: Traditional approach, Modern Approach and Portfolio Risk Return

The twelvlth unit will help us in knowing the The Markowitz Model: Assumptions and Concept, Varying Degrees of Correlation, Simple Diversification,Problems of vast Diversification, Risk and Return with Different Correlation, Markowitz Efficient Frontier, Utility Analysis, Risk Free Asset, Sharpe-The Single Index Model and Sharpe's Optimal Portfolio

The thirteenth unit gives us a broad idea on Capital Asset Pricing Theory: An Introduction,

Security Market Line,Evaluation of Securities, Market Imperfection and SML, Empirical Tests of the CAPM and Present Validity of CAPM

The forteenth unit will help us in understanding the Capital Asset Pricing Model (CAPM), Concepts of Risk-free Asset, Risk-free Lending and Risk-free Borrowing, Efficient Set with Risk Free Lending and Borrowing, Leveraged Portfolio, The CAPM, Assumptions, Security Market Line and Arbitrage Pricing Theory (APT)

The Block is devided into Seven units:

UNIT 8: Valuation of Fixed Income Securities or Valuation of Bonds

UNIT 9: Valuation of Variable Income Securities or Equity Share Valuation

UNIT 10: Portfolio Management

UNIT 11: Portfolio Construction

UNIT 12: The Markowitz Model

UNIT 13: Capital Asset Pricing Theory

UNIT 14: Capital Market Theory and the Arbitrage Pricing Theory

UNIT 8 : VALUATION OF BONDS

UNIT STRUCTURE

8.1 Learning Objectives

8.2 Introduction

8.3 Bond valuation-Terminology

8.3.1 The Time Value Concept

8.3.2 The Present Value

8.4 The Bond Valuation Model

8.5 Bond Yield

8.6 Price-Yield Relationship

8.7 Bond market

8.8 The term structure of interest rate (yield curve)

8.9 Riding the yield curve

8.10 Duration and Immunisation

8.11 Let Us Sum Up

8.12 Further Reading

8.13 Answers to Check Your Progress

8.14 Model Questions

8.1 LEARNING OBJECTIVES

After going through this unit,you will be able to :

- discuss the main characteristics of fixed income instruments.
- describe the time value concept
- discuss basic discounted cash flow valuation model and its application to bonds.

8.2 INTRODUCTION

In the previous unit, we have discussed about 'Efficient Market Hypothesis'. In this unit we rae going to discuss about Bond valuation-Terminology, Valuation model, Bond return, Price-yield relationshi, Bond market and Immunisation. A long-term debt instrument (a legal contract) in

which a borrower agrees to make payments of principal and interest, on specific dates, to the holders of the bond.

Assets can be real or financial; securities like shares and bonds are called financial assets while physical assets like plant and machinery are called real assets. The concepts of return and risk, as the determinants of value, are as fundamental and valid to the valuation of securities as to that of physical assets.

Let us now discuss the concept oin the following ways:

8.3 VALUATION OF BONDS:TERMINOLOGY

Let us first discuss about the terms.

Bond: A long-term debt instrument (a legal contract) in which a borrower agrees to make payments of principal and interest, on specific dates, to the holders of the bond. A bond or debenture is a debt instrument issued by the government or a government agency or a business enterprise.Table 8.1 describes briefly the variety of debt instruments in the Indian market.

Table 8.1. Debt Instruments

Type	Typical features
Central Government Securities	Medium- to long-term bonds issued by RBI on behalf of GOI. Coupon payments are semi-annual.
State Government Securities	Medium- to long-term bonds issued by RBI on behalf of the state government. Coupon payments are semi-annual.
Government-Guaranteed Bonds	Medium- to long-term bonds issued by government agencies and guaranteed by the central government or a state government. Coupon payments are semi-annual.
PSU Bonds	Medium- to long-term bonds issued by public sector companies in which the central or state government has an equity stake of 51 per cent or more.

Corporate Debentures	Short-to medium-term debt issued by private and public sector companies.
Money Market Instruments	Debt instruments like Treasury Bills (issued by GOI), Commercial Paper (issued by corporates) and Certificates of deposits (issued by banks and financial institutions) that have a maturity of less than a year.

Types of Bonds: Bonds with Maturity,Pure Discount Bonds and Bond Yields

Concept of Value : The value can be classified as below:

Book Value- Book value per share is determined as net worth divided by the number of shares outstanding. Book value reflects historical cost, rather than value.

Replacement Value- Replacement value is the amount that a company would be required to spend if it were to replace its existing assets in the current condition.

Liquidation Value- Liquidation value is the amount that a company could realise if it sold its assets, after having terminated its business.

Going Concern Value- Going concern value is the amount that a company could realise if it sold its business as an operating business.

Market Value- Market value of an asset or security is the current price at which the asset or the security is being sold or bought in the market.

In order to understand the valuation of bonds, we need to be also familiar with certain bond-related terms:

Par Value- It is the value stated on the face of the bond. It represents the amount the firm borrows and promises to repay at the time of maturity. Usually the par or face value of bonds issued by business firms is Rs. 100. Sometimes it can be Rs. 1000.

Coupon Rate and Interest- A bond carries a specific interest rate which is called the coupon rate. The interest payable to the bond holder is simply par value of the bond × coupon rate. Most bonds pay interest semi-annually. For example, a GOI security which has a par value of Rs. 1000 and a coupon rate of 11 per cent pays an interest of Rs. 55 every six months.

Maturity Period- Typically, bonds have a maturity period of 1-10 years; sometimes they have a longer maturity. At the time of maturity the par (face) value plus perhaps a nominal premium is payable to the bondholder.

8.3.1 The Time Value Concept

The time value concept of money is that the rupee received today is more valuable than a rupee received tomorrow. The investor will postpone current consumption only if he could earn more future consumption opportunities through investment. Individuals generally prefer current consumption to future consumption. If there is inflation in the economy, a rupee today will represent more purchasing power than a rupee at a future date.

Interest is the rent paid to the owners to part their money. The interest that the borrower pays to the lender causes the money to have a future value different from its present value. The time value of money makes the rupee invested today grow more than a rupee in the future. To quantify this concept mathematically compounding and discounting principles are used.

The one period future time value of money is given by the equation: *Future Value = Present Value (1 + interest rate).* If hundred rupees are put in a savings bank account in a bank for one year, the future value of money will be:

Future Value = Rs. 100 (1.0 + 6%)

100 × 1.06 = Rs. 106.

If the deposited money is allowed to cumulate for more than one time, the period exponent is added to the previous equation.

Future Value = (Present Value) $(1 + \text{Interest Rate})^{t}$

t- the number of time periods the deposited money accumulates as interest. Suppose Rs. 100 is put for two years at the 6% rate of interest, money will grow to be Rs. 112.36.

Future Value = Present value $(1 + \text{interest rate})^2$

$= 100 (1 + 0.06)^2$

= 100 (1.1236)

= 112.36.

To find out the values in a simple manner, the compound sum of Re. 1 at the end of a period FVIF1,/K, n and compound sum of an annuity of Re. 1 per period FVIFA tables are given in the appendix 1 and 2 of the book on- Financial Management for MBA second semester.

8.3.2 The Present Value

The present value of money can be found simply by reversing the earlier equation.

Present value × (1 + interest rate) = Future value

$$\text{Present value} = \frac{\text{Future value}}{1+\text{interest rate}}$$

Here, the discounting principle is used. Today's worth of Rs. 100 to be received after a year at 10 per cent interest would be:

$$\text{Present value} = \frac{\text{Future value}}{1+\text{interest rate}}$$

$$= \text{Rs.}\frac{100}{1+0.10} = \frac{100}{1.1} = \text{Rs. } 90.90$$

The multiple period of present value equation takes into account of the multiple periods**.**

$$\text{Present value} = \frac{\text{Future value}}{(1+\text{interest rate})^{t}}$$



CHECK YOUR PROGRESS

Q1: Define Bond.

.....

.....

Q2: Define Book Value

.....

.....

Q3: What is Time Value Concept

.....

.....

8.4 THE BOND VALUATION MODEL

The value of a bond- or any asset, real or financial- is equal to the present value of the cash flows expected from it. Hence, determining the value of a bond requires:

- An estimate of expected cash flows
- An estimate of the required return.

To simplify the analysis of bond valuation we will make the following assumptions:

- The coupon interest rate is fixed for the term of the bond.
- The coupon payments are made every year and the next coupon payment is receivable exactly a year from now.
- The bond will be redeemed at par on maturity.

Given these assumptions, the cash flow for a non-callable bond comprises an annuity of a fixed coupon interest payable annually and the principal amount payable at maturity. Hence the value of a bond is:

$$P = \sum_{t-1}^{n} \frac{C}{(1+r)^t} + \frac{M}{(1+r)^n}$$

Where P = Present value (in rupees)

n = number of years

C = annual coupon payment (in rupees)

r = periodic required return

M = maturity value

t = time period when the payment is received.

Since the stream of semi-annual coupon payments is an ordinary annuity, we can apply the formula for the present value of an ordinary annuity. Hence the bond value is given by the formula:

$P = C \times PVIFA_{r, n} + M \times PVIF_{r, n}$

To illustrate how to compute the value of a bond, consider a 10-year, 12 per cent coupon bond with a par value of Rs. 1000. Let us assume that the required yield on this bond is 13 per cent. The cash flows for this bond are as follows:

- 10 annual coupon payments of Rs. 120.
- Rs. 1000 principal repayment 10 years from now.

The value of the bond is:

$P = 120 \times PVIFA_{(13\%, 10\ yr)} + 1{,}000 \times PVIF_{(13\%, 10yr)}$

$120 \times 5.426 + 1000 \times 0.295$

651.1 + 295 = Rs. 946.1

Bond values with semi-annual interest

Most bonds pay interest semi-annually. To value such bonds, we have to work with a unit period of six months, and not one year. This means that the bond valuation equation has to be modified along the following lines:

- The annual interest payment, C, must be divided by 2 to obtain the semi-annual interest payment.
- The number of years to maturity must be multiplied by two to get the number of half-yearly periods.
- The discount rate has to be divided by two to get the discount rate applicable to half-yearly periods.

With the above modifications, the basic bond valuation becomes:

$$P = \sum \frac{C/2}{(1+r/2)^{2n}} + \frac{M}{(1+r/2)^{2n}}$$

$P = C/2 * PVIFA(_{r/2,}\ 2n) + M * (PVIF)(_{r/2,\ 2n})$

Where P = value of the bond

C/2 = semi-annual interest payment

r/2 = discount rate applicable to a half-year period

M = maturity value

2n = maturity period expressed in terms of half-yearly periods.



Excercise 8.1:

Consider a 8-year, 12 per cent coupon bond with a par value of Rs. 100 on which interest16 is payable semi-annually. The $\sum$ C/26 ++ 100M required return on this bond is 14 per(1cent+r/2).t^t (1+ r/2)$16^{2n}_{t=1}$ (1.07) (1.07)

Solution.

Applying Eq. 10.2, the value of the bond is:

$$P=\sum \frac{C/2}{(1+r/2)^{2n}}+\frac{M}{(1+r/2)^{2n}}$$

P = 6 ($PVIFA_{7\%, 16\ yr}$) + 100 ($PVIF_{7\%, 16\ yr}$)

Rs. 6 (9.447) + Rs. 100 (0.388) = Rs. 95.5.

Excercise 8.2: At an annual rate of compounding of 9%, how long does it take for a given sum to become double and triple its original value?

Solution.

$P_t = P_0 (1 + r)^n$

When the n value is not given it can be solved by using log ln:

n ln (1 + r) = ln P_t

n ln (1 + 0.09) = ln 2

n. ln 0.0862 = ln 0.6931

n = 8.04 years

To triple

n ln (1 + 09) = ln 3

n. ln 0.0862 = ln 1.0986

= 12.74 years



Excercise 8.3:

Of the following which amount is worth more at 16 per cent; Rs. 1000 today or Rs. 2100 after five years.

Solution.

The present worth of Rs. 2100

$$= \frac{2100}{(1+r)^n}$$

$$= \frac{2100}{(1+0.16)^5}$$

2100 × 0.476 = 999.60

The present worth of Rs. 2100 is Rs. 999.60 which is less than Rs. 1,000. Hence Rs. 2100 after five years is not worthful.

8.5 BOND YIELD

Bond yield is the return an investor realizes on a bond. The bond yield can be defined in different ways. Setting the bond yield equal to its coupon rate is the simplest definition. The current yield is a function of the bond's price and its coupon or interest payment, which will be more accurate than the coupon yield if the price of the bond is different than its face value. More complex calculations of a bond's yield will account for the time value of money and compounding interest payments.

The bond yield is the annualized return of the bond. Thus, bond yield will depend on the purchase price of the bond, its stated interest rate — which is equal to the annual payments by the issuer to the bondholder divided by the par value of the bond — plus the amount paid at maturity. Let us discuss the bond yields in following ways:

- **The current yield** - The current yield is the coupon payment as a percentage of current market prices. Because bonds trade in the secondary market, they may sell for less or more than par value, which will yield an interest rate that is different from the nominal yield, called the current yield, or current return. Since the price of bonds moves in the opposite direction of interest rates, bond prices decrease when interest rates increase, and vice versa.

$$\text{Current yield} = \frac{\text{Annual coupon payment}}{\text{Current market price}}$$

With this measure the investors can find out the rate of cashflow from their investments every year. The current yield differs from the coupon rate, since the market price differs from the face value of the bond. When the bond's face value and market price are same, the coupon rate and the current yield would be the same. For example, when the coupon payment is 8% for Rs. 100 bond with the same market price, the current yield is 8%. If the current market price is Rs. 80 then the current yield would be 10%.

- **Yield to Maturity (YTM)**

The concept of yield-to-maturity (YTM) is one of the widely used tools in bond investment management. Arithmetically, YTM is the single discount factor that makes present value of future cashflows from a bond equal to the current price of the bond. Intuitively, YTM is the rate of return, which an investor can expect to earn if the bond is held till maturity.

The yield to maturity is calculated based on certain assumptions. They are:

1. There should not be any default. Coupon and principal amount should be paid as per schedule.
2. The investor has to hold the bond till maturity.
3. All the coupon payments should be reinvested immediately at the same interest rate as the same yield to maturity of the bond.

Understanding this, is crucial for better investment decisions. For example, if an 11 per cent coupon paying bond with four years to mature has a TYM, of say 13 per cent it would be realised only if two conditions are met: One, the bond is held till maturity (for four years), and two, the interest received from the bond is reinvested for the rest of the period at 13 per cent. Otherwise actual or realised rate of return of the investor will be different from the expected return.

In the above example, if coupon receipts are re-invested at say, 10 per cent for the rest of the period then the realised rate of return will be less than the YTM. Conversely, if the coupon receipts are reinvested at 14 per cent, the realised rate of return will be higher than the YTM.

Any difference in the re-investment rate will cause a difference between the actual return and the YTM. In this sense, the YTM is only a

measure of yield. It cannot be regarded as a measure of return from a coupon-paying bond.

The YTM concept has a slightly different meaning for Zero Coupon Bonds (ZCB), popularly known as Deep Discount Bonds (DDB). ZCBs do not carry any coupon but are issued at a price discounted to the face value. On maturity, these bonds are redeemed at face value. Since thee bonds do not have any coupon payments during the life of the bond, the question of re-investment of coupon payments does nto arise at all. There is no re-investment risk for ZCBs.

To find out the yield to maturity the present value technique is adopted. The formula is,

$$\text{Present value} = \frac{\text{Coupon}_1}{(1+y)^1} + \frac{\text{Coupon}_2}{(1+y)^2} + \ldots + \frac{(\text{Coupon}_n + \text{face value})}{(1+y)^n}$$

Y = The yield to maturity.

Excercise 8.5: A four-year bond with the 7% coupon rate and maturity value of Rs. 1000 is currently selling at Rs. 905. What is its yield to maturity?

Solution. Since all the three values are known out of the four values, it can be found out by using trial and error procedure. Let us try ten per cent.

Cash flow	PV for 10%	PV of CF
70	0.9091	63.64
70	0.8264	57.85
70	0.7513	52.59
1070	0.6830	730.82
		Rs. 904.90

The yield to maturity is 10 per cent

The approximate yield to maturity can be found out by using the following formula too.

$$Y = \frac{C + (P \text{ or } D/\text{years to maturity})}{(Po + F)/2}$$

Y = Yield to maturity

C = Coupon interest

P or D = Premium or discount

P_O = Present value

F = Face value

In the case of previous sum

$$= \frac{70 + (95/4)}{(905 + 1000)/2} = \frac{93.75}{952.5} = 0.098$$

Y = 9.8%

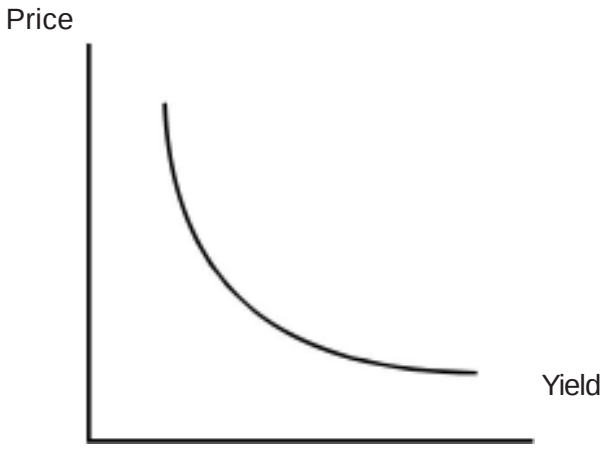
Yield to maturity(YTM)= 9.8%.

8.6 PRICE-YIELD RELATIONSHIP

A basic property of a bond is that its price varies inversely with yield.

The reason is simple. As the required yield increases, the present value of the cash flow decreases; hence the price decreases. Conversely, when the required yield decreases, the present value of the cash flow increases; hence the price increases. The graph of the price-yield relationship for any callable bond has a convex shape as shown in Fig 8.1

Fig 8.1. Price-yield relationship

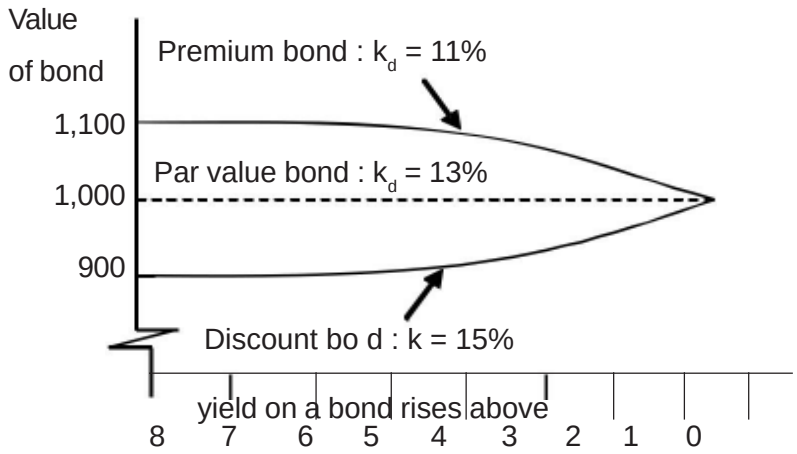


Relationship between bond price and time

Bond prices, generally, change with time as the price of a bond must equal its par value at maturity (assuming that there is no risk of default). For example, a bond that is redeemable for Rs. 1000 (which is its par value) after 5 years when it matures, will have a price of Rs. 1000 at maturity, no matter what the current price is. If its current price is, say, Rs. 1100, it is said to be a premium bond. If the required yield does not change between now and the maturity date, the premium will decline over time as shown by curve A in Fig 8.2. On the other hand, if the bond has a current price of say

Rs. 900, it is said to be a discount bond. The discount too will disappear over time as shown by curve B in Fig 8.2. Only when the current price is equal to par value-in such a case the bond is said to be a par bond-there is no change in price as time passes, assuming that the required yield does not change between now and the maturity date. This is shown by the dashed line in Fig 8.2.

Fig 8.2: Price changes with time



Relationship between coupon rate, required yield, and price

As yields change in the marketplace, prices of bonds change to reflect the new required yield. When the required its coupon rate, the bond sells at a discount. When the required yield

Years to maturity on a bond equals its coupon rate, the bond sells at par. When the required yield on a bond falls below its coupon rate, the bond sells at a premium. We can summarise the relationship between coupon rate, required yield, and price as follows:

Coupon Rate < Required Yield **<=>** Price < Par (Discount bond)

Coupon Rate = Required Yield **<=>** Price = Par

Coupon Rate > Required Yield **<=>** Price > Par (Premium bond)

Realised Yield to Maturity(YTM)

The YTM calculation assumes that the cash flows received through the life of a bond are reinvested at a rate equal to the yield to maturity. This assumption may not be valid as reinvestment rate/s applicable to future

cash flows may be different. It is necessary to define the future reinvestment rates and figure out the realised yield to maturity. How this is done may be illustrated by an example.

For example,consider a Rs. 1000 par value bond, carrying an interest rate of 15 per cent (payable annually) and maturing after 5 years. The present market price of this bond is Rs. 850. The reinvestment rate applicable to the future cash flows of this bond is 16 per cent. The future value of the benefits receivable from this bond, calculated in Exhibit 10.3 works out to Rs. 2032. The realised yield to maturity is the value of r* in the following equation.

Present market price $(1 + r^*)^5$ = Future value

$$850\ (1 + r^*)^5 = 2032$$

$$(1 + r^*)^5 = 2032/850 = 2.391$$

$$1 + r = (2.391)^{1/5}$$

$$r^* = 0.19 \text{ or } 19\%.$$

Table 8.3. Future value of benefits

	0	1	2	3	4	5
● Investment	850					
● Annual interest		150	150	150	150	150
● Re-investment period (in years)		4	3	2	1	0
● Compound factor (at 16%)		1.81	1.56	1.35	1.16	1.00
● Future value of intermediate cash flows		271.5	234.0	202.5	174.0	150.0
● Maturity value						1000
● Total future value	= 271.5 + 234.0 + 202.5 + 174.0 + 150.0 + 1000 = 2032					

8.7 BOND MARKET

Bonds are bought and sold in large quantities. Most trading in bonds, however, takes place over the counter. This means that the transactions

are privately negotiated and they do not take place through the process of matching of orders on an organised exchange. This is a characteristic of bond markets all over the world, not just in India. Because the bond market is largely over the counter, it lacks transparency. A financial market is transparent if you can easily observe its prices and volumes.

The National Stock Exchange has a Wholesale Debt Market (WDM) segment. The WDM segment is a market for high value transactions in government securities, PSU bonds, commercial papers, and other debt instruments. The quotations of this segment mostly reflect over-the-counter transactions that are privately negotiated over the phone or computer and registered with the exchange for reporting purposes.

8.8 THE TERM STRUCTURE OF INTEREST RATE (YIELD CURVE)

The bond portfolio manager is often concerned with two aspects of interest rates; the level of interest rate and the term structure of interest rate. The relationship between the yield and time or years to maturity is called term structure. The term structure is also known as yield curve. In analysing the effect of maturity on yield all other influences are held constant. Usually pure discount instruments are selected to eliminate the effect of coupon payments. The bonds chosen do not have early redemption features. The maturity dates are different but the risks, tax liabilities and redemption possibilities are similar.

The general reception is that the curve will be upward moving up to a point then it becomes flat. This is indicated in the following Figure 8.5.

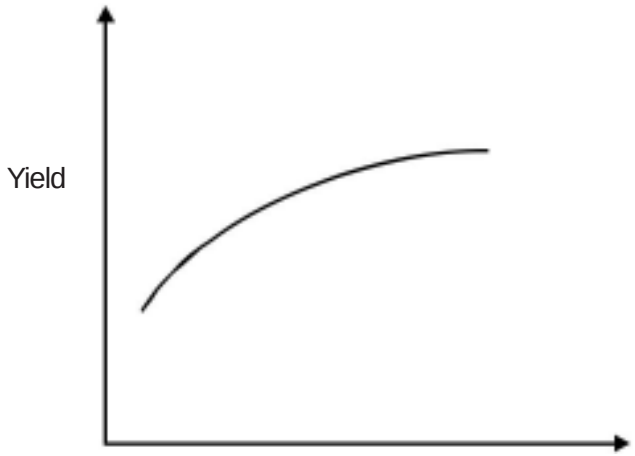


Fig. 8.5. Rising yield curve

There are at least three competing theories that attempt to explain the term structure of the interest rates viz., the expectation theory, liquidity preference theory and preferred habitat or segment theory.

Expectation theory- The theory was developed by J.. Hicks (1939), F. Lutz (1940) and B. Malkiel (1966). According to the expectation theory, the shape of the curve can be explained by the expectations of the investors about the future interest rates. If the short term rates are expected to be relatively low in the future, then the long term rate will be below the short term rate.

There are three reasons for the investors to anticipate the fall in the interest rate.

- Anticipation of the fall in the inflation rate and reduction in the inflation premium.
- Anticipation of balanced budget or cut in the fiscal deficit.
- Anticipation of recession in the economy, and a fall in the demand for funds by the private corporate.

The long term rates will exceed the current short term rates if there is an expectation that the market rates would be higher in the future. Thus the yield curve depends upon the expectations of the investors.

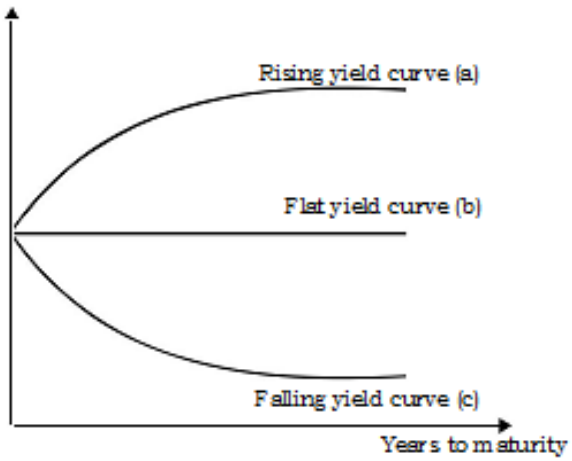


Fig. 8.6

A rising yield curve (a) indicates that the investors' expectation of a continuous rise in interest rate. The flat yield curve (b) means that the investors expect the interest rate to remain constant. The declining yield curve (c) shows that the investor expects the interest rate to decline.

Liquidity preference theory- Keynes' liquidity preference theory as advocated by J.R. Hicks (1939) accepts that expectations influence the shape of the yield curve. In a world of uncertainty, it would be more desirable for the investors to invest in short term bonds than on long term bonds because of their liquidity property. If no premium exists for holding the long term instruments, investors would prefer to hold short term bonds to minimise the possible variation in the nominal value of their portfolio.

The exponents of the liquidity preference theory believe that the investors prefer short term rather than long term. Hence they must be motivated to buy the long term bonds or lengthen the investment horizon. The bond issuing corporate or contributor pay premium to motivate the investors to buy. This liquidity premium theory indicates that in years time the forward rates are actually higher than the projected interest rate.

Sementation theory- Critics of the expectation theory, such as, J. Culbertson (1957) and F.V. Modigliani and R. Sutch (1966) pointed out that the liquidity preferences cannot be the main consideration for all classes of investors. In their view insurance companies, pension funds and even retired persons prefer the long term rather than short term securities to avoid the

possible fluctuations in the interest rate. This can be explained in detail.

Life insurance companies offer insurance policies that do not require any payment for a long time. For example, an insurance policy issued to a 25 year old individual may involve another 20 or more years before the company has to make a payment. Premium payments are fixed by the expected future rate of interest. If the insurance company invests the funds in a long term bond, the interest the bond would earn is certain and if the earned interest rate is higher than the promised interest rate, the company stands to gain and its risk is also reduced. If it invests in one year bonds, the risk of re-investment is there and if there is a fall in the market interest rate, the insurance company stands to lose and it would be difficult for the company to meet its obligation. This leads the insurance companies to prefer the long term bonds rather than short term bonds.

On the other hand, commercial banks and corporates may prefer liquidity to meet their short term requirements and therefore, they prefer short-term issues. Supply and demand for fund are segmented in sub markets because of the preferred habitats of the individuals. Thus the yield is determined by the demand and supply of the funds.

8.9 RIDING THE YIELD CURVE

When the long term coupon rates are higher than the short term rates, the yield curve would have an upward sloping shape. Bond portfolio manager tries to exploit this to his advantage and tries to increase the yield by purchasing the long term bonds. This strategy is known as riding the yield curve. When the long term bond approaches to maturity, the interest rate may get closer to the short term bond but, there would be capital gain. The bond portfolio manager may maintain the long term bond to utilise the capital gains as the bond moves to maturity date and “rides down the yield curve” to the lower interest rate, which will be appropriate when it becomes shorter term bond. Riding the yield curve would be successful only if the market interest rate does not rise. Sometimes the market interest rate may increase or short term end of the yield curve may slope upwards causing capital losses to the bond portfolio manager. To manage the situation

efficiently the bond portfolio manager should be continuously watchful about the shape of the yield curve and the shifts that occur in the market interest rates.

8.10 DURATION AND IMMUNISATION

Duration

Duration measures the time structure of a bond and the bond's interest rate risk. The time structure ways. The common way to state is how many years until the bond matures and the principal money is paid back. This is known as asset time to maturity or its years to maturity. The other way is to measure the average time until all interest coupons and the principal is recovered. This is called Macaulay's duration. Duration is defined as the weighted average of time periods to maturity, weights being present values of the cash flow in each time period. The formula for duration is,

$$D = \frac{\frac{C_1}{(1+r)}}{P_0} x1 + \frac{\frac{C_1}{(1+r)^1}}{P_0} x2 + ... \frac{\frac{C_1}{(1+r)^t}}{P_0} xT$$

This can be summarised as

$$D = \sum_{t=1}^{T} \frac{Pv(C_t)}{P_o} \times t$$

D = Duration

C = Cash flow

R = Current yield to maturity

T = Number of Years

PV (C) = Present value of the Cash flow

P = Sum of the present values of cash flow.



Excercise 8.9.

Calculate the duration for bond A and Bond B with 7 per cent and 8 per cent coupons having maturity period of 4 years. The face value is Rs. 1000. Both the bonds are currently yielding 6 per cent.

Solution.

$$D = \frac{\frac{C_1}{(1+r)}}{P_0} x1 + \frac{\frac{C_1}{(1+r)^2}}{P_0} x2 + \frac{\frac{C_1}{(1+r)^3}}{P_0} x3 + \frac{\frac{C_1}{(1+r)^4}}{P_0} x4$$

C_4 includes principal repayment

Bond 'A' with 7% coupon rate.

Year	Cash flow Ct	$\frac{1}{(1+r)^t}$	PV × CT	$\frac{C_t}{(1+r)^t}{P_0}$	$\frac{C_t}{(1+r)^t}{P_0} \times t$
1.	70	0.943	66.01	0.0638	0.0638
2.	70	0.890	62.30	0.0602	0.1204
3.	70	0.8396	58.77	0.0568	0.1704
4.	1070	0.7921	847.55	0.8191	3.2764
		P_0 = Rs. 1034.63			D = 3.6310

Bond 'B' with 8% coupon rate.

Year	Cash flow Ct	$\frac{1}{(1+r)^t}$	PV × CT	$\frac{C_t}{(1+r)^t}{P_0}$	$\frac{C_t}{(1+r)^t}{P_0} \times t$
1.	80	0.943	75.44	0.0706	0.0706
2.	80	0.890	71.200	0.0666	0.1332
3.	80	0.8396	67.168	0.0628	0.1884
4.	1080	0.7921	855.468	0.8000	3.2000
		P_0 = Rs. 1069.276		D = 3.5922	

Example	'A' Bond	'B' Bond
Face value	Rs. 1000.00	Rs. 1000.00
Coupon rate	7%	8%
Years to maturity	4.0	4.0
Macaulay's duration	3.631 Years	3.592 Years

From the above Excercise 8.9, it is clear that the bond with larger coupon payments has a shorter duration compared to the bond with low coupon rate.

General Rule

1. Larger the coupon rate, lower the duration and less volatile the bond price.
2. Longer the term to maturity, the longer the duration and more volatile the bone.
3. Higher the yield to maturity, lower the bond duration and bond volatility, and vice versa.
4. In a zero coupon bond, the bond's term to maturity and duration are the same. The zero coupon bond makes only one balloon payment to repay the principal and interest on the maturity date.

Importance of Duration- The concept of duration is important because it provides more meaningful measure of the length of a bond, helpful in evolving immunisation strategies for portfolio management and measures the sensitivity of the bond price to changes in the interest rate.

Duration and Price Changes- The price of the bond changes according to the interest rate. Bond's price changes are commonly called bond volatility. Duration analysis helps to find out the bond price changes as the yield to maturity changes. The relationship between the duration of a bond and its price volatility for a change in the market interest rate is given by the following formula.

$$\text{Percentage change in price} = \frac{-MD[BP]}{100}$$

MD = Modified duration

BP = Basis point is 0.01 of 1% (1% = 100).

$$\text{Modified duration MD} = \frac{D}{1+\frac{R}{P}}$$

Where

D = Duration

R = Market Yield

P = Interest payment per year (usually two)

Immunisation:

Immunisation is a technique that makes the bond portfolio holder to be relatively certain about the promised stream of cash flows. The bond interest rate risk arises from the changes in the market interest rate. The market rate affects the coupon rate and the price of the bond. In the immunisation process, the coupon rate risk and the price risk can be made to offset each other. Whenever there is an increase in the market interest rate, the prices of the bonds fall. At the same time the newly issued bonds offer higher interest rate. The coupon can be reinvested in the bonds offering higher interest rate and losses that occur due to the fall in the price of bond can be offset and the portfolio is said to be immunized.

Let us now solve sum excercise which are as follow:



Excercise 8.4:

Determine the price of Rs. 1,000 zero coupon bond with yield to maturity of 18 per cent and 10 years to maturity. What is YTM of this bond if its price is Rs. 220 ?

Solution.

(a) $$\text{Price} = \frac{\text{Face value}}{(1+\text{YTM})^n}$$

$$= \frac{1{,}000}{(1+0.18)^{10}} = \frac{1{,}000}{5.2338}$$

Rs. 191.07

(b) $$\left(\frac{\text{Face value}}{\text{Bond value}}\right)^{1/r} - 1 = \text{YTM}$$

$$\left(\frac{\text{Rs. } 1000}{\text{Rs. } 220}\right)^{1/r} - 1 = \text{YTM}$$

$(4.55)^{0.1} - 1 = \text{YTM}$

$1.163 - 1 = 0.163$

$\text{YTM} = 16.3\%$

Illustration 8.5. Arvind considers Rs. 1000 par value bond bearing a coupon rate of 11% that matures after 5 years. He wants a minimum yield to maturity of 15%. The bond is currently sold at Rs. 870. Should he buy the bond ?

Solution.

$$Po = \frac{Coupon}{(1+Y)} + ... + \frac{Coupon + Face\ value}{(1+Y)^5}$$

(or)

P_0 = (Coupon) (PVIFA, n) + (Principal amount) (PVIF/k,n)

P_0 = 110 (PVIFA 15%, 5 years) + 1000 (PVIF/15%, 5 yrs)

110 (3.352) + 1000 (0.497)

368.7 + 497 = 865.7.

At Arvind's anticipated minimum yield of 15% the price should be Rs. 865.70 but the market price is higher. Hence, he should not buy.

Illustration 7A.6. Anand owns Rs. 1,000 face value bond with five years to maturity. The bond has an annual coupon of Rs. 75. The bond is currently priced at Rs. 970. Given an appropriate discount rate of 10%, should Anand hold or sell the bond?

Solution.

P_0 = Coupon (PVIFA k, n) + Principal amount (PVIF k, n)

75 (PVIFA 10%, 5 yrs) + 1000 (PVIF 10%, 5 yrs)

75 × 3.7908 + 1000 (0.6209)

Rs. 284.31 + 620.9

Rs. 905.21.

The market price Rs. 970 is higher than the estimated price Rs. 905.2. It is better for Anand to sell the bond.



CHECK YOUR PROGRESS

Q1: Define Current Yield

.....

.....

Q2: Define Term Structure

.....

.....



8.11 LET US SUM UP

In this unit we have discussed the following points :-

- Bonds, do have risk. Changes that occur in the market interest rate affect the value of the bond. It is known as interest rate risk. Other than this, there are default risk, marketability risk and callability risk.

  Holding period return

$$= \frac{\text{Price gain or loss during the holding period} + \text{Coupon interest}}{\text{Price at the beginning of holding period}}$$

- Yield to maturity is the single discount factor that makes the present value of future cash flows from a bond equal to current price of the bond.

$$\text{Present value} = \frac{\text{Coupon}_1}{(1+y)^1} + \frac{\text{Coupon}_2}{(1+y)^2} + \ldots + \frac{(\text{Coupon}_n + \text{face value})}{(1+y)^n}$$

- Bond value theorem states that market price affects the yield and vice versa. This leads to convexity in the yield curve.
- The relationship between the yield and time to maturity is the term structure of interest rate. The term structure of interest rate is explained by expectation theory, liquidity theory and segmentation theory.
- When the long term coupon rate is higher than the short term rate, the bond portfolio manager may switch over from short term bond to long term bond and earn capital gains. This is known as riding the yield curve.
- Duration is a measure of the average time until all interest coupons and the principal amount is recovered.

$$D = \left[\sum_{t=1}^{T} \frac{Pv(C_t)}{P_o} \times t \right]$$

- Immunisation is the technique adopted to make the cashflows from holding the bond relatively certain. On the basis of duration immunisation can be done.



8.12 FURTHER READING

- M. Ranganathan and R. Madhumathi: Investment Analysis and Portfolio Management, Pearson Education, New Delhi.
- Punithavathy Pandian: Security Analysis and Portfolio Management, Vikas Publishing House Pvt. Ltd., New Delhi.
- Bharti V. Phathak: Indian Financial System, Pearson Education, Delhi.
- Donald E. Fischer and Ronald J. Jordon: Security Analysis and Portfolio Management, PHI.
- Prasanna Chandra: Investment Analysis and Portfolio Management, TMH, Delhi.



8.13 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q1: A bond is a debt instrument issued by the government or a government agency or a business enterprise.

Ans to Q2: Book value per share is determined as net worth divided by the number of shares outstanding. Book value reflects historical cost, rather than value.

Ans to Q3: The time value concept of money is that the rupee re ceived today is more valuable than a rupee received to morrow.

Ans to Q4: The current yield is the coupon payment as a percentage of current market prices. Because bonds trade in the secondary market, they may sell for less or more than par value, which will yield an interest rate that is different from the nominal yield, called the current yield,

Ans to Q5: The relationship between the yield and time or years to maturity is called term structure.



8.14 MODEL QUESTIONS

1. How would you assess the present value of a bond? Explain the various bond value theorems with examples.
2. Discuss the term structure of the interest rate? How do theories explain the term structure of the interest rate?
3. What is meant by duration? Explain the relationship between duration and price change.
4. How would you immunise the bond portfolio using the immunisation technique?
5. Find out the yield to maturity on a 8 per cent 5 year bond selling at Rs. 105?
6. Determine the present value of the bond with a face value of Rs. 1000, coupon rate of Rs. 90, a maturity period of 10 years for the expected yield to maturity of 8 per cent.
   (ii) If N is equal to 7 years in the above example, determine the present value of the bond. Discuss the effect of the maturity period on the value of the bond.
7. Bond A and B have similar characters except the maturity period. Both the bonds carry 9 per cent coupon rate with the face value of Rs. 10,000. The yield to maturity is 9%. If the yield to maturity is to rise to 11% what will be the respective percentage price change in bond A with 7 years to maturity and B with 10 years to maturity?
8. A bond with the face value of Rs. 1000 pays a coupon rate of 9 per cent. The maturity period is 9 years (a) Find out the approximate yield to maturity (b) current yield and the nominal interest rate.
9. Determine the yield to maturity if a zero coupon bond with a face value of Rs. 1000 is sold at Rs. 300. The maturity period is 10 years.
10. Prem is considering the purchase of a bond currently selling at Rs. 878.50 the bond has four years to maturity, face value of Rs. 1,000 and 8% coupon rate. The next annual interest payment is due after one year from today. The required rate of return is 10%.

a. Calculate the intrinsic value (Present value) of the bond. Should Prem buy the bond?

b. Calculate the yield to maturity of the bond.

11. What is the value of a Rs. 1,000 bond that paying 5 per cent annual coupon rate in semiannual payments over 5 years until it matures if its yield to maturity is 7 per cent?

12. Determine Macaulay's duration of a bond that has a face value of Rs. 1,000 with 10 per cent annual coupon rate and 3 years term to maturity. The bond's yield to maturity is 12 per cent.

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UNIT 9: VALUATION OF VARIABLE INCOME SECURITIES OR EQUITY SHARE VALUATION

UNIT STRUCTURE

9.1 Learning Objectives

9.2 Introduction

9.3 Basic Concepts

9.4 Share valuation

- 9.4.1 Earnings valuation
- 9.4.2 Cash flow valuation
- 9.4.3 Asset valuation
- 9.4.4 Dividend-Discount model

9.5 Let Us Sum Up

9.6 Further Reading

9.7 Answers To Check Your Progress

9.8 Model Questions

9.1 OBJECTIVES

After going through this unit, you will be able to :

- learn valuations of equity instruments
- discuss their interpretation and applicability of valuation in the stock market.
- analyse share valuation through the most often used methods such as earnings valuation, cash flow valuation, book valuation and dividend valuation.

9.2 INTRODUCTION

In this unit we are going to discuss about the share valuation, Earnings valuation, Cash flow valuation, Asset valuation and Dividend-Discount model.

Let us now discuss these concepts in the following sections.

9.3 BASIC CONCEPTS

Equity shares are floated in the market at face value, or at a premium or at a discount. Only companies with a track record or companies floated by other firms/companies with a track record are allowed to charge a premium. The premium is normally arrived at after detailed discussions with the merchant bankers. This is the first exercise involving the valuation of share by the company itself.

After allotment of shares to the shareholders, the company may distribute its surplus profits as returns to investors. The returns to equity shareholders are in the form of distribution of business profits. This is termed as declaration of dividends. Dividends are declared only out of the profits of the company. Dividends are paid in the form of cash and are called cash dividends. When shares are issued additionally to the existing investors in the form of returns, they are called bonus shares. These decisions are taken in the annual general meeting of the shareholders. The announcement of dividend is followed by the book closure dates, when the register of shareholders maintained by the company is closed till the distribution of dividends. The shareholders whose names appear on the register on the date are entitled to receive the dividend payment. Cash dividend payments reduce the cash balance of the company while bonus share payments reduce the reserve position of the company.

Thus, the dividends are a direct benefit from the company to its owners. It is an income stream to the owners of equity capital. Many expectations surround the company's quarterly announcement periods in terms of the dividend declared by the corporate enterprises to its shareholders. The payment of dividend itself is expected to influence the share price of the company. To the extent that cash goes out of the company, the book value of the company stands reduced and it is theoretically expected to lower the market price of the share. This is based on the argument that future expectations are exchanged for current benefits from the company in the form of dividends.

While bonus shares do not reduce the cash flow of the company, they increase the future obligations of the company to pay extra dividend in the future. Bonus shares result in an increase in the number of existing shares. Hence, the company has to pay dividend on its newly issued bonus shares in addition to its existing number of shares. These bonus shares are different from stock splits. Stock splits simply imply a reduction in the face value of the instrument with an increase in the quantity of stock. A stock split does not increase the value of current equity capital. Bonus shares, on the other hand, increase the value of equity capital to the company.

All these exercises by the company call for a renewed valuation of the shares traded in the secondary market. Hence, investment evaluation begins with the computation of the value of securities.

9.4 SHARE VALUATION

Share valuation is the process of assigning a rupee value to a specific share. An ideal share valuation technique would assign an accurate value to all shares. Share valuation is a complex topic and no single valuation model can truly predict the intrinsic value of a share. Likewise, no valuation model can predict with certainty how the price of a share will vary in the future. However, valuation models can provide a basis to compare the relative merits of two different shares.

Common ways for equity valuations could be classified into the following categories:

1. Earnings valuation
2. Cash flow valuation
3. Asset valuation
4. Dividend-discount model

Let us discuss categories in the follwoing sections:



CHECK YOUR PROGRESS

Q1: What is Stock Splits

.....

.....

Q2: Write the ways for equity valuations.

.....

.....

9.4.1 Earnings Valuation

Earnings (net income or net profit) is the money left after a company meets all its expenditure. To allow for comparisons across companies and time, the measure of earnings is stated as earnings per share (EPS). This figure is arrived at by dividing the earnings by the total number of shares outstanding.

Thus, if a company has one crore shares outstanding and has earned Rs. 2 crore in the past 12 months, it has an EPS of Rs. 2.00.

Rs. 20,000,000/10,000,000 shares = Rs. 2.00 earnings per share

EPS alone would not be able to measure if a company's share in the market is undervalued or overalued. Another measure used to arrive at investment valuation is the Price/Earnings (P/E) ratio that relates the market price of a share with its earnings per share. The P/E ratio divides the share price by the EPS of the last four quarters. For example, if a company is currently trading at Rs. 150 per share with a EPS of Rs. 5 per share, it would have a P/E of 30.

The P/E ratio or multiplier has been used most often to make an investment decision. A high P/E multiplier implies that the market has overvalued the security and a low P/E multiplier gives the

impression that the market has undervalued the security. When the P/E multiple is low, it implies that the earnings per share is comparatively higher than the prevailing market price. Hence, the conclusion that the company has been 'undervalued' by the market. Assume a P/E multiplier of 1.0. The implication is that the earnings per share is equal to the prevalent market price. While market price is an expectation of the future worth of the firm, the earnings per share is the current results of the firm. Hence, the notion that the firm has been 'undervalued' by the market. On the other hand, a high P/E ratio would imply that the market is 'overvaluing' the security for a given level of earnings.

Asian paints had a P/E ratio of 25.3 on July 26, 2005. The market price as on that date was Rs. 457.65 and the earnings per share was Rs. 18.1. Zee Telefilms, had a consistent P/E multiplier. ICICI Bank, had a price of Rs. 509.25 and PE ratio of 18.8 on the same date. The interpretation of 'overvaluation' will hold good when the market is expected to adjust towards the real worth of the company. A consistent high ratio, on the other hand, implies that the future returns expectations from the company is consistently good and that the high P/E ratio need not necessarily indicate a 'overvalued' position for the company.

The forward P/E valuation is another technique that is based on the assumption that prices adjust to future P/E multipliers. The assumption is that shares typically trade at a constant P/E and therefore the 'future' value of a share can be calculated by comparing the current P/E with the future P/E (as predicated using analysts' estimated earnings for that year).

The forecasted market price is calculated as [Price* (P/E, current)/ (P/E, future)]. For example, if current market price is Rs. 20, current P/E is 4 and forecasted P/E is 2.5, the forecast price is Rs. 32 $\left(\frac{20 \times 4}{2.5}\right)$. This valuation technique cannot be applied to shares with negative current or future earnings.

The forward P/E ratio is most often used in comparison with the current rate of growth in earnings per share. This is based on the assumption that for a growth company, in a fairly valued situation, the price/earnings ratio ought to be equal to the rate of EPS growth. When the growth rate is not in tune with P/E multiplier, then P/E multiplier can be modified to include the growth ratio.

Assume, for example, that a company's P/E ratio is 15; earnings growth rate of 13 per cent -14 per cent would substantiate the fair valuation of the share in the market price. This can be incorporated in the P/E growth ratio (PEG). The PEG considers the annualised rate of growth and compares this with the current share price. Since it is future growth that makes a company valuable to the investors in the market, the earnings growth is expected to depict the valuation of a company better than the historical earnings per share. If a company is expected to grow at 10 per cent a year over the next two years and has a current P/E multiple of 15, the PEG will be computed as 15/10 = 1.5. The interpretation of PEG is that the market price is worth 0.5 times more than what it really is worth, since the assumption is that the P/E multiplier ought to be equal to the earnings growth rate.

A PEG of 1.0 suggests that a company is fairly valued. That is, in the previous example, if the P/E multiplier is 15 and the earnings growth rate is also 15, then PEG is equal to (15/15) 1.0. Here the company is evaluated as priced correctly by the market. If the company in the above example had a P/E of 15 but was expected to grow at 20 per cent a year, it would have a PEG of (15/20), 0.75. This means the shares are selling for 75 per cent of their real value. This leads to the conclusion that the shares are 'underpriced' in the market.

The PEG measure is useful only for positive growth companies. When the companies are not experiencing a growth opportunity or there is a short spell of negative performance due to various factors, the PEG will not be the right measure to use to

assess the valuation of shares.

The forward P/E and growth ratio (FPEG) can be used for valuing companies with an expected long-term performance. Rather than looking at the current historical price earning multiplier, the measure considers the price earnings multiplier forecast by analysts. This is compared with the expected earnings growth rate to evaluate the fair price of the shares. Assuming the analysts' expectation of the P/E multiplier of a company is 20 and the earnings growth is expected to be 25 per cent over the next five years, the FPEG is computed as 20/5 = 0.8. The interpretation of this number is similar to the interpretation of PEG, that is, the company is evaluated in the market at only 80 per cent of its realistic price. This will be an indicator of 'underpricing' of shares in the market. Similarly, a company that has an expected P/E multiplier of 20 and the growth in earnings in the next five years of 10 per cent will have a FPEG of (20/10) =2.0. This indicates an 'overpricing' of the share by the market by double its fair value.

Although the PEG and FPEG are helpful, they both operate on the assumption that the P/E should equal the EPS rate of growth. In the real market, the assumptions behind the earnings valuation methods need not necessarily hold good. A modification to the P/E multiplier approach is to determine the relationship between the company's P/ E and the average P/E of the stock index. This is called as the price-earnings relative. Price-earnings relative is given by the following formula:

P/E relative = (share P/E) / (Index P/E)

This formula estimates the shares' P/E movement along with the index P/E. A P/E relative of 1.5 implies that the share is sold in the market 1.5 times that of the index price/earnings.

However, these earnings multipliers become inapplicable when the earnings are negative. Negative earnings cannot be used for valuation of shares. However, when negative earnings occur, appropriate alternative estimates may be used for valuation. The

substitute measures would depend on the cause for negative earnings.

There are a number of reasons for a company to have negative earnings. Some of the reasons for negative earnings can be listed as follows:

- Cyclical nature of industry
- Unforeseeable circumstances
- Poor management
- Persistent negative earnings
- High leverage cost

Earnings Forecast

Earnings can be forecast through the forecasts of the rates resulting in the earnings. The variables that can be considered for forecasting earnings can be the future return on assets, expected financial cost (interest cost), the forecasted leverage position (debt equity ratio), and the future tax obligation of the company. The formula for forecasting the earnings could be stated as follows:

Forecasted earnings (value) = (1-t)x[ROA + (ROA-I)x(D/E)]xE

Where,

ROA = Forecasted return on assets

I = Future interest rate

D = Total expected long term debt

E = Expected equity capital

t = Expected tax rate

Alternatively, a forecast of sales and projected profit margin can be made to compute the forecasted earnings. The sales forecast would depend on the market share of the estimated industry sales forecast. The profit margin forecast will depend on the operational and financial expenses of the company. From this information earnings can be forecast using the following formula:

Forecasted Sales = Industry Sales Target x Company's Expected Share in Industry Sales

Forecasted Earnings = Forecasted Sales x Projected Profit

Margin

The third method of forecasting earnings is to identify the individual variables constituting the earnings determination and forecast each of these variables separately. This will involve the forecast of the fixed and variable components of the operational expenses and the financial expense. This method is most applicable when the fixed and variable components of the cost structure of a company do not vary drastically with that of the average industry cost figures.

Consider a company with a high fixed cost relative to that of the industry average. The company will be able to make a positive return only when the projected sales dramatically exceeds this high cost.

The company's total cost far exceeds the industry total cost. Given a sales level and variable cost level, a company whose fixed costs are above the industry average will be able to reach a profit figure at a comparatively higher level of activity. Similarly, any company that is able to minimise its fixed costs will have a better position in terms of profitability than the industry average. Hence the need to forecast the individual variables that constitute profit rather than the overall return on assets.

9.4.2 Cash Flows Valuation

Cash flows indicate the net of inflows less outflows from operations. Cash flows differ from book profits reported by companies since accounting profits identify expenses that are non-cash items such as depreciation. Cash flows can also be used in the valuation of shares. It is used for valuing public and private companies by investment bankers. Cash flow is normally defined as earnings before depreciation, interest, taxes, and other amortisation expenses (EBDIT). There are also valuation methods that use free cash flows. Free cash flows is the money earned from operations that a business can use without any constraints. Free

cash flows are computed as cash from operations less capital expenditures, which are invested in property, plant and machinery and so on.

EBDIT is relevant since interest income and expense, as well as taxes, are all ignored because cash flow is designed to focus on the operating business and not secondary costs or profits. Taxes especially depend on the legal rules and regulation of a given year and hence can cause dramatic fluctuations in earning power. The company makes tax provisions in the year in which the profits accrue while the real tax payments will be made the following year. This is likely to overstate/ understate the profit of the current year.

Depreciation and amortisation, are called non-cash charges, as the company is not actually spending any money on them. Rather, depreciation is an accounting allocation for tax purposes that allows companies to save on capital expenditures as plant and equipment age by the year or their use deteriorates in value as time goes by. Amortisation is writing off a capital expenses from current year profit. Such amortised expenses are also the setting aside of profit rather than involving real cash outflows. Considering that they are not actual cash expenditures, rather than accounting profits, cash profits will indicate the real strength of the company while evaluating its worth in the market.

Cash flow is most commonly used to value industries that involve tremendous initial project (capital) expenditures and hence have large amortisation burdens. These companies take a longer time to recoup their initial investments and hence tend to report negative earnings for years due to the huge capital expense, even though their cash flow has actually grown in these years.

The most common valuation application of EBDIT is the discounted cash flow method, where the forecast of cash flows over a period fo time are made and these are discounted for their present worth.

The formula for computing the value of the firm will be

$$v = \sum_{i=1}^{n} \frac{C}{(1+d)^t}$$

Where C_i = cash flows forecast for year i

d = expected rate of return

n = number of years for which forecasts have been made.

This can be easily computed using software applications such as the spreadsheet. The following table illustrates the computation of the value of firm based on cash flow expectations.

Year	Expected cash flows (Rs. In crore)	Discounted cash flows (Discount rate 12%) (Formula)	Discounted cash flows (Discount Rate 12%)
1	200	= B2/(1.12)^A2	178.57143
2	254	= B3/(1.12)^A3	202.48724
3	236	= B4/(1.12)^A4	167.98014
4	280	= B5/(1.12)^A5	177.94506
5	310	= B6/(1.12)^A6	175.90233
6	324	= B7/(1.12)^A8	164.14848
7	356	= B8/(1.12)^A8	161.03632
8	368	= B9/(1.12)^A9	148.62903
9	375	= B 10/(1.12)^A10	135.22876
10	420	= B11/(1.12)^A11	135.22876
11	451	= B12/(1.12)^A12	129.65172
12	473	= B13/(1.12)^A13	121.40732
13	492	= B14/(1.12)^A14	112.7537
14	520	= B15/(1.12)^A15	106.4023
15	534	= B16/(1.12)^A16	97.5598
16	567	= B17/(1.12)^A17	92.4899
17	591	= B18/(1.12)^A18	86.0758
18	612	= B19/(1.12)^A19	79.5842
19	634	= B20/(1.12)^A20	73.6116
20	657	= B21/(1.12)^A21	68.1090
Cash flows		= SUM (C2:C21)	2614.8032
No. of shares (in crore)		200	200
Value per share		= (C23/C24)	13.0740

Buying a company with good cash flows can yield a lot of benefits to an investor. Cash can fund product development and strategic acquisitions and can be used to meet operational and financial expenditures.

Cash forecasts are made for a limited time duration. However, the shares are valued for their ability to produce an indefinite stream of cash flows. This is referred to as the terminal value of shares. Terminal value usually refers to the value of the company (or equity) at the end of a high growth period. When an indefinite duration of growth is considered, it is normal to assume that a stable growth will follow the high growth. This stable growth rate is expected to remain constant. With this assumption, the terminal value computation can be given by the following formula:

Terminal value in year

n = Cash flow in year (n + 1)/(d – g)

Where,

'd' is the discount rate of the cash flows

'g' is the stable growth rate

This approach also requires the assumption that growth is constant forever, and that the cost of capital (discount rate) will not change over time.

A stable growth rate is one that can be sustained forever. Since no company, in the long term, can grow faster than the economy that it operates in, a stable growth rate cannot be greater than the growth rate of the economy.

This stable growth rate cannot be greater than the discount rate either because of the risk-free rate that is implied in the discount rate. This invariably means that the discount rate has to be fixed after considering the inflation rate, economic growth rate, time value, and so on.

Price to cash flow ratio can also be used as a valuation model. Cash flow multiplier is computed as: market price/cash flow per share. For example, if the current market price is Rs. 60 and cash

flow per share is Rs. 20, the cash flow multiplier would be 3. If the forecasted cash flow per share is Rs. 23, then the market value can be estimated as (23 × 3)= Rs. 69.

Economic value added (EVA) is another modification of cash flow that considers the cost of capital and the incremental return above that cost.

Assuming the after tax return from operations is 18% and the cost of capital is 10%, the incremental return for the company would be 8% (18 – 10). If the face value of the investment in the company is Rs. 100 per share, the economic value added per share will be Rs. 8. If the current market price of the share is Rs. 200, then the EVA multiplier will be (200/8) 25. EVA multiple can then be used to identify the under pricing or over pricing of a share in the market.



CHECK YOUR PROGRESS

Q3: What is Earning.

.....

.....

Q2: How are earning forecasted.

.....

.....

9.4.3 ASSET VALUATION

Expectation of earnings, and cash flows alone may not be able to identify the correct value of a company. This is because the intangibles such as brand names give credentials for a business. In view of this, investors have begun to consider the valuation of equity through the company's assets.

Asset valuation is an accounting convention that includes a company's liquid assets such as cash, immovable assets such as

real estate, as well as intangible assets. This is an overall measure of how much liquidation value a company has if all of its assets were sold off. All types of assets, irrespective of whether those assets are office buildings, desks, inventory in the form of products for sale or raw materials and so on are considered for valuation.

Asset valuation gives the exact book value of the company. Book value is the value of a company that can be found on the balance sheet. A company's total asset value is divided by the current number of shares outstanding to calculate the book value per share. This can also be found through the following method- the value of the total assets of a company less the long-term debt obligations divided by the current number of share outstanding.

The formulas for computing the book value of the share are given below:

Book value = Equity worth (capital including reserves belonging to shareholders)/Number of outstanding shares

Book value = (Total assets – Long-term debt)/Number of outstanding shares

Book value is a simple valuation model. If the investor can buy the shares from the market at a value closer to the book value, it is most valuable to the investor since it is like gaining the assets of the company at cost. However, the extent of revaluation reserve that has been created in the books of the company may distract the true value of assets. The revaluation reserve need not necessarily reflect the true book value of the company; on the other hand, it might be depicting the market price of the assets better.

Book value, however, may not correctly depict the company value, since most companies use different accounting methods. Further, the adjustment to the historical figures in terms of economic inflation or deflation of the asset book values are not incorporated in these value estimations. The book values are also subject to adjustments depending on the tax framework within which the company falls and the consequences relating to the company's tax

planning measures. But, with increased corporate governance practices, the book value concept is becoming more relevant to the investor for valuation purposes.

Another useful measure of asset valuation is the price to book ratio. This ratio is arrived at by relating the current market price of the share to the book value per share. The intention is to compare the prevailing market price with the book value per share and identify if the shares are undervalued or overvalued in the market. The computation of price to book ratio is computed as follows:

Price to book value ratio = Market price/Book value per share.

The undervaluation of shares will be established when the price to book ratio is relatively low. A high price to book ratio, on the other hand, implies that the shares are sold at a price not supported by its asset value in the market.

For example, if a company has total assets less long-term liabilities as Rs. 43950 crores and the number of outstanding shares at 2,000 crores, the book value per share will be (43950/2000) = Rs. 21.975. If the market price is quoted at Rs. 84, the Price to Book multiplier will be (84/21.975) 3.82.

Another use of asset valuation is through the return on equity, (ROE). Return on equity is a measure of how much earnings a company generates in four quarters compared to its shareholder's equity. For instance, if a company earned Rs. 2 crores in the preceding year and has a shareholder's equity of Rs. 10 crores, then the ROE is 20 per cent. Investors might use ROE as a filter to select companies that can generate large profits with a relatively small amount as capital investment. The nature of ROE, however, depends on the type of industry the company belongs to. The ROE figures of trading companies are expected to be comparatively higher than that of heavy manufacturing concerns since trading companies need not necessarily require constant capital expenditure.

The book value computation that includes within its fold the valuation of intangible assets such as brands and patents, are viewed

positively by investors. Investors view brands as valuable and they are assumed to increase the expected future profits of the company. Brands also tend to have a strong market potential since customers prefer a brand exclusively for its name and sometimes, brands convey more meaning than the product quality. Specific value is given to brands that have recently established unshakable credentials. Companies also spend a lot of money on building brands in their product portfolios. Some companies build the brand name around their company name; this has a direct impact on the valuation by companies build the brand name around their company name; this has a direct impact on the valuation by investors. Companies such as Colgate, Intel, Nestle, and Bata have built their company names into brands that give them an incredible edge over their competitors in the market. Intangibles can also sometimes mean that a company's shares can trade at a premium to its historical growth rate. Thus, a company with large profit margins, a dominant market share, consistent performance can trade at a slightly higher multiple than its growth rate would otherwise suggest.

A company can sometimes be worth more in reality than when viewed individually in terms of all the assets in its Balance Sheet. Many times, human resource strength is an intangible that is built inside the organisation and neither the company nor the shareholders give a uniform quantification to such strengths. The book valuation process of a company is hence the exercise of a few investment bankers and consultants who get to know the intricate details of the company. Rather than attempting to make a book valuation of the company individually, investors can rely on such sources to assess the undervaluation or overvaluation of shares in the market.

9.4.4 DIVIDEND DISCOUNT MODEL

The dividend discount model (DDM) is a quantitative method used for predicting the price of a company's stock based on the

theory that its present day price is worth the sum of all of its future dividend payments, when discounted back to their present value. It attempts to calculate the fair value of a stock irrespective of the prevailing market conditions, and takes into consideration the dividend payout factors and the market expected returns. If the value obtained from the DDM is higher than the current trading price of shares, then the stock is undervalued and qualifies for a buy, and vice versa.

In other words, it is used to value stocks based on the net present value of the future dividends. The equation most widely used is called the Gordon growth model (GGM), the Gordon Growth Model, which uses next year's estimated dividend (D), the company's cost of equity capital (r), and the estimated future dividend growth rate (g).

$$Price = \frac{D}{r - g}$$

Let us take an example.

Example 9.1:

Let's assume that a certain stock is expected to pay a $2.00 dividend next year, and its dividend has historically grown by 4% per year, so it's fair to assume this same growth rate going forward. And we'll say that my desired rate of return is 10%. Using these input values, we can calculate the stock's value (to me) using the dividend discount model as:

$$Price = \frac{\$2.00}{1.10 - 1.04} = \$33.33$$

Therefore, according to the dividend discount model, I should pay about $33.33 for the stock based on my required rate of return. If the stock were trading for say, $40, an investor using this model may consider the stock to be overvalued, while a price of $25 might make it look like a buying opportunity.

Some examples of regular dividend paying companies are McDonalds, Procter & Gamble, Kimberly Clark, PepsiCo, 3M, CocaCola, Johnson & Johnson, AT&T, Walmart etc. We can use Dividend Discount Model to value these companies.

There are 3 models used in the dividend discount model:

1. zero-growth, which assumes that all dividends paid by a stock remain the same;
2. constant-growth model, which assumes that dividends grow by a specific percent annually;
3. variable-growth model, which typically divides growth into 3 phases: a fast initial phase, then a slower transition phase that ultimately ends with a lower rate that is sustainable over a long period.

Let us discuss this model in the following ways:

1. Zero-growth model : Zero-growth model assumes that the dividend always stays the same i.e. there is no growth in dividends. Therefore, the stock price would be equal to the annual dividends divided by the required rate of return.

Stock's Intrinsic Value = Annual Dividends / Required Rate of Return

This is basically the same formula used to calculate the Present Value of Perpetuity, and can be used to price preferred stock, which pays a dividend that is a specified percentage of its par value. A stock based on the zero-growth model can still change in price if the required rate changes when perceived risk changes, for instance.

Example 9.2

If a preferred share of stock pays dividends of $1.80 per year, and the required rate of return for the stock is 8%, then what is its intrinsic value?

Sol: Here we use the dividend discount model formula for zero growth dividend,

Dividend Discount Model Formula = Intrinsic Value = Annual Dividends / Required Rate of Return

Intrinsic Value = $1.80/0.08 = $22.50.

2. Constant Growth Model

One of the most popular dividend discount models assumes that the dividend per share grows at a constant rate (g). The value of a share, under this assumption, is:

$$P_0 = \frac{D_1}{(1+r)} + \frac{D_1(1+g)}{(1+r)^2} + \ldots \frac{D_1(1+g)^n}{(1+r)^{n+1}} + ..$$

Applying the formula for the sum of a geometric progression, the above expression simplifies to:

$$P_0 = \frac{D_1}{r-g}$$

Example 9.3: Ramesh Engineering Limited is expected to grow at the rate of 6 per cent per annum. The dividend expected on Ramesh's equity share a year hence is Rs. 2.00. What price will you put on it if your required rate of return for this share is 14 per cent?

$$P_0 = \frac{2.00}{0.14-0.06} \qquad = \text{Rs. } 25.00$$

3. Variable-Growth Model

Variable Growth rate Dividend Discount Model or DDM Model is much closer to reality as compared to the other two types of dividend dicount model. This model solves the problems related to unsteady dividends by assuming that the company will experience different growth phases.

Variable growth rates can take different forms, you can even assume that the growth rates are different for each year. However, the most common form is one that assumes 3 different rates of growth:

a. an initial high rate of growth,

b. a transition to slower growth, and

c. lastly, a sustainable, steady rate of growth.

Variable Growth Model further can be classified as under:

i. *Two stage Dividend Discount Model DDM:* This model is designed to value the equity in a firm, with two stages of growth, an initial period of higher growth and a subsequent period of stable growth. Two-stage Dividend Discount Model is best suited for firms paying residual cash in dividends while having moderate growth. For instance, it is more reasonable to assume that a firm growing at 12% in the high growth period will see its growth rate drops to 6% afterwards.

Assumptions of this model:

a. Higher growth rate is expected the first period.

b. This higher growth rate will drop at the end of the first period to a stable growth rate.

c. The dividend payout ratio is consistent with the expected growth rate.

ii. Three stage Dividend Discount Model DDM: One improvement that we can make to the two-stage DDM Model is to allow the growth rate to change slowly rather than instantaneously.

The three-stage Dividend Discount Model or DDM Model is given by:

First phase: there is a constant dividend growth (g1) or with no dividend

Second phase: there is a gradual dividend decline to the final level

Third phase: there is a constant dividend growth again (g3), i.e. the growth company opportunities are over.

The logic that we applied to two-stage model can be applied to

$$V_0 = \sum_{t=1}^{T} \frac{D_0(1+g_1)^t}{(1+k)^t} + \sum_{t=T+1}^{N} \frac{D_t}{(1+k)^t} + \frac{D_{N+1}}{(k-g_3)(1+k)^N}$$



9.5 LET US SUM UP

In this unit we have discussed the following points:

- Equity shares carry with them ownership rights. They give voting rights to the holders. They have a face value (in monetary terms) at the time of issue and are evaluated at their market value when they are listed on a stock exchange.
- Equity valuation is a complex procedure since there is no consistent definition regarding what constitutes the intrinsic value of a share.
- Different valuation approaches and models with different assumptions and implications are available to investors to assess the true worth of a share. These include earnings approach, cash flow approach and dividend discount approach.
- An investor can choose the appropriate procedure of valuation for shares and make profits from the stock market.
- Asset valuation is an accounting convention that includes a company's liquid assets such as cash, immovable assets such as real estate, as well as intangible assets.



9.6 FURTHER READING

- M. Ranganathan and R. Madhumathi: Investment Analysis and Portfolio Management, Pearson Education, New Delhi.
- Punithavathy Pandian: Security Analysis and Portfolio Management, Vikas Publishing House Pvt. Ltd., New Delhi.
- Bharti V. Phathak: Indian Financial System, Pearson Education, Delhi.
- Donald E. Fischer and Ronald J. Jordon: Security Analysis and Portfolio Management, PHI.
- Prasanna Chandra: Investment Analysis and Portfolio Management, TMH, Delhi.



9.7 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q1: Stock splits simply imply a reduction in the face value of the instrument with an increase in the quantity of stock.

Ans to Q2: Common ways for equity valuations could be classified into the following categories:

1. Earnings valuation
2. Cash flow valuation
3. Asset valuation
4. Dividend-discount model

Ans to Q3: Earnings (net income or net profit) is the money left after a company meets all its expenditure.

Ans to Q4: Earnings can be forecast through the forecasts of the rates resulting in the earnings.



9.8 MODEL QUESTIONS

1. Discuss the assumptions and implications of earnings approach to equity valuation.
2. What are the quantitative models of equity valuation? What are their limitations?
3. Explain zero-growth model, constant-growth model, and variable-growth model dividend discount model.

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UNIT 10 : PORTFOLIO MANAGEMENT

UNIT STRUCTURE

10.1 Learning Objective

10.2 Introduction

10.3 Inputs to Portfolio Management

- 10.3.1 Return and Risk Characteristics of Individual Assets
- 10.3.2 Expected Return and Risk of a Portfolio
- 10.3.3 Diversification of Risk

10.4 Portfolio Analysis and Selection

- 10.4.1 Correlation between Securities and its Impact on Portfolio Risk
- 10.4.2 Portfolio Selection

10.5 Let Us Sum Up

10.6 Further Reading

10.7 Answers to Check Your Progress

10.8 Model Questions

10.1 LEARNING OBJECTIVES

After going through this unit, tyou will be able to:

- explain and illustrate the concepts and measures of return and risk as they apply to individual assets as well as portfolio of assets
- highlight the concept of diversification of risk
- discuss the portfolio selection problem and the process.

10.1 INTRODUCTION

In this unit we are going to discuss about Portfolio Management: Return and Risk Characteristics of Individual Assets,Expected Return and Risk of a Portfolio and Diversification of Risk.

We will also discuss Portfolio Analysis and Selection, Portfolio Risk Selection Problem and election of Optimal Portfolio

In this unit, we will discuss more about how one should go about in

constructing a portfolio that suits specific objectives of the investment. In this unit, we will address some of the basic issues measuring return and risk of the portfolio before addressing the main issue of selecting optimal portfolio.

In this unit, We will begin with the analysis of return -risk characteristics of individual assets, and then proceed to examine how individual assets combine into a portfolio to determine its return and risk attributes.

10.3 INPUTS TO PORTFOLIO MANAGEMENT

The term 'portfolio' generally means a collection or combination and in the context of investment management, it means a collection or combination of financial assets (or securities) such as shares, debentures and government securities. However, in a more wider context the term 'portfolio' may be used synonymously with the expression 'collection of assets', which can even include physical assets (gold, silver, real estate, etc.). What is to be borne in mind is that, in the portfolio context, assets are held for 'investment' purposes and not for 'consumption' purposes.

Portfolio analysis builds on the estimates of future return and risk of holding various combinations of assets. As we know, individual assets have risk return characteristics of their own. Portfolios, on the other hand, may or may not take on the aggregate characteristics of their individual parts. In this section, we will reflect on the assessment of return-risk attributes of individual assets and portfolios.

10.3.1 Return and Risk Characteristics of Individual Assets

Any investment decision requires an estimate of return and risk associated with the investment. However the most difficult task of investment decision is estimation of return and risk. Earlier in the previous units, we spent about discussing how should one estimate the future value of the asset so that return can be measured. If we

are able to estimate a range of expected return, then it is possible to estimate the probabilities associated with the range of expected return to get the risk measure. In practice, however, the return and risk of the securities are estimated based on the historical return and risk of securities. A stock's single period basic return is:

$$\text{Total Return} = \frac{\text{Dividend} + (\text{Market Price}_t - \text{Market Price}_{t-1})}{\text{Market Price}_{t-1}} \times 100$$

There are different measures of historical return. The most elementary form of return measure is holding period yield or return. Here, the dividend received during the holding period is added along with the capital gain and divided by the purchase price. If the holding period is more or less than one year, normally the holding period return is stated for one-year period. This measure is not much useful if one wants to measure the risk associated with the security. There are two other measures of return by which one can measure risk.

a) ***Arithmetic Average:*** The arithmetic average return is equal to sum of returns of period and divided by 'n'. For instance, if the stock has offered a holding period return of 11% in period 1, 12% in period 2 and 16% in period 3, then the arithmetic average return is equal to 13%. Though it is better than holding period return, this measure suffers because of its failure in considering time value of money. Another problem of this measure is differential treatment of positive and negative return. For instance if a stock price increases from Rs. 10 to Rs. 20 in period 1 and declines back to Rs. 10 in period 2, the Arithmetic average return is still positive value of 25% (Period I return is 100% and Period 2 return is -50%; Total return is 50% and hence average return is 25%).

b) ***Geometric Average(GA):*** The geometric average return is based on the compound value and is also called time-weighted average return. It addresses the problem of differential treatment of positive and negative return described above. The

geometric average return is computed as follows:

$$GMR = [(1 + R_1) \times (1 + R_2) \times (1 + R_3) .....x (1 + R_n)]^{1/n} \text{ “1}$$

Example 10.1: Five years back, you have applied and was allotted 100 shares of a company at the rate of Rs. 50 per share (Face Value Rs. 10) . The price at the end of each year along with annual dividend per share received from the stock are as follows:

Year	1	2	3	4	5
Dividend per share (Rs.)	1	1.5	1.5	2	2
Market Price (Rs.)	40	55	70	77	91

Find the Holding Period Return (HPR), Arithmetic Average(AA) and Geometric Average return of the stock (GA).

HPR : [Dividend (Rs. 8) + Capital Appreciation (Rs. 41)] / Investment (Rs. 50)

: 49/50 = 98% for five years or 19.60% per year

AA Return : $[R_1 (-18\%) + R_2 (41.25\%) + R_3 (30\%) + R_4 (12.86\%) + R_5(20.78\%)]/5$ 17.38%

Note: R_1 is equal to [(41-50)/50], R_2 is equal to [(56.5 - 40)/40], etc.

GA Return : $[(1+R_1)x(1+R_2)x(1+R_3)x(1+R_4)x(1+R_5)]^{1/5} - 1$

: $[(.82)x(1.41)x(1.30)x\ l.13)x(1.21\%)]^{1/5}$-1

: 15.47%

As you may observe, for the same set of data, we get different values of return. HPR is the highest and GAR is the lowest. The Geometric Return is lower than other two returns because of compounding. In Table 8.1, the different measures of return of NSE-50 companies are given based on last ten years data. The list contains only for the companies, which have 10-year listed life.

In addition to the above two types of return, a foreign investor or foreign fund would compute dollar-weighted return to adjust differences in the foreign exchanges between the point of

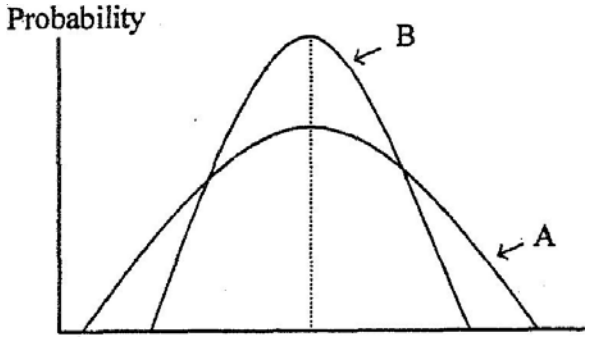
investment and sale. For example, if a foreign fund purchased a stock at Rs. 50 today when the US Dollar - Rupee rate is Rs. 50 per US Dollar and sold the stock at Rs. 55 at the end of one year, the holding period return in Rupee term is 10% [(55-50)/50]. However, if the Rupee depreciates during this period and quotes Rs. 56 per US Dollar, the foreign fund incurs a loss because it can get less than one Dollar with the sale value of the stock. The loss is equal to 1.79% [$1- $(55/56)].

While historical return gives a fair idea about the future return, they are often used to measure the risk. It is true that it is more relevant to use expected risk by measuring the probability associated with various returns, often such measure is not used in practice. Historical data is generally used to measure the risk. The risk associated with an investment in. stocks is measured using variance or standard deviation of the historical return. Table 8.1 also shows the standard deviation of stock return along with different return measures. A question with variance as a measure of risk is: why count 'happy' surprises (those above the average historical return or expected return) at all in a measure of risk? Why not just consider the deviations below the average historical return or expected return (i.e. the downside danger)? Measures to do so have much to recommend them. But if a distribution is symmetric, such as the normal distribution, the result will be the same. Because, left side of a symmetric distribution is a mirror image of the right side. Although distributions of historical or forecasted returns are often not normal, analysts generally assume normality to simplify their analysis.

Company Name	Holding Period Return	Arithmetic Mean Return	Geometric Mean Return	Variance	Standard Deviation
Wipro	2857%	57%	38%	65%	81%
Cipla	646%	42%	19%	58%	76%
Dr. Reddy's Laboratories	517%	40%	24%	44%	66%
Hero Honda Motors	233%	32%	24%	25%	50%
HDFC	202%	31%	22%	26%	51%
Hindustan Lever	126%	26%	24%	7%	27%
Ranbaxy Laboratories	95%	24%	15%	26%	51%
Digital Globalsoft	47%	17%	-4%	55%	74%
I T C	36%	15%	10%	12%	34%
Reliance Industries	36%	15%	8%	16%	40%
Asian Paints (India)	36%	15%	14%	3%	18%
Nestle India	34%	15%	12%	6%	24%
Britannia Industries	28%	13%	3%	19%	43%
Bajaj Auto	26%	13%	4%	22%	47%
Gujarat Ambuja Cements	21%	11%	1%	19%	43%
Castrol India	19%	11%	4%	17%	41%
Hindalco Industries	19%	11%	6%	12%	35%
Procter & Gamble	18%	10%	6%	10%	32%
Glaxosmithkline Pharma	18%	10%	3%	14%	37%
Smithkline Beecham	17%	10%	[illegible]	[illegible]	[illegible]
Indian Hotels Co.	15%	9%	-2%	29%	54%
Mahindra & Mahindra	11%	9%	-2%	22%	47%
Asea Brown Boveri	8%	6%	-4%	20%	44%
BSES	8%	6%	1%	12%	34%
State Bank of India	7%	5%	0%	13%	36%
Tata Power Co.	6%	5%	1%	9%	29%
Larsen & Toubro	6%	4%	3%	4%	20%
Associated Cement Cos.	4%	3%	-5%	22%	46%
Colgate-Palmolive (India)	2%	2%	-3%	13%	36%
Tata Tea	0%	0%	-18%	31%	56%
ICICI	0%	0%	-16%	34%	58%
Novartis India	-1%	-1%	-30%	32%	56%
Grasim Industries	-1%	-1%	-10%	19%	44%
Tata Chemicals	-3%	-4%	-12%	21%	46%
Telco	-4%	-5%	-18%	15%	39%
Tata Iron & Steel Co.	-5%	-8%	-10%	4%	21%

Previously, we had discussed how to compute mean and variance (or standard deviation), so we need not reiterate the procedure here. You may, however, look up appendix at the end of this unit to quickly revise the concepts of portfolio return and risk. We may now refer to Figure 8.1 that depicts the distribution of returns that might be expected for two investments, A and B.

Figure 10.1 : Possible Outcomes of two Independent Investments



The mean or expected return, at the vertical dotted line, is the same for both investments. But, investment B is more risky. With investment A, the distribution of returns (or possible outcomes) is more closely grouped about the mean value. In other words, the variance is smaller than that of investment B. Consequently, it can be said with greater degree of accuracy that our forecast will be close to the actual return from investment A.

When we move from evaluating a single asset in isolation to evaluating a portfolio, our return -risk analysis changes. Return is still the expected return, but for a portfolio the return will be the average return from all the assets held in the portfolio. Risk is still the variance (or standard deviation) of the expected returns from the portfolio. The investor is still concerned with upside potential and downside danger. However, the risk of a combination of assets is very different from a simple average of the risk of individual assets. Most dramatically, the variance of a portfolio of two assets may be less than the variance of either of the assets themselves. We will

examine all these aspects in the discussion that follows.

10.2.2 Expected Return and Risk of a Portfolio

Expected Return of a Portfolio

As a first step in portfolio analysis, an investor needs to specify the list of securities eligible for selection or inclusion in the portfolio. Next he has to generate the risk-return expectations for these securities. These are typically expressed as the expected rate of return (mean) and the variance or standard deviation of the return.

The expected return of a portfolio of assets is simply the weighted average of the return of the individual securities held in the portfolio. The weight applied to each return is the fraction of the portfolio invested in that security.

Let us consider a portfolio of two equity shares P and Q with expected returns of 15 per cent and 20 per cent respectively.

If 40 per cent of the total funds are invested in share P and the remaining 60 per cent, in share Q, then the expected portfolio return will be:

(0.40 x 15) + (0.60 x 20) = 18 per cent

The formula for the calculation of expected portfolio return may be expressed as shown below:

$$R_{(p)} = \sum_{i=1}^{N} X_i R_i$$

Where

R_p = Expected return of the portfolio

X_i = Proportion of funds invested in security i.

R_i = Expected return of security i.

N = Number of securities in the portfolio

Risk of a Portfolio

The variance of return and standard deviation of return are alternative statistical measures that are used for measuring risk in investment. These statistics measure the extent to which returns are expected to vary around an average over time. The calculation of

variance of a portfolio is a little more difficult than determining its expected return.

The variance or standard deviation of an individual security measures the riskiness of a security in absolute sense. For calculating the risk of a portfolio of securities, the riskiness of each security within the context of the overall portfolio has to be considered.

This depends on their interactive risk, i.e. how the returns of a security move with the returns of other securities in the portfolio and contribute to the overall risk of the portfolio.

Covariance is the statistical measure that indicates the interactive risk of a security relative to others in a portfolio of securities. In other words, the way security returns vary with each other affects the overall risk of the portfolio.

The covariance between two securities X and Y may be calculated using the following formula:

$$Cov_{xy} = \sum_{i=1}^{n} \frac{(R_x - \overline{R}_x)(R_y - \overline{R}_y)}{N}$$

Where:

Cov_{xy} = Covariance between x and y.
R_x =Return of security x.
R_y =Return of security y
$\overline{R}_x$ = Expected or mean return of security x.
$\overline{R}_y$ = Expected or mean return of security y.
N = Number of observations.

The calculation of covariance is illustrated below with an example:

Example 10.2

Calculation of Covariance

Year	Rx	Deviation $R_x - \overline{R}_x$	R...3 y	Deviation $R_y - \overline{R}_y$	Product of deviations $(R_x - \overline{R}_x)(R_y - \overline{R}_y)$
1	10	-4	17	5	-20
2	12	-2	13	1	-2
3	16	2	10	-2	-4
4	18	4	8	-4	-16

$$Cov_{xy} = \sum_{i=1}^{n} \frac{(R_x - \overline{R}_x)(R_y - \overline{R}_y)}{N}$$

=-42 / 4= -10.5

The covariance is a measure of how returns of two securities move together. If the returns of the two securities move in the same direction consistently the covariance would be positive. If the returns of the two securities move in opposite direction consistently the covariance would be negative. If the movements of returns are independent of each other, covariance would be close to zero.

Covariance is an absolute measure of interactive risk between two securities. To facilitate comparison, covariance can be standardized. Dividing the covariance between two securities by product of the standard deviation of each security gives such a standardised measure. This measure is called the coefficient of correlation. This may be expressed as:

$$r_{xy} = \frac{Cov_{xy}}{\sigma_x \sigma_y}$$

Where,

r_{xy} = Coefficient of correlation between x and y

Cov_{xy} =Covariance between x and y

σ_x =Standard deviation of x.

σ_y =Standard deviation of y

It may be noted from the above formula that covariance may be expressed as the product of correlation between the securities and the standard deviation of each of the securities. Thus,

$$Cov_{xy} = r_{xy} ó_x ó_y$$

The correlation coefficients may range from - 1 to 1. A value of -1 indicates perfect negative correlation between security returns, while a value of +1 indicates a perfect positive correlation. A value close to zero would indicate that the returns are independent.

The variance (or risk) of a portfolio is not simply a weighted average of the variances of the individual securities in the portfolio. The relationship between each security in the portfolio with every other security as measured by the covariance of return has also to be considered. The variance of a portfolio with only two securities in it may be calculated with the following formula.

$$ó^2_p = x_1^2 ó_1^2 + x_2^2 ó_2^2 + 2x_1x_2(r_{12} ó_1 ó_2)$$

Where

$ó^2_p$ = Portfolio variance.
x_1 = Proportion of funds invested in the first security.
x_2 = Proportion of funds invested in the second security.
$ó_1^2$ = Variance of first security.
$ó_2^2$ = Variance of second security.
$ó_1$ = Standard deviation of first security.
$ó_2$ = Standard deviation of second security.
r_{12} = Correlation coefficient between the returns of first and second security.Portfolio standard deviation can be obtained by taking the square root of portfolio variance.

Let us take an example to understand the calculation of portfolio variance and portfolio standard deviation.

Example 10.2 Two securities P and Q generate the following sets of expected returns, standard deviations and correlation coefficient:

	P	Q
r = 15 percent		20 percent
ó = 50 percent		30 percent
r_{pq} = -0.60		

The first set of factors is parametric to the investor in the sense that he has no control over the returns, risks and covariances of individual securities. The second sets of factors are choice variables in the sense that the investor can choose the proportions of each security in the portfolio.

A portfolio is constructed with 40 per cent of funds invested in P and the remaining 60 per cent of funds in Q.

The expected return of the portfolio is given by:

$$r_p = \sum_{i=1}^{n} X_i r_i$$

=(0.40x15)+(0.60x20)=18

Percent

The variance of the portfolio is given by:

$$ó^2_p = x_1^2 ó_1^2 + x_2^2 ó_2^2 + 2x_1x_2(r_{12} ó_1 ó_2)$$

$$= (0.40)2\ (50)2 + (0.60)2\ (30)2 + 2(0.40)(0.60)(-\ 0.60)(50)(30)$$

$$= 400 + 324 - 432 = 292$$

The standard deviation of the portfolio is:

$$s_p = \sqrt{292} = 17.09 \text{ per cent.}$$

The return and risk of a portfolio depends on two sets of factors (a) the returns and risks of individual securities and the covariance between securities in the portfolio, (b) the proportion of investment in each security.

The first set of factors is parametric to the investor in the sense that he has no control over the returns, risks and covariances of individual securities. The second sets of factors are choice variables in the sense that the investor can choose the proportions of each security in the portfolio.

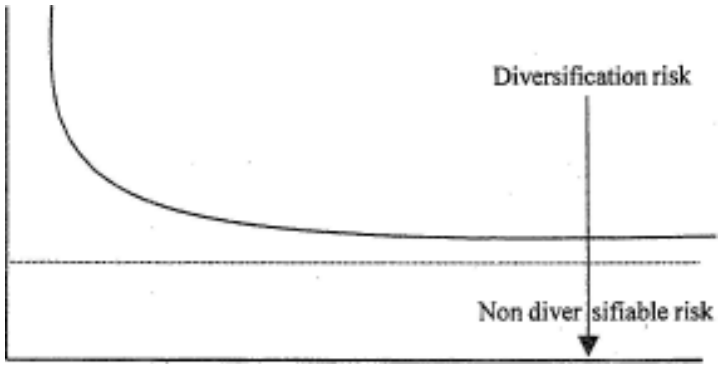
10.2.3 Diversification of Risk

Efforts to spread and minimize portfolio risk take the form of diversification. Most investors prefer to hold several assets rather than putting all their eggs into one basket, with the hope that if one goes bad, the others will provide some protection from extreme loss. Surely enough, there is merit in this approach; although some investors hold a contrary view point that recommends putting all eggs into one basket and then keeping a sharp eye on the basket.

It is not difficult to understand that adding more assets in the portfolio can reduce the overall portfolio risk. Consider the table drawn earlier to compute the portfolio risk and look into the diagonal cells. The diagonal cells contain the variance of securities in the portfolio. In that example, we assumed that an equal investment is made in three stocks. The sum of the diagonal cells is equal to sum of the variance of three securities multiplied by $(1/3)^2$. Suppose, we add one more stock in the portfolio and revise our weights to 0.25 for each stock. The values of diagonal cells is now equal to sum of the variance of four securities multiplied by $(1/4)^2$. We know $(1/4)^2 < (1/3)^2$. Suppose, if the number of securities in the portfolio is increased to 20, then the value of the diagonal cells is equal to sum of the variance of individual securities multiplied by $(1/20)^2$. The value of $(1/20)^2$ is equal to .0025 and close to zero. Since the multiplier is now close to zero, the sum of the diagonal cells will reach close to zero. Thus, when a security is added to the portfolio, the value of diagonal cells is close to zero and thus reduced the variance of the portfolio. However, there is a limitation in adding securities to reduce the risk because the diagonal cells value can not be reduced below

zero (i.e. negative) to reduce the portfolio risk further. Thus, beyond a level, diversification fails to yield further benefit by way of reducing the risk. This is being illustrated in Figure 10.2.

Figure 10.2 : Diversification of Risk



It may be noted that beyond certain portfolio size, the reduction in risk is marginal and insignificant.

We will discuss more about diversifiable and non-diversifiable risk in Unit 12.A word of caution may, however, be urged here. The above discussion would appear to suggest that the overall portfolio risk can be reduced by only increasing number of assets in the portfolio. This is not true. Several empirical studies have indicated that a portfolio comprising a few assets selected carefully for their risk-diversifying characteristics (i.e. nature. and degree of variance and covariance), would be less risky than a portfolio of considerably greater size with assets being selected without regard to risk. Thus, -what matters in diversification is not the number of assets *per se,* but right kinds.



CHECK YOUR PROGRESS

Q1: Define Portfolio.

……

……

……

Q2: What is Arithmetic Average

……

……

……

……

10. 4 PORTFOLIO ANALYSIS AND SELECTION

In the previous section, we have discussed how portfolio risk is measured. Let us summarise important points before discussing how an investor can use the concept in selection of the portfolio. Risk associated with investments can be reduced through diversification and such diversification helps the investors to reduce the risk of the portfolio. Investments in individual securities, risk (variance) associated with individual securities and the relationships (covariance) between the securities are the three variables that affect the risk of the portfolio. While diversification reduces the unsystematic risk of the portfolio, the number of securities required to minimize the portfolio risk is not very large. Finally more than the number of securities, what matters in reducing the risk of the portfolio is the kind of securities included in the portfolio. The last observation is stressed in this section.

10.4.1 Correlation between Securities and its Impact on Portfolio Risk

We have discussed that risk is reduced when the portfolio includes one stock in the portfolio. The above observation is not universal in a sense that if the new stock is perfectly correlated with other securities in the portfolio: In other words, the job of investment analysts or any other persons responsible in constructing the portfolio is to identify stocks or securities that are less related with each other for portfolio construction. The risk of the portfolio can be reduced to zero if the correlation between the assets included in the portfolio is

equal to minus 1. However, such securities are *difficult* to identify in the market. If two securities are perfectly correlated, then there is no diversification benefit and such combination will not reduce the risk of the portfolio. There are only very few securities in the market whose correlation is equal to minus one. What is more prevalent in the market is securities whose return are correlated between minus 1 to plus 1. Depending on the level of correlation, diversification reduces the risk of the portfolios. The relationship between the assets and its impact on portfolio risk is explained below in Figure 8.3 with the help of two securities.

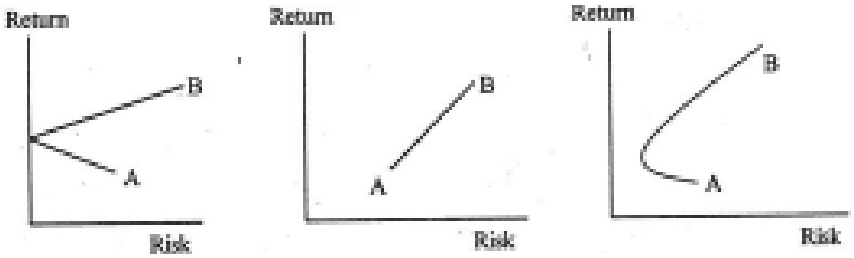


Figure 8.3: Correlation and Portfolio Risk

(a) Correlation = - 1 (b) Correlation = +1 (c) Correlation = 0.72

Figure 10.3 (c) is more relevant for our discussion since the correlation between the securities is often less than 1 and greater than zero. In such a situation, when an investor combine such securities, the risk of the security is initially reduced. We will show you with a real life example in the following Table:

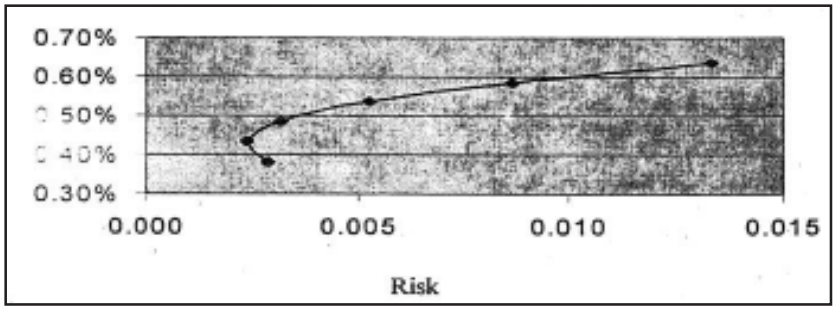
Sl.No.	Proportion of Investment in		Portfolio Return	Portfolio Risk (Variance)
	Hindustan Lever	Infosys		
1	100%	0%	0.38%	0.00289
2	80%	20%	0.43%	0.00240
3	60%	40%	0.48%	0.00320
4	40%	60%	0.54%	0.00529
5	20%	80%	0.59%	0.00867
6	0%	100%	0.64%	0.01334

Note: Correlation between Hindustan Lever Ltd. and Infosys is .0087

The risk and return of the portfolio is plotted below to show how the

graph looks similar to one shown in Figure 8.3 (C)

Figure10 .4: Risk and Return of Portfolios of HLL and Infosys



If there are 10 securities in *the* market, it is possible to draw the diagrams of the above for a number of combination of two-securities.

10.4.2 Portfolio Selection

The objective of every rational investor is to maximise his returns and minimise the risk. Diversification is the method adopted for reducing risk. It essentially results in the construction of portfolios. The proper goal of portfolio construction would be to generate a portfolio that provides the highest return and the lowest risk. Such a portfolio would be known as the optimal portfolio. The process of finding the optimal portfolio is described as portfolio selection. We will discuss this about more in the next unit.



10.5 LET US SUM UP

In this unit we have discussed the following points:

- Portfolio analysis builds on the estimates of future return and risk of holding various combinations of assets.
- The variance or standard deviation of an individual security measures the riskiness of a security in absolute sense.
- The expected return of a portfolio of assets is simply the weighted average of the return of the individual securities held in the portfolio.
- While diversification reduces the unsystematic risk of the portfolio, the

number of securities required to minimize the portfolio risk is not very large.



10.6 FURTHER READING

1) Elton, Edwin J. and Gruber, Matin J., 1987 *Modern Portfolio Theory and Investment Analysis, John Wiley 84 Sons.*

2) Alexander, Gordon J., Sharpe, William F., and Jeffery V. Baibey *Fundamentals of Investments,* (3rd ed.) Prentice-Hall, Inc.



10.7 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q1: The term 'portfolio' generally means a collection or combination and in the context of investment management, it means a collection or combination of financial assets (or securities) such as shares, debentures and government securities.

Ans to Q2: The arithmetic average return is equal to sum of returns of period and divided by 'n'.



10.8 MODEL QUESTIONS

1) The following forecasts have been made for investments A and B.

Investments A		Investments B	
Return (%)	Probability	Return (%)	Probability
10	.05	2	.05
15	.20	12	.25
20	.50	20	.40
25	.20	28	.25
30	.05	38	.05

Calculate the expected rate of return and standard deviation. Which investment has more upside potential and downside danger?

2) If a portfolio's expected return is always equal to the weighted average of the expected return of the component assets, why is not portfolio risk always equal to the weighted average of the component assets' variances? Explain.

3) Suppose an analysts has provided you with the following estimates in respect of equity shares of Century Textiles, Escorts and Hoechst:

	Century	Escorts	Hoechst
Expected monthly Return (%)	5	4	9
Standard Deviation (%)	8	7	17
Correlation coefficients of Returns Between			
Century and Escorts	0.4		
Century and Hoechst	0.6		
Escorts and Hoechst	0.3		

Assuming that equal amounts of the available funds will be invested in the three scrips, estimate the portfolio's mean return and standard deviation.

4) Consider two securities with the following characteristics:

	A	B
Expected Return	12	02
Standard Deviation	08	10

Assuming no correlation between the returns on two securities, calculated expected return and standard deviation for each of the following portfolios:

	Portfolio Weights (X_A)				
Security A	1.0	.75	.50	25	0.0

Plot these portfolios with expected portfolio returns *on x-axis* and standard deviation on y-axis. Locate the efficient frontier' and the portfolio with least risk or standard deviation.

Can you precisely determine X_A corresponding to the portfolio with minimum standard deviation? (Hint: Obtain the equation for σ_p with zero correlation between returns on two securities. To find X_A for which σ_p is minimum, set the first order derivative of σ_p with respect to X_A equal to zero, and then solve for X_A).

UNIT 11 : PORTFOLIO CONSTRUCTION

UNIT STRUCTURE

11.1 Learning Objectives

11.2 Introduction

11.3 Benefits of Portfolios

11.4 Approaches in Portfolio Construction

- 11.4.1 Traditional approach
- 11.4.2 Modern Approach

11.5 Portfolio Risk Return

11.6 Let us Sum Up

11.7 Further Reading

11.8 Answers to Check Your Progress

11.9 Model Questions

11.1 LEARNING OBJECTIVES

After going through this unit, you will be able to will be able to :

- describe the meaning of portfolio construction
- discuss the approaches of portfolio construction
- discuss the reasons and process of portfolio construction of financial instruments.

11.2 INTRODUCTION

Portfolio is a combination of securities such as stocks, bonds and money market instruments. The process of blending together the broad asset classes so as to obtain optimum return with minimum risk is called portfolio construction. Individual securities have risk-return characteristics of their own. Portfolios may or may not take on the aggregate characteristics of their individual parts. Diversification of investment helps to spread risk over many assets. A diversification of securities gives the assurance of obtaining the anticipated return on the portfolio. In a diversified portfolio, some securities may not perform as expected, but others may exceed the

expectation and making the actual return of the portfolio reasonably close to the anticipated one. Keeping a portfolio of single security may lead to a greater likelihood of the actual return somewhat different from that of the expected return. Hence, it is a common practice to diversify securities in the portfolio.

In this unit we will also discuss about approaches in Portfolio Construction like the Traditional approach and Modern Approach.At the end of this unit we shall discuss Portfolio Risk Return and Managing the Portfolio

11.3 BENEFITS OF PORTFOLIOS

Portfolio construction is the process of blending together the broadasset classes to obtain optimum return with minimum risk.

We know that expected return from individual securities carry some degree of risk. Risk was defined as the standard deviation around the expected return. In effect we equated a security's risk with the variability of its return. More dispersion or variability about a security's expected return meant the security was riskier than one with less dispersion.

The simple fact that securities carrying differing degrees of expected risk lead most investors to the notion of holding more than one security at a time, is an attempt to spread risks by not putting all their eggs into one basket. Diversification of one's holdings is intended to reduce risk in an economy in which every asset's returns are subject to some degree of uncertainty. Even the value of cash suffers from the inroads of inflation. Most investors hope that if they hold several assets, even if one goes bad, the others will provide some protection from an extreme loss.

11.4 APPROACHES IN PORTFOLIO CONSTRUCTION

Commonly, there are two approaches in the construction of the portfolio of securities viz. traditional approach and Markowitz efficient frontier approach. In the traditional approach, investor's needs in terms of income and capital appreciation are evaluated and appropriate securities are selected to meet the needs of the investor. The common practice in the traditional

approach is to evaluate the entire financial plan of the individual. In the modern approach, portfolios are constructed to maximise the expected return for a given level of risk. It views portfolio construction in terms of the expected return and the risk associated with obtaining the expected return.

11.4.1 Traditional approach

Traditional approach of portfolio construction is based on thefinancial needs of the individual investors.The traditional approach basically deals with two major decisions.

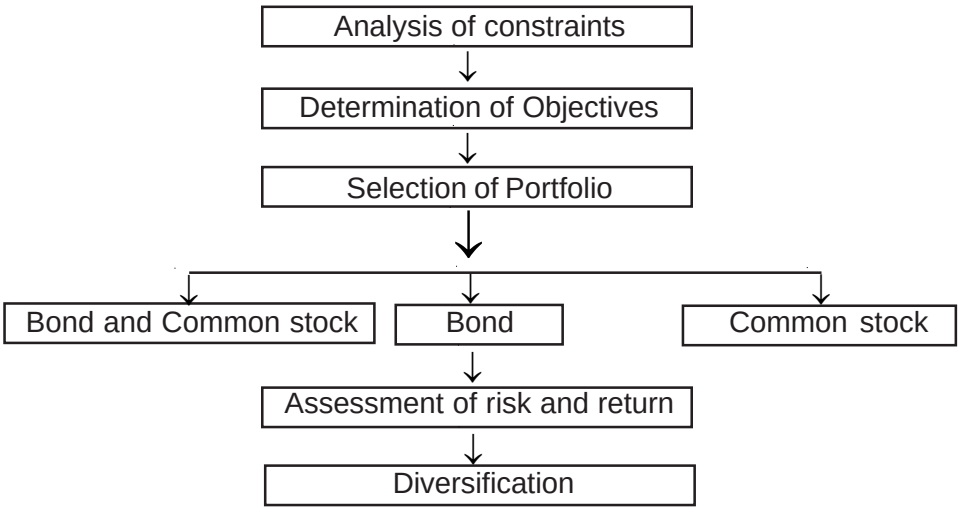
They are:

(a) Determining the objectives of the portfolio.

(b) Selection of securities to be included in the portfolio.

Normally, this is carried out in four to six steps. Before formulating the objectives, the constraints of the investor should be analysed. Within the given framework of constraints, objectives are formulated. Then based on the objectives, securities are selected. After that, the risk and return of the securities should be studied. The investor has to assess the major risk categories that he or she is trying to minimise. Compromise on risk and non-risk factors has to be carried out. Finally relative portfolio weights are assigned to securities like bonds, stocks and debentures and then diversification is carried out. The flow chart Fig 11.1 explains this.

Fig. 11.1 Steps in traditional approach



1. Analysis of constraints : The constraints normally discussed are: Income needs, liquidity, time horizon, safety, tax considerations and the temperament.

Income needs- The income needs depend on the need for incomein constant rupees and current rupees. The need for income in current rupees arises from the investor's need to meet all or part of the living expenses. At the same time inflation may erode the purchasing power, the investor may like to offset the effect of the inflation and so, needs income in constant rupees.

(a) Need for current income: The investor should establish theincome which the portfolio should generate. The current income need depends upon the entire current financial plan of the investor. The expenditure required to maintain a certain level of standard of living and all the other income generating sources should be determined. Once this information is arrived at, it is possible to decide how much income must be provided for the portfolio of securities.

(b) Need for constant income: Inflation reduces the purchasingpower of the money. Hence, the investor estimates the impact of inflation on his estimated stream of income and tries to build a portfolio which could offset the effect of inflation. Funds should be invested in such securities where income from them might increase at a rate that would offset the effect of inflation. The inflation or purchasing power risk must be recognised but this does not pose a serious constraint on portfolio if growth stocks are selected.

Liquidity- Liquidity need of the investment is highly individualisticof the investor. If the investor prefers to have high liquidity, then funds should be invested in high quality short term debt maturity issues such as money market funds, commercial papers and shares that are widely traded. Keeping the funds in shares that are poorly traded or stocks in closely held business and real estate lack liquidity. The investor should plan his cash drain and the need for net cash inflows

during the investment period.

Safety of the principal- Another serious constraint to be consideredby the investor is the safety of the principal value at the time of liquidation, investing in bonds and debentures is safer than investing in the stocks. Even among the stocks, the money should be invested in regularly traded companies of longstanding. Investing money in the unregistered finance companies may not provide adequate safety.

Time horizon- Time horizon is the investment-planning period ofthe individuals. This varies from individual to individual. Individual's risk and return preferences are often described in terms of his 'life cycle'. The states of the life cycle determine the nature of investment. The first stage is the early career situation. At the career starting point assets are lesser than their liabilities. More goods are purchased on credit. His house might have been built with the help of housing loan scheme. His major asset may be the house he owns. His priority towards investments may be in the form of savings for liquidity purposes. He takes life insurance for protecting him from unforeseen events like death and accidents and then he thinks of the investments. The investor is young at this stage and has long horizon of life expectancy with possibilities of growth in income, he can invest in high-risk and growth oriented investments.

The other stage of the time horizon is the mid-career individual. At this stage, his assets are larger than his liabilities. Potential pension benefits are available to him. By this time he establishes his investment program. The time horizon before him is not as long as the earlier stage and he wants to protect his capital investment. He may wish to reduce the overall risk exposure of the portfolio but, he may continue to invest in high risk and high return securities.

The final stage is the late career or the retirement stage. Here, the time horizon of the investment is very much limited. He needs stable income and once he retires, the size of income he needs from investment also increases. In this stage, most of his

loans are repaid by him and his assets far exceed the liabilities. His pension and life insurance programmes are completed by him. He shifts his investment to low return and low risk category investments, because safety of the principal is given priority. Mostly he likes to have lower risk with high interest or dividend paying component to be included in his portfolio. Thus, the time horizon puts restrictions on the investment decisions.

Tax consideration- Investors in the income tax paying group considerthe tax concessions they could get from their investments. For all practical purpose, they would like to reduce the taxes. For income tax purpose, interests and dividends are taxed under the head "income from other sources". The capital appreciation is taxed under the head "capital gains" only when the investor sells the securities and realises the gain. The tax is then at a concessioanl rate depending on the period for which the asset has been held before being sold. From the tax point of view, the form in which the income is received i.e. interest, dividend, short term capital gains and long term capital gains are important. If the investor cannot avoid taxes, he can delay the taxes. Investing in government bonds and NSC can avoid taxation. This constraint makes the investor to include the items which will reduce the tax.

Temperament- The temperament of the investor himself poses aconstraint on framing his investment objectives. Some investors are risk lovers or takers who would like to take up higher risk even for low return. While some investors are risk averse, who may not be willing to undertake higher level of risk even for higher level of return. The risk neutral investors match the return and the risk. For example, if a stock is highly volatile in nature then the stock may be selling in a range of Rs. 100-200, and returns may fluctuate between Rs. 00-100 in a year. Investors who are risk averse would find it disturbing and do not have the temperament to invest in this stock. Hence, the temperament of the investor plays an important role in setting the objectives.

2. **Determination of objectives :** Portfolios have the common objective of financing present and future expenditures from a large pool of assets. The return that the investor requires and the degree of risk he is willing to take depend upon the constraints. The objectives of portfolio range from income to capital appreciation. The common objectives are stated below:
   - Current income
   - Growth in income
   - Capital appreciation
   - Preservation of capital

The investor in general would like to achieve all the four objectives, nobody would like to lose his investment. But, it is not possible to achieve all the four objectives simultaneously. If the investor aims at capital appreciation, he should include risky securities where there is an equal likelihood of losing the capital. Thus, there is a conflict among the objectives.

3. **Selection of Portfolio :** The selection of portfolio depends on the various objectives of the investor. The selection of portfolio under different objectives are dealt subsequently.

Objectives and asset mix- If the main objective is getting adequateamount of current income, sixty per cent of the investment is made on debts and 40 per cent on equities. The proportions of investments on debt and equity differ according to the individual's preferences. Money is invested in short term debt and fixed income securities. Here the growth of income becomes the secondary objective and stability of principal amount may become the third. Even within the debt portfolio, the funds invested in short term bonds depends on the need for stability of principal amount in comparison with the stability of income. If the appreciation of capital is given third priority, instead of short term debt the investor opts for long term debt. The period may not be a constraint.

Growth of income and asset mix- Here the investor requires a certainpercentage of growth in the income received from his

investment. The investor's portfolio may consist of 60 to 100 per cent equities and 0 to 40 per cent debt instrument. The debt portion of the portfolio may consist of concession regarding tax exemption. Appreciation of principal amount is given third priority. For example computer software, hardware and non-conventional energy producing company shares provide good possibility of growth in dividend.

Capital appreciation and asset mix- Capital appreciation means thatthe value of the original investment increases over the years. Investment in real estates like land and house may provide a faster rate of capital appreciation but they lack liquidity. In the capital market, the values of the shares are much higher than their original issue prices. For example Satyam Computers, share value was Rs. 306 in April 1998 but in October 1999 the value was Rs. 1658. Likewise, several examples can be cited. The market capitalisation also has increased. Next to real assets, the stock markets provide best opportunity for capital appreciation. If the investor's objective is capital appreciation, 90 to 100 per cent of his portfolio may consist of equities and 0-10% of debts. The growth of income becomes the secondary objective.

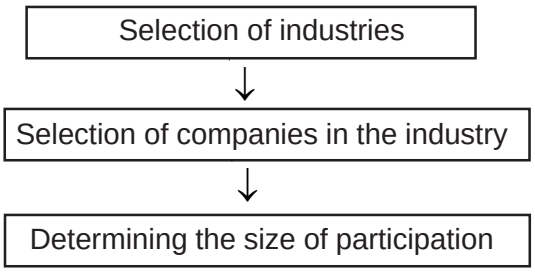
Safety of principal and asset mix- Usually, the risk averse investorsare very particular about the stability of principal. According to the life cycle theory, people in the third stage of life also give more importance to the safety of the principal. All the investors have this objective in their mind. No one like to lose his money invested in different assets. But, the degree may differ. The investor's portfolio may consist more of debt instruments and within the debt portfolio more would be on short term debts.

4. **Risk and return analysis:** The traditional approach to portfoliobuilding has some basic assumptions. First, the individual prefers larger to smaller returns from securities. To achieve this goal, the investor has to take more risk. The ability to achieve higher returns is dependent upon his ability to judge

risk and his ability to take specific risks. The risks are namely interest rate risk, purchasing power risk, financial risk and market risk. The investor analyses the varying degrees of risk and constructs his portfolio. At first, he establishes the minimum income that he must have to avoid hardships under most adverse economic condition and then he decides risk of loss of income that can be tolerated. The investor makes a series of compromises on risk and non-risk factors like taxation and marketability after he has assessed the major risk categories, which he is trying to minimise. The methods of calculating risk and return of a portfolio is classified in following pages of this chapter.

5. **Diversification:** Once the asset mix is determined and therisk and return are analysed, the final step is the diversification of portfolio. Financial risk can be minimised by commitments to top-quality bonds, but these securities offer poor resistance to inflation. Stocks provide better inflation protection than bonds but are more vulnerable to financial risks. Good quality convertibles may balance the financial risk and purchasing power risk. According to the investor's need for income and risk tolerance level portfolio is diversified. In the bond portfolio, the investor has to strike a balance between the short term and long term bonds. Short term fixed income securities offer more risk to income and long term fixed income securities offer more risk to principal. In the stock portfolio, he has to adopt the following steps which are shown in the following figure.

Fig. 11.2 Steps in Stock Portfolio



As investor, we have to select the industries appropriate to our investment objectives. Each industry corresponds to specific goals of the investors. The sales of some industries like two wheelers and steel tend to move in tandem with the business cycle, the housing industry sales move counter cyclically. If regular income is the criterion then industies, which resist the trade cycle should be selected. Likewise, the investor has to select one or two companies from each industry. The selection of the company depends upon its growth, yield, expected earnings, past earnings, expected price earning ratio, dividend and the amount spent on research and development. Selecting the best company is widely followed by all the investors but this depends upon the investors' knowledge and perceptions regarding the company. The final step in this process is to determine the number of shares of each stock to be purchased. This involves determining the number of different stocks that is required to give adequate diversification. Depending upon the size of the portfolio, equal amount is allocated to each stock. The investor has to purchase round lots to avoid transaction costs.

11.4.2 Modern Approach

Modern approach is based on the risk and return analysis.We have seen that the traditional approach is a comprehensive financial plan for the individual. It takes into account the individual needs such as housing, life insurance and pension plans. But these types of financial planning approaches are not done in the Markowitz approach. Markowitz gives more attention to the process of selecting the portfolio. His planning can be applied more in the selection of common stocks portfolio than the bond portfolio. The stocks are not selected on the basis of need for income or appreciation. But the selection is based on the risk and return analysis. Return includes the market return and dividend. The investor needs return and it may be either in the form of market return or dividend. They are assumed to be indifferent towards the form of return.

Among the list of stocks quoted at the Bombay Stock Exchange or at any other regional stock exchange, the investor selects roughly some group of shares say of 10 or 15 stocks. For these stocks' expected return and risk would be calculated. The investor is assumed to have the objective of maximising the expected return and minimising the risk. Further, it is assumed that investors would take up risk in a situation when adequately rewarded for it. This implies that individuals would prefer the portfolio of highest expected return for a given level of risk.

In the modern approach, the final step is asset allocation process that is to choose the portfolio that meets the requirement of the investor. The risk taker i.e. who are willing to accept a higher probability of risk for getting the expected return would choose high risk portfolio. Investor with lower tolerance for risk would choose low level risk portfolio. The risk neutral investor would choose the medium level risk portfolio.



CHECK YOUR PROGRESS

Q1: Define Portfolio Construction.

……

……

Q2: What is Traditional approach of portfolio construction

……

……

Q3: Write the two major decisions. of traditional approach of of portfolio construction.

……

……

11.5 PORTFOLIO RISK/RETURN

As mentioned earlier, an investment decision involves selection of a combination or group of securities for investment. This group of securities is referred to as a portfolio. The portfolio can be a combination of securities irrespective of their nature, maturity, profitability, or risk characteristics. Investors, rather than looking at individual securities, focus more on the performance of all securities together. While portfolio returns are the weighted returns of all securities constituting the portfolio, the portfolio risk is not the simple weighted average risk of all securities in the portfolio. Portfolio risk considers the standard deviation together with the covariance between securities. Co-variance measures the movement of assets together.

The portfolio risk and return using historical data is computed using the following formula:

Portfolio return $= E(r) = \sum_{i=1}^{n} w_i r_i$

Portfolio risk $= \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2} + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}$

Where

ω = weights (percentage value)

r = return on the securities

Portfolio risk is thus the summation of the individual security variance and the co-movement with other securities in the portfolio. The above formula can be split into a spreadsheet showing all the co-movement measures of the securities.

The total variance is the summation of all cells in the following table. The diagonal summation represents the first part. This is the variance of each security individually. The weights of the securities in the portfolio are represented by the variables ω_i. Weights are the market values of the securities held by the investor. When all securities in the portfolio are given equal weights, the ω_i will be simply (1/n). In a two security portfolio with equal weights the value of ω_i is (1/2) 0.5. When there are three securities in a portfolio, the market values being equal for all the three securities, the

weights for each security will be (1/3) 0.33. Similar weights result in the multiplication of ω_i twice.

The second part of the variance computation equation is the summation of all other cells except the diagonal cells. These are the co-variance of one security with another security in the portfolio. The total covariance is computed by considering the weight of each security in the portfolio. When the weight of each security is different the weight of a combination in a portfolio will be ($\omega_i \times \omega_j$); where i and j represent the two securities.

The square root of the variance gives the standard deviation of the portfolio, i.e., the risk of the portfolio. The following table gives the computation of the standard deviation elaborately. The group of individual securities 1,2,3, … n are related with each other to arrive at the co-variance matrix.

$$\sigma_p^2 = \boxed{\sum_{i=1}^{n} w_i^2 \sigma_i} + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i \, w_j \sigma_{ij}$$

$$\sigma_p = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i \, w_j \sigma_{ij}}$$

The computation of co-variance i.e., σ_{ij} when i is not equal to j is as follows:

$$\sigma_{ij} = \frac{1}{n} \sum_{t=1}^{n} (r_{it} -) \times (r_{jt} - \overline{r_j})$$

Co-variance can also be measured in terms of the correlation coefficient. The correlation coefficient is a measure of the relationship between two assets. The correlation coefficient ranges between the value +1 and –1. A correlation coefficient of +1 indicates that two securities returns move perfectly in tandem with each other. A negative correlation coefficient of -1 implies that when one securities' returns increase, the other securities' return reduces by the same quantum.

The computation of the co-variance σ_{ij} through the correlation coefficient is by the application of the following formula:

$$\sigma_{ij=} \sigma_{i\times} \sigma_{j\times} \rho_{ij}$$

ρ_{ij} is the correlation coefficient

The correlation coefficient between two securities can be stated in any of the following formats.

$$\frac{\frac{1}{n}\sum_{t=1}^{N}(r_{it}-\bar{r}_j)\times(r_{jt}-\bar{r}_j)}{\sqrt{\rho ij = \frac{\sum_{t=1}^{n}(r_{it}-\bar{r}_j)^2}{n}}\sqrt{\frac{\sum_{t=1}^{N}(r_{it}-\bar{r}_j)_2}{N}}}$$

$$\rho_{ij}=\frac{\sigma_{ij}}{\sigma_i\times\sigma_j}$$

Illustration - 11.1 Two securities P and Q are considered forinvestment. Compute the risk and return of the portfolio assuming the two securities, whose correlation coefficient of returns is –0.84, are combined in the following proportions in the portfolio: (a) 0: 100,

(b) 10: 90, (c) 20: 80, (d) 50: 50, (e) 80: 20, (f) 90: 10, (g) 100: 0. The historical risk-return of the two securities is as follows:

Table 11.1 - Risk-Return

Security	Risk % (Std. Dev.)	Return %
P	20	15
Q	30	20

Solution. Computation of portfolio return:

(a) 0 : 100 = 20%

(b) 10 : 90 – (0.1*15) + (0.9*20) = 19.5%

(c) 20 : 80 – (0.2*15) + 0.8*20) = 19%

(d) 50 : 50 – (0.5*15) + (0.5*20) = 17.5%

(e) 80 : 20 – (0.8*15) + (0.2*20) = 16%

(f) 90 : 10 – (0.9*15) + (0.1*20) = 15.5%

(g) 100 : 0 = 15%

Computation of portfolio risk:

(a) 0 : 100 = 30%

(b) 10 : 90 = 25.34%

$$\sigma_p = \sqrt{(0.1^2 \times 20^2) + (0.9^2 \times 30^2) + (2 \times 0.1 \times 0.9 \times -0.84 \times 20 \times 30)}$$

(c) 20 : 80 = 20.75%

$$\sigma_p = \sqrt{(0.2^2 \times 20^2) + (0.8^2 \times 30^2) + (2 \times 0.2 \times 0.8 \times -0.84 \times 20 \times 30)}$$

(d) 50 : 50 = 8.54%

$$\sigma_p = \sqrt{(0.5^2 \times 20^2) + (0.5^2 \times 30^2) + (2 \times 0.5 \times 0.5 \times -0.84 \times 20 \times 30)}$$

(e) 80 : 20 = 11.43%

$$\sigma_p = \sqrt{(0.8^2 \times 20^2) + (0.2^2 \times 30^2) + (2 \times 0.8 \times 0.2 \times -0.84 \times 20 \times 30)}$$

(f) 90 : 10 = 15.57%

$$\sigma_p = \sqrt{(0.9^2 \times 20^2) + (0.2^2 \times 30^2) + (2 \times 0.8 \times 0.1 \times -0.84 \times 20 \times 30)}$$

(g) 100 : 0 = 20%

Illustration 11.2 -Compute the risk return characteristic of an equallyweighted portfolio of three securities whose individual risk and return are given in the following table. the correlation between Security A and B is –0.43 and the correlation between security B and C is 0.21 and the correlation coefficient between security A and C is –0.62.

Table 11.2 RiskReturn

Security	Risk	Return
A	15%	12%
B	20%	18%
C	25%	22%

Solution. The portfolio return is computed as follows:

(0.33*12) + (0.33*18) + (0.33*22) = 17.16%

the portfolio risk is computed as follows:

$$\sigma = \sqrt{(0.33_2 \times 15_2) + (0.33_2 \times 20_2) + (0.33_2 \times 25_2)(2 \times 0.33_2 \times 0.43 \times 15 \times 20) + (2 \times 0.33_2 \times 0.21_2 \times 20 \times 25) + (2 \times 0.33 \times -0.62 \times 15 \times 25)}$$

= 8.96%.

When the correlation coefficient ranges between 0 and -1, there is a possibility of minimising the total risk by combining the two securities.

For a two security combination it is possible to find the ratio of investment in the two securities that will result in minimum risk portfolio. The percentage of investment in security (A) can be ascertained using the

following equation.

$$\omega_A = \frac{\sigma_B^2 - \rho_{AB} \times \sigma_A \times \sigma_B}{\sigma_A^2 + \sigma_B^2 - 2 \times \rho_{AB} \times \sigma_A \times \sigma_B}$$

The proportion of investment in security (B) will be 1 - ω_A or can also be computed using the following equation.

$$\omega_B = \frac{\sigma_A^2 - \rho_{AB} \times \sigma_A \times \sigma_B}{\sigma_A^2 + \sigma_B^2 - 2 \times \rho_{AB} \times \sigma_A \times \sigma_B}$$

When the correlation coefficient is -1, the proportion of investment in each security can be given by the following equation.

$$\omega_A = \frac{\sigma_B^2 - \rho_{AB} \times \sigma_A \times \sigma_B}{\sigma_A^2 + \sigma_B^2 - 2 \times \rho_{AB} \times \sigma_A \times \sigma_B} = \frac{\sigma_A(\sigma_A + \sigma_B)}{(\sigma_A + \sigma_B)^2} = \frac{\sigma_B}{\sigma_A + \sigma_B}$$

$$\omega_B = \frac{\sigma_A^2 - \rho_{AB} \times \sigma_A \times \sigma_B}{\sigma_A^2 + \sigma_B^2 - 2 \times \rho_{AB} \times \sigma_A \times \sigma_B} = \frac{\sigma_A(\sigma_A + \sigma_B)}{(\sigma_A + \sigma_B)} = \frac{\sigma_A}{\sigma_A + \sigma_B}$$

Illustration 11.3. From the two securities available for investment opportunity, find the proportion of investment in each security that will minimise the risk for the investor. The correlation coefficient between the two securities is – 0.65. Determine portfolio risk.

Table 11.3 - Risk-return

Security	Risk%	Return%
A	25	18
B	30	22

$$\omega_A = \frac{\sigma_A^2 - \rho_{AB} \times \sigma_A \times \sigma_B}{\sigma_A^2 + \sigma_B^2 - 2 \times \rho_{AB} \times \sigma_A \times \sigma_B}$$

$$\omega_A = \frac{30^2 - (-0.65 \times 25 \times 30)}{30^2 + 25^2 - (2 \times -0.65 \times 25 \times 30)}$$

$$\omega_A = 0.556 \quad \omega_B = 1 - 0.556 = 0.444$$

The risk return composition for a portfolio with these weights are as follows:

Portfolio return

(0.556*18) + (0.444*22) = 19.78%

Portfolio risk

$$\sigma_P = \sqrt{(556^2 \times 25^2) + (0.444^2 \times 30^2) + (2 \times -0.65 \times 25 \times 30)}$$

= 11.40%

Minimal risk is achieved since the correlation coefficient is ranging between 0 and –1. A positive correlation coefficient increases the portfolio risk proportionately. The following table illustrates the risk-return profile of a two security portfolio when the correlation coefficient is 0, 0.5, 1,-0.5 and –1.

Security	Risk%	Return%
A	20	15
B	30	20

ρ	0,100		40,60		50,50		60,40		100,0	
	Risk	Return	Risk	Return	Risk	Return	Risk	Return	Risk	Return
0	30	20	19.7	18	18	17.5	17	17	20	15
0.5	30	20	23.1	18	21.8	17.5	20.8	17	20	15
1	30	20	26	18	25	17.5	24	17	20	15
-0.5	30	20	15.6	18	13.2	17.5	12	17	20	15
-1	30	20	10	18	5	17.5	10	17	20	15

The graph in Figure 11.2 plots all the combinations of securities for different correlation coefficients.

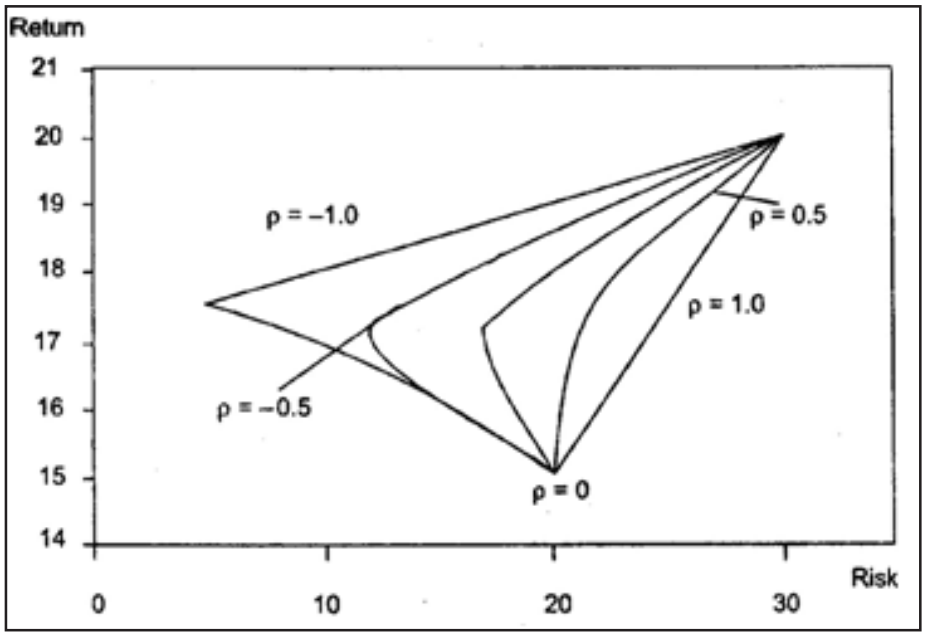


Figure 11.2. Risk-Return impact of different correlations

Plots for a larger number of securities are similar and can be represented through the graphs in Fig 11.4 and Fig 11.5.

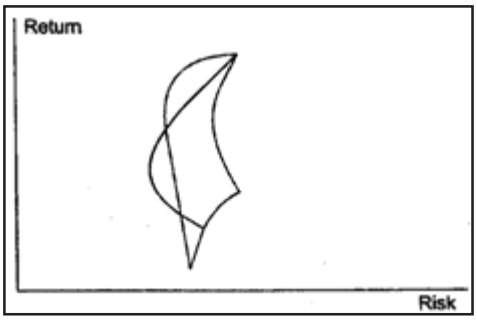


Fig 11.4

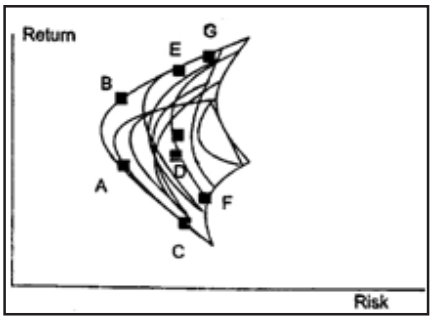


Fig 11.5.

A rational investor, given the above options of portfolios, will tend to select only those portfolios that give the highest return for a given risk or on the other hand, a lowest risk for a given return option. Consider the points A and B in Fig 11.5. Given the same risk level the return from B is higher than A, hence the rational investor will prefer B rather than A. Similarly, consider the points C, D, and E, compared to points C, and D, E gives a higher return for the same level of risk. The preference of investors will be E. Also, as risk level increases between the points F and G, G will be a preferred investment considering the higher return from this investment. The choice between B, E, and G will depend on the risk preference of investors. Given a higher risk preference level the choice of an investor will be towards point G. On the other hand if the investor is averse to risk the preference will be towards B rather than E and G.

The selection of portfolios for the investor is thus made only between the top most points in the feasible portfolio region shown in Figure 11.6. The feasible region is the combination of securities available in the market. The outer layer of the feasible region gives the investor maximum returns for a specific risk. Hence this is called the efficient frontier. An investor can evaluate among the efficient frontier to select the specific risk return portfolio that is preferred. These portfolios provide the highest return for a given level of risk.

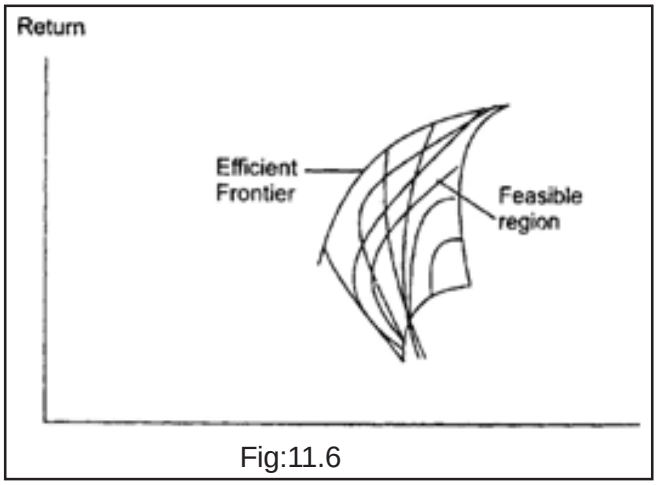


Fig:11.6



11.9 LET US SUM UP

In this unit we have discussed the following :

- Portfolio is a combination of various securities.
- Portfolios can be constructed according to the traditional approach or modern approach.
- In the traditional approach the constraints, investor's need for current income and income in constant rupees are analysed. Liquidity, safety, time horizon of the investment, tax consideration and temperament of the individual investor's are the other constraints to frame the objectives.
- The general objectives of the portfolio are current income, constant income, capital appreciation and preservation of capital.
- According to the objectives the portfolio whether it is a bond portfolio or a stock portfolio or combination of both of bond and stock is decided. After that, the equity component of the portfolio is chosen. The traditional approach takes the entire financial plan of the individual investor.
- In the modern approach, Markowitz model is used. More importance is given to the risk and return analysis.



11.10 FURTHER READING

1) Farrell, J.I., Jr. *Guide to Portfolio Management.* New York: McGraw-Hill, 1983.

2) Harrington, D.R. *Modern Portfolio Theory.* 2d ed. Englewood Cliffs, N.J.: Prentice Hall, 1987.

3) Markowitz, H.M. *Portfolio Selection: Diversificationo f Investments.* New York: John Wiley, 1959.

4) Markowitz, H.M. "Individual versus Institutional Investing." *FinancialServices Review* 1 (1991).

5) Sharpe, W.F. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." *Journal of Finance* (September 1964).

6) Sharpe, W.F. *Portfolio Theory and Capital Markets.* New York: McGraw-Hill, 1970.

7) Statman, M. "How Many Stocks Make a Diversified Portfolio?" *Journalof Financial and Quantitative Analysis* (September 1987).



11.10 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q1: Portfolio construction is the process of blending to gether the broadasset classes to obtain optimum return with minimum risk.

Ans to Q2: Traditional approach of portfolio construction is based on thefinancial needs of the individual investors.

Ans to Q3: The traditional approach basically deals with two major decisions. They are:

(a) Determining the objectives of the portfolio.

(b) Selection of securities to be included in the portfolio.



11. 11 MODEL QUESTIONS

1. What are the steps in the traditional approach?
2. Explain the constraints in the formation of objectives.
3. How would you formulate the asset mix according to the given objectives?
4. What are the differences between the traditional approach and modern approach?
5. State the modern approach in the construction of the portfolio.
6. Consider two situations: a young man X in early twenties and another young man Y in the late thirties X and Y earns same amount of money. Mr. Y has a family, a house, a car and all the encumbrances related with the marital status. Both of them like to invest in securities, what would be their constraints and objectives?
7. Ajay, aged 26 is chalking out an investment program to invest in common stocks. Ajay is married and working in a MNC. He is paid nearly 5 lakhs per year. He is having a well furnished house and a car. He is a member of the life insurance scheme. He has purchased his house on loan scheme. The MNC with whom he is working has given him 15 years of job contract.

   They may or may not renew their contract. Assist him in his investment plan. Advise him about the components of his portfolio worth of 5 lakhs.
8. Compute the risk and return of a portfolio of these securities.

   Assume equal weights.

Security	**S1**	**S2**	**S3**	**S4**	**S5**
Return	12%	10%	8%	15%	18%
Risk (standard deviation)	20%	18%	10%	18%	25%

9. Give the minimum risk portfolio from the combination of the following securities.

Security	**S1**	**S2**
Risk (standard deviation)	15%	20%
Return	20%	30%

10. Select suitable portfolios for an investor who falls in the risk bracket of 40 per cent.

Portfolio	**P1**	**P2**	**P3**	**P4**	**P5**
Standard deviation	15%	16%	18%	12%	19%
Return	16%	18%	22%	19%	23%

11. Use the Sharpe Index Model to select the best combination of securities for a portfolio. The risk free rate is 5% and market

standard deviation is 20%.

Security	**S1**	**S2**	**S3**	**S4**	**S5**
Risk (Beta)	1.5	1.2	1.3	1.4	0.85
Return	12%	15%	10%	16%	8%
Error	20%	15%	12%	24%	22%

12. Compute the beta for the following security:

Security price	410	421	415	417	418	422	420	419
Market price	3282	3285	3286	3290	3285	3290	3294	3298

UNIT 12: THE MARKOWITZ MODEL

UNIT STRUCTURE

12.1 Learning Objectives

12.2 Introduction

12.3 The Markowitz Model: Assumptions and Concept

12.4 Varying Degrees of Correlation

12.5 Simple Diversification

12.6 Problems of vast Diversification

12.7 Risk and Return with Different Correlation

12.8 Markowitz Efficient Frontier

12.9 Utility Analysis

12.10 Risk Free Asset

12.11 Sharpe-The Single Index Model

12.12 Sharpe's Optimal Portfolio

12.13 Let Us Sum Up

12.14 Further Reading

12.15 Answers to Check Your Progress

12.16 Model Questions

12.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- describe Marrkowitz Model: Assumptions and Concept
- describe the Simple Diversification and Problems of vast Diversification
- outline the risk and return with different correlation
- explain Markowitz Efficient Frontier
- discuss Utility Analysis and Risk Free Asset
- describe Sharpe-The Single Index Model

12.2 INTRODUCTION

In this unit we are going to dicuss about the Marrkowitz Model: Assumptions and Concept, Varying Degrees of Correlation, Simple Diversification and Problems of vast Diversification.In this unit we will also discuss the risk and return with Different Correlation, Markowitz Efficient Frontier.

At the end of this you will get some idea about Risk Free Asset,Sharpe-The Single Index Model and Sharpe's Optimal Portfolio

12.3 THE MARKOWITZ MODEL: ASSUMPTIONS AND CONCEPT

Most people agree that holding two stocks is less risky than holding one stock. For example, holding stocks from textile, banking, and electronic companies is better than investing all the money on the textile company's stock. But building up the optimal portfolio is very difficult. Markowitz provides an answer to it with the help of risk and return relationship.

Assumptions

The individual investor estimates risk on the basis of variability of returns i.e. the variance of returns. Investor's decision is solely based on the expected return and variance of returns only.

For a given level of risk, investor prefers higher return to lower return. Likewise, for a given level of return investor prefers lower risk than higher risk.

The Concept

In developing his model, Markowitz had given up the single stock portfolio and introduced diversification. The single security portfolio would be preference if the investor is perfectly certain that his expectation of highest return would turn out to be real. In the world of uncertainty, most of the risk averse investors would like to join Markowitz rather than keeping a single stock, because diversification reduces the risk. This can be shown with the help of the following illustration.

Take the stock of ABC company and XYZ company. The returns expected from each company and their probabilities of occurrence, expected returns and the variances are given. The calculation procedure is given in the table.

	Stock ABC	Stock XYZ
Return %	11 or 17	20 or 8
Probability	.5 each return	.5 each return
Expected Return	14	14
Variance	9	36
Standard deviation	3	6

ABC Expected return	=.5 X 11 + .5 X 17	= 14
XYZ Expected return	= .5 X 20 + .5 X 8	= 14
ABC variance	= .5 (11-14)2 + .5 (17-14)2	= 9
XYZ variance	= .5 (20-14)2 + .5 (8-14)2	= 36
ABC standard deviation	= $\sqrt{}$ Variance	= $\sqrt{}$ 9 = 3
XYZ standard deviation	= $\sqrt{}$ Variance	= $\sqrt{}$ 36 = 6

ABC and XYZ companies stocks have the same expected return of 9%. XYZ company's stock is much riskier than ABC stock, because the standard deviation of the former being 6 and latter 3. When ABC return is high XYZ return is low and vice-versa i.e. when there is 17% return from ABC, there would be 8% return from XYZ. Likewise when ABC return is 11% XYZ return is 20%. If a particular investor holds only ABC or XYZ he would stand to lose in the time of bad performance.

Suppose the investor holds two thirds of ABC and one third of XYZ, the return can be calculated as follows

$$R_p = \sum_{t=1}^{N} X_1 R_1$$

R_p = return on the portfolio

X_1 = proportion of total portfolio invested in security 1.

R_1 = expected return of security 1.

Let us calculate the expected return for the both the possibilities.

Possibility 1 = 2/3 X 11 + 1/3 X 20 = 14

Possibility 2 = 2/3 X 17 + 1/3 X 8 = 14

In both the situations, the investor stands to gain if the worst occurs, than by holding either of the security individually.

Holding two securities may reduce the portfolio risk too. The portfolio risk can be calculated with the help of the following formula.

$$\sigma_p = \sqrt{X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2(r_{12}X_2\sigma_2)}$$

σ_p = portfolio standard deviation

X_1 = percentage of total portfolio value in stock X_1

X_2 = percentage of total portfolio value in stock X_2

σ_1 = standard deviation of stock X_1

σ_2 = standard deviation of stock X_2

r_{12} = correlation co-efficient of X_1 and X_2

$$r_{12} = \frac{\text{covariance of X12}}{\sigma_1\sigma_2}$$

Using the same example given in the return analysis, the portfolio risk can be estimated. Let us assume ABC as X_1 and XYZ as X_2. Now the covariance is: X_{12}

$$\text{Cov of X} = 1/N \sum_{t=1}^{N} (R1 - \bar{R}2)(R1 - \bar{R}2)$$

12

= 1/2[(11-14)(20-14)(17-14)(18-14)

= ½ [(-18) + (-18)] = -36/2 = -18

$$R = \frac{\text{covariance of X12}}{\sigma_1\sigma_2} = -18/3x6 = -1$$

The correlation co-efficient indicates the similarity or dissimilarity in the behavior of X_1 and X_2 stocks. In correlation, co-variance is not taken as an absolute value but relative to the standard deviation of individual securities. It shows, how much X and Y vary together as a proportion of their combined individual variations measured by σ_1 and σ_2. In our example, the correlation

co-efficient is -1.0 which indicates that there is a perfect negative correlation exists between the securities and they tend to move in the same direction. If the correlation is 1, perfect positive correlation exists between the securities and they tend to in the same direction. If the correlation co-efficient is zero, the securities' returns are independent. Thus, the correlation between two securities depends upon the covariance between the two securities and the standard deviation of each security.

Now, let us proceed to calculate the portfolio risk. Combination of two securities reduces the risk factor if less degree of positive correlation exists between them. In our case, the correlation coefficient is -1.

$$\sigma_p = \sqrt{X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2(r_{12}\sigma_1\sigma_2)}$$

$$= \sqrt{(2/3)2 \text{ x } 9 + (1/3)2 \text{ x } 36 + 2 \text{ x } 2/3 \text{ x } 1/3 \text{ (-1 x 3 x 6)}}$$

$$= \sqrt{4 + 4 + (-8)}$$

$$= \sqrt{0}$$

The portfolio risk is nil if the securities are related negatively. This indicates that the risk can be eliminated if the securities are perfectly negatively correlated. The standard deviation of the portfolio is sensitive to (1) the proportions of funds devoted to each stock (2) the standard deviation of each security and (3) co-variance between two stocks.

The change in portfolio proportions can change the portfolio risk. Taking the same example of ABC and XYZ stock, the portfolio standard deviation is calculated for different proportions.

Stock ABC (X1)	Stock XYZ (X2)	Portfolio Standard Deviation
100	0	3
66.66	33.3	0
50.0	50.0	1.5
0	100	6

By skillful balancing of the investment proportions in different securities, the portfolio risk can be brought down to zero. The proportion to be invested in each security can be found out by $X_1 = \sigma_2 \div (\sigma_1 + \sigma_2)$ the

precondition is that the correlation co-efficient should be -1.0. Otherwise it is

$$X_1 = \frac{\sigma_2^{\ 2} - (r_{12}\sigma_1\sigma_2)}{\sigma_1^{\ 2} + \sigma_2^{\ 2} - (2r_{12}\sigma_1\sigma_2)}$$

If the correlation co-efficient is less than the ratio of smaller standard deviation to larger standard deviation, then the combination of two securities provides a lesser standard deviation of return than when either of the security is taken alone. In our example,

-1 < 3/6 i.e. -1 < + .50

If the standard deviation ratio is 4/6 and the correlation co-efficient is + .8, the combination of securities is not profitable because

+ 8 > 4/6 i.e. + 8 > .66

12.4 VARYING DEGREES OF CORRELATION

Here in order to learn more about the relationship between securities, different degrees of correlation co-efficients are analyzed. Extreme cases like +1, 1, intermediate values and no correlation are calculated for two securities namely X and Y. We assume that the investor has specific amount of money to invest and that can be allocated in any proportion between the securities. Security X has an expected rate of return of 5% and a standard deviation of 4%. While for security Y, the expected return is 8% and the standard deviation of return is 10%.

Let us first work out the expected return and the portfolio risk for different values of correlation coefficients for varying proportions of the securities X and Y. Portfolio return is calculated with the equation:

$$R_p = X_x R_x + X_y R_y$$

If there is 75% investment on X and 25% on Y, then Rp = .75(5%) + 0.25 (8%) = 5.75% then the σ_p would be found out by using equation

$$\sigma_p = \sqrt{X_x^{\ 2}\sigma_x^{\ 2} + X_y^{\ 2}\sigma_y^{\ 2} + 2X_x X_y (r_{xy}\sigma_x\sigma_y)}$$

$$= \sqrt{3/4 \text{ x } ¾ \text{ x } 16 + ¼ \text{ x } ¼ \text{ x } 100 + 2 \text{ x } ¾ \text{ x } ¼ (1 \text{ x } 4 \text{ x} 10)}$$

= 5.5

Table gives the values of R_p and σ_p for varying degrees of correlation co-efficients.

Proportion of X security in portfolio X	Proportion of Y security in portfolio 1-X	$R_p$rxy	σ_p Rxy +1	σ_p Rxy -1	σ_p Rxy 0	σ_p +.5
1.00	0.00	5.00	4.0	4.0	4.0	4.0
0.75	0.25	5.75	5.5	0.5	3.9	4.8
0.50	0.50	6.50	7.0	3.0	5.4	6.25
0.25	0.75	7.25	8.5	6.5	7.6	8.1
0.00	1.00	8.00	10.0	10.0	10.0	10.0

12.5 SIMPLE DIVERSIFICATION

Portfolio risk can be reduced by the simplest kind of diversification. Portfolio means the group of assets an investor owns. The assets may vary from stocks to different types of bonds. Some times the portfolio may consist of securities of different industries. When different assets are added to the portfolio, the total risk tends to decrease. In the case of common stocks, diversification reduces the unsystematic risk or unique risk. Analysts opine that if 15 stocks are added in a portfolio of the investor, the unsystematic risk can be reduced to zero. But at the same time if the number exceeds 15, additional risk reduction cannot be gained. But diversification cannot reduce systematic or undiversifiable risk.

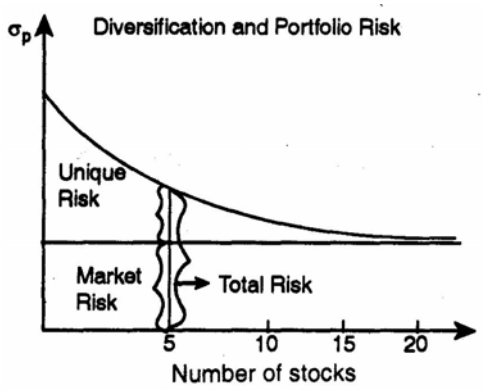
The naive kind of diversification is known as simple diversification. In the case of simple diversification, securities are selected at random and no analytical procedure is used.

Total risk of the portfolio consists of systematic and unsystematic risk and this total risk is measured by the variance of the rates of returns over time. Many studies have shown that the systematic risk forms one quarter of the total risk.

The simple random diversification reduces the total risk. The reason behind this is that the unsystematic price fluctuations are not correlated with the market's systematic fluctuations. The figure shows how the simple

diversification reduces the risk. The standard deviations of the portfolios are given in Y axis and the number of randomly selected portfolio securities in the X axis.

The standard deviation was calculated for each portfolio and plotted. As the portfolio size increases, the total risk line Starts declining. It flattens out after a certain point. Beyond that limit, risk cannot be reduced. This indicates that spreading out the assets beyond certain level cannot be expected to reduce the portfolio's total risk below the level of undiversifiable risk.



12.6 PROBLEMS OF VAST DIVERSIFICATION

Spreading the investment on too many assets will give rise to problems such as purchase of poor performers, information inadequacy, high research cost and transaction cost.They are discussed below:

1. Purchase of Poor Performers: While buying numerous stocks, sometimes the investor may also buy stocks that will not yield adequate return.

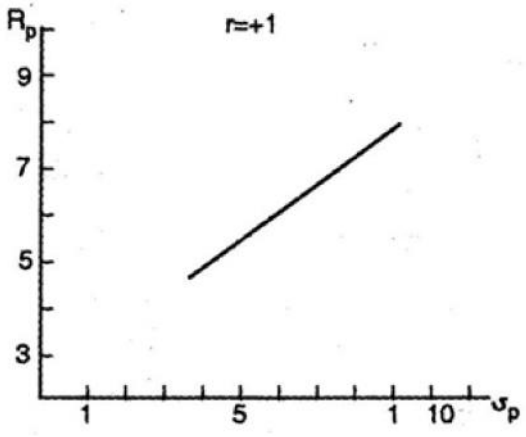
2. **Information Inadequacy:** If there are too many securities in a portfolio, it is difficult for the portfolio manager to get information about their individual performance. The portfolio manager has to be in touch with the details regarding the individual company performance. To get all the information simultaneously is quite High research cost If a large number of stocks are included, before the inclusion itself the returns and

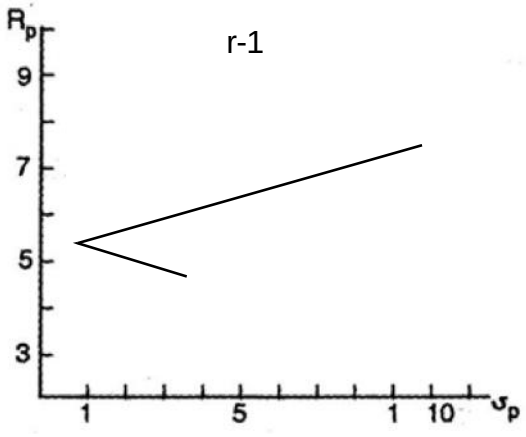
risk of the individual stock have to be analysed. Towards this end, lot of information has to be gathered and kept in store and these procedures involve high cost.

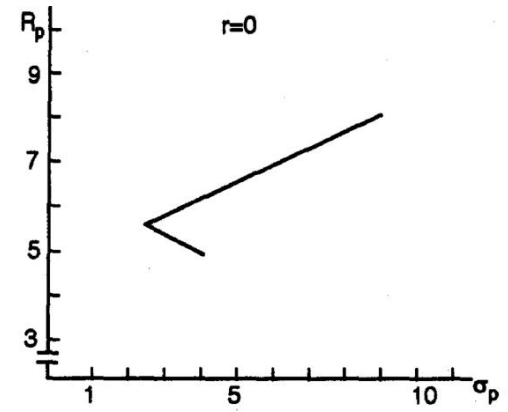
3. High Transaction Cost: When small quantities of stocks are purchased frequently, the investor has to incur higher transaction cost than the purchase of large blocks at less frequent intervals. In spite of all these difficulties big financial institutions purchase hundreds of different stocks. Likewise, mutual funds also invest in different stocks.

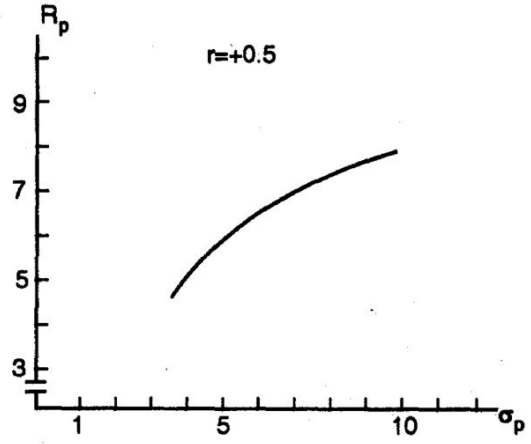
12.7 RISK AND RETURN WITH DIFFERENT CORRELATION

The four figures indicate the relationship between risk and return.



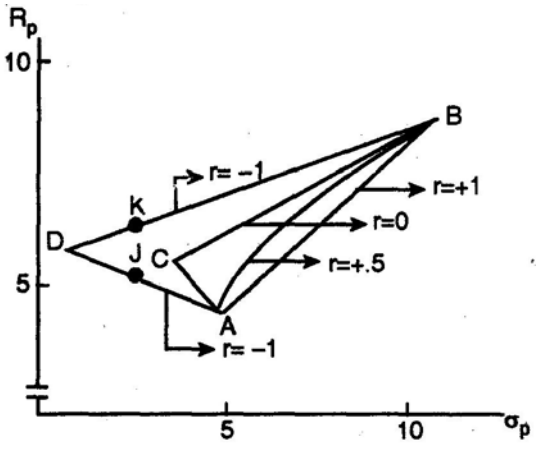






All the graphs show the portfolio risks under varying levels of correlation co-efficients. All the figures can be assembled together and placed in a single figure. The following figure expresses the relationship between expected returns and standard deviations of returns for various correlation coefficients.

Two Security Portfolios with Different Correlation Coefficients



In the figure, portfolio return is given on the vertical axis and portfolio risk on the horizontal axis. Point A represents 100 per cent holdings of X and point B represents 100 per cent holdings of Y. The intermediate points along the line segment AB represent portfolios containing various combinations of two securities.

The straight line r = + 1 shows that the portfolio risk increases with the increase in portfolio return. Here, the combination f two securities could not reduce the portfolio risk-because of their positive correlation. Again, the ratio of smaller standard deviation to larger deviation is less than the correlation coefficient.

1 > 4/10 = 1 > .4 which indicates that benefit cannot be derived by combining both the securities. In this case if an investor wish to minimize his risk, it would be better for him to invest all the money in security X where the risk is comparatively lower.

The r_{xy} = 0 line is a hyperbola. Along the line segment ACB, the r = 0. CB contains portfolios that a superior to those along the line segment AC. Markowitz says that all portfolios along the ACB line segment are feasible but some are more efficient than others. The line segment ADB indicates (r = -1) perfect inverse correlation and it is possible to reduce portfolio risk to zero. Portfolios on the line segment DB provides superior returns than on the line segment AD. For example, take two points on both the line segments K and J. The point K is superior to the point J because with the same level of risk the investor earns more return on point K than on pointy.

Thus, Markowitz diversification can lower the risk if the securities in the portfolio have low correlation coefficients.



CHECK YOUR PROGRESS

Q1: State the Assumption of Markowitz Model

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Q2: Define Portfolio

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12.8 MARKOWITZ EFFICIENT FRONTIER

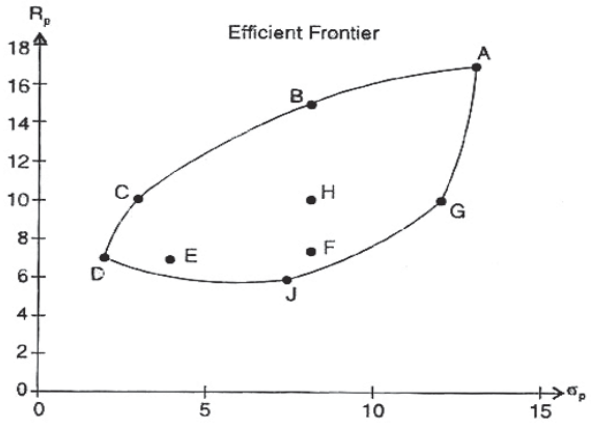
The risk and return of all portfolios plotted in risk-return space would be dominated by efficient portfolios. Portfolio may be constructed from available securities. All the possible combination of expected return and risk compose the attainable set. The following example shows the expected return and risk of different portfolios.

Portfolio Risk and Return

Portfolio	Expected Return (R_P) %	Risk ($ó_P$)
A	17	13
B	15	8
C	10	3
D	7	7
E	7	4
F	7	8
G	10	12
H	9	8
J	6	7.5

The attainable sets of portfolios are illustrated in figure. Each of the portfolios along the line or within the line ABCDEFGJ is possible. It is not possible for the investor to have portfolio outside of this perimeter because no combination of expected return and risk exists there.

When the attainable sets are examined, some are more attractive than others. Portfolio B is more attractive than portfolios F and H because B offers more return on the same level of risk. Likewise, C is more attractive than portfolio G even though same level of return is got in both the points; the risk level is lower at point C. In other words, any portfolio which gives more return for the same level of risk or same return with lower risk is more preferable than any other portfolio.



Among all the portfolios, the portfolios which offer the highest return at particular level of risk are called efficient portfolios. Here the efficient portfolios are A, B, C and D, because at these points no other portfolio offer higher return. The ABCD line is the efficient frontier along which all attainable and efficient portfolios are available. Now the question raised is which portfolio the investor should choose? He would choose a portfolio that maximizes his utility. For that utility analysis has to be done.

12.9 UTILITY ANALYSIS

Utility Analysis

Utility is the satisfaction the investor enjoys from the portfolio return. An ordinary investor is assumed to receive greater utility from higher return and vice-versa. The investor gets more satisfaction or more utility in X + 1 rupees than from X rupee.

If he is allowed to choose between two certain investments, he would always like to take the one with larger outcome. Thus, utility increases with increase in **return.**

The utility function makes certain assumptions about an investors' taste for risk. The investors are categorised into risk averse, risk neutral and risk seeking investor. All the three types can be explained with the help of a **fair gamble.**

In a fair gamble which cost ' 1, the on are A and B events. A event will

yield ‘ 2. Occurrence of B event is a dead loss i.e 0. The chance of occurrence of both the events are 50% and 50%.

The expected value of investment is (1/2) 2 + 1/2 (0) = Rel> the expected value of the gamble is exactly equal to cost. Hence, it is a fair gamble. The position of the investor may, be improved or hurt by undertaking the gamble.

Risk avertor rejects a fair gamble because the disutility of the loss is greater for him than the utility of an equivalent gain. Risk neutral investor means that he is indifferent to whether a fair gamble is undertaken or not.

The risk seeking investor would select a fair gamble i.e. he would choose to invest. The expected utility of investment is higher than the expected utility of not investing. These three different types of investors are shown in figure.

The curves ABC are three different slopes of utility curves. The upward sloping curve A shows increasing marginal utility. The straight line B shows constant utility, and curve C shows diminishing marginal utility. The constant utility, a linear function means doubling of returns would double the utility and it indicates risk neutral situation.

The increasing marginal utility suggests that the utility increases more than proportion to increase in return and shows the risk lover. The curve C shows risk averse investor. The utility he gains from additional return declines gradually. The figures show the utility curves of the different investors.

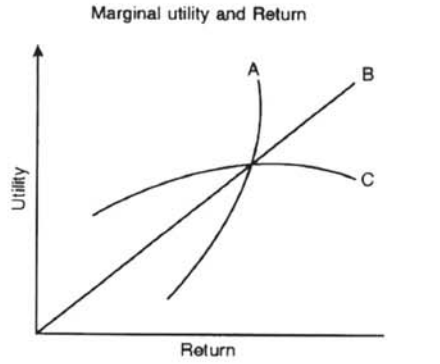


Fig. 17.5

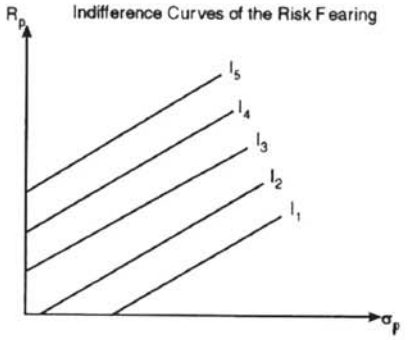
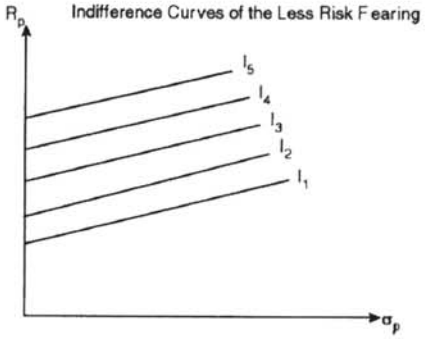


Fig. 17.6





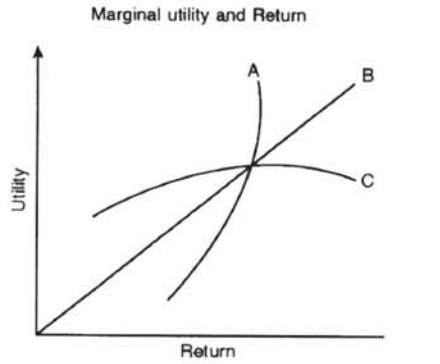


Fig. 17.5

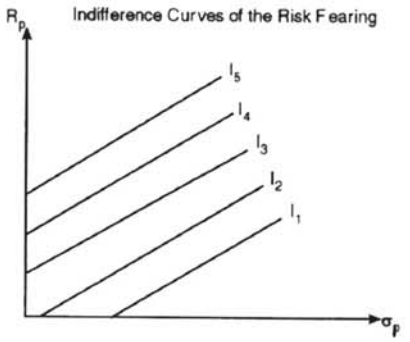
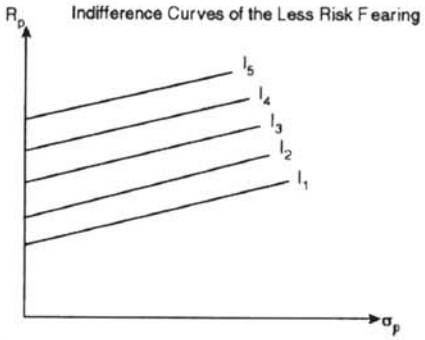


Fig. 17.6



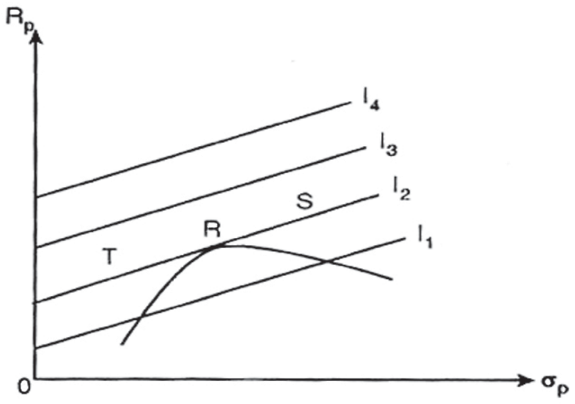


Investors generally like to get more returns for additional risks assumed and the lines would be positively sloped. The risk lover's utility curves are negatively sloped and converge towards the origin. For the risk fearing, lower the risk of the portfolio, happier he would be. The degree of the slope of indifference curve indicates the degree of risk aversion. The conservative investor needs larger return to undertake small increase in risk (Figure) The aggressive investor would be willing to undertake greater risk for smaller return. Even though the investors dislike risk, their trade off between risk and return differs.

Indifference Map and the Efficient Frontier

Each investor has a series of indifference curves. His final choice out of the efficient set depends on his attitude towards risk. The figure shows the efficient frontier and the indifference map.

Indifference Curve and Efficient Frontier



The utility of the investor or portfolio manager increases when he moves up the indifference map from I to 14'He can achieve higher expected return without an increase in risk. In the figure 122 touches the efficient frontier at point R. Even though the points I and S are in the I, curve, R is the only attainable portfolio which maximises the utility of the investor. Thus, the point at which the efficient frontier tangentially touches the highest indifference curve determines the most attractive portfolio for the investor.

Leveraged Portfolios

In the above model, the investor is assumed to have a certain amount

of money to make investment for a fixed period of time. There is no borrowing and lending opportunities. When the investor is not allowed to use the borrowed money, he is denied the opportunity of having financial leverage.

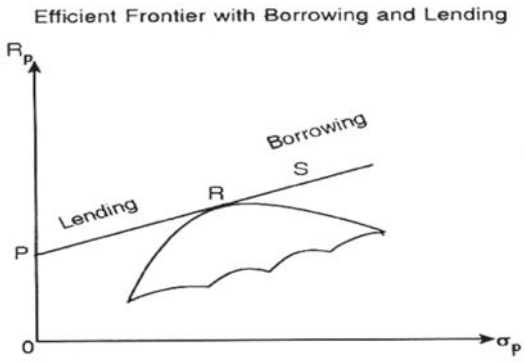
Again, the investor is assumed to be investing only on the risky assets. Riskiess assets are not included in the portfolio. To have a leveraged portfolio, investor has to consider not only risky assets but also risk free assets. Secondly, he should be able to borrow and lend money at a given rate of interest.

12.10 RISK FREE ASSET

The features of risk free asset are: (a) absence of default risk and interest risk and (b) full payment of principal and interest amount. The return from the risk free asset is certain and the standard deviation of the return is nil. The relationship between the rate of return of the risk free asset and risky asset is zero. These types of assets are usually fixed income securities. But fixed income securities issued by private institutions have the chance of default. If the fixed income securities are from the government, they do not possess the default risk and the return from them are guaranteed. Further, the government issues securities of different maturity period to match the length of investors holding period. The risk free assets may be government securities, treasury bills and time deposits in banks.

Inclusion of Risk Free Asset

Now, the risk free asset is introduced and the investor can invest part of his money on risk free asset and the remaining amount on the risky asset. It is also assumed that the investor would be able to borrow money at risk free rate of interest. When risk free asset is included in the portfolio, the feasible efficient set of the portfolios is altered. This can be explained in the Figure.



In the figure, OP is gained with zero risk and the return is earned through holding risk free asset. Now, the investor would attempt to maximise his expected return and risk relationship by purchasing various combinations of riskless asset and risky assets. He would be moving on the line connecting attainable portfolio R and risk free portfolio P i.e. the line PR. When he is on the PR, part of his money is invested in fixed income securities i.e. he has lent some amount of money and invested the rest in the risky asset within the point PR. He is depending upon his own funds. But, if he moves beyond the point R to S he would be borrowing money. Hence the portfolios located between the points RP are lending portfolios and beyond the point R consists of borrowing portfolios. Holding portfolio in PR segment with risk free securities would actually reduces risk more than the reduction in return.

12.11 SHARPE-SINGLE INDEX MODEL

Sharpe- Single Index Model

Casual observation of the stock prices over a period of time reveals that most of the stock prices move with the market index. When the Sensex increases, stock prices also tend to increase and vice-versa. This indicates that some underlying factors affect the market index as well as the stock prices. Stock prices are related to the market index and this relationship could be used to estimate the return on stock. Towards this purpose, the following equation can be used

$$R_i = \sigma_i + \beta_i R_m + e_i$$

where

R_i - expected return on security i

σ_i - intercept of the straight line or alpha co-efficient

β_i - slope of straight line or beta co-efficient

R_m - the rate of return on market index

e_i - error term

According to the equation, the return of a stock can be divided into t components, the return due to the market and the return independent of the market. 13. indicates the sensitiveness of the stock return to the changes in the market return. For example 13 of 1.5 means that the stock returns is expected to increase by 1.5% when the market index return increases by 1% and vice-versa. Likewise, 13.of 0.5 expresses that the individual stock return would change by 0.5 per cent when there is a change of 1 per cent in the market return. 13 of 1 indicate that the market return and the security return are moving in tandem. The estimates of 13.and a are obtained from regression analysis.

The single index model is based on the assumption that stocks vary together because of the common movement in the stock market and there are no effects beyond the market (i.e. any fundamental factor effects) that account the stocks co-movement. The expected return, standard deviation and co-variance of the single index model represent the joint movement of securities. The mean return is

$$R_i = \sigma_i + \beta_i R_m + e_i$$

The variance of security's of return $\sigma^2 = \beta_i{}^2 \sigma_m{}^2 + \sigma_{ei}{}^2$

The covariance of returns between securities i and is

$$\sigma_{ij} + \beta_i \beta_j \sigma^2_m$$

The variance of the security has t components namely, systematic risk or market risk and unsystematic risk or unique risk. The variance explained by the index is referred to systematic risk. The unexplained variance is called residual variance or unsystematic risk. Systematic risk = $\beta_i{}^2$ x variance of market index.

$$= \beta_i{}^2 \sigma_m{}^2$$

Unsystematic risk = Total variance — Systematic risk.

$e_i^2 = \sigma_i^2$ -systematic risk.

Thus, the total risk = Systematic risk + Unsystematic risk.

$' = \beta_i^2 \sigma_m^2 + e_i^2$

From this, the portfolio variance can be derived

σ_2 = variance of portfolio

σ_{2p} = expected variance of index

e_i^{2m} = variation in security's return not related to the market index x_i = the portion of stock i in the portfolio

Likewise expected return on the portfolio also can be estimated. For each security α_1 and β_1 should be estimated. N

$$RP = \sum_{t=1}^{N} xi(ai + \beta_{i}Rm)$$

$$\Sigma i^N = 1^x$$

Portfolio return is the weighted average of the estimated return for each security in the portfolio. The f weights are the respective stocks' proportionsin the portfolio.

A portfolio's alpha value is a weighted average of the alpha values for its component securities using the F proportion of the investment in a security as weight.

$$\sigma_P = \sum_{t=1}^{N} x_i \alpha_j$$

$$\Sigma i^N = 1^x$$

σ_P - Value of the alpha for the portfolio

x_i - Proportion of the investment on security i α_j - Value of alpha for security i N - The number of securities in the portfolio.

Similarly, a portfolio's beta value is the weighted average of the beta values of its component stocks using relative share of them in the portfolio as weights.

$$\sigma = \sum_{t=1}^{N} x\beta$$

p i i

β_p is the portfolio beta.

12.12 SHARPE'S OPTIMAL PORTFOLIO

Sharpe's Optimal Portfolio

Sharpe had provided a model for the selection of appropriate securities in a portfolio.

The selection of any stock is directly related to its excess return-beta ratio.

$$\frac{R_i - R_f}{\beta_i}$$

Where

R_i = the expected return on stock i

R_f = the return on a riskless asset

β_i = the expected change in the rate of return on stock i associated with one unit change in the market return

The excess return is the difference between the expected return on the stock and the riskless rate of interest such as the rate offered on the government security or treasury bill. The excess return to beta ratio measures the additional return on a security (excess of the riskless asset return) per unit of systematic risk or no diversifiable risk this ratio provides a relationship between potential risk and reward

Ranking of the stocks are done on the basis of their excess return to beta. Portfolio managers would like to include stocks with higher ratios. The selection of the stocks depends on a unique cut-off rate such that all stocks with higher ratios of R.-R / B are included and the stocks with lower ratios are left off. The cut-off point is denoted by C*.

The steps for finding out the stocks to be included in the optimal portfolio are given below

1. Find out the "excess return to beta" ratio for each stock under consideration.
2. Rank them from the highest tothe lowest.
3. Proceed to calculate C for all the stocks according to the ranked order using the following formula.

$$C_i = \frac{\sigma^2_m \sum_{t=1}^{N} \frac{(Ri - Rf)\beta_i}{\sigma^2_{ei}}}{1 + \sigma^2_m \sum_{t=1}^{N} \frac{\beta_i^2}{\sigma^2_{ei}}}$$

σ_{m^2} = variance of the market index

σ_{ej^2} = variance of a stock's movement that is not associated with the movement of market index i.e. stock's unsystematic risk.

4. The cumulated values of C start declining after a particular C and that point is taken as the cut-off point and that stock ratio is the dut-off ratio C.

This is explained with the help of an example.

Data for finding out the optimal portfolio are given below:

Security Number	Mean Return R_i	Excess Return $R_i - R_f$	Beta β	Unsystematic Risk σ_{ej^2}	Excess Return to Beta $\frac{R_i - R_f}{\beta_i}$
1	19	14	1.0	20	14
2	23	18	1.5	30	12
3	11	6	0.5	10	12
4	25	20	2.0	40	10
5	13	8	1.0	20	8
6	9	4	0.5	50	8
7	14	9	1.5	30	6

The riskless rate of interest is 5 per cent and the market variance is 10. Determine the cut-off point.

Security Number	$\frac{R_i - R_f}{\beta_i}$	$\frac{(R_i - R_f)x\beta_i}{\sigma e_i^2}$	$\sum \frac{N(R_i - R_f)\beta_i}{i=1\sigma e_i^2}$	$\frac{\beta_i^2}{\sigma e_i^2}$	$\frac{\beta_i^2}{\sigma e_i^2}$	c_i
1	14	0.7	0.7	0.05	0.05	4.67
2	12	0.9	1.6	0.075	0.125	7.11
3	12	0.3	1.9	0.025	0.15	7.60
4	10	1.0	2.9	0.1	0.25	8.29
5	8	0.4	3.3	0.05	0.3	8.25
6	8	0.04	3.34	0.005	0.305	8.25
7	6	0.45	3.79	0.075	0.38	7.90

Calculations are given below

For Security 1

$$C_1 = \frac{10x.7}{1+(10x.05)} = 4.67$$

Here 0.7 is got from column 4 and 0.05 from column 6. Since the preliminary calculations are over, it is easy to calculate the C.

$$C_2 = \frac{10x.1.6}{1+(10x.125)} = 7.11$$

$$C_3 = \frac{10x\ 1.9}{1+(10x.125)} = 7.11$$

$$C_4 = \frac{10x\ 2.9}{1+(10x.25)} = 8.2$$

$$C_5 = \frac{10\,x\ 3.3}{1+(10\,x.3)} = 8.25$$

$$C_6 = \frac{10\,x\ 3.34}{1+(10\,x.305)} = 8.25$$

$$C_7 = \frac{10\,x\ 3.79}{1+(10\,x.38)} = 7.90$$

The highest Ci.value is taken as the cutoff point i.e. C*. The stocks ranked above C* have high excess returns to beta than the cut-off C. and all the stocks ranked below C* have low excess returns to beta. Here, the cut-off rate is 8.29. Hence, the first four securities are selected. If the number of stocks is larger there is no need to calculate C_i values for all the stocks after the ranking has been done. It can be calculated until the C* value is found

and after calculating for one or two stocks below it, the calculations can be terminated.

The C_i can be stated with mathematically equivalent way.

$$C_i = \frac{\beta_{ip}(R_p - R_f)}{\beta_i}$$

β_{ip} - the expected change in the rate of return on stock i associated with 1 per cent change in the return on the optimal portfolio.

R_p - the expected return on the optimal portfolio

β_{ip} and R_p cannot be determined until the optimal portfolio is found. lb find out the optimal portfolio, the formula given previously should be used. Securities are added to the portfolio as long as

$$\frac{R_p - R_f}{\beta_i} > C_i$$

The above equation can be rearranged with the substitution of equation:

$$C_i = \frac{\beta_{ip}(R_p - R_f)}{\beta_i}$$

Now we have,

$$R_i - R_f > \beta_{ip}(R_p - R_f)$$

The right hand side is the expected excess return on a particular stock based on the expected performance of the optimum portfolio. The term on the left hand side is the expected excess return on the individual stock. Thus, if the portfolio manager believes that a particular stock will perform better than the expected return based on its relationship to optimal portfolio, he would add the stock to the portfolio.



CHECK YOUR PROGRESS

Q3: Define Utility Analysis

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……

Q4: Define Portfolio

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12.13 LET US SUM UP

In this unit we have discussed the following :

- Assumptions of Markowitz Model is "The individual investor estimates risk on the basis of variability of returns i.e. the variance of returns.
- Investor's decision is solely based on the expected return and variance of returns only".
- Utility is the satisfaction the investor enjoys from the portfolio return.
- The increasing marginal utility suggests that the utility increases more than proportion to increase in return and shows the risk lover.
- The features of risk free asset are: (a) absence of default risk and interest risk and (b) full payment of principal and interest amount.
- Sharpe had provided a model for the selection of appropriate securities in a portfolio.



12.14 FURTHER READING

- M. Ranganathan and R. Madhumathi(2011); *Investment Analysis and Portfolio Management*, Pearson Education, New Delhi.
- Punithavathy Pandian(2016) *Security Analysis and Portfolio Management,* Vikas Publishing House Pvt. Ltd., New Delhi.
- Bharti V. Phathak(2014; *Indian Financial System,* Pearson Education, Delhi.
- Donald E. Fischer and Ronald J. Jordon(2011); *Security Analysis and Portfolio Management,* PHI.
- Prasanna Chandra(2010);*Investment Analysis and Portfolio Management,* TMH, Delhi.



12.15 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q1: Assumptions: The individual investor estimates risk on the

basis of variability of returns i.e. the variance of returns.Investor's decision is solely based on the expected return and variance of returns only.

Ans to Q2: Portfolio means the group of assets an investor owns.

Ans to Q3: Utility is the satisfaction the investor enjoys from the portfolio return. An ordinary investor is assumed to receive greater utility from higher return and vice-versa. The investor gets more satisfaction or more utility in X + 1 rupees than from X rupee.

Ans to Q4: Spreading the investment on too many assets will give rise to problems such as purchase of poor performers, information inadequacy, high research cost and transaction cost.



12.16 MODEL QUESTIONS

Q1: Explai the concept ofMarkowitz Model with examples

Q2: Discuss the relationship between securities and different degrees of correlation

Q3: Explain simple diversification

Q4: Write the problems of vast diversification

Q5: Explain Markowitz efficient frontier

Q6: What is Utility Analysis

Q7: Define Risk Free Assest

Q8: Explain Sharpe-Single Index Model

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UNIT 13: CAPITAL ASSET PRICING THEORY

UNIT STRUCTURE

13.1 Learning Objectives

13.2 Introduction

13.3 Capital Asset Pricing Theory: An Introduction

13.4 Security Market Line

13.5 Evaluation of Securities

13.6 Market Imperfection and SML

13.7 Empirical Tests of the CAPM

13.8 Present Validity of CAPM

13.9 Let Us Sum Up

13.11 Further Reading

13.12 Answers to Check Your Progress

13.13 Model Questions

13.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- define capital asset pricing theory
- describe the concept of Security Market Line
- outline the evaluation of Securities
- explain Market Imperfection and SML
- discuss Empirical Tests of the CAPM
- describe Present Validity of CAPM

13.2 INTRODUCTION

In this unit we are going to dicuss about capital asset pricing theory, the concept of Security Market Line,the evaluation of Securities

At the end of this you will get some idea about Market Imperfection and SML, Empirical Tests of the CAPM and Present Validity of CAPM.

13.3 CAPITAL ASSET PRICING THEORY(CAPM): AN INTRODUCTION

Investors are interested in knowing the systematic risk when they search for efficient portfolios. They would like to have assets with low beta co-efficient i.e. systematic risk. Investors would opt for high beta co-efficient only if they provide high rates of return. The risk averse nature of the investors is the underlying factor for this behavior. The capital asset pricing theory helps the investors to understand the risk and return relationship of the securities. It also explains how assets should be priced in the capital market.

The CAPM Theory

Markowitz, William Sharpe, John Lintner and Jan Mossin provided the basic structure for the CAPM model. It is a model of linear general equilibrium return. In the CAPM theory, the required rate return of an asset is having a linear relationship with asset's beta value i.e. undiversifiable or systematic risk.

Assumptions

1. An individual seller or buyer cannot affect the price of a stock. This assumption is the basic assumption of the perfectly competitive market.
2. Investors make their decisions only on the basis of the expected returns, standard deviations and co variances of all pairs of securities.
3. Investors are assumed to have homogenous expectations during the decision-making period.
4. The investor can lend or borrow any amount of funds at the riskless rate of interest. The riskless rate of interest is the rate of interest offered for the treasury bills or Government securities.
5. Assets are infinitely divisible. According to this assumption, investor could buy any quantity of share i.e. they can even buy ten rupees worth of Reliance Industry shares.
6. There is no transaction cost i.e. no cost involved in buying and selling of stocks.
7. There is no personal income tax. Hence, the investor is indifferent to the form of return either capital gain or dividend.

8. Unlimited quantum of short sales is allowed. Any amount of shares an individual can sell short.

Lending and Borrowing

Here, it is assumed that the investor could borrow or lend any amount money at riskless rate of interest. When this opportunity is given to the investors, they can mix risk free assets with the risky assets in a portfolio to obtain a desired rate of risk-return combination.

R_p = Portfolio return

X_f = the proportion of funds invested in risk free assets

$1- X_f$ = the proportion of funds invested in risky assets

R_f = Risk free rate of return

R_m = Return on risky assets

The expected return on the combination of risky and risk free combination is:

$$R_p = R_f X_f + R_m(1 – X_f)$$

This formula can be used to calculate the expected returns for different situations, like mixing ri assets with risky assets, investing only in the risky asset and mixing the borrowing with risky assets.

Now, let us assume that borrowing and lending rate to be 12.5% and the return from the risky assets to be 20%. There is a trade off between the expected return and risk. If an investor invests in risk free assets and risky assets, his risk may be less than what he invests in the risky asset alone. But if he borrows to invest in risky assets, his risk would increase more than he invests his own money in the risky assets. When he borrows to invest, we call it financial leverage. If he invests 50% in risk free assets and 50% in risky assets, his expected return of the portfolio would be

$$R_p = R_f X_f + R_m(1 – X_f)$$

$$= 12.5 \times .5 + 20(1 - .5)$$

$$= 6.25 + 10$$

$$= 16.25\%$$

If there is a zero investment in risk free asset and 100% in risky asset, the return is

$R_p = R_f X_f + R_m(1 - X_f)$

$= 0 + 20\%$

$= 20\%$

If - .5 in risk free asset and 1.5 in risky asset, the return is

$R_p = R_f X_f + R_m(1 - X_f)$

$= (l2.5 \times -.5) + 20x\ 1.5$

$= - 6.25 + 30$

$= 23.75$

The variance of the above mentioned portfolio can be calculated by using the equation.

$$\sigma^2_p = \sigma^2_p X^2_f + \sigma^2_m (1 - X_f)^2 + 2Cov_{fm} X_f (1 - X_f)$$

The previous example can be taken for the calculation of the variance. The variance of the risk free asset is in. The variance of the risky asset is assumed to be 15. Since the variance of the risky asset is zero, the 1, rtfolio risk solely depends on the portion of investment on risky asset.

Proportion in risky asset ($1-X_f$)	Portfolio risk
0.5	1.0
1.5	7.5
1.5	22.5

The risk is more in the borrowing portfolio being 22.5% and the return is also high among the three alternatives. In the lending portfolio, the risk is 7.5% and the return is also the lowest. The risk premium is proportional to risk, where the risk premium of a portfolio is defined as the difference between $R_p - R_f$ i.e. the amount by which a risky rate of return exceeds the riskless rate of return.

Risk - Return Trade Off

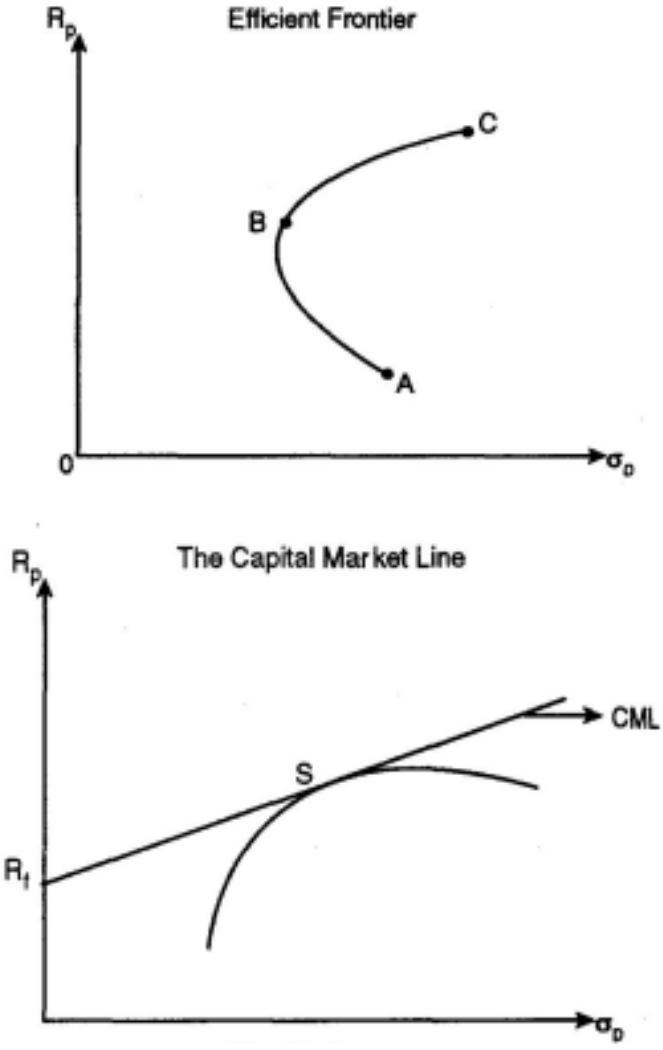
Portfolio Return R_p	Risk-free Return R_f	Risk Premium $R_p - R_f$	Portfolio Risk σ	Factor of Proportionality $(R_p - R_f)\sigma_p$
16.25	12.5	3.75	7.5	0.5
20.0	12.5	7.5	15.0	0.5
23.75	12.5	11.25	22.5	0.5

The risk-return proportionality ratio is a constant .5, indicating that one unit of risk premium is accompanied by 0.5 unit of risk.

The Concept

According to CAPM, all investors hold only the market portfolio and riskless securities. The market portfolio is a portfolio comprised of all stocks in the market. Each asset is held in proportion to its market value to the total value of all risky assets. For example, if Reliance Industry share represents 20% of all risky assets, then the market portfolio of the individual investor contains 20% of Reliance industry shares. At this stage, the investor has the ability to borrow or lend any amount of money at the riskiness rate of interest. The efficient frontier of the investor is given in fig 13.1.

fig 13.1.



The figure shows the efficient frontier of the investor. The investor prefers any point between B and C because, with the same level of risk they face on line BA, they are able to get superior profits. The ABC line shows the investor's, portfolio of risky assets. The investors can combine riskless asset either by lending or borrowing. This is shown in Figure.

The line $R_f S$ represents all possible combination of riskless and risky asset. The 'S' portfolio does not represent any riskless asset but the line RS gives the combination of both. The portfolio along the path RS is called lending portfolio that is some money is invested in the riskless asset or may be deposited in the bank for a fixed rate of interest. If it crosses the point S. it becomes borrowing portfolio. Money is borrowed and invested in the risky asset. The straight line is called capital market line (CML). It gives the desirable set of investment opportunities between risk free and risky investments. The CML represents linear relationship between the required rates of return for efficient portfolios and their standard deviations.

$E(R_p)$ = portfolio's expected rate of return

R_m = expected return on market portfolio

σ_m = standard deviation of market portfolio

σ_p = standard deviation of the portfolio

For a portfolio on the capital market line, the expected rate of return in excess of the risk free rate is in proportion to the standard deviation of the market portfolio. The price of the risk is given by the slope of the line. The slope equals the premium for the market portfolio $R_m - R_f$ divided by the risk or standard deviation of the market portfolio. Thus, the expected return of an efficient portfolio is:

Expected return = Price of time + (Price of risk . Amount of risk)

Price of time is the risk free rate of return. Price of risk is the premium amount higher and above the risk free return.



CHECK YOUR PROGRESS

Q1: What is CAPM Theory

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Q2: What is market portfolio

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13.4 SECURITY MARKET LINE (SML)

The risk-return relationship of an efficient portfolio is measured by the capital market line. But, it does not show the risk-return trade off for other portfolios and individual securities. Inefficient portfolios lie below the capital market line and the risk-return relationship cannot be established with the help of the capital market line. Standard deviation includes the systematic and unsystematic risk. Unsystematic risk can be diversified and it is not related to the market. If the unsystematic risk is eliminated, then the matter of concern is systematic risk alone. This systematic risk could be measured by beta. The beta analysis is useful for individual securities arid portfolios whether efficient or inefficient.

When an additional security is added to the market portfolio, an additional risk is also added to it. The variance of a portfolio is equal to the weighted sum of the co-variances of the individual securities in the portfolio.

If we add an additional security to the market portfolio, its marginal contribution to the variance of the market is the covariance between the security's return and market portfolio's return. If the security i am included, the covariance between the security and the market measures the risk. Covariance can be standardized by dividing it by standard deviation of market portfolio coy i_m / σ_m. This shows the systematic risk of the security. Then, the expected return of the security i is given by the equation:

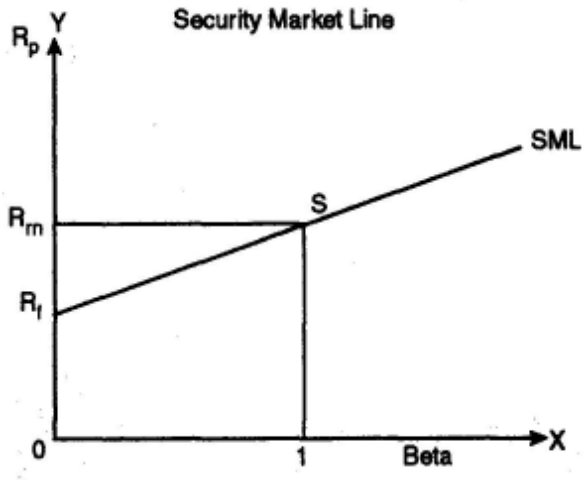
$$R_i - R_f = (R_m - Rf / \sigma_m) Coyi_m / \sigma_m$$

This equation can be rewritten as follows

$$R_i - R_f = Coyi_m / \sigma^2_m (R_m - Rf)$$

The first term of the equation is nothing but the beta coefficient of the stock. The beta coefficient of the equation of SML is same as the beta of the market (single index) model. In equilibrium, all efficient and inefficient portfolios lie along the security market line. The SML line helps to determine

the expected return for a given security beta. In other words, when betas are given, we can generate expected returns for the given securities. This is explained in fig 13.2.



If we assume the expected market risk premium to be 8% and the risk free rate of return tube 7%, we can calculate expected return for A, B and C securities using the formula:

$E(R_i) = R_f + ß_i[E(R_m)-R_f]$

If beta for ß = 1

If beta for = 1

= 7 + 1 (8)

= 15%

Security A

Beta = 1.10

E(R) = 7+1.10(8)

= 15.8

Security B

Beta = 1.20

E(R) = 7 + 1.20(8)

= 16.8 = 16.6

Security C

Beta = .7

E(R) = 7 + .7(8)

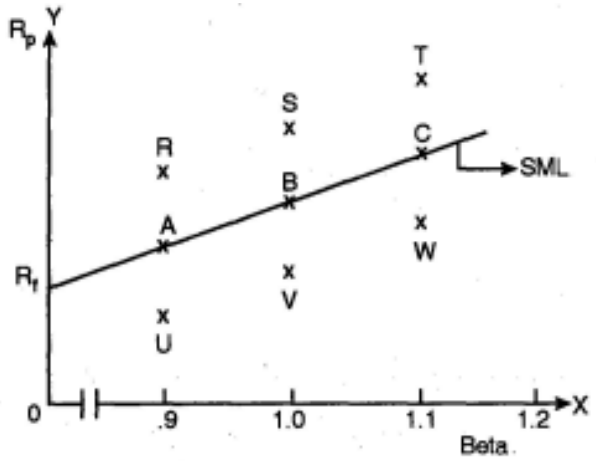
=12.6

The same can be found out easily from the figure too. All we have to do is, to mark the beta on the horizontal axis and draw a vertical line from the relevant point to touch the SML line. Then from the point of intersection, draw another horizontal line to touch the Y axis. The expected return could be very easily read from the Y axis. The securities A and B are aggressive securities, because their beta values are greater than one. When beta values are less than one, they are known as defensive securities. In our example, security C has the beta value less than one.

13.5 EVALUATION OF SECURITIES

Relative attractiveness of the security can be found out with the help of security market line. Stocks with high risk factor are expected to yield more return and vice-versa. But the investor would be interested in knowing whether the security is offering return more or less proportional to its risk.

Fig: 13.3 Evaluation of Securities with SML



The fig 13.3 provides an explanation for the evaluation. There are nine points in the diagram. A, B and C lie on the security market line, R, S and T above the SML and U, V and W below the SML. ARU have the same beta level of, 9. Likewise beta values of SBV = 1.00 and TCW = 1.10. The stocks above the SML yield higher returns for the same level of risk. They are underpriced compared to their beta value. With the simple rate of return formula, we can prove that they are undervalued.

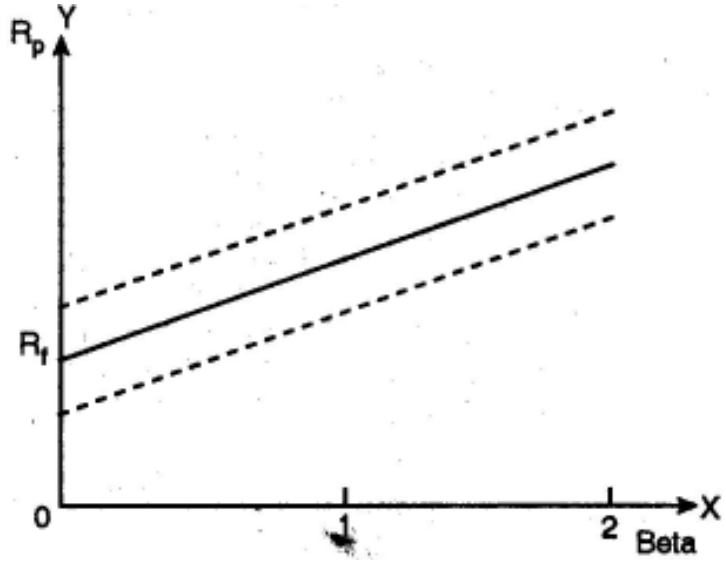
P_i is the present price P_0 - the purchase price and Div - Dividend. When the purchase price is low i.e. when the denominator value is low, the expected return could be high. Applying the same principle the stocks U, V and W can be classified as overvalued securities and are expected to yield lower returns than stocks of comparable risk. The denominator value may be high i.e. the purchase price may be high. The prices of these scripts may fall and lower the denominator. There by, they may increase the returns on securities.

The securities A, B and C are on the line. Therefore considered to be appropriately valued. They offer returns in proportion to their risk. They have average 4oclc performance, since they are neither undervalued nor overvalued.

13.6 MARKET IMPERFECTION AND SML

Information regarding the share price and market condition may not be immediately available to all investors. Imperfect information may affect the valuation of securities. In a market with perfect information, all securities should lie on SML. Market imperfections would lead to a band of SML rather than a single line. Market imperfections affect the width of the SML to a band. If imperfections are more, the width also would be larger. SML in imperfect market is given in fig 13.4.

Fig 13.4 SML in Imperfect Market



13.7 EMPIRICAL TESTS OF THE CAPM

In the CAPM, beta is used to estimate le systematic of the security and reflects the future volatility of the stock in relation to the market. Future volatility of the stock is estimated only through historical data. Historical data are used to plot the regression line or the characteristic line and calculate beta. If historical betas are stable over a period of time, they would be good proxy for their ex-ante or expected risk.

Robert A. Levy, Marshall B. Blume and others have studied the question of beta stability in depth. They calculated betas for both Individual securities and portfolios. Their study results have provided the following conclusions:

(1) The betas of individual stocks are unstable; hence the past betas for the individual securities are not good estimators of future risk.

(2) The betas of portfolios of ten or more randomly selected stocks are reasonably stable, hence the portfolio betas are good estimators of future portfolio volatility. This is because of the errors in the estimates of individual securities' betas tend to offset one another in a portfolio.

Various researchers have attempted to find out the validity of the model by calculating beta and realized rate of return. They attempted to test (1) whether the intercept is equal to i.e. risk free rate of interest or the interest rate offered for treasury bills (2) whether the line is linear and pass through the beta = 1 being the required rate of return of the market. In general, the studies have showed the following results.

(1) The studies generally showed a significant positive relationship between the expected return and t systematic risk. But the slope of the relationship is usually less than that of predicted by the CAPM.

(2) The risk and return relationship appears to be linear. Empirical studies give no evidence of significant curvature in the risk/return relationship,.

(3) The attempts of the researchers to assess the relative importance of the market and company risk have yielded definite results. The CAPM theory implies that unsystematic risk is not relevant, but unsystematic and systematic risks are positively related to security returns. Higher

returns are needed to compensate both the risks. Most of the observed relationship reflects statistical problems rather than the true nature of capital market.

(4) According to Richard Roll, the ambiguity of the market portfolio leaves the CAPM untestable. The practice of using indices as proxies is loaded with problems. Different indices yield different betas for the same security.

(5) If the CAPM were completely valid, i4 should apply to all financial assets including bonds. But, when bonds are introduced into the analysis, they do not fall on the security market line.

13.8 PRESENT VALIDITY OF CAPM

The CAPM is greatly appealing at an intellectual level, logical and rational. The basic assumptions on which the model is built raise, some doubts in the minds of the investors. Yet, investment analysts have been more creative in adapting CAPM for their uses.

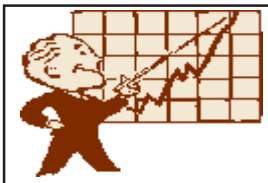
(1) The CAPM focuses on the market risk, makes the investors to think about the riskiness of the assets in general. CAPM provides basic concepts which are truly of fundamental value.

(2) The CAPM has been useful in the selection of securities and portfolios. Securities with higher returns are considered to be undervalued and attractive for buy. The below normal expected return yielding securities are considered to be overvalued and Suitable for sale.

(3) In the CAPM, it has been assumed that investors consider only the market risk. Given the estimate of the risk free rate, the beta of the firm, stock and the required market rate of return, one can find out the expected returns for a firm's security. This expected return can be used as an estimate of the cost of retained earnings.

(4) Even though CAPM has been regarded as a useful tool to financial analysts, it has its own critics too. They point out, when the model is ex-ante, the inputs also should be ex-ante, i.e. based on the expectations of the future. Empirical tests and analyses have used ex-post i.e. past data only.

(5) The historical data regarding the market return, risk free rate of return and betas vary differently for different periods. The various methods used to estimate these inputs also affect the beta value. Since the inputs cannot be estimated precisely, the expected return found out through the CAPM model is also subjected to criticism 4.



CHECK YOUR PROGRESS

Q3: What does standard deviation includes.

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Q4: How is historical data calculated.

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Q5: What is the use of security market line

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13.9 LET US SUM UP

In this unit we have discussed the following :

- Markowitz, William Sharpe, John Lintner and Jan Mossin provided the basic structure for the CAPM model. It is a model of linear general equilibrium return. In the CAPM theory, the required rate return of an asset is having a linear relationship with asset's beta value i.e. undiversifiable or systematic risk.
- According to CAPM, all investors hold only the market portfolio and riskless securities. The market portfolio is a portfolio comprised of all stocks in the market. Each asset is held in proportion to its market value to the total value of all risky assets.
- Price of time is the risk free rate of return. Price of risk is the premium

amount higher and above the risk free return.

- Relative attractiveness of the security can be found out with the help of security market line. Stocks with high risk factor are expected to yield more return and vice-versa.
- Market imperfections would lead to a band of SML rather than a single line. Market imperfections affect the width of the SML to a band. If imperfections are more, the width also would be larger.



13.10 FURTHER READING

- M. Ranganathan and R. Madhumathi(2005); *Investment Analysis and Portfolio Management,* Pearson Education, New Delhi.
- Punithavathy Pandian(2012); *Security Analysis and Portfolio Management,* Vikas Publishing House Pvt. Ltd., New Delhi.
- Bharti V. Phathak (2014); *Indian Financial System,* Pearson Education, Delhi.
- Donald E. Fischer and Ronald J. Jordon (2002): *Security Analysis and Portfolio Management*, Pearson Education India.
- Prasanna Chandra(2017); *Investment Analysis and Portfolio Management,* McGraw Hill Education, Delhi.



13.11 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q No.1: Markowitz, William Sharpe, John Lintner and Jan Mossin provided the basic structure for the CAPM model. It is a model of linear general equilibrium return. In the CAPM theory, the required rate return of an asset is having a linear relationship with asset's beta value i.e. undiversifiable or systematic risk.

Ans to Q No.2: The market portfolio is a portfolio comprised of all stocks in the market.

Ans to Q No.3: Standard deviation includes the systematic and unsystematic risk.

Ans to Q No.4: Historical data are used to plot the regression line or the characteristic line and calculate beta.

Ans to Q No.5: Relative attractiveness of the security can be found out with the help of security market line.



13.13 MODEL QUESTIONS

Q 1: Discuss Capital Asset Pricing Theory

Q 2: Describe the Security Market Line

Q 3: How is Evaluation of Securities is done.

Q 4: What is Market Imperfection and SML

Q 5: Explain Empirical Tests of the CAPM

Q 6: What is Present Validity of CAPM

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UNIT 14 : CAPITAL MARKET THEORY AND THE ARBITRAGE PRICING THEORY

UNIT STRUCTURE

14.1 Learning Objectives

14.2 Introduction

14.3 Capital Asset Pricing Model (CAPM)

14.4 Concepts of Risk-free Asset, Risk-free Lending and Risk-free Borrowing

14.5 Efficient Set with Risk Free Lending and Borrowing

- 14.5.1 Leveraged Portfolio
- 14.5.2 Market Portfolio
- 14.5.3 Capital Market Line

14.6 The CAPM

- 14.6.1 Assumptions
- 14.6.2 Security Market Line
- 14.6.3 Limitations

14.7 Arbitrage Pricing Theory (APT)

14.8 Let Us Sum Up

14.9 Further Reading

14.10 Answers to Check your Progress

14.11 Model Question

14.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- outline the basic tenets and assumptions of Capital Asset Pricing Model (CAPM) define risk free asset, risk free lending, risk free borrowing and leveraged portfolio
- discuss and illustrate the implications of leveraged portfolio for efficient set and Capital Market Line (CML)
- describe 'beta' measure of systematic risk and the Security Market Line (SML) that relates the expected return for an asset to its beta
- discuss the limitations of CAPM and describe alternative theory namely Arbitrage Pricing Theory (APT) Structure.

14.2 INTRODUCTION

In this unit we are going to discuss the concepts of Capital Market Theory, Assumptions,Security Market Line and its limitations. This theory sets the environment in which securities analysis is preformed. Without a well-constructed view of modem capital markets, securities analysis may be a futile activity. A great debate, and great divide, separates the academics, with their efficient market hypothesis, and the practitioners, with their views of market inefficiency. Although the debate appears surreal and unimportant at times, its resolution is immensely critical for conducting effective securities analysis and investing successfully.

In this unit we will geta fair idea about Concepts of Risk-free Asset, Risk-free Lending and Risk-free Borrowing, Leveraged Portfolio,Market Portfolio, Capital Market Line and Arbitrage Pricing Theory (APT).

14.3 CAPITAL ASSET PRICING MODEL(CAPM)

The Capital Asset Pricing Model (CAPM) is commonly confused with portfolio theory. Portfolio theory is simply the use of statistical and mathematical programming techniques to derive optimal tradeoffs between risk and return. Under very restrictive assumptions (rarely found in financial markets), the CAPM is a highly specialized subset of portfolio theory. Even so, the CAPM has become very popular as it provides a logical, common sense tradeoff between risk and return.

In this unit, our endeavor will be to extend the portfolio theory described in the previous two units, to the capital market theory that is concerned with pricing risky assets. In particular, we would like to know if two assets differ with respect to their risk, how will they differ in terms of the price investors are willing to pay or the rate of return investors expect to get from them?

The major implication of the capital market theory is that the expected return of an asset will be related to a measure of risk for that asset, known as ‘beta’. The exact manner in which expected return and beta are related

is specified by the Capital Asset Pricing Model or CAPM, which was developed in mid-1960s. The model has generally been attributed to Williams Sharpe, but similar independent derivations were, made to by John Linter and Jan Mossin. Consequently, the model is often referred to as Sharpe-Linter-Mossin (SLM) Capital Asset Pricing Model. Although the model has been extensively examined, modified and extended in the literature, the original SLM version of the CAPM still remains the central theme in capital market theory as well as in current practices of investment management.

14.4 CONCEPTS OF RISK-FREE ASSET, RISK-EREE LENDING AND BORROWING

Following the development of Markowitz portfolio model, institutional investors and others started realizing the need for considering the relationship between the stocks in constructing the portfolios. Many of these investors started using sophisticated mathematical models to derive optimal portfolio but always found it difficult to measure the same in view of large number of assets traded in the market. CAPM resolves this problem to an extent by considering investments in risk-free asset. As we will see in this Unit, giving investors these new opportunities will have major impact in the shape and allocation of the efficient set and subsequent portfolio selection. But before we proceed to discuss this aspect, let us get acquainted with the terms like 'risk-free asset', 'risk-free lending' and 'risk-free borrowing'.

Risk-free Asset: Risk-free asset is an asset, which has a certain future return. In other words, a risk-free asset is one for which there is no uncertainty regarding the future returns; that is, the investor knows exactly what the value of the asset will be at the end of the holding period. Thus, variance of returns of a risk-free asset is equal to zero. A good example of such asset is government bonds.

Whether all types of government bonds are risk-free asset? It is difficult to say because long-term government bonds are exposed to certain types of risk like interest rate risk and inflation risk. For instance, if the maturity period of a government security is (say) 15 years, while the investment

horizon (or the holding period) of an investor is (say) three-months, then the investor does not really know at what market price he will be able to sell the security at the end of his holding period. Any change in interest-rate structure during the holding period will-influence the market price of the security. To give an idea, upward revision of interest rate will have a tendency to lower the market price, such that yield-to-maturity at market-price-based acquisition of the security of given maturity period compares well with the yield-to-maturity of new issue with similar maturity period. This is an example of what is termed as 'interest-rate-risk'. Thus, normally, the short-term government securities like Treasury Bills are called risk-free securities. Can corporate debentures be treated as risk-free asset? Certainly not, because risk of default is associated with them in addition to interest rate risk and inflation risk. In fact, corporate bonds have more risk like liquidity risk. However, in relative term, they are better than equity on risk.

What is the co-movement of returns of risk-free asset and risky asset (or portfolio of risky assets)? Interestingly, it is always zero. We may recall that covariance between returns of two-assets 'i' and 'j' are given by

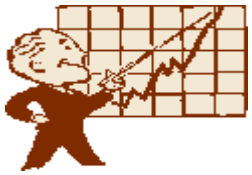
$$\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$$

where ρ_{ij}, σ_i, σ_j are the correlation co-efficient and standard deviation of returns on assets i' and 'j' respectively. If one of the assets is risk-free asset, say asset 'i', then by definition returns on risk-free asset are certain such that the standard deviation (σ_i) is zero and hence the co-variance (σ_{ij}) is also zero.

As we will see later, these two characteristics of risk-free asset, namely, (a) variance = 0; and (b) covariance of returns with any other asset = 0, are quite significant in determining the shape of efficient frontier.

Risk-Free Lending and Borrowing: Investing in a risk-free asset is frequently referred to as 'risk-free lending', since investment in such assets tantamount to giving loan directly to the government. An investor does not have to depend solely on his own wealth to decide how much to invest in assets. She/he can borrow and invest, i.e., the investor can use financial

leverage. However, investor will have to pay interest on borrowed funds and such borrowing is also assumed to have same risk-free interest rate and hence deemed as "risk-free borrowing". Though it may not be practical for an ordinary investor to borrow at risk-free interest rate, it is quiet possible for large funds to borrow at a rate close to risk-free rate.



CHECK YOUR PROGRESS

Q1:What do the following stand for:

a. CAPM ……

b. CML ……

c. SML……

d. SLM……

e. APT……

14.5 EFFICIENT SET WITH RISK-FREE LEADING AND BORROWING

The efficient frontier consists of only risky securities. What happens to the average rate of return and standard deviation of returns when a risk-free asset is combined with a portfolio of risky assets such as exists on the Markowitz efficient frontier?

The expected portfolio return (R_P is given by

$R_P = XR_f + (1- x)R_i$

where,

x = the proportion of the portfolio invested in a risk free asset;

R_f = risk-free rate of return; and

R_i = expected return on risky portfolio 'i' .

Recalling equation (10.2), variance of returns for two-asset portfolio (σ^2_p) is as follows:

$$\sigma^2 = [x^2\sigma^2_f] + [(1-x)^2\sigma^2_i] + [2x(1-x)\sigma_{if}] \quad \text{---- eq 14.1}$$

Where, σ^2_f and σ^2_i are the return variances of risk-free asset and risky portfolio respectively, and σ_{if} is the covariance of returns between risk-free asset and portfolio of risky assets i.

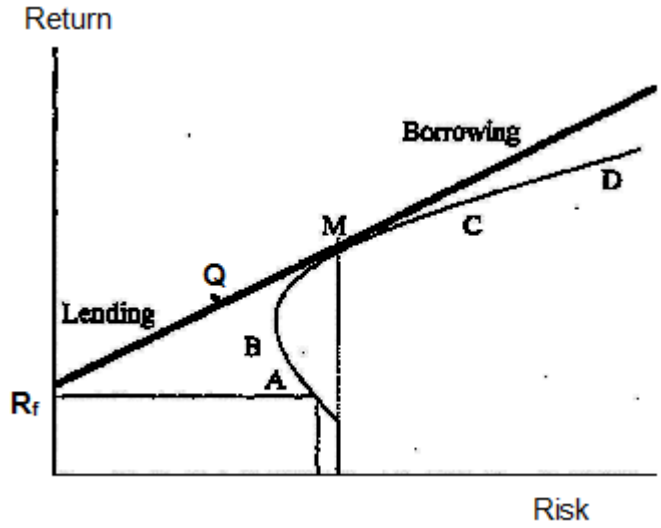
As we have noted earlier, for risk-free asset variance and covariance terms are Zero, i.e., $\sigma^2_f = 0$ and $\sigma_{if} = 0$; and so equation retains only the middle terns and reduces to

$$\sigma^2_p = (1-x)^2\sigma^2_i$$

$$\sigma_p = (1-x)\sigma_i \qquad \text{---- eq 14.3}$$

As the equations (14.1) and (14.2) are both linear, the returns-risk graph for portfolio possibilities, combining the risk-free asset and risky portfolios on Markowitz efficient frontier, is represented by a straight line. Figure 14.1 illustrates the position.

Figure 14.1: Efficient Set of Portfolios with Risk-Free Asset



The set of efficient portfolios marked in the curve A, B, M, C, and D are set of portfolios consisting of risky assets. Suppose there is a risk-free asset offering a return of R_f Now compare an investment in the portfolio of A (consisting of risky assets) and investment in risk-free security. Investment in risk-free security offers a return higher than A but without any risk. Thus, investment in risk-free security is superior to investments in A and in that process A become inefficient portfolio. A tangent line drawn from R_f through the curve A-B-M-C-D is now become efficient portfolio. You may note that only one portfolio marked 'M', which consists of risky assets falls under the

new efficient frontier. Such portfolio is called 'market portfolio' which consists of all risky assets. Investors can now earn any return they like on the efficient frontier by investing a part of money in M, and the rest in R_f For instance, an investor, who is willing to take maximum risk, will invest entire wealth in M whereas an investor, who dislike risk invest the entire wealth in R_f An investor with moderate risk preference will invest 50% in R_f and the balance 50% in M. An investor, who want to go beyond M has to borrow at risk-free rate of interest and invest the amount in M and capture the difference between M and R_f to increase the return.

14.5.1 Leveraged Portfolio

A portfolio that includes at least some securities that were bought with borrowed money. A leveraged portfolio is risky because the securities may result in a loss, which would leave the investor liable to repay the borrowed capital. However, if the securities result in a gain, the investor has essentially made a profit without using his/her own money.

In the foregoing analysis it has been tacitly assumed that investors holding portfolios by combining risk-free asset and risky portfolio M, do so with their own funds. This is not a realistic assumption. In the real world, investors often purchase assets with borrowed funds. We now explore the implications of borrowing.

Assume that an investor is, of course, ready to accept higher level of risk, i.e., the investor is willing to hold portfolio with expected standard deviation of returns σ_p greater than σ_m. One alternative would be to choose a portfolio of risky assets on

Markowitz efficient frontier beyond M, such as the one at point C. A second alternative is to borrow money (i.e., add financial leverage) at risk-free rate and invest the same in the risky asset portfolio at M. By doing so, the investor can move from point M to, say, point Q along the extension of R_f to M line. And as is evident from Figure 14.1, such portfolios as at Q dominate all portfolios below

the line, including the portfolio at C.

To illustrate the point, let us assume that investors can borrow, whatever amount he wants, at a risk-free rate. In other words, we are assuming that risk-free lending and risk-free borrowing rates are the same (we will see the implication of relaxing this assumption later). We may further note that investors would not desire to simultaneously invest in risk-free asset and borrow money at risk-free rate. Now, suppose that an investor borrows an amount equal to 50 per cent of his original wealth of, say, Rs. 10,000. So he has total of Rs. 15,000 which he proposes to invest in portfolio M. What is the proportion of fund being invested in M? It is given by

1 - x = 15,000/10,000= 1.5

However, the sum of proportions being invested in risk-free assets and M must still equal one, which means that

x =-5,000/10,000=-0.5

The negative sign indicates borrowing, on which there will be interest payment at R_f. Thus, restating equation (14.1), we have

$R_P = -0.5R_f + 1.5R_m$

Assuming that R_f = 8% and R_m = 20%, the return on the leveraged portfolio will be = - 0.5 (.08) + 1.5 (0.20) = 0.26 or 26 per cent which is significantly higher than R_m, the expected return of 20 per cent on risky portfolio M. Using equation (14.2), the standard deviation of returns from leveraged portfolio works out to

$$\sigma_p = [1-(-.5)]\sigma_m = 1.5\sigma_m$$

Thus, our investor could increase return along the line R_f - M - Q. Herein lies the advantage of owning a 'leveraged' portfolio. However, leveraging also involves a trade-off; the risk of a leveraged portfolio is always higher than that of tangency portfolio, M (in the instant case it is 1.5 times).

14.5.2 Market Portfolio

In this unit you will learn more about the portfolio. A market portfolio is a theoretical bundle of investments that includes every

type of asset available in the world financial market, with each asset weighted in proportion to its total presence in the market. The discussion may look a bit abstract but necessary to get complete understanding on capital market theory. The portfolio M represents 'optimal combination of risky assets' and is referred to as "market portfolio". It may be explained as follows.

If all investors have homogenous expectations and they all face the same risk-free lending and borrowing rate (R_f), each one of them will generate the same risk-return graph as depicted in figure 14.1. Everyone would obtain the same tangency portfolio M, and invest in this portfolio in conjunction with risk-free lending or borrowing to achieve a personally preferred overall combination of risk and return. An aggressive investor may prefer a leveraged portfolio, which would have a higher risk and return than portfolio M. In contrast, a conservative investor might prefer a lending portfolio, which would have lower risk and return than the portfolio M. The decision to hold a leveraged or lending portfolio is purely a "financial decision" on an investor's risk preference. It has nothing to do with the decision about holding the combination of risky asset (i.e., investment decision) corresponding to the portfolio M. In other words, the composition of risky portfolio M and its inclusion in every investor's portfolio is independent of his or her risk -return preference; this aspect is known as `separation theorem', introduced by James Tobin in 1958.

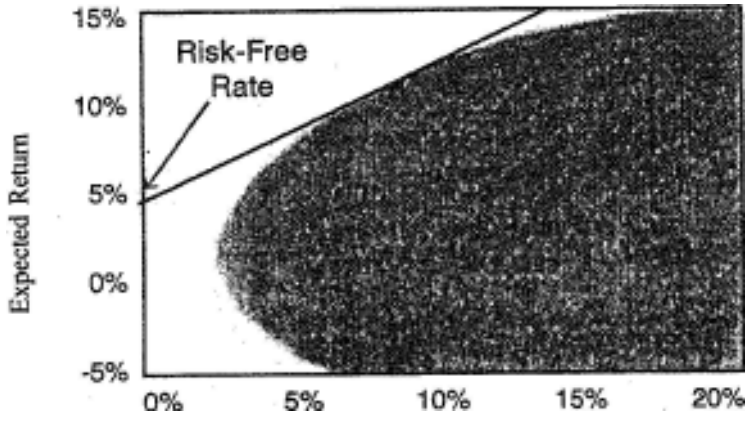
Another important feature of the portfolio M is that it represents a 'market portfolio - a portfolio that is comprised of all risky assets, where the proportion to be invested in each asset corresponds to its relative market value. Why must the portfolio M include some investment in every risky asset? If a risky asset was not in this portfolio, it would mean that nobody is investing in that asset; obviously, the market price of the asset must fall, which in turn would cause the expected return to rise, until it is being included in the portfolio M. In the market portfolio, the asset is held in the proportion that the market value of that asset represents of the total

market value of all risky assets. If, for example, there is a higher proportion of an asset than is justified by its market value, the excess demand for this asset will result in increase in its price until its value becomes consistent with the proportion. Thus, when all the price adjustments are over, i.e., market is brought into equilibrium, tangency portfolio M becomes the market portfolio. Besides, it is the most diversified portfolio, since it contains all the risky assets.

14.5.3 Capital Market Line (CML)

Capital market line (CML) is the tangent line drawn from the point of the risk-free asset to the feasible region for risky assets.With the identification of M as market portfolio, we may define the straight line from R_f through M, as 'capital market line' (CML). This line represents the risk premium as a result of taking on extra risk. James Tobin added the notion of leverage to Modern Portfolio Theory by incorporating into the analysis an asset, which pays a risk-free rate of return. By combining a risk-free asset with risky assets, it is possible to construct portfolios whose risk-return profiles are superior to those of portfolios on the efficient frontier. Consider the diagram below:

Figure 14.2: Capital Market Line



The capital market line is the tangent line to the efficient frontier that passes through the risk free rate on the expected return axis.

The risk -free rate is assumed to be 5%, and a tangent line-

called the capital market line-has been drawn to the efficient frontier passing through the risk-free rate. The point of tangency corresponds to a portfolio on the efficient frontier. That portfolio is called the "super efficient" portfolio. The Capital Asset Pricing Model demonstrates that, given certain simplifying assumptions, the super-efficient portfolio must be the market portfolio.

Using the risk-free asset, investors who hold the super-efficient portfolio may:

- Leverage their position by shorting the risk-free asset and investing the proceeds in additional holdings in the super-efficient portfolio, or
- Deleverage their position by selling some of their holdings in the super-efficient portfolio and investing the proceeds in the risk-free asset.

All types of investors, whether aggressive or conservative, will achieve their desired risk-return levels by combining market portfolio with risk-free lending or borrowing along the CML. Let us re-examine the equation (14.1) of the capital market line to make a few more observations at this stage. The term $(rm - r_m) / \sigma_m$, the slope of capital market line, can be thought of as the market price of risk for all efficient portfolios. It is extra return that can be gained by increasing the level of risk (standard deviation) on an efficient portfolio by one unit. Thus, the entire second term of equation (14.1) represents that element of expected portfolio return that compensates for the risk level accepted. The first term, risk-free rate (or the intercept of CML), is often referred to as the reward for waiting or the return required for delaying potential consumption for one period.

With these two terms, CML sets the expected return on an efficient portfolio as (Price of time) + [(Price of risk) x (Amount of risk)]

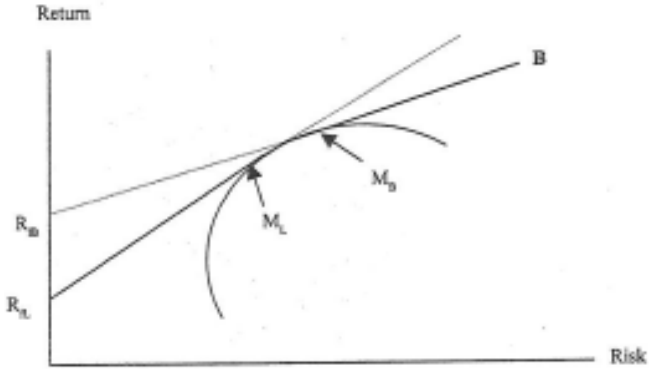
When Risk-Free Rates are different

In the foregoing discussion we assumed that risk-free rates

of lending and borrowing are the same. We now relax this assumption, and consider that where the additional subscripts B and L refer to borrowing and lending respectively.

Figure 14.3 shows the modified efficient set; it consists of three distinct but connected segments, R_{fL} - M_L - M_B - B

Figure 14. 3: Efficient Set of Portfolios with different Risk-free rates



The construction of this efficient set can be explained as follows: If $R_{fL} = R_{fB}$, then the resulting efficient set will be given by the straight line from R_{fL} through M_L. On the other hand, if risk-free lending and borrowings rates are the same, but the rate is set at a higher level equal to R_{fB}, then the efficient set of portfolios will lie on the straight line from R_{fB} through M_B. We may note that M_B is at a higher level than M_L on Markowitz's efficient set, since it corresponds to a tangency point associated with higher risk-free rate, R_{fB}

Now, since the investor cannot borrow at R_{fL}, that part of the line emanating from R_{fL} that extends past M_L is not available to the investors (shown in Figure 14.3 by dotted lines) and can be removed from our consideration. Again, since the investors cannot invest in a risk-free asset that earns a rate equal to R_{fB}, that part of the line from R_{fB} and going through M_B, but lying to the left of M_B, is not available to the investors; and, hence, can be ignored. On the whole, R_{fL} - M_L - M_B - B becomes the relevant efficient set to investors who can lend at R_{fL} and borrow at R_{fB}

14.5 THE CAPITAL ASSET PRICING MODEL (CAPM)

In this section, we turn to the basic Capital Asset Pricing Model developed by Sharpe, Linter and Mossin. We present here a descriptive model of how assets are priced.

The CAPM model describes the relationship between risk and expected return, and serves as a model for the pricing of risky securities. CAPM says that the expected return of a security or a portfolio equals the rate on a risk-free security plus a risk premium. If this expected return does not meet or beat required return then the investment should not be undertaken.

The CAPM builds upon the Markowitz portfolio model and capital market line. Obviously, it pre-supposes all the assumptions stated earlier at appropriate places (including those stated in the previous two Units). Besides, the model itself adds few more assumptions. So, let us begin our discussion of the CAPM by putting together all the assumptions of the model at one place.

14.6.1 Assumptions

1. Investors evaluate portfolios by looking at expected returns and standard deviations of those portfolios over a one-period horizon.
2. Investors, when given a choice, between two otherwise identical portfolios, will choose the one with higher expected return.
3. Investors, when given a choice, between two otherwise identical portfolios, will choose the one with the lower standard deviation or risk.
4. Individual assets are infinitely divisible, meaning that an investor can buy a fraction of a share if he or she so desires.
5. There is a risk-free rate at which an investor may either lend money or borrow money.
6. Taxes and transaction costs are irrelevant.
7. All investors have the same one-period horizon.

8. The risk-free rate is the same for all investors.
9. Information is freely and instantly available to all investors.
10. Investors have homogeneous expectations, meaning that they have the same perceptions in regard to the expected returns, standard deviations and covariance of returns between any two assets.

Needless to say, many of these assumptions are unrealistic, and one may very well wonder how useful a model can be that is based on them. But, then assumptions are necessary in building a model, and we should not be so much concerned about the assumptions as we should be about how well the model explains the relationships that exist in the real world. In fact, several authors have shown that many of the above assumptions can be relaxed with minor impact on the CAPM and no change in the overall concept of the model.

14.6.2 Security Market Line (SML)

The security market line (SML) is a line drawn on a chart that serves as a graphical representation of the capital asset pricing model (CAPM), which shows different levels of systematic, or market, risk of various marketable securities plotted against the expected return of the entire market at a given point in time.

Given the capital market line (CML) and the dominance of the market portfolio, the relevant risk measure for an individual risky asset is its covariance with the market portfolio ($Cov_{i,M}$), or what is known as its 'systematic risk'. When this covariance is standardized by the covariance for the market portfolio, we obtain the well-known 'beta' measure. of systematic risk and a security market line (SML) that relates the expected return for an asset to its beta. Under the CAPM, the postulated relationship is such that higher an asset's beta, the higher its expected return.

To understand the CAPM and computation of beta, let us examine the whole issue intuitively. If the concept of CML is clear,

you will agree that it is not possible for any stock to offer a risk-return relationship below or above the CML. If it is below the CML, such stocks are known as overpriced stocks (meaning they offer lower return for a given level of risk and there is an alternative portfolio on the line of CML, which offer higher return) and investors will start selling the stock until its return increases to the level of CML. The same applies if there is a stock above CML in terms of risk and return and investors will buy such stocks by offering higher price until its return declines to CML. In CML, you can observe only two points namely R_f and M. Since M is an efficient portfolio, we assume that the risk associated with the M is the least. Further it is also a diversified portfolio and hence one can expect no unsystematic risk (Recall the discussion in Unit 9 on how diversification beyond a point fails to yield further results in reducing the risk) . Suppose a stock lies beyond M in the CML line and it means that the stock's risk is higher and hence offer higher return. Now, it is possible to quantify how much that the stock is riskier than M and such a measure is called beta of the stock. If the stock falls on the CML line, it's return (R_s) should satisfy the following equation.

$$R_s = R_f + \beta_s(R_m - R_f)$$

where $\beta_s = \beta_{sm} / \sigma^2 m$

The term β_s, representing covariance of returns between asset 's' and the market portfolio divided by return variance of market portfolio, is known as "beta co-efficient" or simply "beta" for asset. The above equation is the most often written form of the CAPM. If the beta and expected return of stocks are plotted, the line that shows the risk and return of all stocks in the market is called security market line (SLM). Let us now examine some properties of beta.

Beta is a means of measuring the volatility of a security or portfolio of securities in comparison with the market as a whole. Beta is calculated using regression analysis. Beta of 1 indicates that the security's price will move with the market. Beta of greater than 1 indicates that the security's price will be more volatile than

the market. Beta less than 1 means that it will be less volatile than the market.

Many Utilities stocks have a beta of less than 1. Conversely most high-tech stocks have a beta greater than one, they offer a higher rate of return but they are also very risky. For example, if a stock's beta is 1.2, it's 20% more volatile than the market. Beta is a good indicator of how risky a stock is. Beta is the sensitivity of a stock's returns in comparision to the returns on some market index (e.g., BSE Sensex, NSE-50 or BSE-100).

Beta values can be roughly characterized as follows:

1. β less than 0
   Negative beta is possible but not likely. People thought gold should have negative betas but that hasn't been true.
2. β equal to 0
   Cash under your mattress, assuming no inflation or any investments with a guaranteed constant return.
3. β between 0 and 1
   Low-volatility investments (e.g., Utility stocks)
4. β equal to 1
   Matching the index (e.g., any index fund offered by mutual funds)
5. β greater than 1
   Anything more volatile than the index (e.g., small cap. funds)
6. β much greater than 1 (tending toward infinity)
   Impossible, because the stock would be expected to go to zero on any market decline.
   Beta of 2-3 is probably as high as you will get.
   More interesting is the idea that securities may have different betas in up and down markets.

Here is an example showing the inner details of the beta calculation process: Suppose we collected end- of- the-month prices and any dividends for a stock and the BSE sensitive index for 61 months (0 to 60). We need n + 1 price observations to calculate n holding period returns, so since we would like to index the returns

as 1 to 60, the prices are indexed 0 to 60. Also, professional beta services use monthly data over a five-year period.

Now, calculate monthly holding period returns using the prices and dividends. For example, the return for month 2 will be calculated as:

$R_2 = (P_2-P_1+D_2) / P_1$

Here R denotes return, P denotes price, and D denotes dividend.

The following table of monthly data may help in visualizing the process. (Monthly data is preferred in the profession because investors' horizons are said to be monthly.)

Sl. No.	Date	Price	Dividend *	Return
0	31/12/96	45.20	0.00	
1	31/01/97	47.00	0.00	0.0398
2	28/02/97	*46.75*	0.30	0.0011
....	.....	.....	.....	.....
59	30/11/01	46.75	0.30	0.0011
60	31/12/01	48.00	0.00	0.0267

Note: (*) Dividend refers to the dividend paid during the period. They are assumed to be paid on the date. For example, the dividend of 0.30 could have been paid between 01/02/97 and 28/02/97, but is assumed to be paid on 28.02.97.

So now we'll have a series of 60 returns on the stock and the index (1 to 61). Plot the returns on a graph and fit the best-fit line (visually or using least squares process). In Figure 14.4(in the next page), you can see the monthly return of BSE Sensex and ITC over 60 months period (January 1997 - December 2001). In Table 14.1, the beta of stocks forming part of BSE Sensex along with return and total risk measures are listed. You may observe that many new economy stocks like Satyam, Zee Tele have high beta whereas multinational companies like Nestle, Castrol, HLL, Colgate have shown low beta. You may also observe that returns of the new economy stocks were also high compared to other low beta stocks. You may have to periodically revise the beta values since the risk of the stock changes over time based on changes in the economy and industry characteristics.

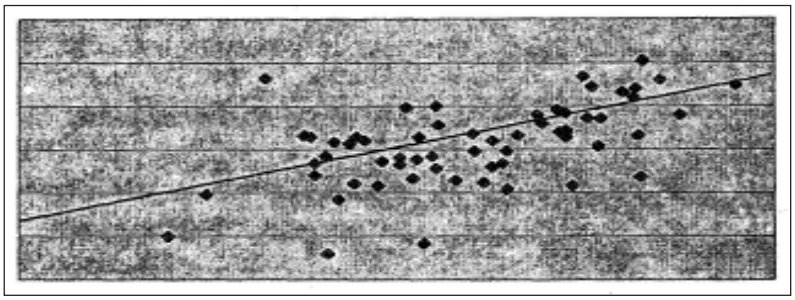
Table 14.1: Return, Variance, SD and Beta of BSE Sensex Stocks

Stock	Return	Variance	SD	Beta
BSE Sensex	0.10	16.99	4.12	1.00
Satyam	2.25	132.18	11.50	1.70
Infosys	1.67	97.61	9.88	1.44
Zee Tele	1.65	138.74	11.78	1.39
MTNL	0.08	61.20	7.82	1.25
L&T	0.18	50.86	7.13	1.18
BHEL	0.10	59.48	7.71	1.17
Telco	-0.18	66.10	8.13	1.16
RIL	0.47	58.62	7.66	1.15
ACC	0.34	60.15	7.76	1.13
ICICI	0.33	80.74	8.99	1.10
NET	0.64	101.07	10.05	1.10
SBI	0.14	47.16	6.87	1.08
M&M.	-0.24	61.47	7.84	1.07
Tisco	-0.02	49.57	7.04	1.03
ITC	0.53	45.90	6.78	0.92
Ranbaxy	0.56	44.92	6.70	0.90
Grasim	0.16	73.30	8.56	0.88
BSES	0.20	45.36	6.74	0.82
Dr. Reddy	1.17	53.77	.7.33	0.82
GA Cement	0.38	39.25	.6.26	0.78
Cipla	1.00	55.28	7.44	0.77
RPL	0.59	44.55	6.67	0.73
Glaxo	0.23	34.16	5.84	0.70
HPCL	0.29	60.29	7.76	0.68
Hindalco	0.19	41.01	6.40	0.67
HLL	0.53	29.88	5.47	0.67
Colgate	0.03	26.21	5.12	0.65
Bajaj Auto	-0.14	33.87	5.82	0.60
Castrol	0.08	22.78	4.77	0.53
Nestle	0.48	29.28	5.41	0.51

Note: Returns represent weekly return of the stock for a period of 1997-2001

If you had a portfolio of beta 1.2, and decided to add a stock with beta 1.5, then you know that you are slightly increasing the riskiness (and average return) of your portfolio. This conclusion is reached by merely comparing two numbers (1.2 and 1.5). That parsimony of computation is the major contribution of the notion of "beta". Conversely if you got cold feet about the variability of your beta = 1.2 portfolio, you could augment it with a few companies with beta less than 1. If you had wished to figure such conclusions without the notion of beta, you would have had to deal with large covariance matrices and nontrivial computations.

Figure 14.4: Monthly Return of BSE Sensex vs. ITC (1997 - 2001)



Hence, beta is the relevant measure of risk for an asset; it measures what is termed as 'systematic or market risk'. It can be shown that the 'total risk' of the asset, as measured by variance of its return, is of the following form

$$\sigma^2_i = \beta^2_i \sigma^2_m + \sigma^2_{ei} \qquad \text{(eq-14.3)}$$

where σ^2_{ei}, is the variance of return for the asset that is not related to the market portfolio. It is also said to measure 'unsystematic or unique risk'. We know that unique or unsystematic risk can be eliminated *in* a completely diversified portfolio such as the market portfolio. So, unsystematic risk is not relevant to investors, and they should not expect to receive added returns for assuming this risk. It is only in the case of assets with greater market risk or betas that

investors should expect higher return.

Second, beta of a portfolio is simply a weighted average of the betas of its component assets (n) where the proportions invested in the assets (x_i) are the weights. Thus, portfolio beta (β_p) is given by

$$\beta_p = \sum_{i=1}^{n} x_i \beta_i$$

we may illustrate this point by taking a stock portfolio comprising seven stocks with their betas and portfolio proportions given as follows:

(1)	(2)	(3)	(4)
Company	Beta	PORTFOLIO PROPORTIONS	WEIGHTED BETA
A	1.50	11.7	.175
B	1.36	22.2	.302
C	1.37	15.7	.215
D	1.07	5.3	.056
E	1.17	26.2	.306
F	1.73	13.9	.240
G	1.09	5.1	.055
		100.0	1.349

The beta of this stock portfolio is 1.35, which is obtained by summing up the multiproduct of (2) and (3) above and shown under (4). It is easy to see the central role played by the beta in the determination of expected return and risk for stocks as well as portfolio and thus in stock selection and portfolio creation and revision.

14.6.3 Limitations

You may be now interested in knowing whether security returns is in fact directly related to beta, as the CAPM asserts. Research results suggest that the CAPM does not reflect the world well at least when tested using ex-post data. Critics have pointed

out that the inadequacy of the model is due to its austerity. The market, in principle includes all stocks, a variety of other financial instruments, and even non-marketable assets such as an individual's investment in education; to which no market index like the SP 500 Index in US or Bombay Stock Exchange National Index (or any other index used to represent the market) can be a perfect proxy. And when we measure market risk using an imperfect proxy, we may obtain a quite imperfect estimate of market sensitivity. Secondly, the CAPM asserts that only a single number- market return - is required to measure risk. The actual returns depend upon a variety of anticipated an unanticipated events. Thus, while systematic factors are the major sources of risk in portfolio return, different portfolios have different sensitivities to these factors. It is the recognition of this phenomenon which lies at the core of an alternative-pricing model called Arbitrage Pricing Theory (APT). Let us briefly discuss APT in the following section.

14.7 ARBITRAGE PRICING THEORY (APT)

As noted above, at the core of APT is the recognition that several systematic factors affect security returns. It is possible to see that the actual return, R, on any security or portfolio may be broken down into three constituent parts, as follows:

$$R = E + b f + e$$

where:

E = expected return on the security

b = security's sensitivity to change in the systematic factor

f = the actual return on the systematic factor

e = returns on the unsystematic factors

The above Equation merely states that the actual return equals the expected return, plus factor sensitivity times factor movement, plus residual risk. The subtler rationale and mathematics of APT are left out here. The empirical work suggests that a three or four - factor model adequately captures the influence of systematic factors on stock - market returns. The APT Equation may thus be expanded to :

$R = E + (b_1)(f_1) + (b_2)(f_2) + (b_3)(f_3) + (b_4)(f_4) + e$

Each of the four middle terms in this equation is the product of the returns on a particular economic factor and the given stock's sensitivity to that factor. What are these factors and separating unanticipated from anticipated factor movements in the measurement of sensitivities is perhaps the biggest problem in APT. Some of the factors empirically found to be useful in measuring risk are:

- Changes in expected inflation, unanticipated changes in inflation, industrial production, default-risk premium and term structure of interest rates (Roll & Ross, J FE, Mar 77)
- Default risk, term structure of interest rates, inflation, long term expected growth rate of profits for the economy, and residual market risk (Berry, FAJ, Mar-Apr 88)

It may be noted that CAPM and APT are different variants of the true equilibrium pricing model. Both are, therefore, useful in supplying intuition into the way security prices and equilibrium returns are established.



CHECK YOUR PROGRESS

Q2: Define Leveraged Portfolio.

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Q3: Define Capital Market line.

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Q4: Define Market Portfolio.

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Q5: Define Security Market line.

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14.8 LET US SUM UP

In this unit we have discussed the following points:

- CAPM is a highly specialized subset of portfolio theory.
- Risk-free asset is an asset, which has a certain future return. In other words, a risk-free asset is one for which there is no uncertainty regarding the future returns; that is, the investor knows exactly what the value of the asset will be at the end of the holding period.
- A leveraged portfolio is risky because the securities may result in a loss, which would leave the investor liable to repay the borrowed capital.
- Capital market line (CML) is the tangent line drawn from the point of the risk-free asset to the feasible region for risky assets.
- The security market line (SML) is a line drawn on a chart that serves as a graphical representation of the capital asset pricing model (CAPM), which shows different levels of systematic, or market, risk of various marketable securities plotted against the expected return of the entire market at a given point in time.



14.9 FURTHER READING

- Fischer, Donald E, and Ronald J. Jordon (1995), *Security Analysis and Portfolio Management, 6th* ed., PHI, New Delhi
- Nancy, Efficient? Chaotic? *What is the New Finance?* Harvard Business Review, March-April, 1993.M. Ranganathan and R. Madhumathi(2005); *Investment Analysis and Portfolio Management,* Pearson Education, New Delhi.
- Punithavathy Pandian(2012); *Security Analysis and Portfolio Management,* Vikas Publishing House Pvt. Ltd., New Delhi.
- Bharti V. Phathak (2014); *Indian Financial System,* Pearson Education, Delhi.

- Donald E. Fischer and Ronald J. Jordon (2002): *Security Analysis and Portfolio Management*, Pearson Education India.
- Prasanna Chandra(2017); *Investment Analysis and Portfolio Management,* McGraw Hill Education, Delhi.



14.10 ANSWERS TO CHECK YOUR PROGRESS

Ans to Q1: a. CAPM -Capital Asset Pricing Model

b. CML -Capital Market Line

c. SML-Security Market Line

d. SLM- Sharpe-Linter-Mossin Capital Asset Pricing Model

e. APT- Arbitrage Pricing Theory

Ans to Q2: A portfolio that includes at least some securities that were bought with borrowed money. A leveraged portfolio is risky because the securities may result in a loss, which would leave the investor liable to repay the borrowed capital. However, if the securities result in a gain, the investor has essentially made a profit without using his/her own money.

Ans to Q3: Capital market line (CML) is the tangent line drawn from the point of the risk-free asset to the feasible region for risky assets.

Ans to Q4: A market portfolio is a theoretical bundle of investments that includes every type of asset available in the world financial market, with each asset weighted in proportion to its total presence in the market.

Ans to Q5: The security market line (SML) is a line drawn on a chart that serves as a graphical representation of the capital asset pricing model (CAPM), which shows different levels of systematic, or market, risk of various marketable securities plotted against the expected return of the entire market at a given point in time.



14.11 MODEL QUESTIONS

1) Define risk free asset. List out two risk free assets.
2) Compare and contrast Capital Market Line (CML) and Security Market Line (SML).
3) What are the basic assumptions underlying Capital Asset Pricing Model?
4) Define efficient frontier. What happens to the Capital Market Line and the choice of an optimal portfolio if borrowing rate is allowed to exceed the lending rate?
5) Define leveraged portfolio and bring out its implications for capital market line.
6) Compare and contrast CAPM and APT. Which of the two is a better model for pricing risky assets and why?
7) Assume the SML is given as $R_i = 0.05 + .06\beta$ and the estimated below on two stocks are $\beta_x = .04$ and $\beta_\gamma = 1.5$. What must be the expected return on two securities in order for one to feel that they are a good purchase?
8) What specifically should a 'true believer' in the CAPM do with her money if she seeks to hold a portfolio with a beta of 1.5?
9) The following data are available to you as a portfolio manager:
   a) Draw a security market line. In terms of the security market line, which of the securities listed above are undervalued? Why?
   b) Assuming that a portfolio is constructed using equal proportions of the five stocks listed above, calculate the expected return and risk of such a portfolio.
10. Compare and contrast standard deviation and beta as measure of stock and portfolio risks.

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